THE STRUCTURE OF THE NEUTRAL WEAK CURRENT

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The structure of the neutral weak current is reviewed from the discovery at CERN in 1973 to the high precision tests of the Standard Model.

* Dedicated to my friend Val Telegdi on his sixty-fifth birthday
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1. INTRODUCTION

In 1973 muon-less neutrino reactions were discovered at CERN in the elastic scattering of anti-muon-neutrinos on electrons [1],

$$\bar{\nu}_\mu e \rightarrow \bar{\nu}_\mu e;$$

the neutral neutrino current $\bar{\nu}_\mu \nu_\mu$ and the neutral electron current $\bar{e}e$ couple by exchanging a neutral intermediate boson. In the reaction [2]

$$\nu_\mu N \rightarrow \nu_\mu \pi^+ \pi^- \pi^0 N$$

muon-neutrinos scatter inelastically on nucleons and transfer part of their energy to the neutral quark currents $\bar{u}u$ and $\bar{d}d$.

Observation of an asymmetry of the cross-section of the scattering of left-handed and right-handed electrons on deuterons at SLAC in 1978 [3] demonstrated that the neutral weak $\bar{e}e$ current is violating parity.

The discovery of parity violation of the weak $\bar{e}e$ current was then confirmed by the observation that the polarization plane of a laser beam passing through bismuth vapour is rotated [4].

The existence of $\bar{\mu}\mu$ and $\bar{\tau}\tau$ neutral currents was deduced from the observation of a weak forward-backward charge asymmetry in the annihilation of electrons and positrons at the PETRA Collider at DESY [5] in 1982. The existence of other neutral quark currents, e.g. $\bar{s}s$, was deduced from the analysis of the inelasticity distribution of deep inelastic neutral current neutrino scattering on nucleons [6]. Also the neutral current $\bar{\nu}_e \nu_e$ has been observed in neutrino experiments at CERN [7].

So far only diagonal neutral currents have been observed which do not change the flavour of the particles involved. With three families of leptons and quarks, the
standard theory [8] predicts the existence of six diagonal neutral lepton currents and six diagonal neutral quark currents.

The neutral current has a more complex helicity structure than the charged current. Experiments, e.g. measurements of the inelasticity distribution of deep inelastic neutral current neutrino scattering [9] have shown that both left-handed and right-handed currents exist. This observation shows directly the existence of a unified, electro-weak force. Its existence was then demonstrated by the observation of interference of amplitudes with $\gamma$ and $Z^0$ interchange, e.g. in the helicity asymmetry of electron-deuteron scattering and in the optical activity of bismuth vapour.

The structure of the current, as deduced from these experiments implies that left-handed fermions transform as doublets under a weak isospin rotation group and right-handed fermions transform as singlets. The Standard Model of the electroweak gauge theory predicted this structure of neutral currents and it successfully describes a large amount of experimental data [10], which are all consistent with universal strength of the forces, $g_2$ and $g_1$, associated with the SU(2) and U(1) symmetry groups, respectively.

The fundamental quantities of the Standard Model, the coupling constants $g_1$ and $g_2$, and the masses of the weak bosons, $m_w$ and $m_z$, are related to the angle $\Theta_w$ that describes the mixing of the two local symmetries by the relations

$$
e = \frac{g_1}{\sqrt{g_1^2 + g_2^2}} = g_2 \sin \Theta_w$$

$$\sin^2 \Theta_w = 1 - \frac{m_w^2}{m_z^2}.$$  

(1)

The value of the mixing angle is not predicted by the Standard Model. Grand Unified Theories predict a value of $\sin^2 \Theta_w = 3/8$ at the unification mass.

The existence of a non-zero mixing angle defines both the structure and the strength of the neutral currents. The left-handed currents of the "up" particles of the
weak isospin doublets couple with a coefficient \((\frac{1}{2} - Q \sin^2 \Theta_w)\), while the left-handed currents of the "down" particles of the doublets couple with a coefficient \((-\frac{1}{2} - Q \sin^2 \Theta_w)\). \(Q\) is the electric charge of the particle. The coupling coefficients of the right-handed currents are the same for "up" and "down" fermions. \(-Q \sin^2 \Theta_w\). Hence the value of \(\sin^2 \Theta_w\) can be deduced from all neutral-current induced processes. In Born approximation, one finds the values \(\sin^2 \Theta_w^{unc}\) obtained before 1985, summarized in Table 1; they are in reasonable agreement with a common value of \(\sin^2 \Theta_w\). The values after radiative correction, also given in Table 1, are not in significantly better agreement, owing to their relatively large errors.

2. Higher Order Corrections

At a higher level of precision we expect that electroweak radiative corrections change this simple picture. The effective values of \(\sin^2 \Theta\) derived from the measurements will be modified differently in each process. It is therefore of great interest to make measurements with improved precision to test whether the theory can correctly predict these radiative shifts. The shifts are in close analogy to the famous Lamb shift of atomic energy levels and to \((g-2)\) of the electron and the muon. The agreement of the high precision measurements and the higher order calculations of the shifts by the theory have given us confidence that Quantum-Electro-Dynamics is a successful theory. The same tests can now be performed for the Electroweak Interaction. While the QED shifts are small, those of the electroweak interaction are large, as we shall see.

