Searches for supersymmetric partners of the bottom and top quarks with the ATLAS detector

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Thesis submitted in partial fulfilment of the requirements for the degree of Doctor of Philosophy at the University of Oxford

Hilary Term, 2014
Abstract

Supersymmetry is a promising candidate theory that could solve the hierarchy problem and explain the dark matter density in the Universe. The ATLAS experiment at the Large Hadron Collider is sensitive to a variety of such supersymmetric models. This thesis reports on a search for pair production of the supersymmetric scalar partners of bottom and top quarks in 20.1 fb$^{-1}$ of $pp$ collisions at a centre-of-mass energy of 8 TeV using the ATLAS experiment. The study focuses on final states with large missing transverse momentum, no electrons or muons and two jets identified as originating from a $b$-quark. This final state can be produced in a $R$-parity conserving minimal supersymmetric scenario, assuming that the scalar bottom decays exclusively to a bottom quark and a neutralino and the scalar top decays to a bottom quark and a chargino, with a small mass difference with the neutralino. As no signal is observed above the Standard Model expectation, competitive exclusion limits are set on scalar bottom and top production, surpassing previously existing limits. Sbottom masses up to 640 GeV are excluded at 95% CLs for neutralino masses of up to 150 GeV. Differences in mass between $\tilde{b}_1$ and $\tilde{\chi}_1^0$ larger than 50 GeV are excluded up to sbottom masses of 300 GeV. In the case of stop pair production and decay $\tilde{t}_1 \rightarrow b + \tilde{\chi}_1^\pm$ and $\tilde{\chi}_1^\pm \rightarrow \tilde{\chi}_0^1 + W^*$ with mass differences $\Delta m = m_{\tilde{\chi}_1^\pm} - m_{\tilde{\chi}_0^1} = 5$ GeV (20 GeV), stop masses up to 580 GeV (440 GeV) are excluded for $m_{\tilde{\chi}_0^1} = 100$ GeV. Neutralino masses up to 280 GeV (230 GeV) are excluded for $m_{\tilde{t}_1} = 420$ GeV for $\Delta m = 5$ GeV (20 GeV). In an extension of this analysis, sbottom quarks cascade-decaying to at least a Higgs boson are searched for in final states with large missing transverse momentum, at least 3 $b$-tagged jets and no electrons or muons, using neural network discriminants.
To my parents.
Acknowledgements

I would like to thank my supervisor, Alan Barr for his precious advice during my whole PhD. He was the one to channel my efforts in the right direction, help me with ideas when I was stuck, and provide feedback for my work. His enthusiasm is contagious. My work on the SCT project has reached its conclusions thanks to the careful supervision of Tony Weidberg, who provided me with invaluable insights about the detector. PhD students in Oxford are lucky to have such a broad spectrum of expertise available locally. I would like to particularly thank Claire Gwenlan for her advice on Monte Carlo simulation related issues. The Oxford SUSY group, including Andrée Robichaud-Veronneau, Chris Young and Mireia Crispín-Ortuzar, has provided indispensable physics input and technical help. Andrée’s work on providing the whole Oxford SUSY group with “slimmed” data and Monte Carlo samples was indispensable for the success of the entire group. The Oxford ATLAS group in general has provided feedback on numerous occasions in weekly meetings. The other DPhil students in Oxford, in particular David Hall and Lucy Kogan, have made every coffee break fun. Much more time would have been spent on bureaucracy, had I not received Sue Geddes’ and Kim Proudfoot’s help. They handled any administrative query or request with impressive speed. The IT staff, in particular Sean Brisbane and Ewan McMahon, have made it possible for me to concentrate on my work rather than on technical issues.

Once at CERN, I have worked closely with very passionate and talented physicists. While working in the SUSY group, I have learnt a lot from Monica D’Onofrio, Iacopo Vivarelli, Xavier Portell-Bueso, Davide Costanzo, Mirjam Fehling-Kaschek, Takashi Yamanaka and others. In particular Kerim Suruliz was an endless resource of wisdom. The atmosphere in the SCT performance meetings at CERN was always welcoming thanks to Steve McMahon, Dave Robinson, Saverio D’Auria, Bruce Gallop and the other members of the SCT team. I am thankful to Dan Short for setting me up in the beginning with my SCT work. While at CERN, I was well catered for, thanks to the STFC, the UK liaison office and the careful attention of the Citadines staff.

I am indebted to Olaf Behnke for guiding and inspiring me during the DESY summer student programme of 2009.

The many hours of work for this DPhil had to be compensated with many hours of dancing. I am grateful to the friends I made through salsa for taking my mind off work when I needed it.

Without the unconditional support of my parents throughout my life journey, during my undergraduate and graduate studies, I would not have reached where I am today. So I thank my parents, Liliana and Nelu, for everything and I dedicate this thesis to them.

Finally, I come to thank Ruxandra for her love and support, and for being my companion on the journey of this PhD.
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Glossary

**ABCD3TA** ATLAS Binary Chip in DMILL, version 3 with TrimDacs, revision A - custom ASIC used to read-out the silicon strips of the SCT modules

**AF** Acceleration factor

**AH** Absolute humidity

**ASIC** Application-specific integrated circuit, for example the ABCD3TA front-end chip used in the binary readout architecture of silicon strip detector modules in the ATLAS SCT

**ATLAS** A Toroidal LHC Apparatus

**BCID** Bunch crossing identification (counter) - 8-bit binary counter which is incremented on every clock cycle. The counter can be zeroed by either a hardware or software reset.

**BER** Bit error rate

**BOC** Back-of-crate card

**BPM** Biphas mark encoding

**CKKW** Jet-parton matching algorithm, Catani, Krauss, Kuhn, Webber

**Dark Matter** Any matter in the Universe which is not observed by the emission of electromagnetic radiation falls under the description of Dark Matter. It can include: failed stars and planets, free or gravitationally bound particles such as neutrinos, or theoretically hypothesised massive particles which are relics of the early Universe.

**DAC** Digital-to-analog conversion

**DAQ** Data acquisition

**DORIC4A** ASIC that decodes the BPM data into the 40 MHz bunch crossing clock and a 40 Mbit/s control data stream, which are further transmitted to the ABCD3TA ASIC

**ID** Inner Detector

**ISR** Initial State Radiation

**L1A** Level 1 accept - signal sent to the ABCD3TA ASIC triggering the read-out of its event FIFO.
LHC Large Hadron Collider

LVL1ID Level 1 trigger identification (counter) - 4-bit binary counter which is incremented each time the ABCD3TA ASIC chip receives a L1 trigger. The counter can be zeroed by either a hardware or software reset.

FSR Final State Radiation.

LSP Lightest Supersymmetric Particle. In scenarios with R-Parity conservation, the LSP is unable to decay and therefore, depending on its mass and couplings, is a good candidate for Cold Dark Matter.

MLM Jet-parton matching method named after Michelangelo Mangano

MSSM “Minimal” Supersymmetric Standard Model. A minimal extension of the Standard Model to include supersymmetric particles and their interactions. It is minimal in the sense that it only contains the particles of the Standard Model and their supersymmetric partners.

MVA Multivariate technique

NN Neural network

PDF Parton Distribution Function. The PDF gives the distribution of the relative fraction of a parton longitudinal momentum compared to that of the proton it originates from. It contributes to the calculation of differential and total cross-sections.

RH Relative humidity

ROD Read-out driver

RX Reception

SCT SemiConductor Tracker. The SCT is part of the Inner Detector.

SEU Single Event Upset

SM Standard Model of Particle Physics

SUSY Supersymmetry

TRT Transition Radiation Tracker

TTC Timing, trigger and control

TX Transmission

VCSEL Vertical-cavity surface-emitting laser. A VCSEL is a type of semiconductor laser diode which emits perpendicularly to its top surface. They are extensively used in the ATLAS SCT (as well as other detector sub-systems), both to transmit data off the detector, as well as to transmit the control, timing and trigger signals to the detector modules.
Preface

The work presented within this thesis was completed by myself as part of the ATLAS collaboration. When a diagram, histogram or exclusion plot in this thesis was not produced by myself, this is clearly stated in the caption. During my DPhil I have worked on several projects, which are listed below in reverse chronological order and with a short description of my contributions:

**Search for the supersymmetric partner of the bottom quark, cascade-decaying to a Higgs boson.** I have designed and performed a search for pair-produced sbottom quarks, cascade-decaying to at least a Higgs boson. The final state targeted has at least 3 $b$-jets, no leptons (electrons or muons) and large missing transverse momentum. Neural network techniques are used to discriminate the potential signal from the background. The search has sensitivity to SUSY scenarios beyond those of other ATLAS searches and is documented in this thesis.

**Search for the supersymmetric partners of bottom and top quarks.** I was involved in the full 2011 and 2012 data set search for pair-produced sbottom and stop quarks in final states with 2 $b$-jets, missing transverse momentum and no electrons or muons. In particular, I led the re-optimisation of the search for the 2012 data set, I assessed theoretical uncertainties for the background and signal, the multijet background and was involved in the estimation of the electroweak backgrounds. The work has resulted in several ATLAS conference notes: the results of analyzing the full 2011 data set with an integrated luminosity of 4.7 fb$^{-1}$ at $\sqrt{s} = 7$ TeV is summarized in [1]. At 8 TeV, [2] and [3] summarise the results obtained with 12.8 fb$^{-1}$ of data, and [4] presents the results with the full 2012 data set amounting to 20.1 fb$^{-1}$. A paper with the final results for the 2012 running was published in JHEP [5].
Measurement of the single event upset rate. I made the first measurement of single event upsets (SEU) in optical links in a collider environment. The SEU measurement for the semiconductor tracker was compared to predictions based on beam tests, for which I developed a prediction model. The results will be published in the SCT performance paper and featured as a separate publication in the proceedings of the TWEPP 2013 conference [6].

Statistical analysis of failure rates of opto-electronic devices in accelerated lifetime tests and in the semiconductor tracker. I have developed the mathematical tools, implemented and validated them to extract failure rates for censored populations of devices. I employed the statistical apparatus to extract the mean time to failure for VCSEL devices in the semiconductor tracker and in accelerated lifetime tests. By comparing the results, it was possible to identify the cause of failure. This work is included in the publications [7] and [8].

Monitoring the optical links of the semiconductor tracker. I have improved an existing monitoring system of the optical transmission links in the semiconductor tracker, allowing for more detailed, as well as better time-resolved monitoring.

Assessing the sensitivity of ATLAS searches to pMSSM models. Even though this work is not described in the thesis, I am the lead author of [9], in which the potential of $E_T^{\text{miss}}$ and $m_{T2}$ based searches is explored at 7 and 14 TeV center of mass energy. Two thousand phenomenological minimal supersymmetric standard model parameter space points that come from a fit to indirect and cosmological data are studied and their discoverability is assessed.

I have taken shifts operating the detector during the period of my D.Phil as an Inner Detector shifter in the ATLAS control room. The SCT related work detailed in chapters 5 and 6 and the shifts qualified me as a member of the long ATLAS collaboration author list.
Chapter 1

Introduction

The Standard Model of particle physics has had an extraordinary success at describing a wide range of experimentally observed phenomena. Nevertheless, it is an incomplete theory, as it fails to describe over 95% of the content of the Universe, and has a series of theoretical problems. The Large Hadron Collider and its experiments were built, commissioned and operated in order to put the Standard Model under stringent tests, to elucidate the mechanism of electroweak symmetry breaking and to search for physics beyond the Standard Model that could explain the composition of the Universe.

This thesis is concerned with two aspects of the above endeavour. It describes searches for physics beyond the Standard Model, in particular searches for supersymmetric partners of third generation quarks with the ATLAS detector. It also reports on several studies which ensured the reliable operation of the semiconductor tracker, an ATLAS sub-detector, and therefore have ensured that the physics aims of the project can be achieved. The thesis is organised as follows.

Firstly, an introduction to the Standard Model, its shortcomings and their potential resolution through the supersymmetric extension are given in chapter 2. The chapter discusses the theoretical motivation for searches for scalar partners of third generation quarks and typical signatures of supersymmetry at collider experiments. Theoretical and computational tools for collider physics are presented in the same chapter.
Chapter 1. Introduction

Chapter 3 outlines the LHC and the ATLAS detector, with which the proton-proton collision data analysed in this thesis was collected in 2012.

Chapter 4 describes in more detail the semiconductor tracker (SCT), one of the ATLAS sub-detectors, and serves as an introduction to the research presented in chapters 5 and 6. The first of these two chapters discusses the effects of environmental conditions on the reliability of off-detector opto-electronic devices. The second of these two chapters reports on the effects of radiation due to the proton-proton collisions on the operation of the SCT front-end electronics. The work presented in these chapters contributed towards ensuring that data is collected with the highest possible efficiency and that the physics aims of the ATLAS project can be pursued.

One of the physics aims of the ATLAS project is the search for supersymmetric (SUSY) particles. If SUSY exists, scalar partners of third generation quarks are theoretically well-motivated to have masses which render them kinematically accessible at the LHC. A search for third generation quarks with the full ATLAS 2012 data set is presented in chapter 7. The assumption of this search is that the sbottom quark decays in one channel only, though nature might not be so kind as to realise such a simple theory. Chapter 8 extends the search by considering further possible decays of the sbottoms, including cascade-decays to the newly discovered Higgs boson, and is the first ATLAS search sensitive to sbottom mixed decays.

Finally, the major results and conclusions from the work outlined in this thesis are summarised in chapter 9.
Chapter 2

Theoretical background

This chapter describes the Standard Model (SM) of particle physics, its problems and the potential for their resolution through a supersymmetric extension.

I start by presenting the SM in section 2.1. The Lagrangian of the theory is discussed, paying particular attention to the mechanism of electroweak symmetry breaking (EWSB). Further in section 2.2 I emphasize a selection of the theoretical problems of the SM, as well as experimental or observational data it cannot explain. Supersymmetry is presented in 2.3 as a possible solution to these problems. The last part of this chapter (sec. 2.4) concentrates on phenomenological aspects of Quantum Chromodynamics (QCD) that are essential for predicting backgrounds and modelling the signal for supersymmetry searches at collider experiments.

2.1 The Standard Model

The Standard Model (SM) of Particle Physics is a renormalisable gauge quantum field theory. It has been extremely successful at describing a large spectrum of experimental data in a broad range of energies. The discussion in this section is based on several books, [10], [11], [12], [13].
### 2.1 The Standard Model

<table>
<thead>
<tr>
<th>Particle type</th>
<th>Name</th>
<th>Symbol</th>
<th>Charge</th>
<th>Spin</th>
<th>Mass</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Quarks</strong></td>
<td>up</td>
<td>$u$</td>
<td>$+2/3$</td>
<td>1/2</td>
<td>2.3 MeV</td>
</tr>
<tr>
<td></td>
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<td>$-1/3$</td>
<td>1/2</td>
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</tr>
<tr>
<td></td>
<td>charm</td>
<td>$c$</td>
<td>$+2/3$</td>
<td>1/2</td>
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</tr>
<tr>
<td></td>
<td>strange</td>
<td>$s$</td>
<td>$-1/3$</td>
<td>1/2</td>
<td>95 MeV</td>
</tr>
<tr>
<td></td>
<td>top</td>
<td>$t$</td>
<td>$+2/3$</td>
<td>1/2</td>
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</tr>
<tr>
<td></td>
<td>bottom</td>
<td>$b$</td>
<td>$-1/3$</td>
<td>1/2</td>
<td>4.18 GeV</td>
</tr>
<tr>
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</tr>
<tr>
<td></td>
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<tr>
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<td>$-1$</td>
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<tr>
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<tr>
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<tr>
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<tr>
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<td>gluon</td>
<td>$g$</td>
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<td>1</td>
<td>0</td>
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<tr>
<td><strong>Higgs boson</strong></td>
<td>Higgs</td>
<td>$h$</td>
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<td>0</td>
<td>125.9 GeV</td>
</tr>
</tbody>
</table>

Table 2.1: SM particle content. Masses are taken from the Particle Data Group [14].

### Matter content

The matter content of the theory is summarised in table 2.1. The observed 3 generations of quarks and 3 generation of leptons are coupled through the gauge bosons, the gluon $g$, the $W^\pm$, $Z^0$ and the photon $\gamma$, while the Higgs boson gives particles mass.

### SM Lagrangian

Using the experimentally observed matter content, various symmetries can be imposed on the theory, from invariance under Poincaré group transformations (i.e. translations, rotations, boosts) to local gauge symmetries. The gauge group of the SM is $SU(3)_c \times SU(2)_L \times U(1)_Y$. A further constraint on the theory is that it is renormalisable, which implies that infinities occurring at loop level in the calculation of various quantities, e.g. the electron charge, mass, and field, cancel each other out up to finite terms. All terms allowed by renormalisability and the imposed symmetries must then appear in the SM Lagrangian:

$$\mathcal{L}_{SM} = \mathcal{L}_{QCD} + \mathcal{L}_{EW,\text{kinetic}} + \sum_{e, \mu, \tau} \mathcal{L}_{EW,\text{interaction}} + \mathcal{L}_{\text{Higgs}}.$$ (2.1)
Strong interactions

The QCD Lagrangian arises by stating the particle content, i.e. the quarks and gluons, and invoking local gauge invariance under $SU(3)_c$ transformations. It is given by

$$\mathcal{L}_{\text{QCD}} = \bar{\psi}(i\gamma^\mu D_\mu - m)\psi - \frac{1}{4} G^a_{\mu\nu} G^{a\mu\nu},$$  

where $\psi(x)$ denotes the quark fermionic field, $m$ is the mass of the quark, $\gamma^\mu$ are Dirac matrices, $D_\mu$ is the covariant derivative and $G^a_{\mu\nu}$ is the gluon gauge field strength tensor. The quark fermionic field $\psi(x)$ is an $SU(3)_c$ triplet comprising the $\psi_R(x)$, $\psi_G(x)$, and $\psi_B(x)$ fields. The QCD Lagrangian is invariant under local $SU(3)$ gauge transformations $\psi(x) \to U(x)\psi$ with $U(x) = \exp(iT^a(x))$, where $T^a$ are the generators of $SU(3)_c$ and $\alpha_a(x)$ are infinitesimal phases; expressed in terms of the Gell-Mann $\lambda$ matrices, the generators are $T^a = \lambda_a/2$ with the commutation relation $[T^a, T^b] = if^{abc}T^c$. The $a$ index runs over the $N_c^2 - 1$ generators, with $N_c$ being the number of colors, in this case three (R, G, B). In order to ensure gauge invariance for the fermion kinetic term, the derivative $\partial_\mu$ has to be promoted to a covariant derivative $D_\mu = \partial_\mu + g_s T^a G^a_\mu$ and gauge vector fields $G^a_\mu$ need to be introduced. In the case of $N_c = 3$ colors, 8 gluon vector fields $G^a_\mu$ are expected, which transform as an octet under $SU(3)_c$ and are singlets under $SU(2)_L$ and $U(1)_Y$ transformations. The new gauge fields have their own, gauge-invariant, kinetic term associated with them, in which the field strength tensor is defined as $G^a_{\mu\nu} = \partial_\mu G^a_\nu - \partial_\nu G^a_\mu + g_s f^{abc} G^b_\mu G^c_\nu$. The term proportional to the structure constant $f^{abc}$ is unique to non-abelian gauge theories and is responsible for three and four-point self-interactions of gluons, not present for photons in QED.

