HALO
A COMPUTER PROGRAM TO CALCULATE MUON HALO

Ch. Iselin

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HALO

A COMPUTER PROGRAM TO CALCULATE MUON HALO

Ch. Iselin
ABSTRACT

HALO is a Monte Carlo computer program which permits the calculation of HALO in muon beams. It may also be used to find the muon background in other charged particle beams.

The beam transport system is entered into the program by describing its cross-section whenever it changes. Various quantities relevant to the beam behaviour, such as, for example, positions, angles, momenta, decay parameters, and many others, may be displayed in the form of one- or two-dimensional histograms. Selected ray histories may be printed out in detail if desired, or saved on disk to be processed later by other programs.
# CONTENTS

1. **INTRODUCTION**

2. **SUMMARY OF THE UNDERLYING THEORY**
   2.1 Parent generation
   2.2 Generation of the decay abscissa
      2.2.1 Unlimited decay length
      2.2.2 Forced decay
   2.3 Tracking of parent particles
      2.3.1 Drift space
      2.3.2 Bending magnet
      2.3.3 Quadrupole
      2.3.4 Beam scraper or magnetized collimator
      2.3.5 Absorber (hadron stopper)
      2.3.6 Collimator
      2.3.7 Apertures
   2.4 Decay
      2.4.1 Unlimited muon momentum
      2.4.2 Limited muon momentum
   2.5 Muon tracking
      2.5.1 Beam muon
      2.5.2 Halo muon
      2.5.3 Absorber (hadron stopper) or collimator
      2.5.4 Apertures
   2.6 Tracking of neutrinos

3. **USE OF THE PROGRAM HALO**
   3.1 Program organization
   3.2 Standard units
   3.3 Card format
   3.4 Beam specification
   3.5 Physical elements
      3.5.1 Drift space
      3.5.2 Bending magnet
      3.5.3 Quadrupole
      3.5.4 Beam scraper
      3.5.5 Magnetized collimator
   3.6 Field maps
      3.6.1 Field map scaling
      3.6.2 Generation of the map file

<table>
<thead>
<tr>
<th>CONTENTS</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>2. SUMMARY OF THE UNDERLYING THEORY</td>
<td></td>
</tr>
<tr>
<td>2.1 Parent generation</td>
<td>1</td>
</tr>
<tr>
<td>2.2 Generation of the decay abscissa</td>
<td>2</td>
</tr>
<tr>
<td>2.2.1 Unlimited decay length</td>
<td>2</td>
</tr>
<tr>
<td>2.2.2 Forced decay</td>
<td>2</td>
</tr>
<tr>
<td>2.3 Tracking of parent particles</td>
<td>3</td>
</tr>
<tr>
<td>2.3.1 Drift space</td>
<td>3</td>
</tr>
<tr>
<td>2.3.2 Bending magnet</td>
<td>3</td>
</tr>
<tr>
<td>2.3.3 Quadrupole</td>
<td>4</td>
</tr>
<tr>
<td>2.3.4 Beam scraper or magnetized collimator</td>
<td>4</td>
</tr>
<tr>
<td>2.3.5 Absorber (hadron stopper)</td>
<td>4</td>
</tr>
<tr>
<td>2.3.6 Collimator</td>
<td>4</td>
</tr>
<tr>
<td>2.3.7 Apertures</td>
<td>4</td>
</tr>
<tr>
<td>2.4 Decay</td>
<td>5</td>
</tr>
<tr>
<td>2.4.1 Unlimited muon momentum</td>
<td>5</td>
</tr>
<tr>
<td>2.4.2 Limited muon momentum</td>
<td>5</td>
</tr>
<tr>
<td>2.5 Muon tracking</td>
<td>6</td>
</tr>
<tr>
<td>2.5.1 Beam muon</td>
<td>6</td>
</tr>
<tr>
<td>2.5.2 Halo muon</td>
<td>6</td>
</tr>
<tr>
<td>2.5.3 Absorber (hadron stopper) or collimator</td>
<td>7</td>
</tr>
<tr>
<td>2.5.4 Apertures</td>
<td>7</td>
</tr>
<tr>
<td>2.6 Tracking of neutrinos</td>
<td>7</td>
</tr>
<tr>
<td>3. USE OF THE PROGRAM HALO</td>
<td></td>
</tr>
<tr>
<td>3.1 Program organization</td>
<td>7</td>
</tr>
<tr>
<td>3.2 Standard units</td>
<td>8</td>
</tr>
<tr>
<td>3.3 Card format</td>
<td>8</td>
</tr>
<tr>
<td>3.4 Beam specification</td>
<td>8</td>
</tr>
<tr>
<td>3.5 Physical elements</td>
<td>9</td>
</tr>
<tr>
<td>3.5.1 Drift space</td>
<td>9</td>
</tr>
<tr>
<td>3.5.2 Bending magnet</td>
<td>10</td>
</tr>
<tr>
<td>3.5.3 Quadrupole</td>
<td>11</td>
</tr>
<tr>
<td>3.5.4 Beam scraper</td>
<td>11</td>
</tr>
<tr>
<td>3.5.5 Magnetized collimator</td>
<td>12</td>
</tr>
<tr>
<td>3.6 Field maps</td>
<td>13</td>
</tr>
<tr>
<td>3.6.1 Field map scaling</td>
<td>13</td>
</tr>
<tr>
<td>3.6.2 Generation of the map file</td>
<td>14</td>
</tr>
</tbody>
</table>
3.7 Aperture specifications
  3.7.1 General remarks on apertures
  3.7.2 Element apertures
  3.7.3 Tunnel, inner limit of cross-section
  3.7.4 Tunnel, outer limit of cross-section
  3.7.5 Absorber
  3.7.6 Non-magnetic collimator
  3.7.7 Material definition

3.8 Output options
  3.8.1 Summary of available output options
  3.8.2 Mnemonics used on output control cards
  3.8.3 Definition of a value
  3.8.4 Simple conditions
  3.8.5 Combined conditions
  3.8.6 Ray histories
  3.8.7 One-dimensional histogram
  3.8.8 Two-dimensional histogram
  3.8.9 Mean value and r.m.s. half-width
  3.8.10 Three-dimensional distributions

3.9 Control codes
  3.9.1 Repetition
  3.9.2 Unit change
  3.9.3 Reset the random generator
  3.9.4 End of the input deck

REFERENCES

APPENDIX A: Selection of material codes by HALO

APPENDIX B: How to access HALO at CERN

APPENDIX C: Field maps
1. **INTRODUCTION**

HALO is a Monte Carlo computer program which permits the calculation of halo in muon beams. It may also be used to find the muon background in other charged particle beams. It uses the same reference system as the programs TRANSPORT$^{1,2}$) and TURTLE$^{3,4}$.

The program is based on ideas contained in TURTLE, written by David C. Carey at NAL, and a program originally written by T. Yamanouchi and modified by R. Clift and J. May. It works at the same speed as the former program, and is faster and more flexible and provides a better accuracy than the latter.

The program HALO follows the procedures outlined below.

i) A parent particle (pion or kaon) is generated according to a given production spectrum and a decay abscissa is sampled from the proper exponential distribution. The standard production is given by the Hagedorn-Ranft formula for a hydrogen target$^{5}$. It may be replaced by the user.

ii) The parent particle is tracked up to the decay point or until it hits an obstacle.

iii) If the parent particle decays, a muon and a neutrino are generated, using the high-energy approximation $W \gg M$ $^{6}$.

iv) The muon is tracked as a member of the beam until it reaches the end of the beam transport system or until it leaves the useful aperture.

v) If the muon leaves the specified central aperture, it is from then on considered as a *halo candidate*. It is tracked through any obstacle (magnet yoke, coil, tunnel wall, etc.) until it reaches the end of the system or until its momentum is reduced to zero. The magnetic deflections are obtained for each magnet from a field map. As a muon traverses material, energy loss and multiple scattering are applied accordingly.

vi) The neutrino is tracked to the end of the system, following a straight line. The neutrino tracking is suppressed if no output is requested for neutrinos.

vii) The above six steps are repeated for a preselected number of parent particles, treating them one at a time.