A more advanced test of the theory, at the level of the radiative corrections of order \(\alpha\), can be performed by a precise determination of \(\sin^2 \Theta_w\) from semileptonic and from leptonic neutrino scattering and from the direct measurements of \(m_w\) and \(m_z\). The Fermi interaction, mediated by the weak bosons, leads to a relation between the fine-structure constant \(\alpha\), the Fermi coupling constant \(G_F\), the boson masses, and the weak mixing parameter \(\sin^2 \Theta_w\):

\[
m_w^2 = m_z^2 \cos^2 \Theta_w = \frac{\pi \alpha}{\sqrt{2} G_F} \sin^2 \Theta_w.
\]

Using \(1/\alpha = 137.035963(15)\), as measured by the Josephson effect, and \(G_F^w = 1.16637(2)\)
\( \times 10^{-5} \text{ GeV}^{-2} \) as determined from the muon lifetime, leads in lowest order to the relations:

\[
\sin^2 \Theta^\text{unc} = (37.281 \text{ GeV}/m_w)^2
\]

\[
\sin^2 2 \Theta^\text{unc} = (74.562 \text{ GeV}/m_Z)^2.
\]

Because of the definition of \( \sin^2 \Theta_w \) in eq. (2) this relation must be radiatively corrected by using values of \( \alpha \) and \( G_F \) at the mass of \( m_w \). The change of the value of \( G_F(m_w^2) \) is negligible, whereas \( \alpha \) is changed to \( \alpha(m_w) = \alpha(0)/(1 - \Delta r) \) with

\[
\Delta r = 0.0713 \pm 0.0013,
\]

assuming \( m_{\text{Higgs}} = 100 \text{ GeV} \) and a top quark mass of \( m_t = 45 \text{ GeV} \) [10]. The predicted corresponding radiative shift of the W mass is about 3 GeV. The radiative correction can be experimentally determined from the measurements of \( m_w \) and \( m_Z \) and of \( \sin^2 \Theta_w \) through the relation

\[
\Delta r = 1 - \left( \frac{\pi \alpha}{m_w^2} \sqrt{2} G_F \sin^2 \Theta_w \right)
\]

and the corresponding relation involving \( m_Z \). A comparison with the calculated radiative shift thus tests the predictive power of the Standard Model at the level of vacuum polarization corrections. The electroweak radiative shift is approximately 50 times larger than \( (g - 2) \) of the muon, owing to the larger mass scale involved.

The weak bosons \( W^\pm \) and \( Z^0 \) which mediate the charged and the neutral weak currents were discovered at the CERN proton-antiproton Collider in 1983 [161]. The measurements of \( m_w \) and \( m_z \) are summarized in Table 2. The predictions of the Standard Model, using \( \sin^2 \Theta_w \) determined from deep inelastic neutrino scattering are given for comparison. The successful prediction of the new mass scale of the intermediate vector bosons has been one of the triumphs of the Standard Model.
3. HIGH PRECISION MEASUREMENTS OF $\sin^2 \Theta_w$

Experimental tests have now, for the first time, reached the precision required to probe the theory at the level of the radiative corrections. The present error on $\sin^2 \Theta_w$ of $\pm 0.004$ corresponds to an error in $m_w$ of 0.2 GeV, whereas the total radiative shift is predicted to be 3.3 GeV.

Future improvements of the luminosity of the CERN SpS Collider using a new antiproton collector ring (ACOL) are aimed at obtaining a data set with $10^4$ nb$^{-1}$ instead of the present integrated luminosity of $10^2$ nb$^{-1}$. These data will then give statistical errors of the masses of

$$\Delta m_w, \Delta m_z \sim \pm 0.15 \text{ GeV}.$$  

The uncertainty in the experimental mass scale will then be the dominant error. Assuming an absolute calibration of the giant calorimeters of the detectors to $\pm 1\%$ one derives a limiting accuracy of

$$\Delta \sin^2 \Theta \sim \pm 0.004.$$  

The scale errors cancel in the ratio of the masses where we expect the full improvement due to the large statistics, corresponding to $\Delta \sin^2 \Theta \sim \pm 0.004$.

Among the low energy experiments, those which use neutrino scattering on nucleons have already been improved to similar precision in $\sin^2 \Theta_w$. A new experiment on neutrino-electron scattering, aiming at a higher precision (CHARM II) is presently being performed at CERN. These experiments and their limitations are described in some detail in chapters 4 and 5. The small theoretical error on the radiative shift requires a measurement of $m_z$ to $\pm 50$ MeV to be matched. This precision can be achieved at LEP.

4. MEASUREMENT OF $\sin^2 \Theta_w$ IN SEMILEPTONIC NEUTRINO REACTIONS

The cross-sections of neutrino scattering on isoscalar targets by the neutral and by the charged-current weak interaction are related by the following expression derived
by Llewellyn-Smith [17]:

\[
\frac{d^2\sigma_{NC}}{dx dy} = (\frac{1}{2} - \sin^2\Theta + \frac{5}{9} \sin^4\Theta) \frac{d^2\sigma_{CC}}{dx dy} + \frac{5}{9} \sin^4\Theta \frac{d^2\sigma_{CC}}{dx dy}.
\]

The simplest measurements are those of the total cross-section ratios

\[
R_{\nu} = \frac{\sigma^{\nu}(NC)}{\sigma^{\bar{\nu}}(CC)}, \quad r = \frac{\sigma^{\bar{\nu}}(CC)}{\sigma^{\nu}(CC)}
\]

which give the relation

\[
R_{\nu} = \frac{1}{2} - \sin^2\Theta + \frac{5}{9} \sin^4\Theta + r\left(\frac{5}{9} \sin^4\Theta\right).
\]

Eqs. (6) and (7) are valid if neutrino interactions with quarks and anti-quarks other than u and d can be neglected and if the Cabibbo angle is set to zero. Higher twist effects have also been neglected; they have been estimated [17] to give uncertainties smaller than \(\Delta \sin^2\Theta \sim \pm 0.005\). Of course, weak isospin symmetry is implied by eq. (6). We know that it is broken by flavour-changing processes which have been observed to contribute to charged-current induced reactions but not to neutral-current reactions.