Electro-weak interactions

The part of the electro-weak (EW) Lagrangian describing the interaction of leptons with
The Standard Model

2.1 The Standard Model

gauge bosons is given by

\[ L_{\text{EW,interaction}} = \bar{\chi}_L \gamma^\mu \left[ i \partial_\mu - g \left( \frac{1}{2} \right) \boldsymbol{\tau} \cdot \mathbf{W}_\mu - \frac{g'}{2} (-1) B_\mu \right] \chi_L + \bar{e}_R \gamma^\mu \left[ i \partial_\mu - \frac{g'}{2} (2) B_\mu \right] e_R. \]  

(2.3)

The notation used is such that

\[ \chi_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}, \quad \bar{\chi}_L = \begin{pmatrix} \bar{\nu}_L \\ \bar{e}_L \end{pmatrix} \]

are a weak isospin doublet and its adjoint, containing a left-handed electron and electron neutrino Dirac spinors. They are acted upon by weak isospin generators \( \tau_i, i = 1, 2, 3 \) in the form of \( 2 \times 2 \) Pauli matrices. The upper and lower elements of the doublet have weak isospin quantum numbers \( T^3 = \pm \frac{1}{2} \), respectively. The \( e_R \) field is an isospin singlet. Each generation of leptons, \( e, \mu, \tau \), will have a similar weak isospin doublet with the same quantum numbers. The hypercharge \( Y \) is defined as \( Y = 2(Q - T^3) \) with \( Q \) being the electric charge, and is the conserved charge corresponding to the \( U(1)_Y \) group. The fermionic currents are coupled to vector fields with a strength corresponding to their weak charge or hypercharge. The \( SU(2)_L \) group is gauged by an isotriplet of vector gauge bosons \( W^i_\mu, i = 1, 2, 3 \) and couples with coupling strength \( g \) to the fermionic fields, the \( U(1)_Y \) group is gauged by a vector boson field \( B_\mu \) which couples to fermions with strength \( g'/2 \). The expressions in the square brackets of eq. (2.3) denote the covariant derivatives, the numbers in round brackets are the weak isospin charge and hypercharge of the fermionic fields. The gauge transformations under which the Lagrangian (2.3) is invariant are

\[ \chi_L \to \chi'_L = \exp \left[ -ig \frac{\tau}{2} \Delta + \frac{i}{2} g' A \right] \chi_L, \]

\[ e_R \to e'_R = \exp(ig' A)e_R, \]  

(2.4)

with \( A(x) \) being the \( U(1)_Y \) gauge transformation and \( \Delta(x) = (\Delta^1(x), \Delta^2(x), \Delta^3(x)) \) the local \( SU(2)_L \) gauge transformations. The gauge fields themselves need to change under a
The Standard Model

2.1 The Standard Model

gauge transformation as:

\[ W'_\mu = W_\mu + g \Delta \times W_\mu + \partial_\mu \Delta \]
\[ B'_\mu = B_\mu + \partial_\mu A. \] (2.5)

The term \( \tau \cdot W_\mu \) in the Lagrangian 2.3 can be rewritten as

\[ \frac{1}{2} (\tau \cdot W) = \frac{\tau^1}{2} W^1_\mu + \frac{\tau^2}{2} W^2_\mu + \frac{\tau^3}{2} W^3_\mu = \frac{1}{\sqrt{2}} \left( \tau^+ W^+_\mu + \tau^- W^-_\mu \right) + \frac{\tau^3}{2} W^3_\mu, \] (2.6)

where \( W^\pm_\mu = \frac{1}{\sqrt{2}} \left( W^1_\mu \mp W^2_\mu \right) \) are the physical states known experimentally as the \( W^\pm \). An expansion of the term involving the \( W_\mu \) fields in eq. 2.3, explicitly performing the multiplications including the \( \tau_i \) matrices, gives expressions of the form \(-\frac{g}{\sqrt{2}} \bar{\nu}_L \gamma_\mu e_L W^{+\mu} + \text{h.c.}\), giving the experimentally known charged currents and showing that the \( W \) bosons only couple to left-handed fermions. The remaining \( W^{3\mu} \) and \( B_\mu \) fields mix to give the physical neutral vector fields \( Z_\mu \) and \( A_\mu \) corresponding to the \( Z \) boson and the photon. The mixing occurs according to

\[
\begin{pmatrix}
W^3_\mu \\
B_\mu
\end{pmatrix} =
\begin{pmatrix}
\cos \theta_w & \sin \theta_w \\
-\sin \theta_w & \cos \theta_w
\end{pmatrix}
\begin{pmatrix}
Z_\mu \\
A_\mu
\end{pmatrix},
\] (2.7)

with \( \theta_w \) being the weak mixing angle. If \( \theta_w \) is chosen such that \( \tan \theta_w = g'/g \), the interaction terms containing the \( W^{3\mu} \) and \( B^\mu \) fields yield after mixing the known interactions of the \( Z \) field and the photon:

\[ \mathcal{L}_{\text{interaction}} = e (\bar{e}_L \gamma_\mu e_L + \bar{e}_R \gamma_\mu e_R) A^\mu \]
\[ - \frac{g}{2 \cos \theta_w} \left[ \nu_\mu \gamma_\mu \nu_L - \bar{e}_L \gamma_\mu e_L + 2 \sin^2 \theta_w (\bar{e}_L \gamma_\mu e_L + \bar{e}_R \gamma_\mu e_R) \right] Z^\mu. \] (2.8)

The kinetic part of the EW Lagrangian, \( \mathcal{L}_{\text{EW,kinetic}} \), is of the form

\[ \mathcal{L}_{\text{EW,kinetic}} = -\frac{1}{4} W_{\mu\nu} \cdot W^{\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu}, \] (2.9)

with \( W_{\mu\nu} = \partial_\mu W_\nu - \partial_\nu W_\mu - g W_\mu \times W_\nu \) and \( B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu \). Upon expansion in terms of the physical fields, terms of the type \((\partial_\mu V)VV\) and \(VVVV\) will appear, which translate into cubic and quartic interactions among the \( W^\pm, Z, \gamma \) fields.
2.1 The Standard Model

Particle masses

Massless fields ensure that the theory is renormalisable and gauge invariant under $SU(2)_L \times U(1)_Y$. However, it is known experimentally that this is not the case, so the theory as described must be incomplete. In principle, one can introduce mass terms of the form

$$\mathcal{L} \propto m_A A^\mu A_\mu + m_f \left( \bar{\Psi}_R \Psi_L + \bar{\Psi}_L \Psi_R \right).$$  \hspace{1cm} (2.10)

The new terms violate gauge invariance, since under the gauge transformation $A^\mu \rightarrow A^\mu + \frac{1}{g} \partial^\mu \theta$, the vector field term will gain a term which depends on the gauge field. The fermion mass term is not gauge-invariant either, as the left and right handed fields transform differently under $SU(2)_L$.

The resolution of this problem is to introduce a complex scalar doublet under the $SU(2)_L$ gauge group, the Higgs doublet $\Theta$, with weak isospin $T = \frac{1}{2}$ and hypercharge $Y = 1$. This introduces 4 additional degrees of freedom. The Lagrangian is enhanced by the Higgs sector

$$\mathcal{L}_{\text{Higgs}} = (D^\mu \Theta)^\dagger (D_\mu \Theta) + y_f \left( \bar{\chi}_L \Theta e_R + \bar{e}_R \Theta^\dagger \chi_L \right) + V \left( \Theta^\dagger \Theta \right).$$  \hspace{1cm} (2.11)

In order for the Lagrangian to be invariant under the gauge transformations 2.4 and 2.5 of the $B_\mu$ and $W_\mu$ fields, the covariant derivative needs to have the form

$$D_\mu = \partial_\mu + ig \frac{\tau_a}{2} W^a_\mu + ig' \frac{Y}{2} B_\mu.$$  \hspace{1cm} (2.12)

The Higgs doublet has weak isospin $T = \frac{1}{2}$ and hypercharge $Y = 1$. The first term in eq. 2.11 couples the Higgs doublet gauge-invariantly to the gauge-bosons. The second term is called the Yukawa term, couples fermions to the Higgs doublet and is manifestly invariant under $SU(2)_L$ transformations. The last term is the Higgs potential

$$V \left( \Theta^\dagger \Theta \right) = \mu^2 \Theta^\dagger \Theta + \lambda \left( \Theta^\dagger \Theta \right)^2.$$  \hspace{1cm} (2.13)

The potential $V$ has a degenerate, continuous set of minima at $|\Theta| = \sqrt{\frac{-\mu^2}{2\lambda}} \equiv \frac{v}{\sqrt{2}}$ when
2.1 The Standard Model

\[ \mu^2 < 0. \]  

The \( SU(2)_L \times U(1)_Y \) symmetry is spontaneously broken by picking the vacuum \( \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} \) from the set of minima, which is referred to as electroweak symmetry breaking (EWSB). The field \( \Theta \) can be expanded around this vacuum, with the following parameterisation making obvious the existence of one radial \( (h) \) and three circular \( (\Delta_i=1,2,3) \) degrees of freedom:

\[ \Theta(x) = \frac{1}{\sqrt{2}} \exp(-i\Delta(x) \cdot \tau/2) \begin{pmatrix} 0 \\ v + h \end{pmatrix}. \]  

(2.14)

Gauge freedom allows us to gauge away the three massless phase fields \( \Delta \), also called Goldstone bosons. This choice is called the “unitary gauge”. Correspondingly, the modes \( \Delta \) must also appear in the gauge transformations of the gauge bosons \( W^{1,2,3} \) and will be absorbed by them. The Goldstone fields disappear as massless degrees of freedom and will reappear as the longitudinal part of the massive vector fields. The remaining perturbation \( h \) around the minimum value corresponds to the physical Higgs boson particle.

The \( (D^\mu \Theta)^\dagger(D_\mu \Theta) \) term in eq. 2.11, upon expansion of the covariant derivative and after EWSB, gives rise to interactions of the EW gauge bosons and the Higgs particle, as well as mass terms for the gauge bosons. Using the same mixing of the gauge fields as in eq. 2.7, the mass matrix for the gauge fields becomes diagonal, and the resulting terms after symmetry breaking are

\[ \mathcal{L}_{Higgs} = \frac{1}{4} g^2 (h^2 + 2vh + v^2) W^+_\mu W^-_\mu + \frac{1}{8} (g^2 + g'^2)(h^2 + 2vh + v^2) Z_\mu Z^\mu - \mu^2 H^2 - \frac{\lambda}{4} (h^2 + 4vh^3). \]  

(2.15)

The masses of the EW gauge bosons and the Higgs boson can be easily read off:

\[ m_W = \frac{1}{2} g v \]  

(2.16)

\[ m_Z = \frac{1}{2} \sqrt{(g^2 + g'^2)} v \]  

(2.17)

\[ m_\gamma = 0 \]  

(2.18)

\[ m_h = \sqrt{2} \mu = v \sqrt{2\lambda} \]  

(2.19)
After EWSB, the Yukawa term in eq. 2.11 simplifies to
\[
\mathcal{L}_{\text{Yukawa}} = -\frac{y_e v}{\sqrt{2}} (\bar{e}_L e_R + \bar{e}_R e_L) - \frac{y_e}{\sqrt{2}} h (\bar{e}_L e_R + \bar{e}_R e_L).
\] (2.20)

The first term is a mass term, predicting the mass of the electron to be \(m_e = y_e v / \sqrt{2}\), the second term describes the interaction of the leptons with the Higgs particle, and predicts that it is proportional to the fermion’s mass.

The quark masses are obtained in a similar fashion as the lepton masses. SU(2)_L isospin doublets can be constructed for quarks, \(\chi^f_L = \begin{pmatrix} U_f \\ D_f \end{pmatrix}\) for the three quark families \(f = 1, 2, 3\). In particular, \(U_1 = u, U_2 = c, U_3 = t, D_1 = d, D_2 = s, D_3 = b\) and the \(\chi^f_L\) denote flavour eigenstates. Experimentally, both transitions \(d \to u\) and \(s \to u\) mediated by the weak force are observed. This leads to the conclusion that the weak interaction eigenstates must be a superposition of the flavour eigenstates, and so the weak interaction eigenstates can be written as
\[
\chi^f_L = \begin{pmatrix} U_f \\ \sum_{f'=1,2,3} V_{ff'} D_{f'} \end{pmatrix},
\] (2.21)

with \(V\) denoting the Cabibbo-Kobayashi-Maskawa (CKM), a 3 \(\times\) 3 unitary matrix. The quark Yukawa term has a similar structure as in the lepton case, but allows for flavour mixing:
\[
\mathcal{L}_{\text{Yukawa,quarks}} = -\sum_{f=1,2,3} \left( \chi^f_L Y^D_{ff'} \Theta D_{f'R} + \chi^f_L Y^U_{ff'} \Theta^c U_{f'R} + \text{h.c.} \right).
\] (2.22)

The \(Y^D_{ff'}, Y^U_{ff'}\) matrices of quark Yukawa couplings can be related to the CKM matrix. Upon EWSB, the Yukawa terms generate the quark masses as well as interactions with the Higgs boson in a similar fashion as in the case of the lepton Yukawa terms.

Experiments with solar, atmospheric, reactor and accelerator neutrinos have provided compelling evidence for oscillations of neutrinos caused by nonzero neutrino masses and neutrino mixing [14]. Currently, there is no experimentally validated theoretical model which provides neutrino masses. The most straightforward extension of the Lagrangian discussed
2.2 Problems of the Standard Model

so far is the introduction of a chiral right-handed neutrino field which is a singlet under $SU(2)_L$. Such an extension permits the introduction of Yukawa terms similar to those in equation \[2.11\]

\[ L'_{\text{mass}} = -y_{\nu,ij} \bar{\nu}_{R,i} \chi_{L,j} \tilde{\Theta}^\dagger + \text{hermitian conjugate}, \] (2.23)

where $\nu_{R,i}$ with $i = 1, 2, 3$ are the newly introduced right-handed neutrino fields, $i$ and $j$ are indices over neutrino flavours, $y_{\nu,ij}$ are Yukawa couplings and $\tilde{\Theta}^\dagger = i \tau_2 \Theta^*$ with $\Theta$ being the Higgs doublet. Upon EWSB, Dirac mass terms emerge:

\[ L'_{\text{mass}} = -\frac{1}{\sqrt{2}} y_{\nu,ij} \bar{\nu}_{R,i} \nu_{L,j} \] + hermitian conjugate. (2.24)

This approach however does not explain the very low experimentally observed masses, as intuitively the Dirac masses $M_{\text{Dirac},ij} = y_{\nu,ij} v/\sqrt{2}$ should be at the electroweak symmetry breaking scale. One of several proposed solutions [15] is to further introduce Lorentz-invariant Majorana mass terms for the neutrinos using charge-conjugate fields $\nu^c$:

\[ L'_{\text{mass}} = -y_{\nu,ij} \bar{\nu}_{R,i} \chi_{L,j} \tilde{\Theta}^\dagger - \frac{1}{2} \bar{\nu}_{R,i} M_{\text{Majorana},ij} \nu^c_{R,j} \] + hermitian conjugate. (2.25)

A Majorana mass term is only allowed for $\nu_R$ fields in order to preserve $SU(2)_L$ gauge invariance. No theoretical constraints exist on the Majorana masses $M_{\text{Majorana},ij}$, and it can be assumed that $M_{\text{Majorana},ij} \gg M_{\text{Dirac}}$. After EWSB, the mass matrix can be diagonalised to extract the physical masses. The three states corresponding to the experimentally observed neutrinos acquire a mass supressed by $1/M_N$, while the remaining neutrino states have masses $O(M_N)$, a process called the type I see-saw mechanism. It is also noted that mass eigenstates are Majorana particles, i.e. the neutrinos are their own anti-particles.

2.2 Problems of the Standard Model

The SM describes a broad spectrum of experimental data in an energy range from sub-eV to TeV. However, broadly speaking, there are four categories of shortcomings of the SM:
Firstly, a global fit of the electroweak sector of the SM to various experimental data gives a goodness-of-fit $p$-value of 7% [16], suggesting a certain degree of tension between the SM and observations.

Secondly, the SM fails to explain certain observed physical phenomena, and is thus not a “theory of everything” yet. The most striking example is that the SM does not even attempt to describe gravity. Another important example is that the SM does not provide a mechanism to generate neutrino masses. Experimentally observed neutrino oscillations imply massive neutrinos, but in the SM, one cannot write a $SU(2)_L$ invariant term in the Lagrangian that couples the $\nu_L$ field with the Higgs field, as the model does not include right-handed neutrinos. Therefore masses cannot be generated through the same mechanism as for the other fermions. A further example is the lack of a dark matter candidate, an issue explained in subsection 2.2.2.

Thirdly, the Standard Model has 19 free parameters, whose values have no theoretical motivations, and which need to be constrained experimentally. In an ultimate theory, one hopes to have a dynamic mechanism that generates the values of the free parameters. At the moment, having established experimentally the masses of all quarks and leptons, there is no theoretical justification for the differences of several orders of magnitude between the mass of the first and the third generation fermions. Similarly, there is no explanation in the theory itself as to why there are three generations of fermions. A further example is that the parameters relating to the CP violation and EW mixing have no theoretical predictions.

Finally, there are inconsistencies within the model, though one might argue that these are more aesthetical in nature. For example, in a general theory, one hopes all fundamental forces unify at a high scale, which is not the case in the SM, as can be seen in figure 2.1. A further such inconsistency, the hierarchy problem, is described in the next subsection.

---

[1] The data the fit is performed to includes the EW precision data measured at the $Z$-pole at the LEP experiments and at SLD, the mass and width of the $W$ boson from LEP and Tevatron, mass of the top quark from Tevatron experiments and the Higgs mass from LHC experiments.
2.2 Problems of the Standard Model

Figure 2.1: Renormalisation group evolution of the inverse gauge couplings $\alpha_a^{-1}(Q)$ as a function of the energy scale $Q$ in the SM (dashed lines) and the MSSM (solid lines). In the MSSM case, the sparticle masses are treated as a common threshold varied between 500 GeV and 1.5 TeV, and $\alpha_S(m_Z)$ is varied between 0.117 and 0.121, giving rise to the red and blue solid lines. Figure taken from [17].

2.2.1 The hierarchy problem

Radiative corrections from Feynman diagrams as those depicted in figure 2.2 need to be included when calculating the physical Higgs mass. The loops giving the largest contributions are those including particles with large couplings to the Higgs boson, like $t, W^\pm, Z^0$ and the Higgs itself. Concretely, the top quark contribution, considering $N_c = 3$ colors, is given by

$$
\Delta m_h^2 = (-1)N_c \int \frac{d^4p}{(2\pi)^4} \text{Tr} \left[ -iy_t \frac{i}{\sqrt{2} p - m_t} - iy_t \frac{i}{\sqrt{2} p - m_t} \right]
$$

$$
\approx -\frac{3}{8\pi^2} y_t^2 \Lambda^2 + \frac{3y_t^2}{8\pi^2} m_t^2 \ln \left( \frac{\Lambda^2}{m_t^2} \right),
$$

Figure 2.2: Radiative corrections to the Higgs mass due to (a) a Dirac fermion $f$, and (b) a scalar $S$. 

\[ \]
where the integral over the 4-momentum of the loop particle was calculated. The integral was performed with a loop particle momentum ranging from zero to a high cut-off scale $\Lambda$ (also called the ultraviolet, or UV scale), which is the scale up to which the SM is expected to be valid. If there is no new physics beyond the SM, then the scale $\Lambda$ will be the Planck scale $M_P \sim 10^{19}$ GeV, at which quantum effects of gravity become strong. Contributions from gauge bosons and the Higgs have a similar dependence on the parameter $\Lambda$ as shown in eq. 2.27. Combined with the bare Higgs mass, these corrections give rise to the physical Higgs mass measured at the electro-weak scale,

$$m_h^2(\text{EW scale}) = m_h^2(\Lambda) + \Delta m_h^2,$$  (2.28)

where $m_h$ is considered to be a function of the energy scale. The above equation is the embodiment of the hierarchy problem: the very high mass of the Higgs at the UV scale and the very large radiative corrections $\Delta m_h^2 \propto \Lambda^2$ are both $O(M_P^2)$, but they need to cancel very precisely in order to give a Higgs mass at the EW scale of only 125.9 GeV.

### 2.2.2 The dark matter problem

Another problem of the Standard Model comes from cosmology. Astronomical observations – like measurements of galactic rotation curves, of gravitational lensing and of collisions of galactic clusters – provide evidence of the existence of a type of matter, called dark matter, which interacts gravitationally, but is non-luminous. Further evidence for its existence come from the power spectrum of the cosmic microwave background, as well as the large scale structure of the universe. The latter requires the dark matter to be “cold”, i.e. non-relativistic. However, up to date no convincing evidence of direct detection of dark matter has been provided. A pedagogical review of all the evidence is provided in [18]. In order to explain the observed effects, dark matter must make up over 80% of the total matter in the universe.