The beam transport system is entered into the program HALO by describing its cross-section whenever it changes. Various quantities relevant to the beam behaviour, as for example positions, angles, momenta, decay parameters, and many others, may be displayed. Selected ray histories may be printed out in detail if desired, or saved on disk to be processed later by other programs.

2. **SUMMARY OF THE UNDERLYING THEORY**

2.1 **Parent generation**

HALO uses a table look-up technique to generate parent particles. Consider the pion (or kaon) distribution per proton interacting in the target:

$$N = N_s(p_\theta, \theta_\phi),$$

(1)

where $p_\theta$, $\theta_\phi$ are the pion momentum and production angle, respectively. Then the probability that a pion has a momentum $p < p_\theta$ is
\[
P(p < p_0) = \frac{\int_{0}^{\infty} \int_{0}^{\infty} N_0(p, \theta) \, d\theta \, dp}{\int_{0}^{\infty} \int_{0}^{\infty} N_0(p, \theta) \, d\theta \, dp} = f(p_0).
\]

Thus the pion momentum can be generated from a uniformly distributed random number \( \xi_1 \) by evaluating the inverse function

\[
p = f^{-1}(\xi_1).
\]

Similarly, when a momentum \( p_0 \) has been selected, the probability that the pion is emitted within \( \theta < \theta_0 \) is

\[
P(\theta < \theta_0 | p = p_0) = \frac{\theta_0}{\int_{0}^{\infty} N_0(p, \theta) \, d\theta} = g(\theta_0 | p = p_0).
\]

Using a second uniformly distributed random number \( \xi_2 \) the production angle may be generated from

\[
\theta = g^{-1}(\xi_2 | p = p_0),
\]

where \( g^{-1}(\xi_2, p = p_0) \) is the inverse function to \( g(\theta_0 | p = p_0) \), taken as a function of \( \theta_0 \) with constant \( p_0 \).

2.2 Generation of the decay abscissa

2.2.1 Unlimited decay length

The decay length for a particle of lifetime \( \tau \) is known to be

\[
\xi_d = c \cdot \tau \cdot \gamma.
\]

The probability that the particle decays within \( \xi < \xi_d \) is

\[
P(\xi < \xi_d) = 1 - \exp \left(-\frac{\xi}{\xi_d}\right).
\]

Thus, a decay abscissa may be generated from

\[
\xi = -\xi_d \cdot \ln \xi_3,
\]

where \( \xi_3 \) is another uniformly distributed random number.

2.2.2 Forced decay

In order to improve the statistics it is often convenient to consider only those particles which decay within a given maximum length \( \xi < \xi_{\text{max}} \). The expression
\[ k_c = -k_d \cdot \ln \left\{ 1 - e_{\nu} [1 - \exp \left( -k_{\max}/k_d \right)] \right\} \]  
\[ \text{(9)} \]

will just generate those pions. The fraction of pions which decay in the limited length \( \ell < \ell_{\max} \) is

\[ \left[ 1 - \exp \left( -k_{\max}/k_d \right) \right]. \]  
\[ \text{(10)} \]

The production spectrum must then be multiplied by this factor in order to find the correct total number of pions generated. This means that the modified parent spectrum

\[ N(p,\theta) = \left[ 1 - \exp \left( -k_{\max}/k_d \right) \right] \cdot N_0(p,\theta) \]  
\[ \text{(11)} \]

must be used.

2.3 Tracking of parent particles

The present version of HALO provides drift spaces and a limited set of different magnetic elements. Parent particles are tracked through any element in one step, using an analytic field model, exactly as is done in TURTLE.

2.3.1 Drift space

For a drift space the transfer matrix is

\[ R = \begin{bmatrix} 1 & \ell & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \ell \\ 0 & 0 & 0 & 1 \end{bmatrix}. \]  
\[ \text{(12)} \]

2.3.2 Bending magnet

HALO accepts at present only uniform-field sector bending magnets. It applies the exact formula set taken from DECAY TURTLE

\[ \sin \theta_2 = \sin (\theta_1 + \alpha) - (x_1 + \rho_0) \cdot (\sin \alpha)/\rho \]
\[ x_2 + \rho_0 = (x_1 + \rho_0) \cos \alpha + \rho \left( \cos \theta_2 - \cos \left( \theta_1 - \alpha \right) \right) \]
\[ y_2 = y_1 + y'_1 \cdot \rho \cos \theta_1 \cdot (\alpha + \theta_1 - \theta_2) \]
\[ y'_2 = y'_1 \cdot \cos \theta_1/\cos \theta_2 \]  
\[ \text{(13)} \]

where

\[ x_1' = \tan \theta_1 \]
\[ x_2' = \tan \theta_2 \]
\( \alpha \) is the bending angle
\( \rho_0 \) is the radius of the reference orbit
\( \rho \) is the bending radius for the actual momentum.
2.3.3 Quadrupole

For a quadrupole the first-order transfer matrix is used:

\[
\begin{pmatrix}
\cos k \xi & \frac{1}{k} \sin k \xi & 0 & 0 \\
-k \sin k \xi & \cos k \xi & 0 & 0 \\
0 & 0 & \cosh k \xi & \frac{1}{k} \sinh k \xi \\
0 & 0 & k \sinh k \xi & \cosh k \xi
\end{pmatrix}
\]  \hspace{1cm} (14)

Since it is evaluated for each particle using its actual momentum, it is exact to all orders in momentum (chromatic aberrations). The geometric error terms are of third and higher order.

2.3.4 Beam scraper or magnetised collimator

For beam scrapers and magnetised collimators, simple analytic models are used. Within their apertures they act like drift spaces. Outside the apertures there is a certain range where the particles encounter iron or copper, and optionally a magnetic field. The difference between a beam scraper and a magnetized collimator lies in the shape of the iron yoke. For details, consult the chapter on the use of HALO and Appendix A.

2.3.5 Absorber (hadron stopper)

Any physical element may be filled with a material that has a sufficient density to absorb the parent particles. Such an element will stop all parents and force decay. This means that HALO will track only those parents which decay before reaching the first absorber.

2.3.6 Collimator

A physical element may also contain an unmagnetized collimator. This is identical to an absorber, but parents will only be stopped if they hit outside the aperture. Decay is not forced. Even though the aperture may be filled with a material, HALO will not consider any absorption, but it will apply multiple scattering and energy loss properly. The method used for tracking parent particles or muons through such an element is explained in Section 2.5.2.

