APPROXIMATE FORMULAE FOR INTRA BEAM SCATTERING CALCULATIONS
APPLICATION TO PROTONS IN THE SPS AND THE LHC, AND HEAVY IONS IN THE
LHC.

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Abstract

The work reported in this paper is based on the theory developed by Piwinski [1]. This theory leads to a numerically complicated triple integral. Evans and Zotter were able to reduce the triple integral to a single one [2]. This allowed a much faster calculation and was implemented in the computer program BBI for the case of protons and electrons [4]. The results of BBI are considered in this paper as the exact results of the Piwinski theory of intrabeam scattering. The aim of this work is to find an acceptable closed analytic approximation for the expressions of beam size growth rates. For future machines it becomes much easier to judge the parameter dependence for the classical case of protons and for the more critical case of heavy ions.

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1. INTRODUCTION

The work reported in this paper is based on the theory developed by Piwinski [1]. This theory leads to a numerically complicated triple integral. Evans and Zotter were able to reduce the triple integral to a single one [2]. This allowed a much faster calculation and was implemented in the computer program BBI for the case of protons and electrons [4]. The results of BBI are considered in this paper as the exact results of the Piwinski theory of intrabeam scattering. The aim of this work is to find an acceptable closed analytic approximation for the expressions of beam size growth rates. For future machines it becomes much easier to judge the parameter dependence for the classical case of protons and for the more critical case of heavy ions.

2. BASIC EXPRESSIONS FOR INTRABEAM SCATTERING BEAM SIZE GROWTH RATES

We will use the same notations as in [2] except for the scattering parameter 'c' which will be replaced by 'C' to avoid confusion with the symbol commonly used for the velocity of light. The second exception is symbol $\sigma_y$ which will be used for transverse RMS beam size. To start we recall the basic expressions from [2]:

The inverse beam size growth rates are:

\[
\frac{1}{\tau_E} = A(1-d^2)F_1 \quad \text{longitudinal}
\]
\[
\frac{1}{\tau_X} = A(F_2 + d^2 F_1) \quad \text{horizontal}
\]
\[
\frac{1}{\tau_Z} = AF_3 \quad \text{vertical}
\]

where A is a function which essentially describes the 3-dimensional particle density, 'd' indicates the relative importance of the momentum spread in the horizontal beam width, and $F_1, F_2, F_3$ are the Piwinski scattering integrals.

\[
A = n r_0^2 E_0 / [\pi \beta^2 \gamma e X Z^2 S^2]
\]

(1)
\[ d = \frac{1}{\sqrt{1 + \left( \beta x E_x \right) \left( 2 \sigma_E D \right)^2}} \]

\[ F_1 = F(a, b, C) \]
\[ F_2 = F(1/a, b/a, C/a) \]
\[ F_3 = F(1/b, a/b, C/b) \]

\[ a = \sigma_E \sqrt{\left[ 1 - d^2 \right] (\gamma \sigma_{X'})} \]
\[ b = \sigma_E \sqrt{\left[ 1 - d^2 \right] (\gamma \sigma_{Z'})} \]
\[ C = \sigma_E \sqrt{\left[ 1 - d^2 \right] \beta \sqrt{4 \sigma_{Z'}/r_0}} \]

where:

\[ n \] particles in one bunch
\[ c \] velocity of light
\[ E_0 \] rest energy
\[ \mu \] permeability in vacuum \([4\pi 10^{-7}]\)
\[ r_0 \] particle radius\(\left(\mu/4\pi\right)\left(e^2c^2/E_0\right)\)
\[ \sigma_{X',Z'} \] normalised transverse emittance \((2\sigma\text{ definition})\)
\[ E_{X,Z} \] transverse emittance
\[ \beta_{X,Z} \] average \(\beta\)-function
\[ \sigma_{X,Z} \] RMS \(\beta\)-angle
\[ \sigma_{X,Z} \] RMS \(\beta\)-position
\[ e_s \] bunch area
\[ D \] average dispersion
\[ \sigma_E \] RMS relative energy spread
\[ \gamma \] ratio of particle mass to rest mass
\[ \beta \] ratio of particle velocity to velocity of light
\[ \frac{\pi}{\gamma} \] Euler's constant 0.5772
The form of the function $F$ is as follows:

$$ F(a,b,C) = 8\pi \int [dx(1-3x^3)/\sqrt{PQ}] \{2\ln[(C/2)(1/\sqrt{P} + 1/\sqrt{Q})] - \gamma \} $$

$$ P = a^2 + (1-a^2)x^2 $$

$$ Q = b^2 + (1-b^2)x^2 $$

Piwinski has shown that the following relations are true:

$$ F_1 + F_2/a^2 + F_3/b^2 = 0 $$

$$ F_1(1,1,C) = 0 $$

3. ANALYTIC APPROXIMATION OF SCATTERING INTEGRALS.

The expressions of the previous section are valid in the most general case and not much can be done in the sense of analytic simplification. Experience with the SPS collider has shown that the beam emittances are comparable in both transverse directions. This same feature has been assumed in the parameter list of the LHC.

Therefore the first hypothesis in this paper will be:

round transverse beams

From now on we will replace the transverse indices $x,z$ by the single index $y$ where applicable.

Let us now look at the consequences of this hypothesis:

$$ a = b $$

$$ P = Q \text{ and } F_2 = F_3 $$

$$ F_2 = -(a^2/2)F_1 $$
The expressions for the growth rates become:

\[
\frac{1}{\tau_E} = A(1-d^2)F \\
\frac{1}{\tau_X} = A(d^2-a^2/2)F \\
\frac{1}{\tau_Z} = -Aa^2/2F
\]  

(2)

where \( F \) has been replaced by \( F_0 \).

The scattering parameters \( a \) and \( C \) can be written:

\[
a = \beta_y d / D \gamma \\
C = (\beta d / D \sqrt{|r_0|}(\epsilon \beta_y / \gamma)^{3/4}
\]

We will now concentrate on the scattering integral \( F \):

\[
F = 8\pi \int [dx(1-3x^2)/(a^2 + (1-a^2)x^2)](2\tau nC/\sqrt{|a^2+(1-a^2)x^2|} - \gamma)
\]

The integration goes from 0 to 1.

Let us look more closely at the factor in curly brackets. It can be written as follows:

\[
f = \tau nC^2 - \tau n[a^2 + (1-a^2)x^2] - \gamma
\]

The function \( f \) consists of a constant (\( \tau n[C^2] - \gamma \)) and a variable part (\( \tau n[a^2 + (1-a^2)x^2] \)). The extreme values of the variable part in the course of the integration are 0 and \( \tau n[a^2] \). If \( |\tau n[a^2]| << |\tau n[C^2]| \) then \( f \) can be replaced by its (constant) average value \( <f> \) in the integral which can now be performed. Remember that \( C \) is a large number since it is inversely proportional to the square root of the particle radius.

The integration is straightforward and the result is:

\[
F = 16\pi / (1-a^2)[\arccos(a)(1+2a^2)/(a\sqrt{1-a^2}) - 3][\tau nC - \gamma/2 + 1 - \arccos(a)a/\sqrt{1-a^2}] 
\]

(3)

The previous approximation is no longer valid when \( a \) becomes a very small number. Then another approximation can be applied. The turning point between the two approximations is taken (somewhat arbitrarily):

\[
3^{\ast}[\tau n[a^2]] = |\tau n[C^2]|
\]
For very small $a$ we neglect $a^2$ with respect to 1 in the scattering integral which becomes:

$$F = 8\pi \int dx (1 - 3x^2)/(a^2 + x^2)[2\ell n C - \overline{\gamma} - \ell n[a^2 + x^2]]$$

The result of the integration is:

$$F = 8\pi [\ell n C^2 - \overline{\gamma}(2\pi/a - 3) - (2 - \pi a) + 2\pi/a + 4\ell n a]$$