Experimental measurements from the Planck collaboration [19] give the following values
for the dark energy density parameter and the physical cold dark matter and baryonic matter density in the Universe, together with their 68% confidence intervals

\[
\begin{align*}
\Omega_{\text{baryons}} &= 0.0489 \pm 0.0003, \quad (2.29) \\
\Omega_{\text{cold dark matter}} &= 0.2651 \pm 0.0017, \quad (2.30) \\
\Omega_{\Lambda} &= 0.686 \pm 0.020. \quad (2.31)
\end{align*}
\]

The Standard Model does not have a suitable dark matter candidate, and so physics beyond the Standard Model is needed to explain cold dark matter. If dark matter is a particle, then it needs to be neutral, stable (or long-lived), and cannot interact with the Standard Model sector through the strong or electromagnetic force. Possible proposals for dark matter include neutrinos and axions. The first are too light and hot to match the observed properties of dark matter. The latter are a suitable dark matter candidate as long as the axion mass is above \(\sim 10\mu\text{eV}\), and also have the advantage of solving the strong CP-problem [20]. In a broad range of the plausible mass range for axions to be dark matter, galactic halo axions may be detected by their resonant conversion into a quasi-monochromatic microwave signal in an electromagnetic cavity permeated by a strong static magnetic field. However, up to date there is no experimental evidence for their existence [14].

**WIMPs as dark matter candidates**

The early Universe was a high energy place, in which dark matter particles \(\chi\) could pair-annihilate to Standard Model particles and vice-versa. If the annihilation rate \(\Gamma\) of \(\chi\bar{\chi}\) is lower than the instantaneous expansion rate of the Universe (the Hubble “constant” at that time) \(H\), then the annihilation of \(\chi\)’s freezes out. The DM relic density is then expected to be

\[
\Omega_\chi h^2 = \frac{m_\chi n_\chi}{\rho_c} \approx 3 \times 10^{-27}\text{cm}^3\text{s}^{-1}\frac{1}{\langle \sigma_A v \rangle} \approx 0.1 \text{ pb} \cdot \text{c} \frac{1}{\langle \sigma_A v \rangle}, \quad (2.32)
\]

where \(h\) is the reduced Hubble constant, \(m_\chi\) is the mass of the DM particle, \(n_\chi\) the relic number density, \(\rho_c\) a critical mass density above which annihilation freezes and \(\langle \sigma_A v \rangle\) is the
velocity-weighted mean annihilation cross-section. The experimentally measured value is $\Omega_\chi h^2 = 0.1196 \pm 0.0031$ [19], which implies a value for $\langle \sigma_A v \rangle$ of the order 1 pb $\cdot$ c.

A proposed solution to the dark matter puzzle is the existence of weakly interacting massive particles (WIMPs), with mass in the approximate range $m_\chi \approx 1 - 1000$ GeV. For a typical weakly interacting particle with mass $m_\chi \approx 100$ GeV, the annihilation cross-section is

$$\langle \sigma_A v \rangle \approx \frac{\alpha_W^2}{m_\chi^2} c \approx 1 \text{ pb} \cdot c$$

(2.33)

with $\alpha_W$ being the weak coupling constant. The annihilation cross-section of WIMPs is the right order of magnitude to give the observed $\Omega_\chi$. This coincidence is also called the “WIMP miracle”. Now a theory is needed that predicts weakly interactive, massive particles.

## 2.3 Supersymmetry

Supersymmetry (SUSY) is an extension of the Standard Model which can provide a dark matter candidate and solve the hierarchy problem. It predicts the existence of super-partners to all Standard Model particles. The discussion in this section is based on the pedagogical treatment in [17], [21] and [22].

A first theoretical motivation for SUSY is provided by the observation that supersymmetry is the only natural extension of the symmetry group of the SM. According to the Coleman-Mandula theorem [23], space-time and internal symmetries like gauge symmetries cannot be unified, as any theory with non-zero commutators of the generators of the internal symmetries with those of the Poincaré group yields a trivial $S$ matrix, i.e. the particle interactions vanish. However, the assumption of the Coleman-Mandula theorem is that all symmetries are bosonic and that their conserved charges have integer spin. Supersymmetry escapes this constraint, as supersymmetry generators transform bosonic fields into fermionic fields and vice-versa. The anti-commutator of SUSY generators yields a spacetime translation, thus intrinsically combining space-time and internal symmetries. The Haag-Lopuszanski-Sohnius theorem [24] proves that the possible symmetries of a consistent 4-dimensional quantum
field theory consist of internal symmetries and Poincaré symmetries, and can also include supersymmetry as a non-trivial extension of the Poincaré algebra, but it is still true that internal gauge symmetries cannot be mixed with spacetime symmetries.

2.3.1 Wess-Zumino Lagrangian: SUSY implies equal masses for superpartners

I start by introducing the simplest four-dimensional supersymmetric, renormalisable field theory, the Wess-Zumino model [25], which includes a real scalar, a real pseudoscalar, and a real fermion field. Supergauge invariance is imposed on the theory and gives rise to relations among the masses and the coupling of the fields. The Wess-Zumino Lagrangian is given by a free part

\[
\mathcal{L}_{\text{free}} = \frac{1}{2} (\partial_\mu A \partial^\mu A) + \frac{1}{2} (\partial_\mu B \partial^\mu B) + \frac{1}{2} \bar{\psi} \gamma^\mu \partial_\mu \psi - \frac{1}{2} (F^2 + G^2),
\]

(2.34)

to which a mass term can be added:

\[
\mathcal{L}_m = m \left( \frac{1}{2} \bar{\psi} \psi - FA - GB \right)
\]

(2.35)

and an interaction term:

\[
\mathcal{L}_i = g \left[ ig \bar{\psi} (A - \gamma_5 B) \psi - F (A^2 - B^2) - 2GAB \right].
\]

(2.36)

The \(A\) and \(B\) fields are a scalar and a pseudoscalar, respectively. \(F\) and \(G\) are auxiliary fields. Applying the Euler-Lagrange equations for the pair of fields \(A, F\), the pair \(B, G\) and the fermion field \(\psi\), equations of motion for the fields can be derived and it can be observed that the fields \(A, B, \psi\) share a common mass \(m\). This finding is intrinsically connected to the fact that only if a common mass exists is the Lagrangian invariant under local supergauge transformations. The infinitesimal SUSY transformations are given by Wess and Zumino [26]
to be

\[
\delta A = i\bar{\alpha}\psi \quad (2.37)
\]
\[
\delta B = i\bar{\alpha}\gamma_5\psi \quad (2.38)
\]
\[
\delta \psi = \partial_\mu (A - \gamma_5 B) \gamma^\mu \alpha + n (A - \gamma_5 B) \gamma^\mu \partial_\mu \alpha + F\alpha + G\gamma_5 \alpha \quad (2.39)
\]
\[
\delta F = i\bar{\alpha}\gamma^\mu \partial_\mu \psi + i(n - \frac{1}{2})\partial_\mu \bar{\alpha}\gamma^\mu \psi \quad (2.40)
\]
\[
\delta G = i\bar{\alpha}\gamma_5 \gamma^\mu \partial_\mu \psi + i(n - \frac{1}{2})\partial_\mu \bar{\alpha}\gamma_5 \gamma^\mu \psi. \quad (2.41)
\]

The parameter \(\alpha(x)\) is an infinitesimal spinor field which anticommutes with itself and with the field \(\psi\) and commutes with all other fields; the parameter \(n\) is arbitrary. In the above equations, the transformation of a scalar into a fermionic field and vice-versa is explicit.

### 2.3.2 Generic Lagrangian and resolution of the hierarchy problem

The Wess-Zumino Lagrangian can be recast in terms of one complex field \(\phi = \frac{1}{\sqrt{2}} (A + iB)\), one left-handed fermion \(\psi_L = (1 - \gamma_5)\psi\) and an auxiliary complex field \(F = \frac{1}{\sqrt{2}} (F + iG)\). The fields \(S, \psi, F\) can be combined in what is called a chiral superfield. A concrete example of such a field would be the left-handed top quark superfield \(T_L = (\tilde{t}_L, t_L, F_t_L)\). The physical fields in the multiplet are the top quark \(t_L\) and stop squark \(\tilde{t}_L\). Similarly, vector superfields can be defined in an analogous way, \(V = (A_\mu, \lambda, D)\), comprising of a gauge boson \(A_\mu\), a gaugino \(\lambda\) and a non-physical auxiliary field \(D\). A concrete example is the gluon superfield, \(G = (g, \tilde{g}, D_g)\). A generic supersymmetric Lagrangian consists of kinetic terms and a function called the superpotential, \(W(\Theta_i)\) depending on the superfields \(\Theta_i\). \(W\) has to be a holomorphic function\(^2\). A generic SUSY Lagrangian has the form\(^3\)

\[
\mathcal{L} = \sum_i \left[ \partial_\mu \phi_i^* \partial^\mu \phi_i - \frac{1}{2} \sum_i \bar{\psi}_{L,i} \gamma_\mu \partial^\mu \psi_{L,i} \right] - \left. \frac{W^2}{\Phi_i} \right|_{\Phi_i = \phi_i} - \frac{1}{2} \sum_{i,j} \left. \frac{\partial^2 W}{\partial \Phi_i \partial \Phi_j} \right|_{\Phi_i = \phi_i} \bar{\psi}_{L,i} \psi_{L,j} + \text{h.c.} \quad (2.42)
\]

\(^2\)i.e. if \(W\) depends on \(\phi\), it should not depend on \(\phi^*\).

\(^3\)The auxiliary fields have been removed by solving the Euler-Lagrange equations.
Let us discuss an example in which the particle content is a Higgs superfield, $H_u = (h_u, \tilde{h}_u)$ and the top left and right superfields, $T_L = (\tilde{t}_L, t_L), T_R = (\tilde{t}_R, t_R)$. The superpotential is taken to be $W = y_t H_u T_L T_R$. Based on (2.42) the interaction term of the Lagrangian is

$$\mathcal{L}_{\text{int}} = - y_t^2 (\tilde{t}_L^* \tilde{t}_L) (\tilde{t}_R^* \tilde{t}_R) - y_t^2 \left[ (\tilde{t}_L^* \tilde{t}_L) + (\tilde{t}_R^* \tilde{t}_R) \right] (\tilde{h}_u^* \tilde{h}_u)$$

$$- y_t \left[ (h_u \tilde{t}_L t_R + \tilde{t}_L \tilde{h}_u t_R + \tilde{t}_R \tilde{h}_u t_L) + \text{h.c.} \right].$$

The Lagrangian contains quartic scalar interactions of the Higgs field with the stop left and stop right, as well as the cubic Yukawa interaction $y_t h_u \tilde{t}_L t_R$ that gave rise to the hierarchy problem. The strength $y_t^2$ of the quartic coupling is exactly the square of the strength of the Yukawa coupling, which leads to an exact cancellation of the quadratic divergences in equation (2.27) thus solving the hierarchy problem.

### 2.3.3 R-parity

Theories like SUSY are constructed by specifying the particle content and symmetries, from which all the different terms of the Lagrangian can be obtained. In the case of SUSY, this procedure gives rise to lepton and baryon number violating terms. The inclusion of all such terms in the model would lead to proton decay, on which very stringent experimental limits exist [27, 28]. The solution of this problem is the introduction of a new symmetry that forbids the offending terms from the Lagrangian, coined “R-symmetry”. The quantum number associated with the symmetry is called R-parity and is multiplicatively conserved. For a particle $p$ with baryon number $B$, lepton number $L$ and spin $S$, R parity is defined as

$$R_p = (-1)^{3B+L+2S}. \quad (2.44)$$

SM particles have $R_p = +1$, SUSY particles have $R_p = -1$. Two important consequences of $R$ parity conservation are the fact that SUSY particles need to be pair-produced, and that the lightest supersymmetric particle (LSP) must be stable. In many models, the LSP is a gaugino and therefore interacts weakly, and is massive, making it a “cold” dark matter candidate.
### 2.3 Supersymmetry

- **Particle type**
  - Fermions, spin 1/2
  - Vector bosons, spin 1
  - $SU(3)_C, SU(2)_L, U(1)_Y$

<table>
<thead>
<tr>
<th>Particle type</th>
<th>Field label</th>
<th>Complex scalars, spin 0</th>
<th>Particle type</th>
<th>Field label</th>
<th>Complex scalars, spin 0</th>
<th>$SU(3)_C, SU(2)_L, U(1)_Y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gluino, Gluon</td>
<td>$\tilde{g}$</td>
<td>$g$</td>
<td>Squarks, Quarks</td>
<td>$Q_L$</td>
<td>$(u_L, d_L)$</td>
<td>$(3, 1, 1/3)$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$\tilde{U}_R$</td>
<td>$u_R^*$</td>
<td>$(3, 1, -2/3)$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Sleptons, Leptons</td>
<td>$L$</td>
<td>$(\nu, \tilde{e}_L)$</td>
<td>$(1, 2, -1/2)$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$\tilde{E}_R$</td>
<td>$e_R^*$</td>
<td>$(1, 1, 1)$</td>
</tr>
<tr>
<td>Wino, $W$ bosons</td>
<td>$\tilde{W}^\pm, \tilde{W}^0$</td>
<td>$W^\pm, W^0$</td>
<td>Higgs, Higgsinos</td>
<td>$H_u$</td>
<td>$(H_u^+, H_u^0)$</td>
<td>$(1, 2, +1/2)$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$H_d$</td>
<td>$(H_d^0, H_d^-)$</td>
<td>$(1, 2, -1/2)$</td>
</tr>
</tbody>
</table>

Table 2.2: Gauge supermultiplets in the MSSM.

Table 2.3: Chiral supermultiplets in the MSSM.

Another dark matter candidate supplied by supersymmetric theories is the superpartner of the graviton, the gravitino, which in many models is the LSP.

#### 2.3.4 Supersymmetric Standard Model

The Supersymmetric Standard Model (SSM) is a supersymmetric theory which consists of three vector superfields, corresponding to the gauge vector bosons of the SM gauge groups $SU(3)_C, SU(2)_L, U(1)_Y$ (table 2.2). The further field content of the SSM is obtained essentially by recognising that each SM fermion field is part of a superfield, essentially doubling the matter content (table 2.3). The only exception is the Higgs doublet: in the SSM, two such doublets are necessary. Using the notation introduced in table 2.3, the most general, $R$-parity conserving superpotential $W_{SSM}$ is given by

$$W_{SSM} = \mu H_u H_d + \sum_{ij} (Y_u)_{ij} H_u Q_{L,i} \tilde{U}_{R,j} + \sum_{ij} (Y_d)_{ij} H_d Q_{L,i} \tilde{D}_{R,j} + \sum_{ij} (Y_L)_{ij} H_d L_i \tilde{E}_{R,j}, \quad (2.45)$$

where $\mu$ is a mass parameter and $Y_u$, $Y_d$, $Y_L$ are Yukawa mass matrices. It should be noted that one key requirement on a superpotential is that it is holomorphic. If only one
Supersymmetry

Higgs doublet were available, the superpotential would have contained $H_u^*$ instead of $H_d$, which would have broken the holomorphy of $W_{SSM}$. A further reason for the necessity of a second Higgs doublet is that the fermion SUSY partner of the Higgs SM scalar would introduce gauge anomalies, which a second Higgs doublet can protect against. The Higgs sector undergoes a similar EW symmetry breaking as discussed for the SM. The theory starts with four complex scalar Higgs fields arranged in two $SU(2)_L$ doublets, accompanied by four fermionic Higgsino fields. The four complex scalar fields imply eight degrees of freedom; after EWSB three degrees of freedom (Goldstone bosons) are absorbed by the gauge fields to give them mass, leaving five physical Higgs bosons. These are two neutral scalars $h, H$ and one pseudoscalar $A$, as well as two charged scalars, $h^\pm$.

A further attractive feature of the MSSM is that the additional superpartners change the running of the coupling constants to the high scale, as can be seen in fig. 2.1. These approximately unify at a high scale $\sim 2 \times 10^{16}$ GeV, which was previously not the case in the SM.

2.3.5 **Soft SUSY breaking and the MSSM**

The SSM provides the simplest supersymmetric extension of the SM. Given its Lagrangian in eq. 2.45 it solves the hierarchy problem exactly, at the cost of almost doubling the particle content of the SM. For supersymmetry to be an exact symmetry of the Lagrangian, the fermion and boson fields in a superfield must have the same mass. Experimentally we are confronted with the fact that no SUSY partners have been observed, so SUSY, should it exist, must be a broken symmetry. Of particular interest is the so-called “soft” SUSY breaking, which can be achieved by the addition of “soft” terms to the Lagrangian that are not SUSY invariant, but preserve the property that superpartner loop contributions to the Higgs mass cancel the quadratic divergences. The soft terms can be of the following types [29]:

- Mass terms for the superpartners of quarks, leptons
- Gaugino mass terms (for the gluino, wino, zino, bino), $M_{\lambda i}$
- Higgs mass terms
• Trilinear couplings of scalar fields $A_{\alpha\beta\gamma}\phi^\alpha\phi^\beta\phi^\gamma$.

A general model including all $R$-parity conserving soft breaking terms is called the “Minimal Supersymmetric Standard Model”, or MSSM. The new terms lead to over 100 free parameters in the theory. In order to reduce the number of parameters, concrete mechanisms for soft SUSY breaking have been proposed; examples are gravity mediated SUSY breaking (mSUGRA) [30–35], gauge mediated breaking (GMSB) and anomaly mediated breaking (AMSB) [36, 37]. Each mechanism leads to a different phenomenology, and the models differ for example in the mass splitting between different particles, or the nature of the LSP.

### 2.3.6 Mass eigenstates and mixing

After EW symmetry breaking, several MSSM particles end up with zero electric charge. The neutral SUSY fermions are called neutralinos, and denoted $\tilde{\chi}_i^0$, $i = 1, 2, 3, 4$. They are mixed states of the neutral wino, bino and higgsino. Similar mixing for charged winos and higgsinos leads to the $\tilde{\chi}_i^\pm$, $i = 1, 2$ states. Mixing can also occur in the quark sector. In principle, with completely arbitrary soft terms, any scalars with the same electric charge, $R$-parity and color quantum numbers can mix with each other. For example, the up-type squarks ($\tilde{u}_L, \tilde{c}_L, \tilde{t}_L, \tilde{u}_R, \tilde{c}_R, \tilde{t}_R$) can in principle mix. However, such mixing would lead to flavour-changing neutral currents due to SUSY loop diagrams, therefore mixing terms are experimentally heavily constrained. Certain assumptions like flavour-blindness of the soft mass and Yukawa terms can be made from the outset in the theory, leading to most mixing angles being very small.

I discuss the mixing mechanism using as an example the stop case. The stop mass term in the Lagrangian has the form

$$\mathcal{L} = \tilde{t}_L^* \tilde{t}_R m^2_{\tilde{t}} \begin{pmatrix} \tilde{t}_L \cr \tilde{t}_R \end{pmatrix}$$

with a mixing matrix

$$m^2_{\tilde{t}} = \begin{pmatrix} m^2_{\tilde{t}L} & m_{\text{soft-breaking}}^2 + m_{\text{EWSB}}^2 \\ m^2_{\text{soft-breaking}} + m_{\text{EWSB}}^2 & m^2_{\tilde{t}R} \end{pmatrix},$$

(2.46)

where the off-diagonal mixing terms are induced by the soft SUSY breaking and by the
2.3 Supersymmetry

existence of the $\mu H_u H_d$ term in the superpotential in eq. 2.45. For the first and second generation squarks, the diagonal terms are much larger than the off-diagonal terms, leading to very little mixing. The large top and bottom quark Yukawa couplings however affect the renormalisation group equation running for the parameters $m_{t_L}, m_{t_R}$, with the consequence that $m_{t_L}, m_{t_R}$ are reduced in magnitude. The off-diagonal terms are enhanced for stops as $m_{EWSB}$ depends on the top Yukawa coupling. Therefore the off-diagonal and diagonal terms in the matrix 2.46 can be comparable in size. The masses of the mass eigenstates are obtained by diagonalising the matrix 2.46 effectively leading to the mixing of the $\tilde{t}_L, \tilde{t}_R$ to give the physical states $\tilde{t}_1$ and $\tilde{t}_2$. In most models, the lightest stop is predominantly $\tilde{t}_R$. A similar mechanism is responsible for the mixing of sbottom $\tilde{b}_L, \tilde{b}_R$ states, giving $\tilde{b}_1, \tilde{b}_2$.