2.3.7 Apertures

Parent particles are assumed to be absorbed if they hit an obstacle. The tracking routine takes the aperture to be the obstacle closest to the reference orbit. Therefore, during parent tracking only one test is made, namely if the particle is inside the aperture limits. If no aperture is specified for a given kind of element, all parents are tracked through that element using the analytical field model and can never be stopped in that element.
2.4 Decaying

2.4.1 Unlimited muon momentum

In the decay process a muon and a neutrino are generated. The decay direction is selected at random using two uniformly distributed random numbers $\xi_1, \xi_2$. The $\cos \theta^*$ is sampled from

$$\cos \theta^* = 2\xi_1 - 1 .$$

(15)

In the high-energy approximation $W \gg M$ one may then calculate the momenta and angles as follows

$$p_\mu = \gamma_\pi (W^* + p^* \cos \theta^*)$$
$$p_\nu = \gamma_\pi \cdot p^*(1 - \cos \theta^*)$$
$$\theta_\mu = \frac{p^*_\mu}{p_\mu} \cdot \sin \theta^*$$
$$\theta_\nu = \frac{p^*_\nu}{p_\nu} \cdot \sin \theta^* ,$$

(16)

where

$$\gamma_\pi = \frac{W}{m_\pi} \approx \frac{p_\pi}{m_\pi}$$
$$W^* = \frac{m^2_\pi + m^2_\mu}{2m_\pi}$$
$$p^* = \frac{m^2_\mu - m^2_\pi}{2m_\pi} .$$

(17)

The polar angle $\phi$ is sampled from

$$\phi = 2\pi \cdot \xi_2 .$$

(18)

2.4.2 Limited muon momentum

Sometimes only a limited range of muon momenta

$$p_{\mu\min} \leq p_\mu \leq p_{\mu\max}$$

(19)

is of interest. In order to get muons within this range, $\cos \theta^*$ must be sampled from the smaller interval

$$c_1 = \max \left( -1, \frac{p_{\mu\min}/\gamma_\pi - W^*}{p^*} \right) \leq \cos \theta^* \leq \min \left( 1, \frac{p_{\mu\max}/\gamma_\pi - W^*}{p^*} \right) = c_2 .$$

(20)

The probability that a pion decays with an angle in this range is

$$P(p_{\mu\min} \leq p_\mu \leq p_{\mu\max}) = \max \left( 0, \frac{c_2 - c_1}{2} \right) .$$

(21)
The pion production spectrum must be corrected with this probability which is a function of the pion momentum. The final spectrum is then

\[ N(p, \theta) = \left[ 1 - \exp\left(-\frac{p_{\text{max}}}{\lambda_d}\right) \right] \cdot N_s(p, \theta) \times \]

\[ \times \left\{ \frac{1}{2} \cdot \max\left(1, \frac{p_{\text{max}}/\gamma - \kappa^*}{p^*} \right) - \max\left(-1, \frac{p_{\text{min}}/\gamma - \kappa^*}{p^*} \right), 0 \right\}. \]  

(22)

2.5 Muon tracking

2.5.1 Beam muon

In the core of the beam, i.e. as long as they do not leave the specified aperture, muons are tracked exactly like parent particles. The difference is that a muon is not stopped when it hits an obstacle, but is tracked until it reaches the end of the beam transport system, or until its momentum becomes zero owing to energy loss.

2.5.2 Halo muon

If a muon hits the specified aperture limit, it becomes by definition a halo candidate. For histogramming purposes, halo candidates are considered to be a kind of particle different from muons. The longitudinal position where a muon hits the specified aperture for the first time is known to the program as the 'HALO POSITION'. From this point on, the tracking continues through all elements in the following steps.

i) The halo muon is assumed to advance along a straight line up to the longitudinal mid-plane of the magnetic element.

ii) The material at the current muon position is determined. See Appendix A for an exact description of how this is done.

iii) The energy loss is computed.

iv) The mean value of the momentum and the field map of the element are used to find the magnetic deflection.

v) Again using the mean value of the momentum, scattering angles and displacements are generated. The Gaussian distributions for the scattering angles have the standard deviations

\[ \sigma_x = \sigma_y = \frac{\gamma_{\text{rad}}}{p_\mu} \cdot \left( \frac{0.015 \text{ GeV/c}}{p_\mu} \right). \]

(23)

The Gaussian distributions for the displacements have the standard deviations

\[ \sigma_x = \sigma_y = \frac{\xi^3}{12 \gamma_{\text{rad}}} \cdot \left( \frac{0.015 \text{ GeV/c}}{p_\mu} \right)^2. \]

(24)

vi) The halo muon is then assumed to advance along a straight line to the exit face of the element.
If desired, the element length can be subdivided into several steps. These will be treated individually in the manner just described. Since in each step the magnetic deflection is computed from the current mean value of the momentum, this will improve the accuracy of the tracking. For halo candidates the magnetic deflections are always taken from the field maps, even though the particle may be deflected back into the central aperture of the system. In other words, the analytic field model is not used for halo candidates.

2.5.3 Absorber (hadron stopper) or collimator

Through an absorber or collimator, HALO tracks muons in the same way as it tracks halo candidates in any element. After having traversed the absorber, the tracking of the muon is resumed in the mode used before encountering the absorber.

2.5.4 Apertures

The element apertures are used to define the maximum beam size. The tracking routine assumes that there are no obstacles inside the element apertures. Thus, if a particle is inside the aperture, no further test for obstacles is made, and a muon can never become a halo muon in an element that has an unlimited aperture, even though it may actually hit a magnet yoke. Otherwise obstacles may overlap freely. The precedences taken are given in Appendix A.

2.6 Tracking of neutrinos

If desired, the neutrinos are also tracked. They will in all cases follow straight lines. If they go through a bending magnet, the curvature of the reference orbit is taken into account by the formula set

\[
\begin{align*}
\theta_2 &= \theta_1 + \alpha \\
(x_2 + \rho_e) \cos \theta_2 &= (x_1 + \rho_e) \cos \theta_1 \\
y_2 &= y_1 + y'_1 (x_1 + \rho_e) \sin \alpha \\
y'_2 \cos \theta_2 &= y'_1 \cos \theta_1,
\end{align*}
\]

where

\[
\begin{align*}
x'_1 &= \tan \theta_1 \\
x'_2 &= \tan \theta_2 \\
\alpha &= \text{the bending angle} \\
\rho_e &= \text{the radius of the reference orbit}.
\end{align*}
\]

3. USE OF THE PROGRAM HALO

3.1 Program organization

If a beam line contains a periodic section, HALO makes it possible to enter one period only and to specify how many times this period shall occur. Such a section is called a repeated section. Repeated sections within repeated sections are permitted to a depth of four.
The storage organization is dynamic, i.e. most data blocks are assigned space in a
bank of 40,000 decimal words. The space restrictions of the program are

i) The total number of elements in the beam line (after expanding repeated sections) must
not exceed 500. Note that some elements are not included in this count, and that the
program may generate some additional elements.

ii) The total number of physical positions (beginning of beam line, end of beam line, and
all points between two physical elements) may not exceed 400.

iii) The total storage is limited by the size of the data bank defined in the program. If
an element occurs within a repeated section, it uses only one block of storage. If
necessary, the size of the data storage available may be increased by use of a control
card (see Appendix B).

3.2 Standard units

Unless something different has been specified, the program HALO uses the following
input/output units:

\[
\begin{align*}
\text{mm} & \quad \text{for transverse length} \\
\text{mrad} & \quad \text{for transverse angles} \\
\text{kG} & \quad \text{for magnetic fields} \\
\text{m} & \quad \text{for path length along the reference orbit} \\
\text{GeV} & \quad \text{for particle masses} \\
\text{GeV/c} & \quad \text{for particle momentum} \\
\circ & \quad \text{for rotations around the longitudinal axis (degrees, called "DEG" in the program).}
\end{align*}
\]

The input/output units may be changed by use of a "UNIT" card (see Section 3.9.2).

3.3 Card format

Throughout the input deck, the data cards have all the same general format. Each card
is divided into 8 fields of 10 columns each, numbered from 1 to 8 starting from the left.
Some of the fields are subdivided into two subfields a and b of 5 columns each.

For floating-point values, HALO always uses a full field. A subfield may contain either
an integer value (right-justified) or a character string (left-justified without quotes).

In the explanations below, the content of each relevant field or subfield is indicated.
Fields not mentioned must be left blank.