The energy threshold of the flavour transition \(s(d)W^+ \rightarrow c\) crosses the peak energy of the neutrino beam used for the measurement. To correct the measured cross-sections we require knowledge of the mass of the charm quark to describe the threshold behaviour. At much higher energy, for instance at HERA, a measurement of the ratio \(R_{\nu}\) does not require a threshold correction. The corrections are applied with the help of the quark model of the nucleon. Using the best knowledge of the charm quark mass, \(m_c = 1.5 \pm 0.3\) introduces an uncertainty of \(\Delta(\sin^2\Theta_w) = \pm 0.004\). Fixing the mass at \(m_c = 1.5\) GeV, the remaining theoretical uncertainty is \(\Delta(\sin^2\Theta_w) = \pm 0.003\). The total theoretical uncertainty is \(\Delta(\sin^2\Theta_w) = \pm 0.005\). A high precision measurement has recently been performed at CERN by the CHARM Collaboration [18] and by the CDHS Collaboration [19]. Ten years after the discovery of the neutral-current interaction by the Gargamelle team at CERN with a signal to background ratio of one to six, events can now be classified as neutral-current (NC) or charged-current (CC) by direct recognition of the muon in the fine-grain calorimeter of the CHARM detector with less
than 0.2% ambiguity. This progress is due to some important new features of this electronic detector:

- Fast timing is used and events occurring upstream are vetoed, thus eliminating the so-called associated neutron background which plagued the Gargamelle experiment;

- The lateral and longitudinal dimensions of the target-calorimeter are more than 10 times larger than the interaction length of hadrons, thus giving clear signatures to neutrino interactions and muon tracks;

- Detector elements of small lateral dimensions (fine-grain) and frequent segmentation of the target plates allow detection of hadron showers with high efficiency and good energy resolution \( \sigma(E_H)/E_H = 0.47/\sqrt{E_H/\text{GeV}} \) and the recognition of muons with momenta as low as 1 GeV/c.

- Nearly equal response of the calorimeter \( E_e/E_H = 1.17 \) to electromagnetic and hadronic showers, allowing the definition of an effectively equal energy threshold in NC and CC events which have different \( \pi^0 \) content.

The feasibility of a precision measurement was discussed by the author at the 1982 Javea Workshop on Weak Interaction [20] and was demonstrated in detail by the CHARM Collaboration at a Physics Workshop at CERN [21].

A photograph of the CHARM detector is shown in figure 1. It is composed of a fine-grained calorimeter and a muon spectrometer. The calorimeter consists of 78 modules, each composed of a target plate of marble of dimensions 3m × 3m and 8cm thickness; a layer of 20 scintillation counters of 15cm width, 3m length and 3cm thickness; a plane of 128 proportional drift tubes (3cm × 3cm × 400cm) oriented at 90° with respect to the scintillation counters and a plane of digital wire chambers with 1 cm wire spacing oriented parallel to the scintillators. The calorimeter was surrounded by magnetized iron frames for the detection and measurement of large angle muons. The orientation of the detector elements alternated from horizontal to vertical in successive modules. A detailed description can be found in reference [22]. Figures 2 and 3 show schematic views of a CC and NC neutrino event, respectively. Scintillation counters and proportional drift tubes which are hit by the event are shown, and the range of a 1 GeV
muon is indicated. Its track can also be recognized close to the hadron shower. Charged-current (CC) events for which the primary muon cannot be identified are classified as neutral-current (NC). Some of these lost CC events have a muon with an energy less than 1 GeV, or a muon that left the detector at the sides before depositing 1 GeV, or a muon that was obscured by the hadronic shower. A correction is required for these CC losses. As this is the largest correction required (11% of the NC events in the CHARM detector and 22% in the CDHS detector) the precision in measuring $R^\nu$ depends in an essential way on the reliability of estimating these losses. The uncertainty of the parent–beam momentum ($\pm 3\%$) and of the muon momentum measurement by the CHARM detector can affect the correction in a systematic way. A simple and beautiful method was used to eliminate them. The correction was calculated relative to the number of events with muon momenta between 3 and 5 GeV/c. All scale errors cancel in this ratio which was then applied by multiplying with the number of events observed in that muon momentum interval. The remaining uncertainty affecting this correction contributed an error of $\Delta R^\nu/R^\nu = \pm 0.32\%$. A summary of all experimental corrections is given in Table 3. A correction was applied for the small deviation from isoscalarity ($N - Z$) of the target material. Selecting events induced by deep inelastic scattering ($E_{\text{hadron}} > 4$ GeV) the result of the ratio is

$$R^\nu = 0.3093 \pm 0.0031.$$  

Table 4 shows the radiative and the various quark model corrections which have to be applied to determine $\sin^2\Theta_w$ in the definition of Sirlin and Marciano. The final result is

$$\sin^2 \Theta_w = 0.236 \pm 0.012 \ (m_c - 1.5) \pm 0.005 \ \text{(exp)} \pm 0.003 \ \text{(theor)}.$$  