In the course of the integration following additional approximation was used:

$$\int dx (\ell n[a^2 + x^2])/(a^2 + x^2) \approx 2(a^2 - 2\ell n a - \pi/a)$$

Keeping only terms in $a^{-1}$ yields finally:

$$F = (8\pi^2/a)(\ell n C - \overline{\gamma}/2 + 2)$$  (4)

In practical cases $a < 1$. We have seen before that $F(1,1,C) = 0$. This means that for $a > 1$ $F$ becomes negative so that the longitudinal and horizontal beam emittances shrink. This interesting phenomenon has not yet been observed and requires typically that the particle energy is less than half the transition energy of the machine. If we exclude this low momentum range then we can approximate both equations (3) and (4), by a single one:

$$F = (\pi/a)(\ell n C - \overline{\gamma}/2 + 2)(1 - a)e^{-4a/\pi}$$  (5)

Figure 1 represents the scattering function $F(a)$ given by (5), together with the functions (3) and (4) each in their respective range of validity. The difference between the two curves is negligible and therefore (5) can be adopted for values of $a$ ranging from 0 to 1. The curves have been drawn for $\ell n C = 14$ but changing the value of $C$ does not alter this result.
4. ANALYTIC APPROXIMATION OF BEAM SIZE GROWTH RATES

The three inverse growth rates all contain the product $A^*F$. With the analytic approximation for $F$, this factor becomes:

$$A^*F = (n/\beta^2\varepsilon_s^2\varepsilon_s d)(D/\beta_\gamma) 8\pi (\mu/4\pi)^2 (ec)^4/E_0 (\gamma nC - \gamma^2/2 + 2)(1 - a)e^{-4a/\pi}$$

where the relation $r_0 = (\mu/4\pi)(e^2c^2/E_0)$ has been used as well as the following substitution for $a$:

$$a = \beta_\gamma d/D\gamma \approx Qd/\gamma$$

where $Q$ is the machine tune.

It is interesting to note that $A^*F$ is only a weak function of beam momentum ($\gamma$) via the parameter $a$ for constant beam conditions in a given machine (no blow-up assumed during acceleration).

To verify the validity of the analytic approximation some checks were performed with the computer program BBI. The results are shown in Table 1. The computer program ZAP [5] which has become recently available is based on another theory developed by Bjorken and Mtingwa [6]. Some results from ZAP are also included in the table.
Table 1: Comparison between results from BBI and analytic formula

<table>
<thead>
<tr>
<th>Machine</th>
<th>SPS</th>
<th>SPS</th>
<th>LHC</th>
<th>LHC</th>
</tr>
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<td>315.</td>
<td>450.</td>
<td>8650.</td>
</tr>
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<td>n</td>
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<td>10</td>
<td>1.341</td>
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<td>479.</td>
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<td>43.0</td>
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<td>78.0</td>
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<td>20.0</td>
<td>5.0</td>
<td>5.0</td>
</tr>
<tr>
<td>εs</td>
<td>0.654</td>
<td>0.540</td>
<td>1.0</td>
<td>2.11</td>
</tr>
<tr>
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<td>2.86</td>
<td>0.273</td>
<td>0.45</td>
<td>0.0727</td>
</tr>
<tr>
<td>d</td>
<td>0.901</td>
<td>0.568</td>
<td>0.845</td>
<td>0.745</td>
</tr>
<tr>
<td>a</td>
<td>0.677</td>
<td>0.035</td>
<td>0.087</td>
<td>0.004</td>
</tr>
<tr>
<td>C</td>
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<td>14364</td>
<td>11654</td>
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<td>9.76</td>
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<tr>
<td>τE</td>
<td>2.608</td>
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<td>0.528</td>
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<tr>
<td>τx</td>
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<td>0.131</td>
<td>0.227</td>
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<td>τz</td>
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<td>-42.74</td>
<td>-29540</td>
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<td>A*F</td>
<td>2.11</td>
<td>22.60</td>
<td>6.44</td>
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<tr>
<td>τE</td>
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</table>