3.4 Beam specification

The standard production spectrum is given by the Hagedorn-Ranft formula for protons
interacting with a hydrogen target\(^1\). This may be changed by the user (see Appendix B).
The input beam is given by 14 parameters on 2 cards. The first card must contain
1a the mnemonic code "BEAM"

1b the particle mnemonic. The following are recognized
"PI+" positive pions
"PI-" negative pions
"KA+" positive kaons
"KA-" negative kaons

2a the target material code. This character string is passed unchanged to the production spectrum routine and may be used to select the proper spectrum parameters. The standard parent production routine ignores this parameter and assumes a hydrogen target.

3 the number of parents to be generated (right-justified). This is the only place where an integer value occupies 10 columns.

4 the lower limit of pion momentum (GeV/c)

5 the upper limit of pion momentum (GeV/c)

6 the step width for the pion momentum generation table (GeV/c)

7 the upper limit for the production angle (mrad).

8 the step size for the production angle generation table (mrad)

The fields of the second card must contain

1 the momentum of incident protons (GeV/c)

2 the target half-width (mm)

3 the target half-height (mm)

4 the lower limit for the muon momentum (GeV/c)

5 the upper limit for the muon momentum (GeV/c).

Example:

<table>
<thead>
<tr>
<th>1</th>
<th>2</th>
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<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>BEAM</td>
<td>PI+</td>
<td>H</td>
<td>10000</td>
<td>200</td>
<td>400</td>
<td>20</td>
<td>10</td>
<td>1000</td>
<td>10</td>
</tr>
</tbody>
</table>

3.5 Physical elements

3.5.1 Drift space

A drift space requires three parameters:

1a the mnemonic code "DRF"

2b the number of subdivisions wanted during tracking of a halo particle

3 the drift length (m).
3.5.2 Bending magnet

A sector bending magnet is specified by six parameters:

1a the mnemonic code "BEND"

1b the name of the field map to be used

2b the number of subdivisions wanted during tracking of a halo particle

3 the effective magnetic length (m)

4 the central magnetic field strength (kG). A positive field bends the beam to the right (towards negative x), a negative field to the left (towards positive x).

5 the design momentum, determining the curvature of the reference orbit (GeV/c).

The reference system is the same as in the programs TRANSPORT and TURTLE. The yoke of a C-type bending magnet is normally placed to the right-hand side of the reference orbit (towards negative x, see Fig. 1). This may be changed by rotating the magnet. If the yoke is to be placed to the left-hand side, rotate the magnet by 180° and change the sign of the field (refer to Section 3.7.2).

Example:
3.5.3 Quadrupole

A quadrupole requires six parameters:

1a the mnemonic code "QUAD"

1b the name of the field map to be used

2b the number of subdivisions wanted during tracking of a halo particle

3 the effective magnetic length (m)

4 the value of the magnetic field on the reference radius (KG). A positive field means horizontal focusing.

5 the size of the reference radius (mm).

Note that the data are entered like in TRANSPORT. The reference radius has no other use than to allow the calculation of the gradient. Quadrupole apertures are never defined through the reference radius, but always taken from the relevant aperture card (see Section 3.7.2).

Example:

<table>
<thead>
<tr>
<th>QUAD</th>
<th>RPL</th>
<th>2</th>
<th>2.1</th>
<th>10.5</th>
<th>100.</th>
</tr>
</thead>
</table>

3.5.4 Beam scraper

For a beam scraper, HALO uses the analytical model shown in Fig. 2. It needs seven parameters:

1a mnemonic code "SCR"

2b the number of subdivisions wanted during tracking of a halo muon

3 the over-all length (m)

4 the strength of the magnetic field in the iron part (B₀ in KG, see Fig. 2). A positive field bends particles away from the reference orbit.

5 the aperture half-width x₁ (mm)

6 the aperture half-height y₁ (mm)

7 the over-all half-height y₂ (mm).

Example:

| SCR | 1 | 1. | 20. | 400. | 50. | 440. |
3.8.6 *Magnetised collimator*

The only difference to the beam scraper lies in the shape of the iron yoke. The analytical model for a magnetised collimator is shown in Fig. 3. It requires eight parameters:

1a mnemonic code "COLM"

2b the number of subdivisions wanted during tracking of a halo muon

3 the over-all length (m)

4 the magnetic field strength in the vertical part of the frame (B₀ in kG, see Fig. 3). A positive field bends particles away from the reference orbit.

5 the aperture half-width x₁ (mm)

6 the aperture half-height y₁ (mm)

7 the over-all half-width x₂ (mm)

8 the over-all half-height y₂ (mm).

Example:

<p>| | | | | | | | | | | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>COLM</td>
<td>2</td>
<td>2.</td>
<td>20.</td>
<td>20.</td>
<td>100.</td>
<td>100.</td>
<td>1500.</td>
<td>1500.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
3.6 Field maps

Since the field maps need a comparatively large amount of data, they are read from a separate data file, the map file. This file must contain all the field maps which may be referenced during a particular run of HALO. The program will load field maps automatically as required.

3.6.1 Field map scaling

A field map may be scaled linearly. A scaled field map must be entered before it is referenced. If the name of a scaled field map duplicates the name of an already defined map, the already existing map is used. In order to define a scaled field map, four parameters must be entered:

1a the mnemonic code "MAPS"
1b the name to be given to the scaled field map
2a the name of the original field map
3 the scale factor.

Example:
The magnet "MCC" will have its linear dimensions doubled with respect to the magnet "MKM".

3.6.2 Generation of the map file

The map file is generated and maintained using a separate program. Appendix C indicates how to proceed in order to define new maps.

3.7 Aperture specifications

3.7.1 General remarks on apertures

All aperture codes remain in effect until they are redefined. If the aperture for a given kind of element has not been defined, HALO assumes it to be unlimited.

If an aperture is changed within a repeated section of the beam line, HALO will make the system fully periodic. It will add aperture and tunnel specifications, such that before re-entering the repeated section all apertures are reset to the values they had at the first entry to the section. This remark is, however, not true for absorbers or collimators (see Sections 3.7.5 and 3.7.6).

The aperture codes use the following shape mnemonics:

"RECT" rectangular aperture
"ELL" elliptic aperture
"CIRC" circular aperture
blank no aperture limits

The geometric parameters for an aperture (see Fig. 4) are the following:

\[ x_m \] horizontal half-aperture (RECT) (mm)
horizontal half-axis (ELL)
circle radius (CIRC)

\[ y_m \] vertical half-aperture (RECT) (mm)
vertical half-axis (ELL)
unused (CIRC)

\[ \Delta x \] horizontal displacement of the element for which the aperture applies (mm)

\[ \Delta y \] vertical displacement of the element (mm)

\[ \psi \] tilt of the element around the longitudinal axis (degrees).

HALO recognizes the following pre-defined material codes:

<table>
<thead>
<tr>
<th>Mnemonic</th>
<th>Meaning</th>
<th>Radiation length</th>
<th>Energy loss</th>
</tr>
</thead>
<tbody>
<tr>
<td>&quot;VAC&quot;</td>
<td>Vacuum</td>
<td>312.4 m</td>
<td>0.0</td>
</tr>
<tr>
<td>&quot;AIR&quot;</td>
<td>Air</td>
<td>33.8 cm</td>
<td>0.23 MeV/m</td>
</tr>
<tr>
<td>&quot;BE&quot;</td>
<td>Beryllium</td>
<td>15.0 cm</td>
<td>0.315 GeV/m</td>
</tr>
<tr>
<td>&quot;DIRT&quot;</td>
<td>Concrete or earth</td>
<td>8.86 cm</td>
<td>0.46 GeV/m</td>
</tr>
<tr>
<td>&quot;AL&quot;</td>
<td>Aluminium</td>
<td>1.8 cm</td>
<td>0.448 GeV/m</td>
</tr>
<tr>
<td>&quot;FE&quot;</td>
<td>Iron or steel</td>
<td>1.47 cm</td>
<td>1.16 GeV/m</td>
</tr>
<tr>
<td>&quot;CU&quot;</td>
<td>Copper</td>
<td>0.51 cm</td>
<td>1.51 GeV/m</td>
</tr>
<tr>
<td>&quot;PB&quot;</td>
<td>Lead</td>
<td></td>
<td>1.27 GeV/m</td>
</tr>
</tbody>
</table>
Other materials can be defined by the use of a "MAT" card (see Section 3.7.7). Existing material codes, except "VAC", can be redefined.