This result, obtained by the CHARM Collaboration, is compared with other recent high statistics results from semi-leptonic neutrino scattering experiments in Table 5, assuming a charm mass of $m_c = 1.5$ GeV/c$^2$. The agreement between the experiments is good and significant in view of the fact that different experimental methods have been used, as indicated in Table 4. A combined value for $m_c = 1.5$ GeV/c$^2$ is

$$\sin^2 \Theta_w = 0.233 \pm 0.004 \pm 0.003.$$
Table 6 shows the averaged value of $\sin^2\theta_w$ from the neutrino experiments and from the $m_w$ and $m_Z$ measurement, with and without the radiative corrections applied. The difference between the values from the two types of experiments shows good agreement for the corrected results, but a difference at the level of three standard deviations for the uncorrected values (the error in the difference is calculated by combining the statistical and theoretical/systematic errors quadratically). The radiative correction determined by the experiments

$$\Delta r \ (\text{exp}) = 0.077 \pm 0.025 \ (\text{exp}) \pm 0.038 \ (\text{syst})$$

agrees with the calculated value of $\Delta r \ (\text{theor}) = 0.0713 \pm 0.0013$ assuming $m_t = 45$ GeV and $m_H = 100$ GeV [10].

Assuming therefore the validity of the radiative corrections, a precise value of the $\rho$ parameter can be obtained by combining our measured value of $R^\nu$ with the value of $\sin^2\theta_w$, as determined in the collider experiments from the $W$ mass. Propagating the errors quadratically, we find

$$\rho = 0.990 - 0.013 \ (m_c - 1.5) \pm 0.009 \pm 0.003$$

in good agreement with the minimal Standard Model.

New physics appendages to the Standard Model or a different input value for $m_t$ would modify the prediction of $\Delta r$. We can therefore use the good agreement with experiment as a constraint. For example, if $m_t$ were 250 GeV/$c^2$, $\Delta r$ would become zero, whereas $\sin^2\theta_w$ hardly changes [25]. From the present experimental value of $\Delta r$, Marciano and Sirlin [26] find the constraint

$$m_t \leq 180 \text{ GeV}.$$
companion the corresponding bound is [25],

\[ m_L \leq 300 \text{ GeV}. \]

Future high-precision experiments should determine \( m_w \) and \( m_z \) to ± 0.1 GeV and ± 0.02 GeV, respectively and, combined with the result of a determination of \( \sin^2 \Theta_w \) from \( \nu_\mu \nu \) and \( \bar{\nu}_\mu \nu \) scattering in the CHARM II experiment to ± 0.005 without theoretical uncertainty, should thus provide even better constraints or perhaps a hint of new physics.

A final comment is concerned with the comparison of \( \sin^2 \Theta_w \) predicted by Grand Unified Theories. The experimental result presented here contradicts the prediction of the minimal SU(5) model [10], \( \sin^2 \Theta_w = 0.214 \pm 0.004 \) [SU(5)], which has already been ruled out by the experimental lower limit of proton decay [27]. It can be used to constrain the additional mass scales of non-minimal Grand Unified Theories. For example, in the case of supersymmetric theories (SUSY), the experimental result is in good agreement with the mass constraint \( M_{\text{SUSY}} \gtrsim 1 \text{ TeV} \) [10].

5. MEASUREMENT OF \( \sin^2 \Theta_w \) IN LEPTONIC NEUTRINO REACTIONS

The first experimental observation of a weak neutral-current phenomenon was one event (reproduced in figure 4) in which an \( \bar{\nu}_\mu \) scattered on an electron, found in 1972 at CERN in the Gargamelle bubble chamber [1]. Now, 15 years later, massive electronic detectors have achieved remarkable progress in this field. The CHARM Collaboration has collected about 83 events of \( \nu_\mu \nu \) and 112 events of \( \bar{\nu}_\mu \nu \) scatterings [11]. A US-Japan Collaboration working at Brookhaven has collected 107 events of \( \nu_\mu \nu \) and 45 events of \( \bar{\nu}_\mu \nu \) scattering [12, 28]. An event observed in the fine-grain calorimeter of the CHARM detector is reproduced in figure 5. Compared to the bubble chamber event in figure 4 it looks quite coarse. In spite of this, the events can be separated from the background and the cross-section determined.
In contrast to semileptonic neutrino scattering there are no theoretical uncertainties to extract the weak neutral-current coupling constants of the electron, $g_V^e$ and $g_A^e$, and $\sin^2\Theta_W$ from the cross-sections of neutrino electron scattering:

$$g_V^2 + g_A^2 = \frac{3\pi}{4G_F^2 m_e} (\sigma^{\nu}/E + \sigma^{\bar{\nu}}/E)$$

$$g_V^e \times g_A^e = \frac{3\pi}{4G_F^2 m_e} (\sigma^{\nu}/E - \sigma^{\bar{\nu}}/E).$$