In many models, while all squarks might start off in the renormalisation group equation running with a common mass parameter at a high scale, the large top Yukawa coupling drives the mass of the stop down as the RGE running is performed, making it the lightest squark. The diagonalisation of the mass matrix in eq. 2.46 can reduce the mass of the lightest stop further. The $\tilde{b}_L$ squark is expected to also have a low mass, as it is part of the same $SU(2)_L$ doublet that contains $\tilde{t}_L$. Therefore it is promising to search for light stop and sbottom quarks.

2.3.7 Natural SUSY

In the MSSM, the following tree-level relation holds:

$$\frac{-m_{Z}^2}{2} = |\mu|^2 + m_{H_u}^2$$ (2.47)

If the superpartner particles are too heavy, the terms on the right hand side must be tuned against each other in order to give the correct $Z$ boson mass. In particular, Higgsinos cannot be too heavy, as their mass is controlled by the $\mu$ parameter. The stop and gluino give contributions to $m_{H_u}^2$ at the one- and two-loop level and must therefore also be light. The remaining SUSY particles can have a much higher mass, without materially affecting
“Naturalness” \cite{38} is the concept that SUSY is not fine-tuned in the sense of equation \ref{2.47}. There is debate as to what is an acceptable degree of fine-tuning. The minimum requirements on a natural SUSY spectrum are generally that the two stops and the left-handed sbottom are below 1 TeV, that the two higgsinos (giving the mass states of one chargino + two neutralinos) are below 500 GeV and a gluino below about 2 TeV. There is thus another reason to believe the lightest strongly-interacting SUSY particles are stops and sbottoms.

\subsection*{2.3.8 NMSSM}

The $\mu$ parameter arising in the MSSM superpotential (eq. \ref{2.45}) needs to be of order 100 GeV to 1 TeV to give the correct Higgs vacuum expectation value and avoid fine-tuning type cancellations with soft breaking terms. The natural scale of the $\mu$ parameter is however the Planck scale. The next-to-minimal MSSM (NMSSM) introduces a mechanism of dynamically generating the mass term and naturally giving $\mu$ the correct value. The NMSSM introduces a new gauge-singlet, chiral supermultiplet $S$, giving a more general superpotential

$$W_{\text{NMSSM}} = W_{\text{MSSM}} + \lambda S H_u H_d + \frac{1}{3} \kappa S^3,$$

where $S$ denotes the superfield containing a fermion $\hat{S}$ and a complex scalar field $\tilde{S}$. An effective $\mu$ term for $H_u H_d$ arises during EWSB as the scalar component of the superfield obtains a vacuum expectation value $\braket{\tilde{S}}$, leading to $\mu_{\text{eff}} = \lambda \braket{\tilde{S}}$. After EWSB, the NMSSM has compared to the MSSM an additional real $P_R = +1$ scalar, a real $P_R = +1$ pseudo-scalar and a $P_R = -1$ Weyl fermion singlino. They have no gauge couplings with the SM sector, and can only interact with it if they are mixed with other neutral MSSM particles. An interesting phenomenology arises under the assumption that the only kinematically accessible states are the $\tilde{b}_1 \approx \tilde{b}_R$ and the lightest two neutralinos. If the lightest neutralino, $\tilde{\chi}_1$ is mostly singlino, and the $\tilde{\chi}_2$ is a mixture of bino, singlino and higgsinos, the $\tilde{\chi}_2^0$ will then decay predominantly to $H + \tilde{\chi}_1^0$ or $Z + \tilde{\chi}_1^0$. These particular mixing behaviour can be obtained in a NMSSM case by a suitable choice of the soft breaking parameters with corresponding renormalisation group
equation evolution. The resulting mixings will enforce decays \( \tilde{b}_1 \to b + \tilde{\chi}_2^0, \tilde{\chi}_2^0 \to H/Z + \tilde{\chi}_1^0 \).

Simplified models capturing this phenomenology are studied in chapter 8.

### 2.3.9 Simplified models

Since scanning the over 100 parameters of softly broken SUSY is computationally impractical, various SUSY models have been considered that reduce the number of degrees of freedom by fixing certain relations between the parameters. For example, the constrained MSSM (CMSSM), assumes gravity mediated SUSY breaking and imposes gaugino mass unification at a high scale, with universal scalar masses and universal trilinear couplings, thus reducing the SUSY parameter space to only 5 degrees of freedom. There has been an industry in the past to interpret experimental constraints in this framework, though with accumulating observations, large regions of the parameter space of this model have been excluded, and attention has turned to more general models.

The phenomenological MSSM defines the SUSY breaking parameters at the breaking scale rather than the unification scale. The pMSSM is constrained to have the same flavour structure as the SM, which leads to 19 free parameters.

The opposite extreme in SUSY model complexity is that of “simplified models”, and has gained more popularity in recent years. Very strong assumptions are made in such models by placing the physical masses of most SUSY particles at a high scale of a few TeV, with only the LSP and one or two other particles kinematically accessible. The gaugino or stop/sbottom mixings are ignored, and branching fractions for the decays of interest are typically fixed at 100%. The simplified model framework thus provides the possibility of studying the production of a certain particle and one of its decays only. While it is unlikely that a SUSY theory will exhibit the same degree of simplicity, the approach enables one to optimise searches for particular particles and particular decay chains.

The relationship between the mass parameters in such simplified models and the choice of branching fraction is sometimes motivated by concrete SUSY breaking mechanisms. I
discuss here several features of SUSY breaking models related to the mass spectra of the gauginos:

- **Gaugino universality**: In the gravity mediated models (mSUGRA), and most of GMSB models, gaugino masses unify at high scale. This leads the masses of the bino ($M_1$), wino ($M_2$) and gluino ($M_3$) to be at the EW scale in the ratio $M_1 : M_2 : M_3 \approx 1 : 2 : 6$. This motivates simplified models in which $\tilde{\chi}_1^0 \approx \tilde{B}, \tilde{\chi}_1^\pm \approx \tilde{W}^\pm$ and where $m_{\tilde{\chi}_1^\pm} = m_{\tilde{\chi}_2^0} = 2 \times m_{\tilde{\chi}_1^0}$.

- In the case of AMSB however, the lightest neutralino and chargino are both wino-like. The mass splitting between the neutral and charged winos belonging to the same $SU(2)_L$ triplet is induced by EWSB, and is $\delta m \approx \frac{M_2}{2} \left( \frac{M_W}{\mu} \right)^4$, leading to mass differences $\delta m(\tilde{\chi}_1^+, \tilde{\chi}_1^0) < \mathcal{O}(\text{GeV})$.

- More generally, if the SUSY breaking occurs such that $\mu \ll M_1, M_2$, after neutralino and chargino mixing the lowest mass eigenstates $\tilde{\chi}_1^0, \tilde{\chi}_2^0, \tilde{\chi}_1^\pm$ will correspond to higgsino weak eigenstates and will be nearly mass-degenerate.

Chapter 7 presents a search that is optimised for a simplified model which assumes that only the $\tilde{b}_1$ and the $\tilde{\chi}_1^0$ are accessible, and that the branching ratio $\text{BR}(\tilde{b}_1 \rightarrow b + \tilde{\chi}_1^0) = 1$. This particular simplified model is motivated by naturalness arguments. In the same chapter, section 7.1 discusses a simplified model for stop $\tilde{t}_1$ pair production, with exclusive decay to $b$ and lightest chargino $\tilde{\chi}_1^\pm$. Small mass differences between the chargino and the LSP can be motivated in the AMSB framework or the higgsino LSP scenarios as discussed ($\Delta m = 5$ or 20 GeV are considered in the analysis), and the chargino is assumed to decay to a virtual $W$ and the LSP, $\tilde{\chi}_1^0$. Finally, chapter 8 introduces a simplified models where the particle content is $\tilde{b}_1, \tilde{\chi}_2^0, \tilde{\chi}_1^0$. In a NMSSM context, the $\tilde{b}_1$ can be found to decay exclusively to $b + \tilde{\chi}_2^0$, with further $\tilde{\chi}_2^0 \rightarrow h + \tilde{\chi}_1^0$ or $\tilde{\chi}_2^0 \rightarrow Z + \tilde{\chi}_1^0$ decays. Invoking gaugino universality, another of the simplified models requires $m_{\tilde{\chi}_2^0} = 2 \times m_{\tilde{\chi}_1^0}$, effectively reducing the mass parameter count of the model to two.
2.3.10 Constraints on SUSY

Many different types of experiments can constrain generic SUSY in different ways.

**Low energy constraints:** The SUSY breaking terms discussed in section 2.3.5 show a very rich spectrum of models that can be achieved given the over 100 SUSY soft breaking parameters. However, many soft breaking terms would introduce flavour changing neutral currents or lepton flavour-violating terms. The former are constrained by $K^0/\bar{K}^0$, $B^0_d/\bar{B}^0_d$, $B^0_s/\bar{B}^0_s$ mixing or meson decays involving processes such as $b \to s + \gamma$, $b \to s\ell^+\ell^-$, $c \to u\ell^+\ell^-$, $s \to d\ell^+\ell^-$ [39]. The non-observation of $\mu \to e + \gamma$ [40] restricts the lepton flavour-violating terms.

SUSY particles could contribute to the $B_s \to \mu^+\mu^-$ branching ratio, which was however measured to be in very good agreement with the SM expectation [41]. The anomalous magnetic moment of the muon [42] shows a deviation from the SM prediction, which might be explained by loop contributions of SUSY particles.

**Dark matter:** $R$-parity conserving SUSY models have a natural DM candidate in their LSP. For each model, depending on the mass and the couplings of the LSP, the annihilation cross-section of two LSPs to SM particles can be calculated, which can be translated into an estimation of the relic dark matter density. If this estimate is larger than the measured DM density, the model is considered to be excluded. However, the annihilation cross-section is allowed to give a lower DM density estimate than the observed one, as other non-SUSY particles might partly contribute to the DM density. These constraints cannot be applied directly to simplified models, as the annihilation cross-section will depend for example on the existence of sfermions in the model, which would increase the cross-section via $\tilde{\chi}\tilde{\chi}$ annihilation with a sfermion $t$-channel exchange. There is also a dependence on e.g. the gaugino mixing.

SUSY models can also be constrained through direct dark matter searches, which look for the signal produced by a WIMP as it interacts with a detector. If no such signal is detected,
the experiments are able to set limits on the WIMP mass. Experiments Cogent [43] and DAMA/Libra [44] claim to have observed annual modulation signals consistent with the detection of a WIMP with mass between 6 and 20 GeV. However, the constraints from the more sensitive direct detection experiment LUX [45] refute these observations. Indirect dark matter searches aim to detect the signature of the annihilation or decay of dark matter particles in the fluxes of cosmic rays. This can be done by observing excesses in fluxes of either charged particles, photons or neutrinos, in channels and energy ranges in which the backgrounds from astrophysical processes are low and well understood. An analysis [46] of Fermi [47] data has identified a spatially extended excess of 1-3 GeV gamma rays from the region surrounding the Galactic Center, consistent with the emission expected from annihilating dark matter. The signal is very well fit by a 31-40 GeV dark matter particle annihilating to $b\bar{b}$. A further excess was identified in the Fermi data-set in the form of a gamma-ray line at an energy of about 130 GeV in a region close to the Galactic Center [48]. If interpreted in terms of dark matter particles annihilating into a photon pair, the observation implies a dark matter mass of about 130 GeV. However, it will take a few years of additional data to confirm the existence and clarify the nature of either of these excesses.

**Non-LHC collider searches:** The CERN LEP $e^+e^-$ collider conducted searches with varying center of mass energy up to a maximum of 209 GeV [49]. No signs of SUSY were observed, and limits were set at roughly half the center of mass energy on various particles. For example, the 95% confidence level lower limit on the mass of $\tilde{e}_R$ is at 99 GeV, while on $\tilde{\mu}_R$ it is at 95 GeV. These limits assume $\Delta m(\tilde{l}_R, \tilde{\chi}_1^0) > 10$ GeV, 100% branching fraction for the decay $\tilde{l}_R \rightarrow l\tilde{\chi}_1^0$ and a bino LSP. The LEP chargino mass bound is $m_{\tilde{\chi}_1^\pm} > 103$ GeV for mass differences to the LSP larger than 3 GeV and heavy sneutrinos. The invisible width of the $Z$ requires the sneutrino masses to be above 45 GeV.

The Tevatron experiments CDF and D0 searched for SUSY in events with multiple high $p_T$ jets, missing transverse momentum, and the requirement of no electrons or muons. They have set exclusion limits on the masses of squarks and gluinos in the context of mSUGRA, and exclude masses in the range 200 - 400 GeV depending on the model assumptions [50], [51].
The LEP experiments, the CDF [52] and D0 [53] experiments at the Tevatron collider all searched for pair-produced sbottoms and stops. Different searches were devised for the different allowed stop decay modes, which depend on the mass difference $\Delta m$ between the $\tilde{t}_1$ and the $\tilde{\chi}^0_1$ masses:

- $\Delta m > m_{\text{top}}$: the decay $\tilde{t}_1 \to t + \tilde{\chi}^0_1$ can occur;
- $m_W + m_b < \Delta m < m_{\text{top}}$: the 3-body decay $\tilde{t}_1 \to b + W + \tilde{\chi}^0_1$ can occur;
- $m_c/m_b + m_f + m_f' < \Delta m < m_W + m_b$: the decay $\tilde{t}_1 \to c + \tilde{\chi}^0_1$ dominates, the 4-body decay $\tilde{t}_1 \to b + f + f' + \tilde{\chi}^0_1$ can occur.

A different allowed decay mode is $\tilde{t}_1 \to b + \tilde{\chi}^\pm_1$, which can occur if the mass difference $\Delta m(\tilde{t}_1, \tilde{\chi}^\pm_1) > m_b$. There is however an experimental limit on the chargino mass from LEP, $m_{\tilde{\chi}^\pm_1} > 103.5$ GeV [49]. No similar constraints exist on the mass of the $\tilde{\chi}^0_1$. The $\tilde{\chi}^\pm_1$ can further decay to $W + \tilde{\chi}^0_1$ or undergo a 3-body decay to $f + f' + \tilde{\chi}^0_1$.

The LEP limits extend up to stop and sbottom masses of 100 GeV, obtained for pair production followed by decay through either $\tilde{t}_1 \to c + \tilde{\chi}^0_1$ or $\tilde{b}_1 \to b + \tilde{\chi}^0_1$ and assuming mass splitting $\Delta m(\tilde{t}_1/\tilde{b}_1, \tilde{\chi}^0_1) > 6$ GeV. The most stringent Tevatron limits on sbottom pair production come from D0, which excluded $m_{\tilde{b}_1}$ between 100 GeV and 247 GeV for $m_{\tilde{\chi}^\pm_1} < 70$ GeV. Both CDF and D0 searched for the $\tilde{t}_1 \to c + \tilde{\chi}^0_1$ decay, excluding stop masses lower than 180 GeV [54], [55]. CDF has also searched for the stop decay $\tilde{t}_1 \to b + \tilde{\chi}^\pm_1$ with further decay $\tilde{\chi}^\pm_1 \to \tilde{\chi}^0_1 \ell^\pm \nu$ [56], excluding stop masses up to 180 GeV for $m_{\tilde{\chi}^\pm_1} = 105.8$ GeV and $m_{\tilde{\chi}^\pm_1} = 45$ GeV, under the assumption of 100% BR of $\tilde{\chi}^\pm_1 \to \tilde{\chi}^0_1 \ell^\pm \nu$.

**LHC searches:** As this thesis is particularly concerned with sbottom pair production, the state of sbottom searches before the author joined ATLAS is discussed. The last paper from ATLAS on a sbottom search [57], before the author joined this effort, searched in 2.05 fb$^{-1}$ of $\sqrt{s} = 7$ TeV data and excluded sbottom masses up to 390 GeV for neutralino masses up to 60 GeV. Neutralino masses up to 125 GeV had been excluded for sbottom masses between 300 - 340 GeV. In particular, the search showed no sensitivity to scenarios
with $\Delta m(\tilde{b}_1, \tilde{\chi}_1^0) < 150$ GeV. These limits have been greatly improved through the work documented in this thesis, as shown in chapter 7.

Evaluating dark matter and low energy constraints require fully-fledged models, as such constraints are sensitive to more than just 2 or 3 parameters of a model. The simplified models that are addressed in this thesis therefore only have previous constraints from collider searches.

### 2.3.11 Typical signatures

I discuss the typical signature of $R$-parity conserving SUSY in a collider environment. If $R$-parity is assumed to be conserved, SUSY particles need to be pair-produced. Similarly due to $R$-parity conservation, the lightest SUSY particle is expected to be stable. Since it is very weakly interacting it has a negligible probability to interact with the detector. Escaping LSPs will lead to large transverse momentum imbalances in the event. Jets originating from SUSY particle decays can be expected to have higher transverse momenta compared to Standard Model jets. In the rest frame of a heavy particle $X$ decaying to two daughters labelled with $i = 1, 2$, one of the daughter particle’s energy, $E_1$, will be

$$E_1^* = \frac{m_X^2 + m_1^2 - m_2^2}{2m_X}, \quad (2.49)$$

where $m_X$ is the mass of the decaying particle and $m_1, m_2$ are the masses of the two daughters. In the case of a $\tilde{q}$ decaying to the light SM $q$ and a massless LSP, the energy of the quark and therefore its maximal $p_T$ will be $m_X/2$. This is to be contrasted with SM processes, for example the decay of a stationary top quark decay, for which the $b$-jet energy can only achieve 68 GeV according to equation 2.49. High SUSY particle masses will thus lead to high jet $p_T$ and large missing transverse momentum due to the large momenta imparted to the LSPs.
2.4 QCD theoretical aspects relevant for searches for supersymmetry

Monte Carlo (MC) generators are powerful tools which provide detailed modelling of high-energy collisions. Several ingredients are relevant for such simulations, spanning different scales of the processes involved: from the very high scale of the hard-scattering process, well modelled in perturbation theory, to the much lower scale of quark hadronisation and hadron decay. Since the latter is not described by perturbative physics, several empirical models have to be employed.

A review on Monte Carlo methods for High Energy Physics can be found in [14]. In the following, I discuss in some detail one particular aspect of the simulations relevant for SUSY searches.

2.4.1 Parton showers

Parton showers (PS) refer to the algorithms of MC generators responsible for the evolution of the initial partons from the scale of the proton to the scale of the hard scattering process, and conversely from that scale to the hadronisation scale. During this evolution, colored partons emit QCD radiation in the same way that accelerated electric charges emit photons. Such parton showers represent higher-order corrections to the hard subprocess. As it is not feasible to calculate such corrections exactly to all orders in perturbation theory, only the dominant contributions to the cross-section are considered, which come from collinear parton splitting or soft gluon emission. The differential cross-section for a given final state which includes one additional soft or collinear radiation can be shown to be that of the initial process, multiplied by a factor:

\[ \Delta_i(t, t') \frac{\alpha_s(t)}{2\pi} P_{i,jk}(z) \frac{dt}{t} \frac{dz}{2\pi}, \]

(2.50)
where $t'$ is the virtuality of the parton splitting, $t$ is the final virtuality after the splitting, $\Delta_i(t, t')$ is called the Sudakov factor and is equal to the probability for a parton not to split as it evolves from virtuality $t'$ to $t$. $\alpha_S$ is the strong coupling constant, $\phi$ is the azimuthal angle of the splitting and $P_{i, jk}$ is the $i \to jk$ splitting function, which describes the distribution of the fraction $z$ of the energy of parton $i$ being carried by parton $j$. The same considerations apply to initial state showers, where the incoming partons might split as they evolve back from the scale of the hard interaction. The Sudakov factor is given by

$$\Delta_i(t, t') = \exp \left[-\int_t^{t'} \frac{dq^2}{q^2} \frac{\alpha_S(q^2)}{2\pi} \sum_{jk} P_{i, jk}(z) dz \frac{d\phi}{2\pi} \right], \quad (2.51)$$

where the integral is performed over the virtuality $q^2$ of the splitting parton. In a Monte Carlo simulation, given the initial scale $t$, one solves the equation $\Delta_i(t, t_1) = R_1$, where $R_1$ is a random number uniform on the interval $[0, 1]$, for the scale $t_1$ of the first splitting. If $t_1$ is smaller than the hadronisation scale $t_{had}$, then the splitting was unresolvable and the showering of that parton is terminated. Otherwise, if the splitting was successful, the procedure is repeated on the daughter partons until the scale $t_{had}$ is reached. At each splitting, the variables $z$ and $\phi$ are chosen according to the distribution $P_{i, jk}(z, \phi)$ using the Monte Carlo method.