3.7.2 Element apertures

Element apertures are defined by eight parameters:

1a mnemonic code "APE"
1b aperture shape mnemonic code
2a mnemonic of the physical element type to which the aperture applies
3-7 $x_m, y_m, \Delta x, \Delta y, \psi$.

It is important that an element aperture is specified which does not exceed the ideal field region, i.e. the region where the analytical field model is valid and where there are no obstacles if meaningful results are expected.

Example:

| APE | RECT | BEND | 100. | 50. | 10. | 6. | 30. |
All subsequent bending magnets will be shifted by 10 mm to the left (positive x) and then rotated clockwise by 90°. This means that any C-type magnets will have their yokes downwards. The aperture of bending magnets is set to 200 mm (in the bending plane) by 100 mm (gap).

3.7.3 Tunnel, inner limit of cross-section

Eight parameters are needed to specify the inner limits of the tunnel cross-section:
1a mnemonic code "TUN"
1b aperture shape mnemonic code
2a material mnemonic code
3-7 \( x_m, y_m, \Delta x, \Delta y, \psi \).

This element defines or redefines the inner limits of the tunnel cross-section. If it is omitted, HALO assumes that no tunnel is present, even if an outer limit of tunnel cross-section is specified (see Section 3.7.4).

Example:

```
<p>| | | | | | | | | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>TUN</td>
<td>RECT</td>
<td>DIAT</td>
<td>4000.</td>
<td>2000.</td>
<td>-1000.</td>
<td>0.</td>
<td>0.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
```

The tunnel aperture is -5 m < x < 3 m and -2 m < y < 2 m.

3.7.4 Tunnel, outer limit of cross-section

The outer limits of the tunnel cross-section are specified by seven parameters:
1a mnemonic code "TUNO"
1b aperture shape mnemonic
2-6 \( x_m, y_m, \Delta x, \Delta y, \psi \).

This element defines the outer limits of the tunnel cross-section. If it is omitted, the tunnel is assumed to have no outer limit. If the inner limit of the tunnel cross-section is omitted, HALO assumes that there is no tunnel.

Examples:

```
<p>| | | | | | | | | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>TUNO</td>
<td>RECT</td>
<td>4000.</td>
<td>10000.</td>
<td>0.</td>
<td>-1000.</td>
<td>0.</td>
<td>0.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
```
Two walls with no roof may be simulated as in the following example:

|   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| TVM | TUNO | RECT | DIAB | 2000. | 4000. | 200. | 0. | -2000. | 0. | 0. | 0. |

3.7.5 Absorber

An absorber is placed within the next following physical element ("DRF", "BEND", "QUAD", "SCR", or "COLL") and is turned off automatically after stepping through that element. HALO will force decay when it first encounters an absorber, i.e. all pions will be forced to decay within the portion of the beam transport system from the target up to the first absorber. If an "ABS" is used immediately preceding a repeated section, it applies to the first element of the first repetition. If it appears within a repeated section, it is repeated like any other element. An absorber requires nine parameters:

1a mnemonic code "ABS"
1b aperture shape mnemonic code
2a material to be used within the aperture of the relevant "APE" card
2b material to be used outside the aperture of the relevant "APE" card, but within the aperture of the "ABS" card. Outside the latter aperture, the data of the relevant field map are used. Consult Appendix A for details.

3-7 $x_m, y_m, \delta x, \delta y, \psi$.

Example:

|   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| APE | ABS | BEND | BEND | PE | 50. | 20. | 0. | 0. | 0. | 0. |
| BEND | MBP | PE | 180. | 2.07 | 1.94 | 2.00 |

Within the bending magnet gap, there is an iron block of 300 by 110 mm cross-section with a hole of 100 x 40 mm. The hole is filled with beryllium.

3.7.6 Non-magnetic collimator

A collimator is placed within the next following physical element ("DRF", "BEND", "QUAD", "SCR", or "COLL") and is turned off automatically after stepping through that element. It is identical in effect to the absorber, except that it does not force decay. HALO does not take into account any pion interactions inside the aperture, but the pions will scatter and lose energy properly, if the aperture contains any material. It is up to the user to specify a material, to be placed inside the aperture, which does not absorb the pions, if meaningful results are expected. If a "COLL" is used immediately preceding a repeated section, it applies to the first element of the first repetition. If it appears within a repeated section, it is repeated like any other element. A collimator requires nine parameters:
1a mnemonic code "COLL"
1b aperture shape mnemonic code
2a material to be used within the aperture of the relevant "APE" card
2b material to be used outside the aperture of the relevant "APE" card, but within the aperture of the "COLL" card. Outside the latter aperture, the data of the relevant field map are used. Consult Appendix A for details.

3-7 $x_m, y_m, \Delta x, \Delta y, \psi$.

Example:

```
APE    CIRC    DRF
COLL   RECT    VAC
FE    1000.0  2000.0  2000.0  2000.0  2000.0  2000.0
```

This is an iron block of 10 m length with a cross-section of $4 \times 4$ m. It contains a circular vacuum hole with a radius of 50 mm.

3.7.7 Material definition

A material code is defined or redefined through four parameters:

1a mnemonic code "MAT"
1b the name to be given to the defined material
2 the radiation length (m)
3 the energy loss per unit length (GeV/m).

A redefinition of the "VAC" material code has no effect.

Example:

```
MAT    W   1.0035  2.26
```

3.8 Output options

3.8.1 Summary of available output options

The most complete output available from HALO consists of tables, each of which represents the curve described in phase space by one particle and by one or the other of its daughters. Such a table is referred to as a ray history. A choice among the possible ray histories is made by specifying the daughter to be traced, and by imposing further conditions on the particles. This output option is known as the "TRACE" option (see Section 3.8.6). The "SAVE" option is identical, except that the ray histories are written on disk instead of being printed.
A large variety of distributions can be displayed in the form of histograms. The values to be used must in all cases be defined prior to their use by a "VAL" element (see Section 3.8.3). The definition of a value and the display of its distribution have been separated in order to gain in flexibility. Examples of one-dimensional histograms ("HIST1" see Section 3.8.7) are:

- distribution of pion momentum at the target,
- distribution of the horizontal position of muons at a given longitudinal position,
- distribution of cos $\theta$ at decay.

Examples of two-dimensional histograms ("HIST2", see Section 3.8.8) are:

- distribution in the horizontal phase plane $(x,x')$ at a fixed longitudinal position,
- distribution of muon momentum versus pion momentum at decay.

It is possible to initiate several one-dimensional histograms of the same quantity in a set of equidistant points along the beam line. The resulting set of histograms is printed as a single two-dimensional display (see Section 3.8.8 on "HIST2"). Example:

- distribution of the horizontal position of halo muons as a function of their longitudinal position.

Quasi-Gaussian distributions may be represented by their mean value and their r.m.s. half-width. HALO will plot these two quantities as a function of the longitudinal position "PLOT2", see Section 3.8.9). The r.m.s. half-width is optionally multiplied by a form factor in order to suit the actual distribution and to give, for example, the 90% width of the distribution. Example:

- central position and three times the r.m.s. half-width of the muon beam, plotted as a function of the longitudinal position.

The "PLOT3" option (see Section 3.8.10) consists of a plot, giving the mean value and the r.m.s. half-width of a quantity as a function of two other quantities. Example:

- mean value and r.m.s. half-width of the muon momentum distribution, plotted as a function of $x$ and $y$, all three quantities taken at a fixed longitudinal position.