5. INTRABEAM SCATTERING FOR HEAVY IONS.

The theory of Piwinski [1] is general as far as the particle mass and charge are concerned. The parameters that are directly influenced by mass and charge are the restmass $E_0$ and the particle radius $r_0$ in formula (1). The ion rest mass and its charge can be written in terms of proton rest mass and charge:
\[ E_i = AE_0 \]

\[ q_i = Ze \]

Formula (6) can be generalized to ions by replacing e by Ze and \( E_0 \) by \( AE_0 \). Remark that also \( C \) is a function of \( r_0 \) hence of \( A \) and \( Z \). The modified formula (6) can be put in a much more handy form by introducing the second hypothesis of this paper:

*the real beam emittance for a given magnetic field is independent of the ion species*

This leads to the following formula in which all parameters are related to the equivalent proton beam:

\[ A^*F = (n/\beta^2 \gamma d)(D/\beta_s)8\pi(\mu/4e)^2(ec)^4(AZ)/E_0(\xi n C - \eta/2 + 2)(1-a)e^{-4a/\pi} \]  \( (7) \)

This formula is now used to calculate the growth rates in the LHC for beams of oxygen, lead and gold ions. We have chosen a particle intensity of \( 10^9 \) ions, which is the ion intensity per bunch in the proposed heavy ion collider RHIC in Brookhaven [3]. The results are shown in Table 2.

The lifetimes for the heavier ions are very short. This is no surprise since the particle radius \( r_0 \) is so much larger and beam scattering becomes more efficient. The importance of this effect can be judged from the factor \( AZ \) in formula (7).
<table>
<thead>
<tr>
<th>Ion</th>
<th>O</th>
<th>Pb</th>
<th>Au</th>
<th>Gev/c</th>
</tr>
</thead>
<tbody>
<tr>
<td>E</td>
<td>8650.</td>
<td>8650.</td>
<td>8650.</td>
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<tr>
<td>A</td>
<td>16</td>
<td>207</td>
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</tr>
<tr>
<td>Z</td>
<td>8</td>
<td>82</td>
<td>79</td>
<td></td>
</tr>
<tr>
<td>n</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>$10^9$</td>
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<tr>
<td>$\gamma$ (equivalent proton)</td>
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<td>9219.</td>
<td>9219.</td>
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<td>D</td>
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<td>1.585</td>
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<tr>
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<tr>
<td>$\epsilon_y$ (equiv.prot.)</td>
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<td>5.0</td>
<td>5.0</td>
<td>m</td>
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<td>2.11</td>
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<td>0.0727</td>
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<td>0.01</td>
<td>0.01</td>
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<td></td>
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<tr>
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</tr>
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6. EVOLUTION OF BEAM SIZE GROWTH RATES AND LUMINOSITY OF ION BEAMS

When the intrabeam scattering growth rates are large the emittances change quickly. Hence the particle density changes (see equation (1)) and this causes the growth rates to change. The factors d,a and C change also. For a given starting condition it is certainly possible to take all these relations into account in a computer program. However with a few reasonable assumptions it becomes possible to calculate the various growth rates and lifetimes analytically which conveys a lot more information at the price of a slightly reduced accuracy. We will now discuss these assumptions.
• the factor $d$ does not change during a collider run:

This only means that the relative contribution of betatron oscillations and momentum spread to the horizontal beam size remain constant. The quadratic sum which is involved assures that developing imbalances between the two components are somewhat attenuated. As a consequence factor $a$ also remains unchanged during the process.

• the beams stay round:

The reason for this stems from following arguments. We will be interested in the luminosity lifetime. The luminosity is inversely proportional to the product of the transverse emittances for identical beams. According to the theory the vertical emittance changes very little or even shrinks! So it is the horizontal emittance alone which will contribute to the luminosity lifetime. In practice of course the vertical emittance grows as well, due to coupling, and at about the same rate as the horizontal emittance but twice as slow as calculated, so the result is the same! However when the beams stay round, then the ratio $\tau_E/\tau_x$ remains constant (see equations (2)) and this becomes useful later.