The virtuality $t$ is given by

$$t = E^2 z (1 - z) (1 - \cos(\theta)) \approx \frac{z(1 - z)}{2} E^2 \theta^2 \approx \frac{p_T^2}{2z(1 - z)}, \quad (2.52)$$

where the approximation is valid in the collinear limit, with $E$ being the energy of the splitting parton, $\theta$ the angle between the products, $z, 1 - z$ the fraction of energy carried by each of the resulting partons and $p_T$ the transverse momentum of the final partons relative to the direction of the incoming one. The evolution in virtuality can thus be recast in terms of the splitting angle $\theta$ or the transverse momentum $p_T$. If an angular ordering of the splittings is imposed, such that the angle at which partons are emitted become increasingly smaller, interference effects between different Feynman diagrams giving the same final state are well
approximated.

Whether the shower is virtuality- or angular-ordered, partons coming from such shower algorithms are constrained to be softer, i.e. have lower transverse momentum, than other particles created in the hard interaction. This is a limitation that can affect searches for new physics, both because of a misunderstanding of the signal topology, as well as an underestimate of the background. For example, one important background for many SUSY searches is the production of a $Z$ or a $W^{\pm}$ boson in association with jets. In such a process, the additional jets will typically have $p_T$’s lower than that of the boson, since the parton shower algorithm will only allow splittings below a scale $\mathcal{O}(m_Z)$. Hard or widely separated jets are described unsatisfactorily in the parton shower paradigm.

### 2.4.2 Matrix element - parton shower merging

In order to overcome the difficulties with parton shower calculations described at the end of the last subsection, one can use algorithms to merge fixed order multi-leg matrix element (ME) calculations with parton shower calculation. While the ME approach can provide accurate predictions in the case that the emitted parton is high-$p_T$ or well separated from other partons, it is computationally expensive, and since it is a fixed order calculation, it will fail in the soft and collinear region. The PS approach alone on the other hand does resum logarithmic terms to all orders in perturbation theory, is computationally cheap, has no limit on the particle multiplicity, is a necessary step towards the modelling of hadronisation, but fails to describe hard or large angle parton emission. Thus a merging procedure is needed to combine the predictions of ME with those of PS calculations. The procedure should ensure the correct description is used in the relevant region of phase space, and that double counting is avoided. Several such procedures exist [58]. I describe in this section the MLM matching scheme as implemented in the MadGraph/Pythia 6 generators. MadGraph [59] is a $2 \to n$ matrix level, leading order generator. Pythia 6 [60] provides the parton shower in this context, using $p_T$-ordering. The MLM matching for final state radiation is explained below, following closely the description in [61].
When MLM matching is enabled, MadGraph restricts all final state, light partons to be above a cut-off scale \( x_{q\text{cut}} \). After the MadGraph generation, the matrix-element level information is passed on to Pythia, which performs the showering. After this, but before hadronization and decays, Pythia clusters the final-state partons into jets using the \( k_T \) algorithm [62] with a cut-off scale \( q_{\text{cut}} \), chosen such that \( q_{\text{cut}} > x_{q\text{cut}} \). The \( k_T \) algorithm is based on clustering into jets partons that are close together, as defined by the distance

\[
k_{T,ij}^2 = 2 \min \{p_{\perp,i}, p_{\perp,j}\}^2 [\cosh(\eta_i - \eta_j) - \cos(\phi_i - \phi_j)] ,
\]

where \( i, j \) labels two distinct partons with transverse momenta \( p_{\perp,i}, p_{\perp,j} \) at pseudo-rapidities \( \eta_i, \eta_j \) and azimuthal angles \( \phi_i, \phi_j \). The jets are compared to the partons at the matrix element level. A jet is considered to be matched to the closest parton if the jet measure \( k_T(\text{parton}, \text{jet}) \) is smaller than the cutoff \( q_{\text{cut}} \). The event is rejected unless each jet is matched to a parton, except for the highest multiplicity sample, where extra jets are allowed below the \( k_T \) scale of the softest matrix element parton in the event. For events from lower-multiplicity samples, the event is also rejected if the first emission occurs at a scale \( q_1 \) which is above the matching scale \( q_{\text{cut}} \), while events from the highest multiplicity sample are rejected if \( q_1 > q_{\text{ME,low}} \), the scale of the softest matrix element parton in the event.

The algorithm effectively ensures that the soft-jet region is simulated by the parton shower by rejecting events in which soft jets originate from the matrix element. Similarly, the algorithm ensures the hard-jet region is populated by jets originating from partons from the matrix element calculation.

### 2.4.3 Effects of MLM merging on sbottom signal samples

I demonstrate the importance of using the correct modelling of initial state radiation (ISR) in this subsection. In chapter [7] a search for the SUSY scalar partner of the bottom quark is presented. The signal investigated is sbottom pair production, with each sbottom decaying as \( \tilde{b}_1 \rightarrow b + \tilde{\chi}_1^0 \) with 100% branching ratio, from which one expects 2 \( b \)-jets and missing transverse momentum in the final state. Further jets can, however, be created from quark
or gluon splitting, either in the final state (final state radiation) or in the evolution of the initial partons from the scale of the hard scattering to the scale of the proton (initial state radiation). In the case of a small mass splitting between the sbottom and the neutralino masses, the $b$-jets from the $\tilde{b}_1$ decays are expected to have low transverse momenta and the $E_T^{\text{miss}}$ is expected to be relatively low, thus making the final state look similar to SM multi-jet production. However, if a high $p_T$ ISR jet is emitted, it can boost the $\tilde{b}_1\tilde{b}_1^*$ system and increase the $b$-jet $p_T$ and $E_T^{\text{miss}}$. Such high $p_T$ splittings are more likely to occur in the case of $\tilde{b}_1\tilde{b}_1^*$ production compared to SM multi-jet production, as the probability of emission of an ISR jet and the scale at which it occurs depends on the scale of the hard scatter process, which in the SUSY case is $\cal O(2 \times m_{\tilde{b}_1})$ and much larger than the typical multi-jet production scale. Due to the hard ISR jets, the production of sbottoms nearly mass-degenerate with the neutralino can be distinguished from SM multi-jet production.

Three types of signal samples are investigated in which the ISR/FSR is modelled differently. The samples were produced by the author in the context of the $\sqrt{s} = 7$ TeV sbottom analysis [1], using MadGraph 5 interfaced with Pythia 6 for the parton shower. In a first set of samples, called MGP0j, only the $\tilde{b}_1\tilde{b}_1^*$ hard scatter process is simulated and ISR is modelled by convolving the parton distribution functions with Sudakov factors, which give the probability of (non-)splitting. This limits the ISR to occur at a smaller scale than the scale of the hard process. In a second set of samples, MadGraph can be asked to explicitly produce an extra parton in the matrix element, therefore including high $p_T$ ISR as part of the hard scattering process. Therefore in the MGP1j samples, the hard-scatter processes $\tilde{b}_1\tilde{b}_1^*$ and $\tilde{b}_1\tilde{b}_1^* + j$ are included. A third set set of samples contains the same processes as the second set and in addition to these, sbottom pair production with two extra partons in the matrix element (MGP2j). For the MGP1j and MGP2j, additional final state partons apart from those included in the matrix element are obtained through the parton shower.

The three types of samples were produced for several points in the $\tilde{b}_1 - \tilde{\chi}_1^0$ mass plane, with a range of mass-splitting $\Delta m \equiv m_{\tilde{b}_1} - m_{\tilde{\chi}_1^0}$ from extremely small ($m_{\tilde{b}_1} = 200$ GeV, $m_{\tilde{\chi}_1^0} = 195$ GeV) to large ($m_{\tilde{b}_1} = 400$ GeV, $m_{\tilde{\chi}_1^0} = 1$ GeV).

\footnote{Also called a degenerate spectrum.}
In order to avoid double-counting between configurations produced both at the matrix element level by MadGraph and at parton shower level by Pythia, the MLM merging scheme described in 2.4.2 was employed for MGP1j and MGP2j samples, with a qcut placed at a quarter of the sbottom mass. In each of the samples, the largest parton multiplicity slice was matched inclusively, while the other slices were matched exclusively.

Kinematic distributions for the leading and sub-leading jets, missing momentum and jet multiplicity were compared between the different samples. Expected differences are observed between the MGP0j and MGP2j samples, the latter exhibiting the anticipated large “ISR tails” in the $E_T^{\text{miss}}$ and jet $p_T$ spectra, as well as larger jet multiplicities for jets with $p_T > 20$ GeV (figure 2.3). The difference between the MGP1j and MGP2j was found to be negligible even in the case of small $\Delta m$.

For the $\sqrt{s} = 7$ TeV search for sbottom squarks, the signal grid was generated using Herwig++ [63, 64], a 2 → 2, leading order generator with parton shower. The ISR and FSR were therefore modelled only through the parton shower, as in the MGP0j samples. From 2.3 it becomes apparent that this model underestimates the $E_T^{\text{miss}}$, the $p_T$ of jets and the jet multiplicity in the case of small $\Delta m$. Therefore, for the $\sqrt{s} = 8$ TeV search, I have simulated the signal grid using MadGraph + Pythia 6 instead, including one extra parton in the matrix element. The new samples served as input to an optimisation procedure, from which a new dedicated signal region emerged, targeting scenarios with low $\Delta m$ and relying on the presence of a high $p_T$ ISR jet. An accurate modelling is essential for obtaining the highest possible sensitivity. Systematic uncertainties related to the use of these generators and the MLM matching scheme are discussed in 7.8.4.
Figure 2.3: Comparison of MGP0j and MGP2j samples (see text). From left to right: $E_T^{\text{miss}}$, $p_T$ of leading jet, jet multiplicity for jets with $p_T > 25$ GeV. Top row: $m_{\tilde{b}_1} = 400$ GeV, $m_{\tilde{\chi}_1^0} = 1$ GeV. No differences are observed between the MGP0j and MGP2j samples. Middle row: $m_{\tilde{b}_1} = 250$ GeV, $m_{\tilde{\chi}_1^0} = 200$ GeV; bottom row: $m_{\tilde{b}_1} = 200$ GeV, $m_{\tilde{\chi}_1^0} = 195$ GeV. For small $\Delta m(\tilde{b}_1, \tilde{\chi}_1^0) \lesssim 50$ GeV, the MGP2j sample has considerable higher $E_T^{\text{miss}}$ and leading jet $p_T$, as well as higher jet multiplicity.
While the Standard Model of particle physics is a very successful theory, describing observed experimental observation in a broad range of energies, it has certain shortcomings. The SM can not explain over 80% of the observed matter in the Universe, or why the Higgs boson has a physical mass close to the electro-weak scale, to name a few.

Supersymmetry is the only possible extension of the symmetries of the Poincaré group which gives a non-trivial quantum field theory with particle interactions. Following the existence of this new symmetry to its logical conclusion, super-partners of the existing SM particles must be introduced. Radiative corrections to the Higgs mass involving the new SUSY particles cancel the quadratic divergences that led to the hierarchy problem. The new super-partners also provide natural dark matter candidates.

If supersymmetry is a symmetry of nature, it must be softly broken in order to ensure the masses of the superpartners are larger than the SM particles, otherwise they would have been observed already. However, in order for SUSY to be “natural” and to be a viable solution to the hierarchy problem, supersymmetric states should be kinematically accessible at the energies of the Large Hadron Collider. In particular, the stop, sbottom, and the gauginos must be light. Amongst these, due to the high production cross-section of strongly interacting particles at a pp machine like the LHC, superpartners of the top and bottom quarks are expected to be amongst the first SUSY particles discovered. In a collider environment, $R$-parity conserving SUSY is expected to make its presence in final states with large missing transverse momentum and energetic jets. Therefore in this thesis I present searches for the pair-production of the partners of the top and bottom quarks in final states with large missing trasverse momentum and energetic jets.
Chapter 3

The ATLAS experiment

ATLAS, “A Toroidal LHC Apparatus” is one of the experiments at the Large Hadron Collider (LHC) at the European particle physics laboratory CERN near Geneva, Switzerland. This chapter describes the LHC, after which it focuses on the design of individual sub-components in the ATLAS detector and on the algorithms used to reconstruct and identify physics objects.

3.1 The Large Hadron Collider

The Large Hadron Collider [65] is as of 2014 the world’s most powerful accelerator, colliding proton beams with a centre-of-mass energy ($\sqrt{s}$) of 8 TeV at unprecedented instantaneous luminosities of up to $7.73 \times 10^{34}$ cm$^{-2}$s$^{-1}$. The 27 km-long, circular LHC tunnel crosses the French-Swiss border, lies at an average depth of 100 m underground and was previously occupied by the Large Electron-Positron (LEP) collider.

Protons for the LHC are produced by stripping electrons off hydrogen atoms in a high electric field. A series of smaller facilities including linacs and synchrotrons accelerate the protons to an energy of 450 GeV, after which they are injected in the LHC in two counter-circulating beams. The accelerator complex is shown schematically in figure 3.1. The proton beams are brought in collision at four different points at which detectors are installed with
3.1 The Large Hadron Collider

![The LHC accelerator complex, from [67].](image)

The ATLAS detector [66], where the studies presented in this thesis were performed, being one of them. The other main detectors are CMS, which like ATLAS is a general-purpose experiment, LHCb, designed for heavy-flavour physics, and ALICE, specialised in recording heavy ion collisions.

The LHC proton beams are not continuous, but arranged in short bunches, which in turn are grouped in so-called bunch trains. Each beam can contain up to 2808 bunches, which intersect at the various interaction points. The nominal bunch spacing is 25 ns, however for most of 2011 and 2012 data taking, the bunch spacing was 50 ns. In 2010 and 2011, the LHC accelerated each beam of protons from 450 GeV to 3.5 TeV, in 2012 the energy per beam reached 4 TeV. A total of 5.61 fb$^{-1}$ were delivered in 2011, and 23.3 fb$^{-1}$ of 8 TeV data were delivered in 2012 (figure 3.2). Throughout operation, the maximal instantaneous luminosity has increased, reaching a peak of $7.73 \times 10^{33}$ cm$^{-2}$s$^{-1}$ in 2012.
3.1 The Large Hadron Collider

The number of interactions $N_i$ that give rise to a particular final state $i$ depends on the cross-section $\sigma_i$ and the luminosity $L$ of the machine, $N_i = \sigma_i \times L$. In order to observe rare processes, on the one hand $\sigma_i$ was increased at the LHC by colliding at a high center-of-mass energy compared to previous experiments, and on the other hand high luminosity was delivered. The latter presents a new challenge to the detector, as several primary interactions occur in any given bunch crossing, a term coined as \textit{in-time pile-up}. The short bunch spacing of 50 ns and the longer calorimeter response time can lead to signals from the previous bunch crossing being recorded at the same time as those from the current bunch crossing, an effect known as \textit{out-of-time pile-up}. Figure 3.2 shows the distribution of the mean number $\mu$ of interactions per bunch crossing, which corresponds to the mean of the Poisson distribution on the number of interactions per crossing for each bunch. It is calculated from the instantaneous per bunch luminosity as $\mu = L_{\text{bunch}} \times \sigma_{\text{inel}} / f_r$ where $L_{\text{bunch}}$ is the per bunch instantaneous luminosity, $\sigma_{\text{inel}}$ is the inelastic cross-section of 71.5 mb for 7 TeV collisions and 73 mb for 8 TeV collisions, $n_{\text{bunch}}$ is the number of colliding bunches and $f_r$ is the LHC revolution frequency.

Figure 3.2: Left: delivered luminosity to ATLAS; right: luminosity-weighted distribution of the mean number $\mu$ of interactions per crossing for the 2011 and 2012 data. The integrated luminosities and the mean $\mu$ values are given in the figure. Figures from [68].
3.2 ATLAS detector

3.2.1 Coordinate system

ATLAS uses a right-handed, orthogonal coordinate system with its origin at the nominal interaction point (IP) in the center of the detector. The $z$-axis points along the beam pipe, the $x$-axis points towards the centre of the LHC ring, and the $y$-axis points upwards. In the transverse plane $x - y$, cylindrical coordinates are used, with $r$ denoting the radius and $\phi$ the azimuthal angle around the beampipe. The pseudorapidity is defined in terms of the polar angle $\theta$ as $\eta = -\log \tan (\theta/2)$. In the massless limit, pseudo-rapidity and rapidity\footnote{The rapidity $y$ is defined as $y = \frac{1}{2} \ln \frac{E + p_z}{E - p_z}$, with $E$ being the energy and $p_z$ the momentum of the particle along the $z$-axis.} are identical for a particle. The distance $\Delta R$ is defined in the $\eta - \phi$ space as $\Delta R = \sqrt{(\Delta \eta)^2 + (\Delta \phi)^2}$. The advantage of this metric is that in the massless limit, the separation $\Delta R$ between two particles is invariant under boosts in the $z$-direction.

At a hadron collider, projections in the transverse plane are essential. In collisions, the $z$-component along the beampipe of the total momentum of the parton-parton interaction is unknown, due to the intrinsic ignorance about the fraction of momentum carried by each of the interacting partons from the proton. However, since the initial total momentum in the transverse plane is zero, one can use momentum conservation in this plane to describe the event. Physics objects are therefore often described in terms of the projection of their momentum in the transverse plane, $p_T = |p| \sin \theta$, a similar projection of the energy, $E_T = E \sin \theta$ and the azimuthal angle $\phi$.

3.2.2 ATLAS detector overview

ATLAS is a general purpose hermetic detector, designed with a forward-backward symmetric cylindrical geometry. It follows a conventional multi-layer design for collider detectors (figure 3.3), starting with the inner detector (ID) with coverage up to $|\eta| = 2.5$ which provides tracking information due to its pixel detector, semiconductor silicon tracker (SCT) and
transition radiation tracker (TRT). The ID is installed within a superconducting solenoid providing a magnetic field of 2T, which bends the paths of charged particles, allowing the particle momentum to be inferred. Further out are situated the electromagnetic calorimeter (ECAL) based on liquid argon technology and, in the barrel region only, the hadronic calorimeter (HCAL) consisting of steel absorbers and scintillator tiles. The end-cap and forward regions are instrumented with electromagnetic and hadronic calorimeters, both based on liquid argon technology. A barrel and two end-cap toroidal superconducting magnets support the large muon spectrometer, which is the outermost layer of the detector. The dimensions of the detector are 25m in height and 44m in length, weighing 7000 tonnes.

The detector layout and performance are described in more detail in references [66,69]. The figures included in this chapter are from these references.
3.2 ATLAS detector

3.2.3 Inner Detector

The Inner Detector comprises the pixel detector, the semiconductor tracker, and the transition radiation tracker and has coverage up to $|\eta| = 2.5$. Its main purpose is to provide tracking information of charged particles, from which the momenta of the particles is inferred, and which can aid particle identification. The ID is also used to measure accurately the distance from the primary interaction vertex of tracks associated with a jet and to identify secondary decay vertices, which are essential input to so-called $b$-tagging algorithms used to identify whether a jet stems from a $b$-quark decay. An overview of the ID and its layout are given in figure 3.4.

Pixel Detector

The pixel detector provides very high-granularity, high-precision measurements close to the interaction point. It consists of a barrel with three layers and two end-cap regions with three disks each, spanning radii between 5 cm and 13 cm. The system comprises 140 million detector elements with a minimum size of $50 \times 400 \mu m$. A total of 61440 pixel elements are found within a module, and those modules are further assembled together to form a barrel layer or an endcap disk. There are 1456 modules in the barrel and 288 modules in the end-caps. Each layer is designed to have a small thickness of 1.7% radiation lengths at normal incidence. The pixel detector is essential in providing accurate determination of the
impact parameter of tracks. In doing so, it aids in the detection of short-lived particles like $B$ hadrons or $\tau$ leptons.

**Semiconductor tracker**

The semiconductor tracker (SCT) provides eight precision measurements per charged particle track in the radial range between 30 and 51 cm. It contributes to the measurement of momentum, impact parameter and vertex position. In contrast to the pixel detector, the SCT modules contain silicon strips rather than pixels. A total of 768, 12.8 cm long strips are found on each module side. In the barrel, they are positioned approximately parallel with the beam-pipe and provide the positional measurement in the $r - \phi$ plane with a high precision of 16 $\mu$m. In the end-caps, the strips point radially outwards. The two sides of a module are off-set at a small stereo angle of 40 mrad to provide in the barrel a $z$-position measurement and in the end-cap an $r$ measurement with a precision of 580 $\mu$m. In the barrel, the modules are assembled together in four layers, while each end-cap contains nine disks. A total of 61.2 m$^2$ of silicon detectors is employed, with 6.2 million readout channels.