In all the above cases it is possible to impose further conditions (see Sections 3.8.4 and 3.8.5) on the particles. Example:

- use only those particles which are inside or outside a given aperture at a given position,
- use only those particles for which decay occurs within a certain range of longitudinal position,
- use only those particles whose momentum is within a specified range.

Entries are made into histograms or plots only if all conditions specified are true. Conditions must all be defined prior to their use.

3.8.8 Mnemonics used on output control cards

The kinds of particles to be used are specified as follows:

"PI", "KA" parent particles (either is accepted, but in the print-out the correct code is given)
"MJ" beam muons (which have never left the aperture before reaching the point in question)
"HALO" halo muons (which are outside the aperture or have been outside at a position upstream of the point in question)
"NJ" neutrinos.

The values to be used are selected by
"X" horizontal position
"X" horizontal angle
"Y" vertical position
"Y" vertical angle
"Z" longitudinal position
"P" momentum
"R" radius
"PHI" polar angle \((-\pi \leq \phi \leq \pi)\)
"THETA" emittance angle
"N" position number (N refers to the entrance of physical element number N)
"COSTH" cosine of the decay angle in the centre-of-mass system.

The position where a value is to be taken is selected by
"HERE" at the point where the card is inserted
"DECAY" at the decay point
"HALO" at the point where the muon leaves the central aperture for the first time
"LOSS" at the point where the particle is lost because of interaction or owing to its momentum becoming zero

blank if this is consistent with the output request, the value is taken as a function of longitudinal position, otherwise "HERE" is assumed. Refer to Sections 3.8.7 to 3.8.9 for details.

3.8.3 Definition of a value

For all histograms and plots, the value to be displayed must be defined by a "VAL" element. The "VAL" element must appear in the input deck before any histogram or plot request that refers to it. It has the purpose of selecting the particle kind, the value, and the position where the value is to be taken. It also gives a name to the value and defines the scales for any histogram of the value. By itself it produces no output. It requires eight parameters:

1a the mnemonic code "VAL"
1b the name to be given to the value
2a a particle mnemonic
2b a value mnemonic
3a a position mnemonic
4 lower limit for histogram
5 upper limit for histogram
6 bin size for histogram.
If the bin size is zero, it is adjusted such that 60 bins are generated. If two or more "VAL" elements have the same name, the one last defined is used when the name is referred to. If a "VAL" element occurs within a repeated section, the value last set is used.

Example:

| VAL  | XHN | MU | HERE | -100. | 100. | 0 |

This defines the value "XHN" to be the horizontal position of the muon at the longitudinal position of the card. The bins for this value will be of 5 mm each from -100 mm to 100 mm.

3.8.4 Simple conditions

Two kinds of conditions can be imposed on a particle. They must be defined before they are referred to by another element. The "RANGE" condition is true if the particle specified passed through the given position and had the value indicated within the given range. It requires seven parameters:

1a the mnemonic code "RANGE"
1b the name to be given to the condition
2a a particle mnemonic
2b a value mnemonic
3a a position mnemonic
4 the lower limit of the range
5 the upper limit of the range.

Examples:

| RANGE | XHN | MU | HERE | 100. | 200. |

This condition is true, if the muon passed through the position of the card as a halo candidate, and if its momentum was between 100 and 200 GeV/c.

| RANGE2D | PI | Z | DECAY | 0. | 100. |

This condition is true, if the decay occurred within the first 100 m of the beam line.
The "FLAG" condition is true if the particle specified passed through the position indicated and was at that position inside (or outside) the aperture specified on the "FLAG" card. Ten parameters are required:

1a the mnemonic code "FLAG"

1b the name to be given to the condition

2a a particle mnemonic

2b an aperture mnemonic. The following are accepted:
   blank no limit
   "RECT" inside rectangle
   "CIRC" inside circle
   "ELL" inside ellipse
   ",RECT" outside rectangle
   ",CIRC" outside circle
   ",ELL" outside ellipse

3a a position mnemonic

4-8 \( x_m, y_m, \Delta x, \Delta y, \psi \), the same parameters as for an aperture (Section 3.7.1).

Examples:

This condition is true, if at the position of the card the beam muon was inside a circle of 10 mm radius.

This condition is true, if the halo muon was outside the rectangle given.

If a "FLAG" or "RANGE" appears within a repeated section, it is considered as true, if it has been set in at least one of the repetitions.

3.8.6 Combined conditions

Conditions can be constructed by logical operation on simpler conditions. The "AND" condition is true if all its arguments are true. The "OR" condition is true if at least one of its arguments is true. They both require four to sixteen parameters:
the mnemonic code "AND" or "OR"

the name to be given to the condition

at least two names of previously defined conditions. These may be of type "RANGE", "FLAG", "AND", "OR", or "NOT" (see below).

Example:

The combined condition "COMB" is true, if at least one of "C1", "C2", "C3", or "C4" is true. The "NOT" condition is true if its argument is not true. It takes three parameters:

the mnemonic code "NOT"

the name to be given to the condition

the name of a previously defined condition (of type "RANGE", "FLAG", "AND", "OR", or "NOT").

Note that

is not equivalent to

In the first example, "YF" is also set when the muon did not pass through that position, since "XF" is not true in this case. In the second example, "YF" is true only if the muon passed through that position, but missing the rectangle.

3.8.6 Ray histories

A selection of ray histories is made by two to sixteen parameters:

mnemonic code "TRACE" ... the ray histories are printed

mnemonic code "SAVE" ... the ray histories are written on disk

a particle mnemonic code
2a  blank or the word "LOSS"
2b-8b optionally, up to 13 condition names (see Sections 3.8.4 and 3.8.5).

Output occurs for a particle if the following three conditions all hold:

i) when tracking stops (i.e. a particle is stopped or reaches the end of the beam line),
   the last particle seen is of the specified kind;

ii) subfield 2a is blank and the particle reached the end of the system, or subfield 2a
    contains the word "LOSS" and the particle was stopped before reaching the end of
    the beam line;

iii) if any conditions are given, all must be true.

If more than one "TRACE" or "SAVE" card is read for a given kind of particle, output occurs
whenever the above conditions hold for at least one of these cards.

Example:

```
+---+---+---+---+---+---+---+---+---+---+---+---+---+---+---+---+---+
|   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
+---+---+---+---+---+---+---+---+---+---+---+---+---+---+---+---+---+
| TRACER | LOSS | C1 | C2 |
+---+---+---+---+---+---+---+---+---+---+---+---+---+---+---+---+---+
```

Pions are traced whenever they are lost before decaying. Muons are traced when they reach
the end of the beam line and conditions "C1" and "C2" are both true.

3.8.7 One-dimensional histogram

A one-dimensional histogram is initiated by entering two to sixteen parameters:

1a  the mnemonic code "HIST1"
1b  the name of the value to be histogrammed
2a-8b optionally, up to 14 condition names.

An entry is made into the histogram, if the value requested is available for the current
particle and if all conditions are true.

Example:

```
+---+---+---+---+---+---+---+---+---+---+---+---+---+---+---+---+---+
|   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
+---+---+---+---+---+---+---+---+---+---+---+---+---+---+---+---+---+
| HISTXVAL | COND |
+---+---+---+---+---+---+---+---+---+---+---+---+---+---+---+---+---+
```

This is a histogram of value "XVAL" for all particles for which condition "COND" is true.