The most precise determination of the value of $\sin^2\Theta$ in the leptonic sector has been obtained by the CHARM Collaboration [11], making use of the direct relation between the ratio of $\sigma(\nu, e)$ and $\sigma(\bar{\nu}, e)$ and $\sin^2\Theta_W$:

$$R = \frac{\sigma(\nu, e)}{\sigma(\bar{\nu}, e)} = 3 \frac{1 - 4 \sin^2\Theta_W + \frac{16}{3} \sin^4\Theta_W}{1 - 4 \sin^2\Theta_W + 16 \sin^4\Theta_W}$$

In the vicinity of $\sin^2\Theta_W = 1/4$ this relation gives $\Delta\sin^2\Theta_W \sim 1/8 (\Delta R/R)$ and, hence, a very precise determination of the mixing angle. The detection efficiency cancels in the ratio, many systematic uncertainties are reduced, and no absolute neutrino flux measurement is required. Radiative corrections also cancel to a large extent in the ratio and the value of $\sin^2\Theta_W$ determined by eq. (9) is therefore practically equal to the value determined by the boson masses, as defined by Sirlin and Marciano [26]:

$$\sin^2\Theta_W = 1 - M_W^2/M_Z^2.$$ 

Several experimental problems have to be solved [29], namely:

1) EVENT RATE requiring a large fiducial tonnage and high selection efficiency over a wide window of recoil electron energies;

2) BACKGROUND, dominantly due to quasi-elastic scattering of electron-neutrinos and to coherent $\pi^0$ production has to be reduced by a factor of about $10^3$ by precise measurements of the shower direction and efficient $e/\pi$ discrimination;
3) MONITORING of the relative flux of the different beam components $\nu_\mu, \bar{\nu}_\mu, \nu_e$ and $\bar{\nu}_e$ is required to determine the cross-section ratio $\sigma(\nu_e)/\sigma(\bar{\nu}_\mu, e)$.

It has been demonstrated by the CHARM Collaboration [11] that these problems can be solved. In two different exposures of the fine-grain CHARM detector to the horn-focused wide-band neutrino beam of the CERN 400 GeV SPS, 83 events of $\nu_\mu e$ scattering and 112 events of $\bar{\nu}_\mu e$ scattering have been recorded.

Criteria based on shower properties in low Z material (marble) were used to select these events:

i) the energy $E_F$ deposited in the first scintillator plane following the vertex was required to be $E_F < 50$ MeV, corresponding to less than 7 minimum ionizing particles;

ii) the number of wires hit in the first proportional and in the streamer tube planes following the vertex were required to be less than 2 or 7, respectively;

iii) the width of the shower was required to be of the order of the Molière radius to distinguish between electromagnetic and hadronic showers (see figure 6). Background processes were thus rejected by a factor of about $10^3$.

Measured distributions of the shower direction ($E^2\Theta^2$) for the remaining events are shown in figures 7a and b. The kinematics of elastic neutrino-electron scattering limits the electron angle to

$$E\Theta^2 \leq 2 \ m_e$$

whereas background processes due to neutrino scattering on nucleons have much broader angular distributions. According to the measured angular resolution of the detector, the bulk of the neutrino-electron scattering events (87%) is expected in the interval $E^2\Theta^2 < 0.12 \ GeV^2$ (forward region). Neutrino-electron scattering events were obtained by subtracting, in the forward region, the background measured in a reference region ($0.12 < E^2\Theta^2 < 0.54 \ GeV^2$) and extrapolated according to the hypothesis of a two-component
background:
(a) quasi-elastic charged-current events on nucleons induced by the $\nu_e (\bar{\nu}_e)$ contamination ($\sim 2\%$) of the beam,

(b) neutral current events with a $\pi^0$ in the final state produced by coherent $\nu_\mu (\bar{\nu}_\mu)$ scattering on nuclei.

The relative amount of the two background components was evaluated from the study of the $E_F$ distribution shown in figure 8; they are different for showers initiated by electrons, photons or neutral pions. Since the $E^2\theta^2$ distributions of the two background components are quite similar, an error in determining the composition of the background has little effect on the number of the neutrino-electron scattering events obtained. The neutrino and antineutrino fluxes were monitored by recording events induced by quasi-elastic charged-current scattering on nucleons and the events induced by inclusive neutral and charged-current processes on nucleons. By taking the average of both flux determinations the ratio of the normalized numbers of $\nu_\mu e$ and $\bar{\nu}_\mu e$ events is found to be

$$ R_{\text{exp}} = \frac{N(\nu_\mu e)}{N(\bar{\nu}_\mu e)} \times F = \frac{\sigma (\nu_\mu e)}{\sigma (\bar{\nu}_\mu e)} = 1.20 \pm 0.41 - 0.28 $$

The relation between $R_{\text{exp}}$ and $\sin^2 \Theta_w$ given by eq. (9) is shown in figure 9. The measured value of $R_{\text{exp}}$ agrees with the predictions of the standard model for

$$ \sin^2 \Theta_w = 0.211 \pm 0.35 \text{ (stat)} \pm 0.011 \text{ (syst)} \quad (12) $$

From the $E^2\theta^2$ distributions of the events with $E_F < 8$ MeV corresponding to a single minimum ionizing particle (see figures 8c and d) a ratio of $\nu_\mu (\bar{\nu}_\mu)e$ events with $E_F < 8$ MeV and with $E_F < 50$ MeV of $0.30 \pm 0.08$ was found, in very good agreement with the relative detection efficiency of $0.32 \pm 0.05$ for electrons, as measured in a calibration beam. This agreement is an experimental proof of the hypothesis that the selected events are due to neutrino-electron scattering.

