Model-independent $\gamma/\phi_3$ measurement with $B \to DK$, $D \to K^0_S\pi^+\pi^-$ at LHCb

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On behalf of the LHCb collaboration
Why $\gamma$

$\gamma = \arg \left( -\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*} \right)$

$\gamma = \arg(-V_{ub}^*)$ in Wolfenstein parametrisation.

Can be measured from tree-level processes (interference of $b \to u$ and $b \to c$ trees), with negligible loop contribution.

Irreducible theory error $< 10^{-7}$

Good Standard Model reference for other UT measurements.

[Brod, Zupan, JHEP 1401 (2014) 051]
How to measure $\gamma$: time-independent approach

Interference of $b \rightarrow c\bar{u}s$ and $b \rightarrow u\bar{c}s$ amplitudes in the final state. Reconstruct $D^0$ and $\bar{D}^0$ in the same final state.

$$ V_{cb} V_{us}^* \sim A\lambda^3 $$

- Counting analyses: ADS ($D \rightarrow K\pi$), GLW ($D \rightarrow KK, \pi\pi$)
  $$ A_{B\pm} = A_D + e^{\pm i\gamma} \bar{A}_D $$

- Dalitz plot analyses: GGSZ (multibody decays):
  $$ A_{B\pm}(D) = A_D(D) + e^{\pm i\gamma} \bar{A}_D(D) $$

Sensitivity $\propto 1/r_B$, where $r_B = |\bar{A}_D/A_D| \sim 0.1$ (for $B^\pm$), $\sim 0.3$ (for $B^0$)

Similar approach with $B \rightarrow D\pi$ decays, but interference smaller by a factor 10 (although $Br$ larger by a factor $\sim 15$).
$B^+ \rightarrow DK^+$, $D \rightarrow K^0_S \pi^+ \pi^-$ Dalitz analysis

[Bondar, Belle Dalitz analysis meeting (2002)]

Look at $D \rightarrow K^0_S \pi^+ \pi^-$ Dalitz plot:

$$d\sigma(m_+^2, m_-^2) \sim |A|^2 dm_+^2 dm_-^2$$

where $m_\pm^2 = m^2_{K_S\pi\pm}$

Flavor $D$ amplitude: $A_D(m_+^2, m_-^2)$

$CP$-conservation in $D$ decays:

$$\overline{A}_D(m_+^2, m_-^2) = A_D(m_-^2, m_+^2)$$

Amplitude of $D$ from $B^+ \rightarrow DK^+$:

$$A_B(m_+^2, m_-^2) =$$

$$+ r_B e^{i\delta_B \pm i\gamma}$$

Dalitz plot variation with phase $\delta_B + \gamma$

$r_B = 0.1$
The amplitude contains $O(10)$ resonant contributions in $K\pi$ ($K^*, K_0^*, K_2^*$) and $\pi\pi$ ($\rho, \omega, f_0, f_2$ etc.) channels.

In flavour-tagged $D^* \to D\pi$ decays used to obtain the $D \to K_S^0 \pi^+ \pi^-$ amplitude, only $|f_D|^2$ is observable $\Rightarrow$ Model uncertainty.
Solution: use binned Dalitz plot, and deal with numbers of events in bins.
Relations between yields in bins of $B \rightarrow DK$ and flavour-specific $D$ phase space.

$$N_{\pm i}(B^+) = h_{B^+} [K_i + r_B^2 K_{-i} + 2\sqrt{K_i K_{-i}} (x_c i + y_s i)]$$

$$N_{\pm i}(B^-) = h_{B^-} [K_{-i} + r_B^2 K_i + 2\sqrt{K_i K_{-i}} (x_c i + y_s i)]$$

$x_{\pm} = r_B \cos(\delta_{B} \pm \gamma)$, $y_{\pm} = r_B \sin(\delta_{B} \pm \gamma)$ are free parameters.

$c_i = \langle \cos \Delta \delta_D \rangle$, $s_i = \langle \sin \Delta \delta_D \rangle$ are measured by CLEO

[PRD82 (2010) 112006]
Solution: use binned Dalitz plot, and deal with numbers of events in bins. Relations between yields in bins of $B \to DK$ and flavour-specific $D$ phase space.

\[
N_{\pm i}(B^+) = h_{B^+}[K_i + r_B^2 K_{-i} + 2\sqrt{K_i K_{-i}}(x_i c_i + y_i s_i)]
\]

\[
N_{\pm i}(B^-) = h_{B^-}[K_{-i} + r_B^2 K_i + 2\sqrt{K_i K_{-i}}(x_i c_i + y_i s_i)]
\]