A more detailed description of the SCT and of aspects relevant for the analyses presented in this thesis are described in detail in chapter 4.

**Transition radiation tracker**

The Transition Radiation Tracker (TRT) uses straw detector technology. The straws are 4 mm in diameter, contain a 30 $\mu$m diameter wire and a Xe/CO2/O2 gas mixture. They are arranged parallel with the beampipe in the barrel and radially in the end-caps. The detection of particle passage through the detector is based on two physical principles. In the first case, as charged particles pass through the straws, the gas is ionised and the ionisation electrons are collected on the wire, providing a drift-time measurement which can be translated in information about the impact parameter of the track with respect to the wire. A threshold of 300 eV is used to discriminate tracking hits. The second physical principle is the emission
3.2 ATLAS detector

of transition radiation as relativistic electrons pass through a radiator situated between straw tubes, consisting of multiple layers with different dielectric constants. The transition radiation is in the X-ray spectrum, and can in turn ionize the Xenon gas in the straws. The intensity of the transition radiation is proportional to the Lorentz $\gamma$ factor, therefore electrons with their low mass give by far the highest contribution. A second threshold is placed at 6 keV to detect transition radiation and is aimed at identifying electrons, whereas low threshold hits are expected to be produced by heavy hadronic particles. The TRT consists of a total of 50,000 straws in the barrel and 320,000 radial straws in the endcaps, with typically 30 TRT hits being registered on a well reconstructed track. The resolution of a single straw is 170 $\mu$m in its transverse plane. The straws are read-out on one end only and do not provide a $z$-position measurement.

3.2.4 Calorimetry

An overview of the ATLAS calorimetry is presented in figure 3.5. The ATLAS calorimeters are as follows: closest to the interaction point, an electromagnetic calorimeter (EMCAL) in the pseudorapidity range $|\eta| < 3.2$; further out, a hadronic barrel calorimeter extending to $|\eta| = 1.7$; at high pseudorapidity values, hadronic end-cap calorimeters in $1.5 < |\eta| < 3.2$ and forward calorimetry spanning $3.1 < |\eta| < 4.9$. The EM calorimeter is based on lead/liquid argon (LAr) technology with accordion geometry and is divided in two half-barrels in the central region and two end-caps. Each half-barrel covers the pseudorapidity range $|\eta| < 1.475$ and consists of 1024 pairs of lead converters and Kapton electrodes, with the gaps in between filled with liquid argon. The converters and electrodes are accordion-shaped, with the accordion spanning wedges in $r - \eta$ to ensure azimuthal uniformity. The barrel was split in projective segments in $\eta$, $r$ and $\phi$ by etching the kapton electrodes at the desired values of $\eta$ and $r$ and by adding together the signal from several electrodes at different $\phi$ values. The barrel region is sub-divided in three radial layers, with the first layer consisting of narrow strips of $\Delta \eta \times \Delta \phi \approx 0.003 \times 0.1$, providing precise measurement of position in $\eta$. The following two sections consist of rectangular towers pointing back to the interaction point. The most
Figure 3.5: Overview of ATLAS Calorimetry [66].

Granular towers have a size of $\Delta \eta \times \Delta \phi = 0.025 \times 0.025$, equivalent to $4 \times 4 \text{ cm}^2$ at $\eta = 0$. The end-cap calorimeters are based on the same technology and are mechanically divided in two coaxial wheels, covering the range $1.375 < |\eta| < 3.2$. The EM calorimeter has a total thickness of over 24 radiation lengths in the barrel, and 26 radiation lengths in the end-caps. The total material a particle incident at $\eta = 0$ traverses before encountering the calorimeter is about 2.3 radiation lengths, and increases with $\eta$ as more material is traversed due to the incidence angle. In the region $|\eta| < 1.8$ in which the thickness of the detector up to the calorimeter exceeds two radiation lengths, a presampler consisting of an active layer of LAr is used just before the calorimeter to correct for the energy lost by electrons and photons. The design resolution of the calorimeter is given by $\sigma_E/E \simeq 10%/\sqrt{E[\text{GeV}]} \oplus 0.7%$ [66].

The ATLAS hadronic calorimeters cover the range in pseudorapidity up to $|\eta| < 4.9$. The range up to $|\eta| < 1.7$ is covered by the barrel hadronic calorimeter, which is divided in three sub-sections, a central barrel and two extended barrels. The calorimeter is based on a sampling technique, with plastic scintillator tiles placed radially and embedded in an iron
3.2 ATLAS detector

absorber. Each row is offset from the previous one for better coverage. The two sides of the 3 mm thick tiles are read out by wavelength shifting fibres into two separate photomultipliers. The thickness of the calorimeter is chosen such that the calorimeter provides good containment of hadronic showers. At $\eta = 0$, the thickness is 11 interaction lengths. It is essential that the punch-through of jets in the muon system is kept to a minimum in order to guarantee good resolution for the jet energy and provide a good $E_{\text{miss}}$ measurement. The granularity is chosen to be mostly cells of dimension $\Delta \eta \times \Delta \phi = 0.1 \times 0.1$.

With increasing pseudo-rapidity, the radiation environment becomes harsher, and so do the requirements on radiation hardness. Therefore the hadronic end-cap calorimeters and the forward calorimeters are based on the intrinsically radiation-hard LAr technology. The hadronic end-cap calorimeters at $1.5 < |\eta| < 3.2$ are copper/LAr detectors with parallel plate geometry, and each consists of two independent wheels. The copper plates are 25 mm and 50 mm thick in the two wheels, with a LAr filled gap of 8.5 mm between plates. The forward LAr calorimeter at $3.1 < |\eta| < 4.9$ is contained within the LAr hadronic end-cap calorimeter and consists of three regions, with the first one containing copper converters for EM calorimetry, and the other two tungsten converters for hadronic calorimetry. The hadronic calorimeter was designed to provide in the range $|\eta| < 3$ an intrinsic jet energy resolution of $\Delta E/E = 50% / \sqrt{E\text{[GeV]}} \pm 3%$ [66].

3.2.5 Muon spectrometer

The muon systems in ATLAS have been designed to ensure efficient muon identification and precise momentum measurement over a wide range of momentum, from 3 GeV to 1 TeV, in a wide range of pseudorapidity up to $|\eta| = 2.7$. They are shown schematically in figure 3.6. The muon systems consist of Monitored Drift Tubes (MDTs) and Cathode Strip Chambers (CSCs) for precision momentum measurements, while Resistive Plate Chambers (RPCs) and Thin Gap Chambers (TGCs) mostly provide input to the trigger. The trigger muon systems have coverage up to $|\eta| = 2.4$. The different types of detectors are assembled such that they form three cylinders concentric with the beam axis up to $|\eta| < 1$ and four disks on each
end-cap side covering $1 < |\eta| < 2.7$. The muon systems are based on the deflection of muon tracks as they pass through the magnetic field of the large superconducting toroid magnets. In the barrel region, the trigger function is provided by the RPCs. A RPC unit consists of two parallel resistive bakelite plates with a narrow gas gap in between. As muons pass through, an ionisation avalanche can form and give a signal with an excellent time resolution. TGCs provide the trigger function in the end-cap disks and are similar in design to multiwire proportional chambers. In order to uniquely identify a signal with the LHC bunch crossing, the time resolution of these detectors is better than the LHC bunch spacing of 25 ns.

Precise position measurements in the principal bending direction ($\eta$) is obtained using MDTs, which contain tubes filled with gas that is ionised when traversed by the charged muon. The ionisation electrons drift in an electric field towards a central wire. At high pseudorapidity $2 < |\eta| < 2.7$, the muon flux is higher and CSCs are used for the inner layer instead. The CSCs consist of four layers of radial multi-wire proportional chambers which read out on cathode strips. Positional information is obtained by having the cathodes segmented into strips perpendicular to the wires.
3.2 ATLAS detector

The precision measurement in the MDTs and CSCs is made in the $r - z$ projection, which is the plane in which the toroidal magnetic field bends the muon tracks. The coordinate $z$ is measured in the barrel, whereas $r$ is measured in the end-cap region. The single-wire resolution of the MDT is about 80 $\mu m$. The RPCs and TGCs also provide some tracking information by measuring the azimuthal coordinate with a resolution of 5-10 mm. The better position resolution of the MDTs is obtained at the cost of a longer response time, with the maximal drift time around 700 ns.

3.2.6 Magnet systems

The magnetic field for ATLAS is provided by four superconducting magnets cooled by liquid helium to a temperature of 4.5 K. The inner-most magnet is a solenoid containing the inner detector and producing a field of magnitude 2T. It was designed to have a small material budget, which ensures a good ECAL performance. A further system of three large air-core toroidal magnets can store current of 20.5 kA for which it provides a magnetic field between 0.5 and 1 T to the muon spectrometer. The three magnets comprise a barrel and two end-cap sections, and their configuration is such that their field is mostly azimuthal and orthogonal to the muon trajectories, providing maximal bending power.

3.2.7 Trigger

For most of 2011 and 2012 data-taking, bunch crossings occurred every 50 ns. If all the detector output for every single bunch crossing were to be written to tape, given the event size of approximately 1.3 MegaBytes, ATLAS would require a storage capacity of 30 TeraBytes per second. The processing power required to reconstruct all events would also be prohibitive. However, the overwhelming majority of the collisions at the LHC yield processes that are well understood and have been studied before at other colliders. Also, processes like elastic or diffractive scattering which account for about half of the total proton-proton cross-section can be studied to very high precision even if the detector only samples a very small subset
Figure 3.7: Inclusive cross-sections for $b$, jet, $W$, $Z$, $t$ and Higgs production as indicated on each curve at the Tevatron and the LHC versus centre of mass energy $\sqrt{s}$ [70]. The cross-sections on the left hand side of the discontinuity at $\sqrt{s} = 4$ TeV are calculated for the proton-antiproton initial state, while the cross-sections on the right hand side are calculated for the proton-proton initial state.

of the data available. Figure 3.7 shows the relative contribution of different processes to the total proton-proton cross-section. For example only about 1 in $10^{10}$ collisions produces a Higgs boson. The role of the trigger is to select a small fraction of such interesting events for further processing.

ATLAS employs a 3-level trigger system to reduce the number of events written to tape to a manageable rate. The first level (L1) is hardware-based and operated in 2012 with an input rate of 20 MHz, and an output rate of 70 kHz, allowing on average 2.5 $\mu$s for a decision. The L1 electronics are located in the ATLAS cavern in order to reduce the latency in the trigger decision. Calorimeters and muon detectors (RPCs/TGCs) at reduced granularity are used to search for high momentum objects like electrons, photons, jets, taus and muons. The calorimeter is sub-divided in approximately 7000 trigger towers of coarse granularity.
(size $\Delta \eta \times \Delta \phi = 0.1 \times 0.1$). The analog output of each tower is digitised and matched to the bunch crossing clock, and hadronic jets are searched for. At this stage, also the vector and scalar sums of the transverse energy are calculated and electron, photon and tau candidates are identified. The central trigger processors combine the information from the calorimeter trigger with that coming from the muon systems and takes a decision whether to keep or discard the event. If an event is accepted, it is passed on to the second level (L2) trigger. At the L2 stage, enough processing time (40 ms) is available per event to employ software trigger algorithms and to include information from the inner detector. The L2 trigger further reduces the event rate from 70 kHz down to 3.5 kHz. The final stage in the triggering is the event filter (EF), which is also software-based. The time budget at the EF level is about 4 seconds / event, which allowes for the full information from all detector sub-systems to be incorporated in the decision and offline-style reconstruction algorithms to be employed. The data rate is reduced to the level of about 200 Hz, corresponding to a volume rate of 300 MB/s, with an additional $\sim 500$ Hz of data not being immediately processed, but being written to tape for later analysis during the shutdown period.

For certain physics objects that ATLAS triggers on, the production rate might be too high to record every single event passing the trigger. In such cases, *prescaled* triggers exist, which record only a certain fraction of the events that would normally pass the trigger, effectively reducing the luminosity recorded. Usually searches for new physics or the Higgs boson need to employ the full dataset to boost their sensitivity, for which an unprescaled trigger with the lowest possible thresholds should be employed. In the search described in this thesis for example, the EFxe80_tclcw_loose was used, which requires 80 GeV of missing transverse momentum at EF level, and is seeded by a trigger at L2 with threshold at 45 GeV, and at L1 with threshold at 40 GeV.

Data events that have passed a logical OR of several triggers are recorded in a so-called stream, for example the JetTauEtmiss, Egamma and Muon streams.
3.3 Physics Objects

3.3.1 Tracks

While tracks themselves are not used as a physics object in the analysis described in this thesis, they are essential in defining other objects. In a first stage of track-finding, the SCT cluster information is translated into three-dimensional space points and is collected together with the Pixel cluster information and the data on TRT drift circles. In the second stage, several tracking algorithms are employed. Spacepoints from the pixel detector and the first SCT layer are used to find track seeds, which are then extended through the rest of the SCT and the TRT. At this stage quality cuts are applied to limit the number of pixels or SCT modules that are traversed by a fitted track but present no hit, or to reject outlier clusters. The tracks serve as input to the vertex finder algorithm used to reconstruct primary and secondary vertices.

3.3.2 Jets

Jets are reconstructed using the infrared and collinear safe anti-$k_t$ jet algorithm [71, 72] with a radius parameter of 0.4, which takes as inputs three-dimensional calorimeter energy clusters. The radius parameter is chosen large enough such that it contains most particles produced in the parton shower and subsequent hadronisation of a sufficiently high $p_T$ colored particle, but small enough that contamination from pileup does not significantly degrade the energy measurement. A larger radius parameter would lead to a too coarse picture in the event, for example a $R = 1.0$ jet may contain all decay products of a boosted top quark (i.e. a $R = 0.4$ $b$-subjet and two further subjets from the $W$ decay). The calorimeter cells are calibrated using the local cluster (LC) weighting method [73], which attempts to correct for inhomogeneities and for the non-compensating nature of the calorimeter. In this method, cells with large signal-to-noise ratio above $r_{\text{seed}} = 4$ grow into topological clusters by iteratively adding neighboring cells with a signal-to-noise ratio above $r_{\text{neighbor}} = 2$ and
3.3 Physics Objects

finally adding all direct neighbor cells on the outer perimeter with signal-to-noise above \( r_{\text{outer cell}} = 0 \). From the characteristics of the calorimeter response in the clusters, the nature of the shower (electromagnetic or hadronic) can be inferred, and the energy of the cluster can be weighted accordingly to compensate for the energy lost in hadronic decays. Jets are then built from the clusters, and a further multiplicative correction, the jet energy scale (JES), dependent on \( p_T \) and \( \eta \) is applied to their four-momenta to improve the agreement between truth jet energy and the calorimeter response. The effect of additional collisions in the same or consecutive bunch crossings is taken into account by subtracting the mean expected extra energy deposited in the jet cone, which is parametrised as a function of the average number of interaction per event and the number of primary vertices. The jet direction is corrected such that it points back to the point of primary interaction. Jet candidates are required to have transverse momentum \( p_T > 20 \, \text{GeV} \) and are reconstructed in the range \( |\eta| < 4.9 \).

3.3.3 \( b \)-tagging

Jets within the acceptance of the Inner Detector (\( |\eta| < 2.5 \)) and with \( p_T > 20 \, \text{GeV} \) can be tested to determine if they may have originated from a \( b \)-quark. Such \( b \)-tagging is performed for the analysis described in this thesis using the MV1 algorithm, which is based on a neural network having as inputs the weights of the JetFitter+IP3D, IP3D and SV1 algorithms (\cite{74}). For all algorithms that serve as input to MV1, a likelihood ratio method is employed. The measured value \( x_i \) of a discriminating variable is compared to existing smoothed and normalised distributions of that variable extracted from light or \( b \)-jets in simulation, \( u(x_i) \) and \( b(x_i) \). If the variable \( x_i \) refers to a track or vertex quantity, a ratio of probabilities \( b(x_i)/u(x_i) \) can be built to define the track or vertex weight \( W_i \). The individual weights can be combined to form a jet weight \( w_{\text{jet}} \),

\[
  w_{\text{jet}} = \sum_{i=1}^{N_T} w_i = \sum_{i=1}^{N_T} \log \frac{b(x_i)}{u(x_i)},
\]  

(3.1)
Figure 3.8: Left: Transverse impact parameter significance with respect to primary vertex for tracks associated with jets. Middle: distribution of the number of two-track vertices found by the SV1 tagging algorithm. Right: distribution of the vertex energy fraction (ratio of the sum of the energies of the tracks participating to the secondary vertex to the sum of the energies of all tracks in the jet) of the inclusive secondary vertex found by the SV1 tagging algorithm. From [73].

with the sum performed over all tracks or vertices associated with the jet. A cut value on \( w_{\text{jet}} \) will give a working point with a particular \( b \)-tagging efficiency and light- and \( c \)-jet rejection rates. IP3D uses two-dimensional histograms of the longitudinal versus transverse impact parameter significance of tracks associated with a jet. The significance is defined as the ratio of the impact parameter to the \( 1\sigma \) uncertainty with which it was measured. The distribution of the transverse impact parameter significance is shown in figure 3.8.

SV1 is an algorithm based on the existence of a secondary vertex. It makes use of the 2D distribution of the invariant mass of all tracks associated with the vertex and the ratio of energies of the tracks participating in the vertex to the sum of energies of all tracks in the jet and a 1D distribution of the number of two-track vertices. Figure 3.8 shows the distributions of the vertex energy fraction distribution and the number of two-track vertices.

The JetFitter algorithm finds the vertices of \( b \) and \( c \)-quark decays inside a jet. The discriminator between jet flavours is based on a likelihood using similar variables as the SV algorithm, with additional intra-jet variables.

The output weights from the JetFitter and IP3D are further combined using neural network techniques to form the JetFitter+IP3D algorithm, whose output together with IP3D and SV1 are fed into the MV1 algorithm. The purpose of the neural network is to combine the
input variables in such a way that for a given signal efficiency the mis-tag rate is minimised. For the presented analysis, a working point is used which has 60% $b$-tagging efficiency and rejection\(^2\) of 580, 8 and 23 against light quarks, $c$-quarks and $\tau$ leptons respectively.

### 3.3.4 Electrons

Electrons are reconstructed from energy clusters in the electromagnetic calorimeter which match a track in the inner detector. The clusters are obtained as the result of a sliding window algorithm which scans regions of size $5 \times 5$ calorimeter cells. Each cell has a size $\Delta \eta \times \Delta \phi = 0.025 \times 0.025$. If the total transverse energy in the window is above a threshold of 3 GeV, the window is used as a cluster seed. Tracks from the ID are then matched to the resulting cluster, and the electron candidate $\eta$ and $\phi$ are taken from the ID if at least 4 silicon hits are present, or from the calorimeter cluster position otherwise. Several requirements based on ID and EMCAL information can be applied to increase the purity of the electrons, at the cost of efficiency. These come in the three flavours “LoosePP”, “MediumPP” and “TightPP”\(^7\). The type of requirements refer to the number of hits recorded in different ID subdetectors, the shower shape, the amount of shower leakage into the HCAL, the track impact parameter, track quality and track-cluster match quality. Further requirements can be applied on the isolation of the electron by limiting the amount of activity allowed in the vicinity of the electron candidate, either in the ID or in the calorimeter.

### 3.3.5 Muons

Muons are identified using one of several reconstruction strategies:

- **Stand-alone** muons are reconstructed using information from the muon systems only ($|\eta| < 2.7$).

- **Combined** muons: a $\chi^2$ match is performed between the tracks reconstructed independently in the ID and muon systems ($|\eta| < 2.5$).

\(^2\)The rejection is defined as the reciprocal of the tagging efficiency.
3.3 Physics Objects

- **Segment-tagged** muons: a track in the ID is extrapolated to the muon systems, where it can be associated with at least one track segment in the precision chambers ($|\eta| < 2.5$).