• The scattering parameter $C$ depends on the horizontal beam size and the scattering integral depends on $\ell n C$. In what follows we will neglect the changes in $\ell n C$. (see end of section 1.3)

We define a characteristic time $t_0$:

$$t_0^{-1} = 0.5(\tau_{E0}^{-1} + \tau_{x0}^{-1})$$

where $\tau_{E0}$ and $\tau_{x0}$ are the growth rates at time $= 0$. Time and lifetimes will be expressed in units of $t_0$. For simplicity we keep the same symbols $t$ and $\tau$. We define also an average growth rate $\tau^{-1}$:

$$\tau^{-1} = 0.5(\tau_{E}^{-1} + \tau_{x}^{-1})$$

(8)

This average growth rate is one (in units of $t_0$) at $t = 0$.

From equations (2) and (8) we can calculate the increase in growth rate for a time increment $\Delta t$:

$$(\tau_E + \Delta \tau_E)^{-1} = (\tau_E)^{-1} \exp(-4\Delta t/\tau)$$

$$(\tau_x + \Delta \tau_x)^{-1} = (\tau_x)^{-1} \exp(-4\Delta t/\tau)$$

Adding the two equations yields:

$$(\tau + \Delta \tau)^{-1} = (\tau)^{-1} \exp(-4\Delta t/\tau)$$
or \[ \tau + \Delta \tau = \tau \exp(4\Delta t/\tau) \]

This difference equation can be expressed in terms of a differential equation:
\[ \frac{d\tau}{dt} = 4 \]

and the solution is simply:
\[ \tau = 1 + 4t \]

When the ratio \( \tau_E/\tau_X \) is constant then the following ratios are also constant:
\[ \frac{\tau_E}{\tau} = \mu = 0.5(1-a^2/2)/(1-d^2) \]
\[ \frac{\tau_X}{\tau} = \nu = 0.5(1-a^2/2)/(d^2-a^2/2) \]

hence \[ \tau_X = \nu(1+4t) \]

As was pointed out before the luminosity is inversely proportional to the horizontal emittance. The instantaneous luminosity lifetime is therefore \( \tau_X/2 \). Again a difference equation can be written:
\[ L + \Delta L = L \exp(-2\Delta t/\tau_X) = L \exp[-2\Delta t/(\nu \tau)] \]

And this can be transformed into a differential equation:
\[ dL = -2L dt/[(\nu(1+4t)] \]

with solution: \[ L/L_0 = 1/(1+4t)^{1/\nu} \]. These solutions are shown in figure 2 for some values for \( \nu \).

7. OPTIMISATION OF AVERAGE LUMINOSITY OF TWO IDENTICAL COLLIDING BEAMS

The highest performance of a collider is achieved when its average luminosity is maximum. When the luminosity is rapidly decaying then this maximum performance is theoretically obtained by continuously refilling. This of course is not practical because some time is needed for preparation of the filling, injection, acceleration and preparation for data taking for physics. Hence we consider 2 dead times where the luminosity is zero. The first dead time is the time that it takes after filling to establish good data taking conditions \( t_g \). The second dead time is the time that elapses between the drop of the
previous stack and the filling of the next stack \( t_e \). It includes injection preparation, injection and acceleration. Time \( t = 0 \) at the end of filling, i.e., when the luminosity reaches its maximum value.