$x_{\pm} = r_B \cos(\delta_B \pm \gamma)$, $y_{\pm} = r_B \sin(\delta_B \pm \gamma)$ are free parameters.

\[c_i = \langle \cos \Delta \delta_D \rangle, \quad s_i = \langle \sin \Delta \delta_D \rangle\] are measured by CLEO [PRD82 (2010) 112006]
LHCb measurement: \( D \rightarrow K_S^0 \pi^+ \pi^- \)

LHCb, 3 fb\(^{-1}\) (2011+2012)

Sample split by \( K_S^0 \) type:
Top: "LL" (decay inside vertex detector)
Bottom: "DD" (outside)

<table>
<thead>
<tr>
<th>Mode</th>
<th>Yield</th>
</tr>
</thead>
<tbody>
<tr>
<td>( B \rightarrow D \pi, \text{ LL } K_S^0 )</td>
<td>10338 ± 106</td>
</tr>
<tr>
<td>( B \rightarrow D \pi, \text{ DD } K_S^0 )</td>
<td>22779 ± 166</td>
</tr>
<tr>
<td>( B \rightarrow DK, \text{ LL } K_S^0 )</td>
<td>702 ± 18</td>
</tr>
<tr>
<td>( B \rightarrow DK, \text{ DD } K_S^0 )</td>
<td>1555 ± 39</td>
</tr>
</tbody>
</table>

\( B \rightarrow DK / B \rightarrow D \pi \) ratio fixed to be the same.
LHCb measurement: $D \to K^0_SK^+K^-$

LHCb, 3 fb$^{-1}$ (2011+2012)  

[JHEP 10 (2014) 097]

Sample split by $K^0_S$ type:  
Top: "LL" (decay inside vertex detector)  
Bottom: "DD" (outside)

<table>
<thead>
<tr>
<th>Mode</th>
<th>Yield</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B \to D\pi$, LL $K^0_S$</td>
<td>1501 ± 38</td>
</tr>
<tr>
<td>$B \to D\pi$, DD $K^0_S$</td>
<td>3338 ± 57</td>
</tr>
<tr>
<td>$B \to DK$, LL $K^0_S$</td>
<td>101 ± 4</td>
</tr>
<tr>
<td>$B \to DK$, DD $K^0_S$</td>
<td>223 ± 7</td>
</tr>
</tbody>
</table>

$B \to DK / B \to D\pi$ ratio fixed to be the same.
Combined fit of $B$ mass in all Dalitz plot bins, $x_{\pm}, y_{\pm}$ as free parameters

\[ x_{+} = -7.7 \pm 2.4 \pm 1.0 \text{(syst)} \pm 0.4 \text{(c, s)} \]
\[ x_{-} = +2.5 \pm 2.5 \pm 1.0 \text{(syst)} \pm 0.5 \text{(c, s)} \]
\[ y_{+} = -2.2 \pm 2.5 \pm 0.4 \text{(syst)} \pm 1.0 \text{(c, s)} \]
\[ y_{-} = +7.5 \pm 2.9 \pm 0.5 \text{(syst)} \pm 1.4 \text{(c, s)} \]

$\gamma = (62^{+15}_{-14})^\circ$,

$r_B = 0.080^{+0.019}_{-0.021}$,

$\delta_B = (134^{+15}_{-14})^\circ$.

Best single measurement of $\gamma$. 
Efficiency variations across Dalitz phase space are large, but (mostly) cancel if they are the same for $D \rightarrow K^0_S h^+ h^-$ from $B$ and for flavour-tagged channel.

- Use LHCb data, $B \rightarrow D^{*\pm} \mu^\mp \nu \mu$, $D^{*\pm} \rightarrow D \pi^{\pm}$, $D \rightarrow K^0_S \pi^+ \pi^-$ semileptonic channel for normalisation. $> 100000$ decays, stat. error is not an issue.
- Trigger and selection are not absolutely the same. Use MC to correct for the difference.
- Consider systematic error for this correction by comparing SL and $B^{\pm} \rightarrow D \pi^{\pm}$ and special binning which emphasises eff. variations. Largest systematic error.
Separate effect: difference in efficiency shape between LHCb and CLEO-c. Good cancellation: in

\[ c_i = \frac{\int \sqrt{\varepsilon p \bar{\varepsilon} p} \cos \Delta \delta_D dD}{\sqrt{\int \varepsilon pdD} \sqrt{\int \bar{\varepsilon}pdD}} \]

\( \varepsilon \) factors out with good precision by construction of the optimal binning.

Error of \( c_i, s_i \) is calculated separately.