A special case is given if the "VAL" element specifies a blank position code and refers
to z, the longitudinal position. In this case the histogram shows the population of the
specified particle as a function of z.
Example:

<table>
<thead>
<tr>
<th>VAL</th>
<th>FLAG</th>
<th>MU</th>
<th>Z</th>
<th>HERE</th>
<th>0.</th>
<th>1000.</th>
<th>0.</th>
<th>40.</th>
<th>0.</th>
<th>6.</th>
<th>0.</th>
</tr>
</thead>
<tbody>
<tr>
<td>HIST</td>
<td>MU</td>
<td>GOOD</td>
<td>ZMU</td>
<td>ZMU</td>
<td>GOOD</td>
<td>HERE</td>
<td>0.</td>
<td>1000.</td>
<td>0.</td>
<td>40.</td>
<td>0.</td>
</tr>
</tbody>
</table>

If these cards are inserted at the end of the beam line a histogram is constructed, which gives, at each longitudinal position, the number of muons present which reached the end of the beam line.

3.8.8 Two-dimensional histogram

A two-dimensional histogram is initiated by entering three to sixteen parameters:

1a the mnemonic code "HIST2"

1b the name of a value to be used as the horizontal coordinate. The "VAL" element should not specify more than 100 bins.

2a the name of a value to be used as the vertical coordinate

2b-8b optionally, up to 13 condition names.

An entry is made into the histogram if both values are available for a particle, and if all specified conditions are true.

Example:

<table>
<thead>
<tr>
<th>HISTZXVAL</th>
<th>YVAL</th>
<th>COND</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

This is a histogram of "XVAL" (horizontal) against "YVAL" (vertical) under the condition "COND".

A special case is given if both "VAL" elements have a blank position code and if the second (subfield 2a) refers to z. In this case the histogram will represent the set of one-dimensional histograms in equidistant positions along the beam line. The positions are the boundaries of the bins given on the second "VAL" card. The particle kind is taken from the first "VAL" element.

Example:

<table>
<thead>
<tr>
<th>VALXH</th>
<th>VALZH</th>
<th>VALXH</th>
<th>ZH</th>
<th>MALO</th>
<th>X</th>
<th>-9000.</th>
<th>4000.</th>
<th>1000.</th>
<th>40.</th>
</tr>
</thead>
<tbody>
<tr>
<td>HISTXH</td>
<td>ZH</td>
<td>HISTXH</td>
<td>ZH</td>
<td>MALO</td>
<td>X</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
3.8.9 Mean value and r.m.s. half-width

A plot of the mean value and r.m.s. half-width as a function of z is available. Such a plot is only meaningful for distributions which are sufficiently close to Gaussian (for example beam distributions, but usually not halo distributions). Optionally, the r.m.s. half-width may be multiplied by a form factor. Such a plot is initiated by entering four to fourteen parameters.

1a the mnemonic code "PLOT2"

1b the name of the value to be plotted. The "VAL" element must have a blank position code.

2a the name of a "VAL" element referring to z. Its position code must also be blank.

3 the form factor to be used. If it is 0.0, 1.0 is taken.

4a-8b optionally, the names of up to 10 conditions to select particles to be considered.

A particle is taken into account if it has the specified type and is present within the given z-range, and if all specified conditions are true.

Example:

<table>
<thead>
<tr>
<th>VAL</th>
<th>XWU</th>
<th>M</th>
<th>H</th>
<th>J</th>
<th>K</th>
<th>L</th>
<th>M</th>
<th>N</th>
<th>O</th>
<th>P</th>
<th>Q</th>
<th>R</th>
<th>S</th>
<th>T</th>
<th>U</th>
<th>V</th>
</tr>
</thead>
<tbody>
<tr>
<td>VAL</td>
<td>2XU</td>
<td>M</td>
<td>U</td>
<td>I</td>
<td>H</td>
<td>-100</td>
<td>100</td>
<td>10</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PLOT3</td>
<td>XWU</td>
<td>2XU</td>
<td>2</td>
<td>-6.</td>
<td>1000</td>
<td>10</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

3.8.10 Three-dimensional distributions

A three-dimensional distribution can be represented by the mean value and the r.m.s. half-width of a value, both plotted as a function of two other values. Such a plot is initiated by entering four to sixteen parameters:

1a the mnemonic code "PLOT3"

1b the name of a value whose mean and r.m.s. half-width are to be plotted

2a the name of a value to be used as the horizontal coordinate in the plot. This "VAL" element should not request more than 40 bins.

2b the name of a value to be used as the vertical coordinate in the plot

3a-8b optionally, up to 12 condition names.

All values will always be taken at a fixed location, i.e. it is not possible to take them as functions of z. A particle is taken into account if all three values are available, and if all conditions entered are true.

Example:

<table>
<thead>
<tr>
<th>VAL</th>
<th>PNU</th>
<th>M</th>
<th>HERE</th>
<th>J</th>
<th>K</th>
<th>L</th>
<th>M</th>
<th>N</th>
<th>O</th>
<th>P</th>
<th>Q</th>
<th>R</th>
<th>S</th>
<th>T</th>
<th>U</th>
<th>V</th>
</tr>
</thead>
<tbody>
<tr>
<td>VAL</td>
<td>KNU</td>
<td>M</td>
<td>HERE</td>
<td>-200</td>
<td>200</td>
<td>20</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>VAL</td>
<td>VNU</td>
<td>M</td>
<td>HERE</td>
<td>-200</td>
<td>200</td>
<td>20</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PLOT3</td>
<td>PNU</td>
<td>XWU</td>
<td>YNU</td>
<td>0</td>
<td>100</td>
<td>20</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
This will display the mean value and the r.m.s. half-width of the muon momentum distribution as a function of x and y, all three values taken at the same fixed position.

3.9 Control codes

3.9.1 Repetition

A section of the beam line may be repeated an arbitrary number of times. Repetitions within repetitions are accepted to a depth of four. It is not recommended to place "VAL" elements within repeated sections, since in such a case HALO will use the last value which has been set during tracking, which is not always at the same position if the particle is lost. A repeated section is sandwiched between two cards. The first card must contain

la the mnemonic code "REP"

2b the number of repetitions.

Example:

```
REP
```

The second card signals the end of the repeated section and reads:

```
REP
```

3.9.2 Unit change

The standard units may be changed to different values by means of a card with four entries:

la the mnemonic code "UNIT"

1b the name of the standard unit to be replaced

2a the new unit name

3 the factor by which a value given in the new unit must be multiplied in order to find its value in standard units.

If subfield 1b is blank, the standard units are reset.

Example:

```
UNIT MM M .001
```
This will set the transverse length units to metres. They may be reset to millimetres by

<table>
<thead>
<tr>
<th>UNIT</th>
<th>m</th>
<th>mm</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>2</td>
</tr>
</tbody>
</table>

3.9.3 Reset the random generator

The random number generator may be reset in order to produce two statistically independent program runs. Two parameters must be entered for this purpose:

1a the mnemonic code "SET"

2-3 a 20-digit octal number. For a subsequent run this value should be the number printed at the end of the preceding run.

Example:

<table>
<thead>
<tr>
<th>SET</th>
<th>1717235221667251288</th>
</tr>
</thead>
</table>

3.9.4 End of the input deck

The end of the input deck is signalled by a card with "END" in subfield 1a. If HALO reads an End-of-File, it assumes that an End card is present.

Acknowledgements

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REFERENCES

1) K.L. Brown, A first- and second-order matrix theory for the design of beam transport systems and charged particle spectrometers, SLAC 75, Rev. 3 (1972).

2) K.L. Brown, D.C. Carey, Ch. Iselin and F. Rothacker, TRANSPORT, A computer program for designing charged particle beam transport systems, Published simultaneously as CERN 73-18, NAL-91, and SLAC 91 (1973).

3) D.C. Carey, TURTLE, Trace unlimited rays through lumped elements, A computer program for simulating charged particle beam transport systems, NAL-54 (1971).