The reconstruction of these muon types can be performed using two different algorithms. The first algorithm (STACO) combines statistically the track parameters obtained from the ID and the MS using the covariance matrices of the two measurements. The second algorithm (MuID) performs a global refit using hits in both the ID and the muon systems. The muon efficiency for combined and segment-tagged STACO muons above 20 GeV was measured in 8 TeV data to be above 98% for most of the $\eta$ range, apart from at $\eta \sim 0$, where there is a gap in the muon spectrometer used for the ID and calo data links and services [76]. The analysis presented in this thesis makes use of STACO muons exclusively. A muon candidate can be either combined, or segment-tagged. The transverse momentum resolution is given by $\sigma_{p_T}/p_T \simeq a + b \times p_T$, with the $a$ term being due to multiple scattering and the $b$ term describing the intrinsic resolution. For the ID, the $a$ term is between 1.6% and 3.4% and the $b$ term between 0.49 and 1.39 TeV$^{-1}$, depending on $\eta$, while the MS has poorer resolution [77].

3.3.6 Missing transverse momentum

The missing transverse momentum, $p_T^{\text{miss}}$ and its magnitude $E_T^{\text{miss}}$ is calculated as the negative vector sum of all objects in the event, projected in the transverse plane [78]. In particular, all pre-selected muons before overlap removal with $p_T > 10$ GeV, pre-selected electrons before overlap removal with $E_T > 10$ GeV and jets with $E_T > 20$ GeV are used as input. The calibrated energy of these objects is used in the calculation. In addition, calibrated calorimeter energy clusters with $|\eta| < 4.9$ not associated with any of these objects are also included in the calculation.
Chapter 4

The ATLAS semiconductor tracker

The semiconductor tracker (SCT) is a silicon strip detector which provides precision three-dimensional space-point measurement for charged particle tracks. It therefore plays an essential role in track reconstruction, charged particle identification and $b$-tagging. In this chapter, I start by presenting the system architecture in section 4.1. Later sections focus in particular on the optical communication used for read-out and control. These sections serve as an introduction for the research reported in chapters 5, 6 which deal with issues that affected SCT detector operation in 2011 and 2012.

One such issue is the large number of failures among the $\sim 4000$ lasers that transmit timing, trigger and control commands from the counting room to the SCT front-end electronics. Chapter 5 presents the statistical analysis of failure rates of such opto-electronic devices in accelerated lifetime tests and in the semiconductor tracker. Therefore section 4.2 of the current chapter provides an account of the optical links of the SCT.

A further issue is the possibility of radiation-induced bit errors in the optical links, an introduction to which is provided in section 4.3. Chapter 6 presents a first measurement of such single event upsets in optical links in a collider environment.
4.1 SCT architecture

The SCT extends up to \(|\eta| < 2.5\) and forms the intermediate layer of the so-called Inner Detector, which also comprises the Pixel Detector and the Transition Radiation Tracker. Geometrically, the SCT is divided into a central barrel region, constituted out of four concentric cylindrical layers around the beam pipe, and two endcaps, consisting of nine disks each, assembled perpendicularly to the beam direction. The design is such that at any \(|\eta| < 2.5\), a track is expected to hit 4 barrel layers and / or endcap disks.

Table 4.1 summarises the position of each barrel layer and endcap disk and the number of modules. Each module has two planes of silicon with 768 AC coupled active strips, glued back-to-back, but offset at a small stereo angle. The 40 mrad stereo angle ensures that each module can provide a space-point resolution of 17 \(\mu\)m perpendicular and 580 \(\mu\)m parallel to the strips. Not all modules are always operational — for example in May 2010, out of the total 4088 modules, 30 were disabled due to a leaking cooling loop in one of the endcaps and due to a variety of high and low voltage errors. Nevertheless, the total number of disabled detector modules was less than 1% throughout operation.

<table>
<thead>
<tr>
<th>Barrel Layer</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Radius (mm)</td>
<td>299</td>
<td>371</td>
<td>443</td>
<td>514</td>
<td></td>
</tr>
<tr>
<td>Nr. Modules</td>
<td>384</td>
<td>480</td>
<td>576</td>
<td>672</td>
<td>2112</td>
</tr>
</tbody>
</table>

Table 4.1: Radius and number of modules for each SCT barrel layer.

<table>
<thead>
<tr>
<th>Barrel Layer</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>(</td>
<td>z</td>
<td>) (mm)</td>
<td>847</td>
<td>934</td>
<td>1084</td>
<td>1262</td>
<td>1377</td>
<td>1747</td>
<td>2072</td>
<td>2462</td>
</tr>
<tr>
<td>Nr. Modules</td>
<td>92</td>
<td>132</td>
<td>132</td>
<td>132</td>
<td>132</td>
<td>92</td>
<td>92</td>
<td>52</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 4.2: Longitudinal position and number of modules for the disks on each SCT endcap.

4.2 The optical links of the ATLAS SCT

The SCT microstrips are read out by the custom-made ABCD3TA ASIC [80], which provides the functions of front-end amplification, discrimination and pipelining the data. Each such chip reads out 128 channels out of the 768 strips of a module side, therefore 6 chips are needed
4.2 The optical links of the ATLAS SCT

The optical links of the ATLAS SCT are shown in Figure 4.1. The system architecture is based on [77].

The data (TTC) links are displayed in the top (bottom) half of the diagram. The data acquisition (DAQ) system configures the ASICs, transmits the first level trigger information, and reads out data from the chips. The communication between the ABCD3TA and the DAQ is achieved by opto-links, shown schematically in Figure 4.1. The opto-links are driven by GaAs Vertical Cavity Surface Emitting Lasers (VCSELs), emitting at a wavelength around 850 nm. The VCSEL devices are described in more detail in Section 5.1. The binary data from each of the 128 channels, corresponding to the information of “hit” or “no hit”, is stored in the pipeline memory of the ABCD3TA ASIC. Event data are stored for 132 clock cycles, awaiting the global level 1 trigger decision. When a level 1 trigger accept is received, data from each ABCD3TA chip on a module side are read out and collected by a master ABCD3TA. In normal operation mode, the data from the master ABCD3TA on each of the two sides are transmitted to the two input channels of another custom-made ASIC, called VDC, which translates the information into the drive signal which operates two VCSELs. The VCSEL output is sent through step index multi-mode optical fiber in non-return-to-zero format to the counting room, where the signal is eventually decoded by the SCT Readout Driver (ROD). Each ROD forms a pair with a complementary Back-Of-Crate (BOC) board, which performs the input / output...
tasks to and from the front-end modules and the central DAQ system. On the BOC cards in the counting room, epitaxial silicon $p-i-n$ diodes convert the optical signal from the front-end modules to electrical form, which is then further processed by the DRX-12 ASIC. Data sent from the front end modules must be uniquely identifiable with a particular LHC bunch crossing and first-level trigger. Therefore the hit data is stored in the ABCD3TA pipeline, and transmitted to the ROD, alongside with a trigger counter (LVL1ID, with a size of 4 bits) and with a counter of the number of clocks (BCID, with a size of 8 bits). Counter resets which clear the ABCD3TA pipelines are issued periodically from the TTC system to remedy potential inter-module desynchronisations. A ROD can become “busy” during operation, for example in the unlikely case in which the instantaneous occupancy is too high and the ROD cannot process all data at the necessary rate. No more triggers will be issued to that ROD until reset signals are issued. Similar optical links are used to send the trigger, timing and control (TTC) signal from the RODs in the counting room to the front-end modules. Custom-made BPM-12 ASICs convolve the TTC signal at 40 MBit/second with the LHC bunch crossing clock at 40 MHz using biphase mark encoding (BPM). The output of these chips is used to drive arrays of 12 VCSELs, each coupled to an optical fiber. Each channel transmits TTC information for both sides of one module. The signal is received on-detector and converted from optical to electrical form by an epitaxial silicon $p-i-n$ diode. The on-detector $p-i-n$ diodes have the same operation principle as those receiving the data on the BOCs, however they are made by a different manufacturer and are suitable for operation in a radiation environment. The DORIC4A custom ASIC decodes the BPM data to the 40 MHz clock and the 40 Mbit/s control data, which is further transmitted to the 12 ABCD3TA ASICs. A redundancy system is in place such that if a module loses its TTC signal, for example due to a TX VCSEL failure, an electrical link can be established with a neighboring module and the TTC signal shared.

Physically, the RODs are based in an underground room called USA15. The VCSEL arrays / opto-boards are part of the ROD/BOC pairs mounted in the slots of 8 VME crates. Pairs of two crates are mounted in a rack. In terms of environmental control, each rack has fans that force airflow through them, and the heat is removed by a water cooling system.
4.3 Bit Error Rates and Single Event Upsets

Further details on the SCT optical links and the data acquisition system can be found in [79], [81].

4.3 Bit Error Rates and Single Event Upsets

One of the factors that might affect detector performance is the rate of radiation-induced single bit errors in the data and the TTC opto-links. Errors in the data link will lead to creation of spurious hits or loss of true hits. In the TTC link, bit errors could lead to a module missing a level 1 trigger accept (L1A), which in turn would de-synchronise the affected module from the rest of the SCT, until a reset signal is issued. In general, the bit error rate (BER) refers to the fraction of transmitted bits that have been corrupted. Single event upsets (SEU) refer to the single event in which for example the L1A word was missed, which led to desynchronisation and therefore to all following data from that module to be erroneous. The SCT specification allows for a maximum Bit Error Rate of $10^{-9}$, which would give a negligible detector inefficiency. Indeed in beam tests an upper limit on the bit error rate was estimated at $3.6 \times 10^{-10}$, which is below the ATLAS SCT specification of $10^{-9}$ [82]. Chapter 6 presents the first measurement of SEUs in the SCT in a collider environment and shows that the level of data-corruption in 2011 and 2012 data is indeed negligible.

4.4 SCT summary

The SCT is a complex detector with over 6 million individual channels, which achieved an outstanding operational fraction of 99.3% in Run I. I studied two aspects which were relevant in this context. In chapter 5 I report in more detail on the failure of VCSELs transmitting timing, trigger and control information from the counting room to the front-end SCT modules. This is an important issue, since a large failure rate can interfere heavily with efficient operation. VCSELs are not only employed in the SCT, but also in other ATLAS and LHCb sub-detectors, where failures have also been observed, and the failure mechanisms are believed to be the same. Chapter 6 presents a first measurement of the single event upset (SEU) rate in the SCT front-end modules.
Chapter 5

Reliability engineering of SCT TX VCSELs

Vertical cavity surface-emitting lasers (VCSELs) are employed in the SCT to transmit timing, trigger and control commands from the counting room to the SCT front-end electronics. After half a year of reliable operation, the VCSEL devices started failing at an alarming rate. In the subsequent two years, over 20% of the initial ∼4000 such lasers have failed. In order to maintain high detector efficiency and good data quality, the non-operational devices had to be replaced continuously. Given the alarming failure rate, the replacement policy was not a viable long-term solution, and several studies were performed to understand the failure mechanism. Humidity was hypothesized to be the cause of death and accelerated lifetime tests at extreme temperature and humidity conditions were performed to confirm this hypothesis.

I begin this chapter with a general description of VCSEL architecture and the monitoring of the failure rate in ATLAS (sections 5.1, 5.2). This is followed in 5.3 by a discussion of reliability engineering and the mathematical apparatus I developed for the extraction of the mean time to failure of a population of devices. In order to confirm whether humidity is indeed the cause of failure for the SCT devices, environmental tests at extreme humidities and temperatures were commissioned, as described in 5.4. Qualitatively, if humidity is indeed
5.1 VCSEL architecture

A VCSEL is a type of semiconductor laser diode that emits laser light perpendicular to its top surface. The different components of a VCSEL can be seen in the schematic representation 5.1. The active GaAs region in which the laser light is generated consists of several quantum wells. On each side of the active region, a distributed Bragg reflector (DBR) mirror, parallel to the active area, provides the laser resonator. The principle of a DBR mirror is that alternating layers of material create a region with periodically varying refractive index. At each boundary, light is partially reflected. If the thickness of the layer is chosen to be $\lambda/4$, a contributing factor to device failure, the mean time to failure is expected to be smaller with increasing humidity. Quantitatively, the acceleration model preferred in the literature can be used to fit the data from the environmental tests and make a prediction for the SCT VCSEL failure times, as presented in section 5.6. Particular care needs to be taken to use the correct values of humidity and temperature relevant for the hypothesised damaging process, as discussed in section 5.5. If the prediction for the SCT VCSEL failure rate from the fitted model and the observed failure rate do not match, it might be indicative of the fact that an additional accelerating factor is at play. The agreement between predictions and measurements and the consistency between different measurements is discussed in section 5.7. Further investigations performed at microscopic scale on failed devices clarified the failure mechanism and are reported in section 5.8. Finally, I conclude in section 5.9.

Figure 5.1: Schematic representation of the GaAs VCSEL driving the SCT opto-links.
where \( \lambda \) is the wavelength of the emitted light, the many reflections from each boundary layer will interfere constructively, effectively reflecting the light. The advantage of GaAs is that technologically it is easy to grow epitaxial layers of GaAs and aluminum gallium arsenide on top of each other, with the two materials having different refractive indices. In common VCSELs, the upper and lower mirrors are doped as \( p \)-type and \( n \)-type materials, forming a diode junction. The VCSELs initially installed in USA15 for SCT operation are of type TSD-8A12 and are supplied by Truelight, Taiwan. They have an oxide aperture, which confines the current and provides optical index guiding. Oxide VCSELs are known to be particularly vulnerable to humidity, therefore single VCSELs are typically integrated in a hermetically sealed package (called a TO\(^1\) can) that protects them against mechanical damage and humidity. The TO cans are very robust, but cannot be employed for large arrays of VCSELs like those used for the SCT, which consist of 12 VCSELs. The VCSEL arrays can be instead protected by applying an epoxy layer on the array. The epoxy is not impermeable to water vapour, but it does delay the humidity ingress. Cracks or holes in the epoxy can further compromise the protection against water vapour. Some manufacturers apply an impermeable dielectric layer on top of the VCSEL DBR to seal the device, which can further be integrated in a TO can, or have an epoxy layer applied. The VCSELs provided by TrueLight do not have the additional dielectric layer, only the epoxy coating. While the epoxy does not provide the same protection as a dielectric layer, epoxy coated arrays are more robust than bare ones.

### 5.2 Monitoring of SCT optical links

In the period 2010-2012, out of the over 4000 VCSEL devices operating in the counting room, almost 20% failed. Due to this high failure rate, a monitoring system of the optical links was put in place in 2010 by Daniel Short, a previous student, to quickly identify failed devices [83]. The current recorded through the on-detector \( p-i-n \) diode, \( I_{PIN} \), is proportional to the optical power received from the VCSEL, therefore an \( I_{PIN} \) of zero is

\(^1\)TO = transistor outline
indictive of a failed device. Each night, the monitoring system would e-mail an alert if the daily minimum $I_{PIN}$ of a device falls from its usual value to less than 0.04 mA from one day to another. I have worked on several improvements to the monitoring system. For example, once Truelight devices started failing, they were replaced by VCSELs from the manufacturer AOC, as the latter were believed to be more robust. It was essential to know if any new failures were Truelight or AOC devices, and the monitoring of the optical links was split by manufacturer. Another improvement was to produce plots that would help identify the cause of failure and eliminate false positives, and make them available in the e-mailed report. In particular, the shape of the $I_{PIN}$ current in the hours just before it reaches 0 mA can be used to discriminate between a true failure and a broken electrical connection, as can be seen in figure 5.2. Devices that are indeed failing show no degradation in performance prior to their failure, followed by a catastrophic decrease in optical power from its nominal value to zero within less than an hour. VCSELs with a broken electrical connection exhibit an instantaneous drop in the corresponding $I_{PIN}$.

![Figure 5.2: $I_{PIN}$ (mA) versus time (UTC - Coordinated Universal Time) for two different VCSELs. The $I_{PIN}$ value is indicative for the VCSEL optical power output. Left: the $I_{PIN}$ decreases from its nominal value to zero within less than one hour. This is typical of a true failure. Right: the $I_{PIN}$ decreases from its nominal value to zero instantaneously. At a later time, it might increase again to its usual value. This behaviour is typical of a deteriorated electrical connection.](image-url)
5.3 Reliability engineering, mathematical apparatus

Any device will, after a certain number of operational hours, fail due to wear and tear. When looking at a large sample of devices of the same type that operate in the same conditions, the failure rate versus time is expected in general to have the shape of the “bathtub curve” in figure 5.3. In particular for the VCSELs operating in ATLAS, the left-hand side of this curve associated with infant mortality and the random failure rate were observed to be negligible. The wear-out part of the failure rate curve starts after a useful time of operation denoted $\gamma$. The probability for a device to fail between time $t$ and $t + dt$ is $f(t)dt$, and all devices in a population are assumed to obey the same $f(t)$. The starting assumption is that all devices are identical and that the operation or failure of one device will not affect the lifetime of other devices. A possible choice for $f(t)$ in the modeling of failure times is the translated Weibull distribution [84].

5.3.1 Translated Weibull distribution

The translated Weibull distribution [84] is fully described by 3 parameters and is given by

$$f(t|\beta, \eta, \gamma) = \frac{\beta}{\eta^\beta} (t - \gamma)^{\beta-1} \exp \left[ - \left( \frac{t - \gamma}{\eta} \right)^\beta \right]$$  \hspace{1cm} (5.1)
for \( t > \gamma \), while \( f(t) = 0 \) otherwise. For \( \gamma = 0 \), one recovers what is known simply as the “Weibull distribution”. The Weibull distribution is widely used in the industry to describe a broad range of reliability data for the end of life period, once wear-out failures start occurring. The translation parameter \( \gamma \) gives the useful life period of the devices, in which no wear-out failures are expected. The Weibull distribution is a versatile distribution: for \( \beta = 1, \gamma = 0 \) it simplifies to the exponential distribution, for \( \beta = 2, \gamma = 0 \) it gives the Rayleigh distribution. In reliability terms, \( \beta = 1 \) can be interpreted as a constant failure rate. A larger \( \beta \) represents a failure rate that is increasing with time, representative for aging processes in the end-of-life period. A \( \beta \) value smaller than 1 is typical for the infant mortality period. The different cases are illustrated in figure 5.3. The Mean Time To Failure (MTTF) of one device is given by

\[
\text{MTTF} = \int_{0}^{\infty} t f(t) \, dt = \gamma + \frac{\eta \Gamma(1/\beta)}{\beta},
\]

where \( \Gamma \) is the Gamma function. This enforces the interpretation of the parameter \( \gamma \) as the length of time before the end-of-life period. The second term gives the mean time to failure once the device enters the end-of-life period. The parameter \( \eta \) mostly dominates the second term in the MTTF.

### 5.3.2 MTTF extraction

If one had a large population of devices operated under the same conditions until the last one failed, one could fit equation 5.1 to the failure times of all devices to extract the parameters \( \beta, \eta, \gamma \). Alternatively, one might only be interested in the MTTF for the whole population, which would be obtained by averaging the individual time to failures of all devices. The extraction of the parameters \( \beta, \eta, \gamma \) and MTTF is complicated however in the case in which not all devices are operated until they fail, so not all failure times are available, or in the case in which the exact times of failure are unknown. For example in the case of the SCT, the detector has to be kept operational throughout the year for data-taking. The devices are part of 12-channel arrays, out of which 4, 10, 11 or 12 devices might be operated. If too many devices fail on a board, the whole set has to be replaced, even if the remaining devices on
the board are still operational. Furthermore, during technical shutdowns, fully operational boards were replaced with devices from a different manufacturer (AOC) thought to be more robust, in order to prevent altogether the risk of failing devices. The only information available from such devices is that they have operated without failure for a certain amount of time, without knowing their failure time. MTTF extraction is therefore no longer a trivial average of failure times.