Four pairs of values of the neutral-current coupling constants, $g_A^e$ and $g_y^e$, can be obtained from these measurements. The limits from other measurements in the lepton
sector, those from the forward-backward asymmetry in the reaction \( e^+e^- \rightarrow t^+t^- \) at PETRA and PEP [30] and from the \( \bar{\nu}_e e \) scattering cross-section [31], shown in figure 10 select a unique solution

\[
g_A^e = -0.53 \pm 0.03 \text{ (stat)} \pm 0.05 \text{ (syst)} \tag{13}
\]

\[
g_V^e = -0.08 \pm 0.07 \text{ (stat)} \pm 0.02 \text{ (syst)}.
\]

A value of \( g_A^e = -\frac{1}{2} \) is predicted by the Standard Model [1], in agreement with the experiment.

The relative strength of the neutral and charged-current coupling constants is found to be

\[
\rho = 1.06 \pm 0.07 \text{ (stat)} \pm 0.11 \text{ (syst)}, \tag{14}
\]

in agreement with the prediction of the Standard Model, \( \rho = 1 \), provided the Higgs fields form an isospin doublet. At a higher level of precision small deviations of \( \rho \) from one are expected, due to fermion doublets with large mass differences between the up and down components.

Can the present technique used for studies of neutrino-electron scattering be sufficiently improved to match the aim of \( \Delta \sin^2 \theta = \pm 0.005 \)?

A new detector, dedicated to this task (CHARM II) [32], has been built and is presently taking data at CERN. The technique of low Z material calorimetry is used again together with fine-grain detection systems.

The limiting accuracy of shower direction measurements is given by the Z number of the target material

\[
\sigma(\theta) \sim \text{const.} \frac{Z}{\sqrt{E}}. \tag{15}
\]

The mean Z of the present marble target is 13, with glass (SiO\(_2\)) a value of \( \sim 11 \) is
achieved. The accuracy depends further upon the sampling frequency (plate thickness), the grain size of the detector and the detection method.

The structure of the new, dedicated detector is sketched in figure 11. It consists of 420 modules of $3.7 \times 3.7$ m$^2$ surface area, each composed of a 4.8 cm thick target plate (glass) and of a plane of streamer tubes with 1 cm wire spacing, read out by the wires and by crossed ($90^\circ$) cathode strips of 2 cm width. Using pulse height measurements the centroid position of a track can be reconstructed with $\pm$ 3 mm accuracy from the cathode strips, whereas the wires are read out digitally to obtain unambiguous information about the track multiplicity near the vertex. A simulation of this detector by Monte Carlo methods gives an electron shower angular resolution of $\sigma(\Theta) \sim 18$ mrad/$\sqrt{E}$/GeV, about equal to the natural angular spread of recoil electrons. A photograph of this new detector is shown in figure 12. Two typical events, one due to charged-current neutrino interaction (a), the other due to $\bar{\nu}_\mu e \to \nu_e$ scattering (b) are shown in figure 13.

Figure 14 shows the measurements of elastic neutrino scattering at low $Q^2$ [33]. In the forward direction, at $Q^2$ values less than 0.03 GeV$^2$ the $\nu_\mu$ and $\bar{\nu}_\mu$ distributions differ because of contributions from the reaction $\nu_\mu e \to \mu^- e^-$. At larger values of $Q^2$ the cross sections differ by the V, A interference term proportional to $Q^2/E$, which can be corrected for. The detection efficiencies and background contributions from delta resonance production differ by small amounts. A total flux ratio error of $\pm$ 2% is estimated, leading to a total error of $\pm$ 4% on $R$, and a corresponding experimental error of $\Delta \sin^2 \Theta = \pm 0.005$. There is no theoretical uncertainty in extracting this result from eq. (9).

CONCLUDING REMARKS

With the recent advances in experiments measuring $\sin^2 \Theta_{w}$ in semi-leptonic and leptonic neutrino scattering, our understanding of the neutral weak current has progressed. Improvements from further measurements in semi-leptonic neutrino processes are likely to be limited by theoretical uncertainties. Neutrino-electron scattering is a
field that is expected to progress, following the demonstration that experiments at high energy using low Z fine-grain calorimeters can detect these rare events and separate them reliably from the dominant semi-leptonic reactions. Future results from the CERN \( \bar{p}p \) collider on \( m_w \) and \( m_z \) and from neutrino electron scattering are expected to match each other in precision of \( \sin^2 \Theta_w \) and, hence, to test the theory of electroweak interaction at the level of vacuum polarisation.
REFERENCES


A. Salam and J. Ward, Phys. Lett. 13 (1963) 168


G. Costa et al, CERN-TH/4675-87


FIGURE CAPTIONS

1. Photograph of the CHARM detector

2. Schematic view of a CC neutrino event recorded by the CHARM I detector

3. Schematic view of a NC neutrino event recorded by the CHARM I detector

4. First event of $\tilde{\nu}_\mu e$ scattering observed in Gargamelle [1]

5. Neutrino-electron scattering events observed in the CHARM I detector

6. Width $\Gamma$ of showers induced by electrons and pions in the CHARM I detector

7. $E^2\Theta^2$ distributions for (a) neutrino, (b) antineutrino events; in (c) and (d) the additional condition $E_F < 8 \text{ MeV}$ is applied