5) H. Grote, R. Hagedorn and J. Ranft, Atlas of particle spectra, CERN 1970. (The parameters used are those contained in the CERN ISR program ISPRC).

6) G. Brianti, Summary of relativistic mechanics applied to two-body collisions at high energies, CERN/Lab II/EA/74-1 (1974).
Selection of material codes by HALO

1. Is current element a drift space?
   - Yes
     - Assume air
   - No
     - Use element data to find material code

2. Is material found air?
   - Yes
     - Assume air
   - No
     - Are we outside inner tunnel cross-section?
       - Yes
         - Are we outside outer tunnel cross-section?
           - Yes
             - Assume outer "COLL" or "ABS" material code
           - No
             - Are we outside "APE" size?
               - Yes
                 - Assume inner "COLL" or "ABS" material code
               - No
                 - Assume vacuum
         - No
           - Use tunnel material
       - No
         - Are we outside "COLL" or "ABS" size?
           - Yes
             - Assume outer "COLL" or "ABS" material code
           - No
             - Assume inner "COLL" or "ABS" material code

3. Is there a collimator or absorber?
   - Yes
     - Are we outside outer tunnel cross-section?
       - Yes
         - Assume inner "COLL" or "ABS" material code
       - No
         - Assume vacuum
   - No
     - Apply multiple scattering
How to access HALO at CERN

Normally, the program HALO is accessed by the following control cards:

job, Tnm.
ACCOUNT (name, div, number)
FIND (PROFIL, PRODUCTIONMPSISELIN, ID = MPSISELIN)
BEGIN (HALO)
  end-of-record card
  data deck
  end-of-file card
If the data bank size of 40,000 words is insufficient, a control card

RFL(L = nnn)

must be inserted after the account card. The number nnn gives the required data bank size in total thousands (total multiple of 512 words) and must be at least 120. The program limits nnn to at most 400, but the SCOPE 2.0 operating system may impose a smaller limit.

HALO is available on the following permanent files:

HALOLMPSISELIN, ID = MPSISELIN : UPDATE OLDPL
HALOMPSISELIN, ID = MPSISELIN : binary version

As described in Section 3.4, the standard production spectrum is the Hagedorn-Ramft formula for protons interacting in a hydrogen target). Any other production spectrum may be used by replacing the function PRODN. The calling sequence is

\[ X = \text{PRODN}(P, \text{THETA}, \text{PP}, \text{PCL}, \text{TGT}) \]

where \( P \) is the parent momentum
\( \text{THETA} \) is the production angle
\( \text{PP} \) is the proton momentum
\( \text{PCL} \) is the particle mnemonic in Hollerith as read on the beam card
\( \text{TGT} \) is the target material code in Hollerith as read on the beam card.
\text{PRODN} must return the parent density according to the input parameters.
Appendix C

Field maps

The first time the user refers to a field map name, HALO reads this field map on a binary disk file, called the "map file". For this purpose, the standard call deck given in Appendix B attaches the permanent file

HALOMAPMPSISELIN, ID = MPSISELIN.

This file holds field maps for most of the magnets to be used in the CERN North and West Experimental Areas.

The first logical record of the map file contains a title to identify the version of the file and an index of all field maps stored on the file. Both the title and the index are printed by HALO before beginning execution. The subsequent logical records each contain the data for one field map.

An auxiliary program, MAKEMAP, is provided to generate a map file or to add new field maps to an existing file. The first card read by MAKEMAP becomes the title of the newly created or updated map file. The title card is followed by data for one or more field maps. A blank card terminates input. After execution of MAKEMAP, the file TAPE12 will contain the field maps read on cards. If TAPE11 is a properly formatted map file, TAPE12 will also contain copies of the field maps found on TAPE11.

Each field map to be read must be headed by three cards in the following format:

<table>
<thead>
<tr>
<th>Card No.</th>
<th>Format</th>
<th>Symbol</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>I5,5X,</td>
<td>N</td>
<td>Number of different names to be given to the field map</td>
</tr>
<tr>
<td></td>
<td>14A5</td>
<td>KEY</td>
<td>Up to 14 different names to be given to the field map</td>
</tr>
<tr>
<td>2</td>
<td>A5,5X,</td>
<td>KODE</td>
<td>Element type mnemonic (see Section 3.5)</td>
</tr>
<tr>
<td></td>
<td>15</td>
<td>KS</td>
<td>Shape code (see below)</td>
</tr>
<tr>
<td></td>
<td>15</td>
<td>NS</td>
<td>Number of values needed to describe the geometric shape of the magnet (see below)</td>
</tr>
<tr>
<td></td>
<td>15</td>
<td>NX</td>
<td>Number of columns in the field map</td>
</tr>
<tr>
<td></td>
<td>15</td>
<td>NY</td>
<td>Number of rows in the field map</td>
</tr>
<tr>
<td></td>
<td>EI10.0</td>
<td>Δx</td>
<td>Table step in metres between columns</td>
</tr>
<tr>
<td></td>
<td>EI10.0</td>
<td>Δy</td>
<td>Table step in metres between rows</td>
</tr>
<tr>
<td>3</td>
<td>8E10.0</td>
<td>d</td>
<td>NS lengths d₁ ... dₙₕ in metres, describing the shape of the magnet (see below)</td>
</tr>
</tbody>
</table>

The header cards are followed by the normalized field values. For dipole magnets, the fields are divided by the field B₀ in the centre of the aperture. For quadrupoles, B₀ is the value of B₀ on the x-axis at 80% of the pole-tip radius. The normalized field values are entered pairwise (Bₓ/B₀, Bᵧ/B₀) in the format 12F6.3, i.e. six pairs per card, and Nₓ values per row in order of increasing x. The Nᵧ rows follow each other in order of increasing y. Each new row begins on a new card.
The geometric shape of the iron yoke and of the coil is defined by the shape code $KS$ and the NS values $d_1 \ldots d_{NS}$. The shape codes known to HALO at the time of writing are represented in Figs. 5 to 9 and are explained below. The rectangle for which the field values must be entered is shaded in the figures.

i) $KS = 1$: Window-frame bending magnet or Panofsky quadrupole (Fig. 5)

This shape needs six parameters for its description (for bending magnets, $d_2 = d_4$). The scalar magnetic potential is antisymmetric to the x-axis. Depending on the element type, it is symmetric or antisymmetric to the y-axis. The field values must be entered for the first quadrant.

![Fig. 5](image)

ii) $KS = 2$: Symmetric H-type bending magnet (Fig. 6)

This is described by seven lengths. The scalar potential is symmetric to the y-axis and antisymmetric to the x-axis. Field values are entered for the first quadrant.

![Fig. 6](image)
iii) $KS = 3$: Quadrupole magnet (Fig. 7)

This is described by six lengths. The scalar potential is antisymmetric to both $x$- and $y$-axes, but it need not be symmetric to the first bisectrix. Field values are entered for the entire first quadrant. Note that for simplicity HALO ignores the coil, but replaces the copper by iron. This has no effect on the magnetic deflections, but it will slightly change the multiple scattering and energy loss. The normalizing field $B_0$ is $B_y$ at $x = 0.8 \cdot d_3$, $y = 0.0$.

iv) $KS = 4$: C-type bending magnet with flat poles (Fig. 8)

This is described by six lengths. Since the coil need not be symmetric, this case also covers septum magnets. The scalar potential is antisymmetric to the $x$-axis. Field values must be entered for the upper half-plane. For reasons internal to HALO, the rows must cover the range $-d_5 \leq x \leq d_5$. 

**Fig. 7**

**Fig. 8**
v) \( KS = 5 \): C-type bending magnet (Fig. 9)

This is described by seven lengths. The scalar potential is antisymmetric to the \( x \)-axis. Field values must be entered for the upper half-plane. Again the rows must cover the interval \(-d_x \leq x \leq d_x\).