<table>
<thead>
<tr>
<th>Source</th>
<th>( \sigma(x_+) )</th>
<th>( \sigma(x_-) )</th>
<th>( \sigma(y_+) )</th>
<th>( \sigma(y_-) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Statistical</td>
<td>2.4</td>
<td>2.5</td>
<td>2.5</td>
<td>2.9</td>
</tr>
<tr>
<td>Efficiency corrections</td>
<td>0.9</td>
<td>0.9</td>
<td>0.2</td>
<td>0.2</td>
</tr>
<tr>
<td>Mass fit PDFs</td>
<td>0.2</td>
<td>0.2</td>
<td>0.1</td>
<td>0.2</td>
</tr>
<tr>
<td>Shape of ( D\pi^\pm ) mis-identified as ( DK^\pm )</td>
<td>0.1</td>
<td>0.1</td>
<td>0.0</td>
<td>0.1</td>
</tr>
<tr>
<td>Shape of partially reconstructed backgrounds</td>
<td>0.1</td>
<td>0.3</td>
<td>0.1</td>
<td>0.2</td>
</tr>
<tr>
<td>( c_i, s_i ) bias due to efficiency</td>
<td>0.0</td>
<td>0.0</td>
<td>0.1</td>
<td>0.0</td>
</tr>
<tr>
<td>Migration</td>
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<td>0.1</td>
<td>0.2</td>
<td>0.2</td>
</tr>
<tr>
<td>Bias correction</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
</tr>
<tr>
<td>Total experimental</td>
<td>1.0</td>
<td>1.0</td>
<td>0.4</td>
<td>0.5</td>
</tr>
<tr>
<td>Strong-phase-related uncertainties</td>
<td>0.4</td>
<td>0.5</td>
<td>1.0</td>
<td>1.4</td>
</tr>
</tbody>
</table>
Combination of many modes:

- $B^+ \rightarrow Dh^+, D \rightarrow hh'$ (ADS, GLW): 1 fb$^{-1}$
- $B^+ \rightarrow Dh^+, D \rightarrow K3\pi$ (ADS): 1 fb$^{-1}$
- $B^+ \rightarrow DK^+, D \rightarrow K^0_S h^+ h^-$ (GGSZ): 3 fb$^{-1}$
- $B^+ \rightarrow DK^+, D \rightarrow K^0_S K^\pm \pi^\mp$ (GLS): 3 fb$^{-1}$
- $B^0 \rightarrow DK^*0, D \rightarrow hh'$ (ADS, GLW): 3 fb$^{-1}$
- $B_s \rightarrow D_s K$ time-dep., 1 fb$^{-1}$.

"Robust" combination (using only $B \rightarrow DK$):

$$\gamma = (73^{+9}_{-10})^\circ$$

95% interval:

- $[54.6, 91.4]^\circ$ "Full"
- $[52.0, 90.5]^\circ$ "Robust"
Obtaining $c_i, s_i$

Current error of $\gamma$ coming from precision of $c_i, s_i$ is $20 - 50\%$ of the statistical error (evaluated for $x_\pm, y_\pm$).

The effect of $c_i, s_i$ is not a simple systematic error, it also scales with statistics.

- $B \rightarrow DK$ alone has sensitivity to $c_i, s_i$, too.
- Could make $c_i, s_i$ free parameters, but precision drops by $\sim 10$ times. [A. Bondar, AP, EPJ C47 347 (2006)]. But this was calculated for suboptimal square binning, possibly with optimal it's better?
- Will BES-III sample be sufficient for precision $\gamma$ measurement?

Binning with 8 bins was $\sim$optimal for the CLEO/Belle statistics. We've shown it provides $\sim 90\%$ sensitivity of the unbinned method. Should we consider a finer one to gain another 10%?

In principle, $\psi(3770) \rightarrow D\bar{D}$ is not the only system sensitive to $D \rightarrow K^0_S\pi^+\pi^-$ phase. Can use other coherent $D - \bar{D}$ systems:

- $B$ decays with larger $r_B$, e.g. $B^0 \rightarrow DK^{*0}$. See in the next slides.
- Charm mixing. $c_i, s_i$ sensitivity with large $D^0$ sample and external input of $x, y$ gives precision similar or somewhat better than current CLEO-c. [G. Wilkinson, C. Thomas, arXiv:1209.0172]
Corrections for degree-level precision

Various corrections of the order $1^\circ$ in $B \to DK$ measurement:

- $D$ mixing. Can be corrected for.
  - Linear in $x, y$ if time acceptance if different for $D$ from $B \to DK$ and flavour $D$. Becomes an issue already now for $B \to D\pi$.
    
    [Matteo Rama, PRD 89 014021 (2014)]

- Another effect for GGSZ: correction $\sim r_B \times (x, y) \sim 0.2^\circ$ due to use of $c_i, s_i$ from charm threshold.
  