We express the time dependent luminosity in units of initial luminosity \( \ell = L/L_0 \). The average luminosity can then be written:

\[
<\ell> = \left[ \int \ell \, dt \right]/(t + t_e)
\]

the integration goes from \( t_s \) to \( t \)

Maximising this expression gives:

\[
\ell(t + t_e) = \int \ell \, dt
\]

This equation has a simple significance: for given dead times \( t_e \) and \( t_s \), \( t \) is such that the luminosity at the end of the run is equal to the average luminosity. The relation between \( t_e, t_s \) and \( t \) (the duration of the run which yields the highest average luminosity) follows from the computation of the extremum of \( <\ell> \). After some algebra we find:

\[
y = \left[ x/(2\nu - 1) \right] \left[ 1 - 2\nu^2 (1 - 2\nu)^{2\nu} \right]
\]

with

\[
x = (4t + 1)/(4t_s + 1)
\]

\[
y = (4t_e - 1)/(4t_s + 1)
\]

The curves shown in figure 3 are for various values of \( \nu \). For given dead times \( t_s \) and \( t_e \), \( y \) can be computed. Then the value of \( x \) can be read from a curve of figure 3, depending on the value of \( \nu \), which is known. Finally \( t \) can be computed from the value of \( x \).

A few examples may clarify the use of these relations. The first three columns of Table 3 show the case of Au ions in the LHC at 8.65 TeV/c for a bunch intensity of \( 10^7, 10^8 \) and \( 10^7 \) ions. The fourth column is the case of the SPS for protons based on the data in table 1.

From the table it is evident that the maximum performance is achieved for the maximum intensity that can be obtained from the injectors or can be tolerated by the machine or by the physics experiments. Given this maximum intensity and given the beam and machine conditions an optimum run time can be determined which maximises the real average luminosity.
**Table 3: Optimisation run time**

<table>
<thead>
<tr>
<th>Machine</th>
<th>LHC</th>
<th>LHC</th>
<th>LHC</th>
<th>SPS</th>
</tr>
</thead>
<tbody>
<tr>
<td>E</td>
<td>8650</td>
<td>8650</td>
<td>8650</td>
<td>315</td>
</tr>
<tr>
<td>n</td>
<td>1</td>
<td>.1</td>
<td>.01</td>
<td>100</td>
</tr>
<tr>
<td>t_0</td>
<td>10</td>
<td>100</td>
<td>1000</td>
<td>1470</td>
</tr>
<tr>
<td>v</td>
<td>0.9</td>
<td>0.9</td>
<td>0.9</td>
<td>1.56</td>
</tr>
<tr>
<td>deadtime T_3</td>
<td>30</td>
<td>30</td>
<td>30</td>
<td>10</td>
</tr>
<tr>
<td>t_3</td>
<td>3</td>
<td>0.3</td>
<td>0.03</td>
<td>0.0086</td>
</tr>
<tr>
<td>deadtime T_ε</td>
<td>120</td>
<td>120</td>
<td>120</td>
<td>120</td>
</tr>
<tr>
<td>t_ε</td>
<td>12</td>
<td>1.2</td>
<td>0.12</td>
<td>0.081</td>
</tr>
<tr>
<td>y</td>
<td>3.61</td>
<td>3.39</td>
<td>-0.46</td>
<td>-0.655</td>
</tr>
<tr>
<td>x</td>
<td>9</td>
<td>8.7</td>
<td>2.85</td>
<td>3.09</td>
</tr>
<tr>
<td>t</td>
<td>29</td>
<td>4.53</td>
<td>0.55</td>
<td>0.543</td>
</tr>
<tr>
<td>runtime T</td>
<td>290</td>
<td>453</td>
<td>548</td>
<td>800</td>
</tr>
<tr>
<td>&lt;μ&gt; = &lt;L&gt;/L_0</td>
<td>0.071</td>
<td>0.19</td>
<td>0.52</td>
<td>0.69</td>
</tr>
<tr>
<td>LHC performance</td>
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<td>0.0267</td>
<td>0.00073</td>
<td></td>
</tr>
</tbody>
</table>

**REFERENCES**


[3] Conceptual design of the relativistic heavy ion collider RHIC Brookhaven National Laboratory,BNL 51932,May 1986


Figure 1

Figure 2
Run time optimisation curves

\[ X = \frac{(4T+1)}{(4T_s+1)} \]

\[ Y = \frac{(4T_e+1)}{(4T_s+1)} \]

Figure 3