8. Distribution of $E_F$ (first scintillator plane following the vertex) for (a) electrons and (b) $\pi^0$

9. Relation between $R = \sigma(\nu_\mu e)/\sigma(\tilde{\nu}_\mu e)$ and $\sin^2\Theta$ with the measured value of $R$ and the corresponding value of $\sin^2\Theta$

10. Values of $g_A^e$ and $g_V^e$, the neutral current coupling constants obtained from the measurements of $\sigma(\nu_\mu e)$ and $\sigma(\tilde{\nu}_\mu e)$ by the CHARM Collaboration [11]. The limits from forward-backward asymmetry in $e^+e^- \rightarrow t^+t^-$ [30] and from $\sigma(\tilde{\nu}_e e)$ measurements [31] select a unique solution.

11. Sketch of the structure of the CHARM II detector [32]

12. Photograph of the CHARM II detector

13. Typical neutrino events in the CHARM II detector,
   (a) $\nu_\mu$ charged-current interaction
   (b) a candidate of $\nu_\mu e \rightarrow \nu_\mu e$ scattering

14. Observed $Q^2$ dependence of quasi-elastic $\nu_\mu (\mu^-)$ and $\tilde{\nu}_\mu (\mu^+)$ events
Fig. 7

Fig. 8

Fig. 9
VALUES OF $\sin^2 \Theta_W$ OBTAINED FROM VARIOUS PROCESSES AND THE VALUES AFTER ELECTROWEAK
RADIATIVE CORRECTIONS HAVE BEEN APPLIED

<table>
<thead>
<tr>
<th>Experiment</th>
<th>$\sin^2 \Theta_W^{\text{unc}}$</th>
<th>Condition</th>
<th>$\sin^2 \Theta_W^{\text{corr}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parity violation in Cs atoms [4]</td>
<td>0.221 ± 0.027</td>
<td></td>
<td>0.228 ± 0.027</td>
</tr>
<tr>
<td>eD scattering asymmetry [3]</td>
<td>0.224 ± 0.020</td>
<td>$\rho = 1$</td>
<td>0.218 ± 0.020</td>
</tr>
<tr>
<td>$\nu_\mu (\bar{\nu}_\mu) e$ scattering [11]</td>
<td>0.215 ± 0.034</td>
<td>Independent of $\rho$</td>
<td>0.215 ± 0.034</td>
</tr>
<tr>
<td>$\nu_\mu (\bar{\nu}_\mu) \mu$ scattering</td>
<td>0.209 ± 0.032</td>
<td>independent of $\rho$</td>
<td>0.209 ± 0.032</td>
</tr>
<tr>
<td>$\nu_\mu (\bar{\nu}_\mu) p$ scattering [13]</td>
<td>0.220 ± 0.016</td>
<td>$M_A = 1.05 \text{ GeV}$</td>
<td>0.220 ± 0.016</td>
</tr>
<tr>
<td>$\nu_\mu N$ deep inel. scattering [14]</td>
<td>0.232 ± 0.014</td>
<td>$\rho = 1$</td>
<td>0.223 ± 0.014</td>
</tr>
<tr>
<td>$\nu_\mu N$ deep inel. scattering [15]</td>
<td>0.239 ± 0.012</td>
<td>$\rho = 1$</td>
<td>0.226 ± 0.012</td>
</tr>
</tbody>
</table>
TABLE 2

The W and Z masses. The first error is statistical and the second systematic, mainly due to energy calibration.