  [Bondar et.al, PRD 82 034033 (2010)]

- CPV in $D$. Can affect $D \to \pi\pi, KK$. $A_{CP}$ can be taken into account.

- CPV and matter interaction of neutral kaons. Currently treated as systematics for GGSZ.

Relative magnitude of these effects is enhanced by a factor 10 for $B \to D\pi$. $B \to D\pi$ thus serves as a testbed for precision $\gamma$ measurement already with the current sample.
Other modes to consider for the precision measurement:

- $D \rightarrow$ four-body.
  - $D \rightarrow 4\pi$, $D \rightarrow KK\pi\pi$ — interesting for LHCb (no $K^0_S$ in the final state)
  - $D \rightarrow K^0_S\pi^+\pi^-\pi^0$ — possibly interesting for Belle II

  Should be possible to do in a model-independent way. 5D phase space, but still divide in a few bins. Optimal binning still needs the amplitude model.

- $B \rightarrow DK^*$ — larger $r_B \simeq 0.3$, but lower yield.
  - $B \rightarrow DK^*$ interferes with other structures in $B \rightarrow DK\pi$ amplitude. Use coherence factor $\kappa$ to take it into account.
  - Considering the full amplitude provides additional sensitivity.
Flavor-specific amplitude $\bar{B}^0 \to D_{2^+}^* K^-$:

Smaller yields than $B \to DK$ but larger CPV: $r_B \sim 0.3$.

Now, Dalitz analysis of three-body $B$ decay.

Can extract $\gamma$ by comparing $D_{CP^+} \to hh$ and $D_{flavor} \to K\pi$ states, using $D_{2^+}^* K^-$ as a reference.

\[
\frac{A(\bar{B}^0 \to D_{CP}^+ K_{*0}^0)}{A(\bar{B}^0 \to D_{2^+}^* K^-)} = \rho e^{i\Delta} \\
\frac{A(\bar{B}^0 \to D_{CP}^+ K_{*0}^0)}{A(\bar{B}^0 \to D_{2^+}^* K^-)} = \rho e^{i\Delta} (1 + r_B e^{i(\delta_B - \gamma)})
\]

This approach is inherently model-dependent.
Moving to higher dimensions: double Dalitz plot analysis


An attempt to make a model-independent analysis with $B^0 \to DK^+\pi^-$.

$B^0 \to DK^+\pi^-$ and $D \to K_S^0\pi^+\pi^-$ Dalitz plots are correlated:

$$A = \overline{A}_B \overline{A}_D + e^{i\gamma} A_B A_D,$$

Strong phase varies over both $D \to K_S^0\pi^+\pi^-$ and $B^0 \to DK^+\pi^-$ phase spaces.

After binning:

$$M_{\alpha i} = h\left\{N_\alpha K_i + N_\alpha K_{-i} + 2\sqrt{N_\alpha K_i N_\alpha K_{-i}} \times \right.$$  
$$\left[\left(\kappa_\alpha c_i - \sigma_\alpha s_i\right) \cos \gamma - \left(\kappa_\alpha s_i + \sigma_\alpha c_i\right) \sin \gamma\right]\right\}.$$  

Phase terms $c_i, s_i$ are known from CLEO. $N_i$ and $\overline{N}_i$ are numbers of events in bins of flavour-specific $B^0 \to D^0 K^+\pi^-$ and $B^0 \to \overline{D}^0 K^+\pi^-$ decays.

Can solve it (even without CLEO $c_i, s_i$ input). Remember Yuval’s point: $n + k < n \times k$.

Sensitivity depends a lot on the structure of the $B^0 \to DK^+\pi^-$ amplitude. Will make a more reliable estimate after $B^0 \to DK^+\pi^-$ amplitude analysis.
Model-independent $B^+ \rightarrow DK^+$, $D \rightarrow K_S^0 h^+h^-$ analysis provides the best single constraint on $\gamma (\sigma = 15^\circ)$.

Combined LHCb sensitivity is better than $10^\circ$.

Expected statistical sensitivity:

- $\sim 4^\circ$ for LHCb Run 2.
- $\sim 2^\circ$ for Belle II ($D \rightarrow K_S^0 \pi^+\pi^-$ only)
- $< 1^\circ$ for upgraded LHCb (many modes combined).

No principal systematic uncertainty known so far, but several factors need to be carefully taken into account: charm mixing and CPV, neutral kaon effects.

Model-independent GGSZ measurement depends on the input from charm threshold. Current CLEO-c sample won’t be enough. Have to encourage BES-III to take as much data at $\psi(3770)$ as possible, or build super-$c\bar{c}$ factory.