<table>
<thead>
<tr>
<th>Group</th>
<th>$M_W$</th>
<th>$M_Z$</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>UA2 (CERN)</td>
<td>$80.2 \pm 0.8 \pm 1.3$</td>
<td>$91.5 \pm 1.2 \pm 1.7$</td>
<td>R. Ansari et al., Phys. Lett. 186B (1987) 440</td>
</tr>
<tr>
<td>UA1 (CERN)</td>
<td>$83.5^{+1.1}_{-1.0} \pm 2.7$</td>
<td>$93.0 \pm 1.4 \pm 3.0$</td>
<td>G. Arnison et al., Phys. Lett. 166B (1986) 484</td>
</tr>
<tr>
<td>UA1 + UA2 combined</td>
<td>$80.9 \pm 1.4$</td>
<td>$92.1 \pm 1.8$</td>
<td></td>
</tr>
<tr>
<td>Prediction from deep inelastic ν scattering (with radiative corrections; $\sin^2\Theta_W = 0.233 \pm 0.006$)</td>
<td>$80.2 \pm 1.1$</td>
<td>$91.6 \pm 0.9$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>NC</td>
<td>CC</td>
<td></td>
</tr>
<tr>
<td>----------------------</td>
<td>-----------------------</td>
<td>-----------------------</td>
<td></td>
</tr>
<tr>
<td>Uncorrected data sample</td>
<td>$39239 \pm 198$</td>
<td>$108472 \pm 329$</td>
<td></td>
</tr>
<tr>
<td>Trigger + filter efficiency</td>
<td>$7 \pm 4$</td>
<td>$0 \pm 0$</td>
<td></td>
</tr>
<tr>
<td>Scan correction</td>
<td>$40 \pm 40$</td>
<td>$60 \pm 44$</td>
<td></td>
</tr>
<tr>
<td>Corrected raw data sample</td>
<td>$39286 \pm 202$</td>
<td>$108532 \pm 332$</td>
<td></td>
</tr>
<tr>
<td>WB and cosmic correction</td>
<td>$-2310 \pm 87$</td>
<td>$-4311 \pm 119$</td>
<td></td>
</tr>
<tr>
<td>- of which WB</td>
<td>$-1998 \pm 88$</td>
<td>$-4308 \pm 119$</td>
<td></td>
</tr>
<tr>
<td>- of which cosmic</td>
<td>$-312 \pm 8$</td>
<td>$-3 \pm 1$</td>
<td></td>
</tr>
<tr>
<td>Clean NBB data sample</td>
<td>$36976 \pm 225$</td>
<td>$104220 \pm 361$</td>
<td></td>
</tr>
<tr>
<td>Possible difference in energy cut for NC and CC</td>
<td>$-$</td>
<td>$0 \pm 129$</td>
<td></td>
</tr>
<tr>
<td>Lost muons</td>
<td>$-3737 \pm 105$</td>
<td>$3735 \pm 105$</td>
<td></td>
</tr>
<tr>
<td>$\pi$ and $K$ decay</td>
<td>$1893 \pm 50$</td>
<td>$-1835 \pm 50$</td>
<td></td>
</tr>
<tr>
<td>$K_{e3}$ CC</td>
<td>$-1768 \pm 68$</td>
<td>$-106 \pm 6$</td>
<td></td>
</tr>
<tr>
<td>$K_{e3}$ NC</td>
<td>$-532 \pm 20$</td>
<td>$-33 \pm 2$</td>
<td></td>
</tr>
<tr>
<td>Corrected event numbers</td>
<td>$32831 \pm 283$</td>
<td>$105981 \pm 408$</td>
<td></td>
</tr>
</tbody>
</table>
TABLE 4

Corrections to $\sin^2 \Theta_w$ ($E_h > 4$ GeV)

<table>
<thead>
<tr>
<th>Source</th>
<th>$\Delta \sin^2 \Theta_w$</th>
<th>Theoretical uncertainty</th>
</tr>
</thead>
<tbody>
<tr>
<td>Muon mass</td>
<td>$+ 0.0011$</td>
<td>$\pm 0.0001$</td>
</tr>
<tr>
<td>$W^2$ thresholds, $F_L$</td>
<td>$+ 0.0005$</td>
<td>$\pm 0.0005$</td>
</tr>
<tr>
<td>K-M mixing matrix</td>
<td></td>
<td>$\pm 0.0010$</td>
</tr>
<tr>
<td>Strange sea for $m_c = 0$</td>
<td>$- 0.0074$</td>
<td>$\pm 0.0010$</td>
</tr>
<tr>
<td>Charm sea for $m_c = 0$</td>
<td>$+ 0.0015$</td>
<td>$\pm 0.0010$</td>
</tr>
<tr>
<td>Radiative corrections</td>
<td>$- 0.0092$</td>
<td>$\pm 0.0020$</td>
</tr>
<tr>
<td>Total uncertainty (fixed $m_c$)</td>
<td></td>
<td>$\pm 0.0030$</td>
</tr>
<tr>
<td>Charm mass ($m_c = 1.5$ GeV/c$^2$)</td>
<td>$+ 0.0140$</td>
<td></td>
</tr>
<tr>
<td>Total ($m_c = 1.5$ GeV/c$^2$)</td>
<td>$+ 0.0005$</td>
<td>$\pm 0.0030$</td>
</tr>
</tbody>
</table>

TABLE 5

Values of $\sin^2 \Theta_w$, derived from semi-leptonic neutrino scattering experiments. The common theoretical error is $\pm 0.003$.

<table>
<thead>
<tr>
<th>Experiment</th>
<th>$\sin^2 \Theta_w$</th>
<th>Method</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>FMMF</td>
<td>$0.246 \pm 0.016$</td>
<td>event-by-event</td>
<td>[23]</td>
</tr>
<tr>
<td>CCFR</td>
<td>$0.239 \pm 0.010$</td>
<td>event length</td>
<td>[24]</td>
</tr>
<tr>
<td>CDHS</td>
<td>$0.225 \pm 0.005$</td>
<td>event length</td>
<td>[19]</td>
</tr>
<tr>
<td>CHARM</td>
<td>$0.236 \pm 0.005$</td>
<td>event-by-event</td>
<td>[18]</td>
</tr>
</tbody>
</table>
TABLE 6

Comparison of values for $\sin^2 \Theta_w$, measured in semi-leptonic neutrino scattering and from the masses of the intermediate bosons, with and without radiative correction

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Corrected $\sin^2 \Theta_w$</th>
<th>Uncorrected $\sin^2 \Theta_w$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\nu N$</td>
<td>$0.233 \pm 0.004 \pm 0.005$</td>
<td>$0.242 \pm 0.004 \pm 0.005$</td>
</tr>
<tr>
<td>$m_w, m_z$</td>
<td>$0.226 \pm 0.008$</td>
<td>$0.210 \pm 0.008$</td>
</tr>
<tr>
<td>Difference</td>
<td>$0.007 \pm 0.010$</td>
<td>$0.032 \pm 0.010$</td>
</tr>
</tbody>
</table>