It is useful to define the reliability function $R(t)$ of a device, which gives the probability that a device survives for a stated amount of time, i.e. there is no failure between time 0 and $t$:

$$R(t) = 1 - \int_0^t f(t')dt' = \begin{cases} \exp \left[-\left(\frac{t-\gamma}{\eta}\right)^\beta\right], & \text{if } t > \gamma. \\ 1, & \text{otherwise.} \end{cases} \quad (5.3)$$

The likelihood of the data, which is composed of VCSEL failure times and replacement times, under a particular $(\beta, \eta, \gamma)$ hypothesis, can be expressed as follows:

$$L = \prod_{\text{failures}} f(t_i|\beta, \eta, \gamma) \prod_{\text{survivals, removals}} R(t_i|\beta, \eta, \gamma) \quad (5.4)$$

The values of the parameters $(\beta, \eta, \gamma)$ that maximize the likelihood can be found, and the MTTF can be calculated according to equation 5.2. Errors on MTTF can be calculated by recognising that the log-likelihood ratio is asymptotically $\chi^2$-distributed with three degrees of freedom,

$$-2 \log \left(\frac{L(\beta, \eta, \gamma)}{L_0}\right) \sim \chi^2_{d.o.f.} \quad (5.5)$$

where $L_0$ is the maximal value of the likelihood function. The 68% confidence region is given by the contour which is $\Delta \log L = 3.53/2$ away from the best fit point $(\beta_0, \eta_0, \gamma_0)$. The contour is scanned in the $\beta, \eta, \gamma$ space and the maximum and minimum values of the MTTF are quoted as the error on the MTTF.

After it was independently developed, the statistical apparatus used for MTTF extraction was found to be consistent with that taught in advanced statistical inference courses, e.g. [85]. The notation and nomenclature used in this work were adapted to match the prevalent ones.
5.3 Reliability engineering, mathematical apparatus

5.3.3 Validation of method

\( fTTF \) versus \( MTTF \)

Several, simpler techniques were proposed initially by others in the ATLAS SCT collaboration to extract the mean time to failure for the sample of failing VCSELs. A first approach suggested that one should find the time at which 50%, or some other fraction \( f \) of the devices, have failed, and use this as a value for the MTTF. The problem with such an approach is that it does not correctly take into account the replacement policy of the devices. Also, if the failure rate is indeed described by a Weibull distribution, the ratio of MTTFs of two different populations described by two different sets of parameters \( \beta, \eta, \gamma \) will not equal the ratio of 50%\( TTF \). The time to a fraction \( f \) of failures is given by

\[
\eta \left[ \ln \left( \frac{1}{1 - f} \right) \right]^{\frac{1}{\beta}} + \gamma.
\]  
(5.6)

Ratios of \( fTTF \)s calculated for different samples can vary by \( \mathcal{O}(10\%) \) from the correct values of ratios obtained by using the correct MTTFs given by equation 5.2 in the range of interest of the parameters. Lifetime acceleration factors between experiments run in different environmental conditions will later be calculated based on model parameters extracted from ratios of MTTFs. Therefore a mathematical formalism which correctly evaluates such ratios is important. Also, in the case of device replacement, it is not obvious how to correctly calculate \( fTTF \)s and whether the fraction \( f \) should refer to the number of originally installed devices, or the number of operating devices.

Population censoring

The extraction of MTTF in the case of the SCT VCSELs is complicated by the population censoring, which refers to the fact that while some devices have failed, other operational devices were removed from the system or operated successfully until the end of the monitoring period. Therefore only impartial information is available about the system and the mean time to failure for the whole population can no longer be calculated as a simple average of failure times, since not all of these are available. An early, naive proposal from other
members of the collaboration for assessing the mean time to failure of the devices was to attempt the extraction of the MTTF from the first failure on each board only, ignoring entirely the surviving devices. I investigate therefore to what extent using only the failure data, and ignoring completely the survival data, biases the extraction of the MTTF. In a simulation, 4080 failure times were sampled from a Weibull distribution with the parameters $\beta = 2.9, \eta = 780$ days, $\gamma = 0$ days. The black curve in figure 5.4 shows all failure times of these hypothetical devices. Assuming a similar replacement policy as in the SCT, the devices are grouped in batches of 12 and only the first failure in each batch is shown in red in the same figure. It is assumed in this particular case that the pseudo-experiment has run for very long until all devices have failed in case of the black curve, or at least one device on each opto-board has failed in case of the red curve.

![Figure 5.4](image)

Figure 5.4: Distribution of number of failed channels versus time to failure (in days) for all devices of a sample or for the first failure in a group of 12 for a sample characterised by Weibull parameters $\gamma = 0, \beta = 2.9, \eta = 780$ days and analytical MTTF = 695.5 days. The MTTF calculated as the average time to failure is 691.9 days for the black curve and 288.7 days for the red curve.

The MTTF calculated from the failure times shown in the red curve underestimate the MTTF by a factor of 2-3. One can show in fact from the formalism developed in the previous section that if one extracts the MTTF from the first failure data only, this would be related to the MTTF of the whole population by a factor $n^{1/\beta}$, where $n$ is the number of devices on one array. This is complicated however by the fact that in the SCT one has boards with 12, 11, 10 or 4 devices each, and that devices are not replaced immediately after the first death. Indeed, sometimes a second death occurs on the same board before replacement. Therefore
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the complication of an analytical formula connecting the two MTTFs justifies using the fit approach from equation 5.4 instead.

**Weibull fit performance for (right-)censored data**

The usefulness of the fit is tested by using the fit prescription of equation 5.4 to the pseudo-data simulated in the previous subsection. Either the full data set can be used, or the data set of first failures, considering the rest as survivals. Furthermore, the experiment can be considered as having been stopped after a cutoff time of 1000, 800 or 600 days of operation. Any information about later deaths or survivals are considered not to be available in this case, and these devices are considered survivors with a survival time equal to the cutoff time of the experiment. The data is referred to as being “right-censored”, as failure data is missing from the right hand side of the Weibull distribution. Table 5.1 shows how the extracted parameters from the fit and the corresponding MTTF change with the degree of censoring.

<table>
<thead>
<tr>
<th></th>
<th>$\beta$</th>
<th>$\eta$</th>
<th>$\gamma$</th>
<th>MTTF</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Nominal input values</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Simulated data</td>
<td>2.9</td>
<td>780</td>
<td>0</td>
<td>695.5</td>
</tr>
<tr>
<td>All failures (100% failures)</td>
<td>2.85</td>
<td>776</td>
<td>1</td>
<td>692.5$^{+26.6}_{-20.5}$</td>
</tr>
<tr>
<td>First failures (8.3% failures)</td>
<td>2.85</td>
<td>776</td>
<td>1</td>
<td>690.7$^{+52.6}_{-24.2}$</td>
</tr>
<tr>
<td>stop at 400 days (7% failures)</td>
<td>2.85</td>
<td>774</td>
<td>1</td>
<td>683.8$^{+52.6}_{-20.3}$</td>
</tr>
<tr>
<td>stop at 300 days (4.6% failures)</td>
<td>2.875</td>
<td>766</td>
<td>1</td>
<td>548.8$^{+69.9}_{-45.5}$</td>
</tr>
<tr>
<td>stop at 200 days (2% failures)</td>
<td>3.375</td>
<td>610</td>
<td>1</td>
<td>548.8$^{+69.9}_{-45.5}$</td>
</tr>
</tbody>
</table>

Table 5.1: Mean time to failure extracted from Weibull fit to pseudo-data, depending on whether one uses the full failure information, or right-censors some of it. The “Nominal” values for $\beta$, $\eta$, $\gamma$ refer to the Weibull distribution the pseudo-data failure times were sampled from, all other values are the maximum likelihood estimates (MLE). The MTTF from the “Simulated data” row is a simple average of the simulated failure times, the other MTTFs are obtained from the analytical expression 5.2 using the MLEs.

The interesting observation is that even when only the first failure on a board is recorded and the other devices are treated correctly as survivals, the extracted MTTF is the same as in the case in which all failures are recorded, however with a larger error. The earlier the experiment is stopped, the less constrained will the upper error on the extracted MTTF be, with a reasonable lower error. The best fit MTTF does not differ by more than 2% from the MTTF of the simulated data, even when only information about 5% of failed devices
Reliability engineering, mathematical apparatus

was used in the fit, with the rest of the devices being censored. Only when the number of failed devices used in the fit is 2%, does the extracted MTTF differ by a significant amount. The more information about failed devices is used in the fit, the more accurate the extracted MTTF will be. In the case of the SCT devices in USA15, the fit has been split in two sub-populations, with the percentage of failed devices in each being 16.8% and 15.4%. In the case of accelerated lifetime tests, at least 50% devices have failed in each test.

Absence of in-situ measurement (interval censoring)

Another difficulty in extracting the MTTF is that in some populations of devices, one does not have in situ measurement of failure rates. Only at discrete times can the number of failures be counted, which will affect the accuracy with which the MTTF can be determined. This is also known as “interval censoring”. For example, the accelerated lifetime tests described in section 5.4 suffer from this limitation. To assess the impact of this effect, 60 failure times are sampled from a Weibull distribution characterized by the parameters $\eta = 484$ hours, $\beta = 1.3$, $\gamma = 211$ hours. These values were extracted from the log-likelihood fit to an accelerated lifetime test performed at 85°C, 85% relative humidity ($RH$). The MTTF calculated from the analytical formula, given the Weibull parameters, is 658 hours. A large number of $10^5$ simulations is performed, with each simulation consisting of a set of 60 failure times. For each simulation, the 50%TTF and the MTTF are calculated based on the failure times, and figure 5.5 shows their distribution for the ensemble of all simulations. The calculation of the 50%TTF and MTTF is then repeated with the extra assumption that the number of failed devices is only counted at discrete times. For the purpose of calculating an average time to failure, the time of failure of a device is taken to be in the middle of the interval between two measurements. The times of measurements considered are those from the accelerated lifetime test from which the extracted Weibull parameters stem from, namely (in hours): 0, 170, 267, 455, 769, 1009, 1200, 1400, 1600, 1900.

From the results in figure 5.5 it can be concluded that in the case in which all failure times are available, the average time to failure is a good approximation to the analytical MTTF even in the case of interval censoring.
5.3 Reliability engineering, mathematical apparatus

Figure 5.5: \( N = 10^5 \) simulations for failures of 60 devices with theoretical MTTF = 658 h. Different ways of calculating the time to failure are considered: either the mean time to failure (MTTF) or the time to 50% failures (50TTF) is considered, for the case in which the time to failure of each individual device is known with the precision of an hour (called continuous measurement), or only every O(100h) (called discrete measurement).

The difficulty with the available data is that it is both right-censored and interval-censored. I have shown in the current subsection that the second constraint does not affect the estimate of MTTF if it is calculated as a simple average of failure times. However, the first constraint implies that a simple average of failure times is not appropriate as it would underestimate the real MTTF. It is more appropriate to perform a Weibull log-likelihood fit to the data, as the procedure provides good estimates of the real MTTF in the case of no interval censoring (as demonstrated in subsection 5.3.3). Equation 5.4 can be adapted to interval censored populations by substituting terms of the type \( f(t_{\text{failure},i}) \) with terms \( [R(t_{i\mid}) - R(t_{i\mid})] \) with \( t_{i\mid} \) being the beginning and \( t_{i\mid} \) the end of the interval in which the device failed. The validity of this approach was tested using MC simulations. Two Weibull parameter sets were investigated, \( \beta = 1.5, \eta = 534, \gamma = 0 \) and \( \beta = 8.3, \eta = 2106, \gamma = 0 \). These values are representative for two of the experiments described in section 5.4. For each parameter set, \( 10^3 \) toy MC datasets are simulated for 26 devices. The failure pseudo-data is right-censored at 2000h and interval-censored every 200 hours, the latter being typical for the experiments discussed later in this chapter. The maximum likelihood estimators \( \hat{\eta}, \hat{\beta} \) are found for each MC pseudo-dataset. Using the average \( < \hat{\eta} >, < \hat{\beta} > \) of all pseudo-samples, an average fitted MTTF is determined for each set of true Weibull parameters using eq. 5.2. The bias due to the interval-censoring is found to be less than 1% and consistent with the
expected statistical uncertainty due to the limited number of pseudo-data samples.

5.4 Experiments

The apparatus for MTTF extraction developed in the previous section and based on a maximum likelihood fit was shown to be robust in all cases of population censoring (right-censoring, the absence of in-situ monitoring of the devices and removal of whole array after first failure). These features will be relevant for the MTTF extraction in the experiments described below.

5.4.1 SCT USA15

The sample of this “experiment” is constituted from the SCT transmission (TX) VCSELs initially installed in the USA15 cavern in the crates hosting the ROD/BOC pairs. The study is performed on devices in the period starting July 27\textsuperscript{th} 2009, when most devices were installed for SCT operation, until February 28\textsuperscript{th} 2012. Out of the 945 days in this period, only on 759 days was the SCT turned on, while in the remaining days the detector, including the VCSELs, were turned off for maintenance purposes. The periods of shutdown were not imposed by the VCSEL failures, but this opportunity was taken to perform VCSEL array replacements. The SCT shutdown times for most of the period of this study are shown in figure 5.6. The times when the VCSELs were off are not considered to be contributing to the aging of the devices. This was shown to be true in accelerated lifetime tests in which the devices were exposed to high temperatures and humidities while being turned off. The exposure did not affect their reliability. The failure analysis is split by crates 0-3 and 4-7, as it was observed that different environmental conditions were present in the two sets of crates. The airflow in the crates serving the endcap modules (0-3) could not be turned on at full capacity, leading to slightly higher ambiental temperature and therefore lower relative humidity. The environmental and operational conditions are detailed in table 5.2. It was observed that the failure rates between the two sets of crates were also different.
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High levels of operability have to be maintained in the SCT to ensure good quality of the data collected. If only one laser on a TX array fails, the previously described redundancy system for the TTC links kicks in. If the VCSEL sending the control data to the neighbor which shares the TTC signal with the affected module also fails, or if the electrical connection between the two modules is broken, the whole TX opto-board needs to be replaced. Therefore opto-boards with failed devices were replaced once two neighboring VCSELs on them failed.

Figure 5.6: Left: Cumulative number of SCT VCSEL failures and removals in crates 0-3 and 4-7 versus number of days of operation. The total number of VCSEL devices is just over 4000. In crates 0-3, 363 VCSELs failed, 1799 were removed and 108 operated until the end of the monitoring period. In crates 4-7, 393 devices failed, 1647 were replaced and 83 survived until the end of the monitoring period. The devices have operated for 228 days with no failures before day 1 shown on the x-axis. Days in which the SCT was shut down were removed from the time axis. Right: SCT status versus time. A value of 1 on the y-axis corresponds to the SCT being on, while a value of 0 indicates the SCT was off.

Figure 5.6 shows the cumulative number of failures and removals in the crates 0-3 and 4-7, with respect to the initial population of installed VCSELs only. Affected opto-boards can be replaced whenever there is a longer break between physics runs. The jumps in the number of removals correspond to technical stops (typically around the winter holidays) in which large numbers of plug-ins were removed and replaced. At the end of the monitoring period, only a few hundred original channels were still in operation, and they are shown as the last jump in the removals graphs.

For simplicity it was decided to only use in the fit the failure and survival data of the initial installs. This already provided a large enough sample. Also the most constraining...
power on the MTTF determination comes from the failed devices. The replacement opto-boards would most often have to operate for at least a period $\gamma$ before they start failing. Therefore the data from the replacement opto-boards is not expected to improve the fit. Another issue is that the devices from the initial installs are guaranteed to have experienced the same environmental conditions even if those are varying with time. Therefore their time to failure is described by the same probability distribution. The replacement devices would have started operation with a considerable time lag, when the environmental conditions were somewhat different.

It can be noticed in figure 5.6 that after each large replacement campaign, the slope in the cumulative distribution of failures is reduced. This is an effect of the fact that only initially installed devices were tracked. By replacing functioning devices, the pool of initially installed devices that can in principle fail is effectively reduced.

5.4.2 ULM tests

The high failure rate in USA15 prompted several accelerated lifetime tests at high temperature and relative humidity to be performed at U-L-M photonics by collaborators. The mean time to failures from accelerated lifetime tests can be used to predict the failure rates expected for USA15 conditions using a temperature and humidity acceleration model. The prediction can then be compared with the measured USA15 MTTF, and any discrepancy might point towards an additional accelerating factor beyond temperature and humidity. A subset of the ULM tests with their different humidities, temperatures and operating conditions is detailed in table 5.2. The tests were performed in environmental chambers to ensure a constant temperature and humidity. Failures were recorded every few 100s of hours, with the tests having a length between 1000 and 2200 hours. Not all devices in all tests have failed, therefore the statistical treatment developed in the previous sections had to be applied for the correct extraction of MTTF for these tests. The same table shows in the last row the MTTF extracted from the Weibull fit. Figure 5.7 shows the cumulative number of failures, normalised to the total number of tested devices, as well as the distributions predicted by
5.4 Experiments

the best Weibull fits and errors thereon.

<table>
<thead>
<tr>
<th>Name of sample</th>
<th>85C/85%</th>
<th>60C/85%</th>
<th>85C/60%</th>
<th>Crates 0-3</th>
<th>Crates 4-7</th>
<th>SR1</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_{\text{ambient}}$ (°C)</td>
<td>85</td>
<td>60</td>
<td>85</td>
<td>26.1</td>
<td>23.7</td>
<td>25</td>
</tr>
<tr>
<td>$RH_{\text{ambient}}$ (%)</td>
<td>85</td>
<td>60</td>
<td>40.9</td>
<td>48.0</td>
<td>44.5</td>
<td></td>
</tr>
<tr>
<td>Calculated $AH$ (g/m$^3$)</td>
<td>304.0</td>
<td>111.1</td>
<td>214.6</td>
<td>10.01</td>
<td>10.26</td>
<td>10.24</td>
</tr>
<tr>
<td>Current (mA)</td>
<td>DC 10</td>
<td>DC 10</td>
<td>DC 10</td>
<td>AC 10</td>
<td>AC 10</td>
<td>AC 10</td>
</tr>
<tr>
<td>Duty cycle</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
<td>50%</td>
<td>50%</td>
<td>50%</td>
</tr>
<tr>
<td>Nr devices</td>
<td>31</td>
<td>26</td>
<td>24</td>
<td>2054</td>
<td>1957</td>
<td>48</td>
</tr>
<tr>
<td>Environmental chamber</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>×</td>
<td>×</td>
<td>×</td>
</tr>
<tr>
<td>Forced airflow</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>×</td>
</tr>
<tr>
<td>Extracted MTTF (hours)</td>
<td>658 $^{+82}_{-61}$</td>
<td>1986.8 $^{+85}_{-57}$</td>
<td>1193.2 $^{+132}_{-79}$</td>
<td>11022$^{+311}_{-238}$</td>
<td>9595$^{+246}_{-107}$</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 5.2: Environmental and operational conditions for the accelerated lifetime tests at U-L-M photonics as well as for the SCT crates and the SR1 setup. $RH$ ($AH$) refers to relative (absolute) humidity. The SCT extracted MTTFs have been corrected by a factor 2 to account for the fact that the devices only operate half the time (50% duty cycle).

Figure 5.7: Cumulative number of failures for the ULM tests versus time of operation. The filled circles indicate the times at which device failures were counted and the fraction of failed devices. The continuous lines are the distributions predicted by the best results of the Weibull fits. Dotted lines indicate the 1σ uncertainty on the best fit results and are obtained as explained in equation 5.5.

All tests described in table 5.2 were performed with VCSEL arrays coated with a layer of epoxy, i.e. devices of the same type as those used in USA15. Two more experiments were performed at 85C/85% conditions with devices lacking this coating. The devices in the two additional tests were driven at a current of 5 mA and 10 mA respectively. They both had a MTTF significantly shorter than the similar test with epoxy-coated devices (346 and 297 hours, compared with 658 hours for epoxy-coated VCSELs). This supports the hypothesis that humidity plays a role in the failure mechanism. The devices operated at a higher driving current had the shorter lifetime, which is expected to be due to the higher heat dissipation
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and therefore the higher junction temperature for these devices.

5.4.3 SR1 test

I anticipate the results of section 5.6 in which the MTTFs observed in USA15 and the ULM tests are fitted to a lifetime acceleration model that depends on the test temperature and humidity. The conclusion of section 5.6 is that the MTTFs observed in USA15 are not consistent with the MTTFs predicted from the acceleration model, the observed MTTFs being several times lower than the predicted ones. This can be interpreted in either of the following two ways. In the first interpretation, temperature and humidity are assumed to be the only damaging factors, but the functional form of the acceleration model is considered not to be valid for the particular type of devices under study, and any discrepancy between the measured USA15 MTTF and the prediction based on the acceleration model is therefore meaningless. Alternatively, in the second interpretation, the temperature and humidity-related terms in the acceleration model are correct, but an additional damaging agent is present beyond temperature and humidity. As this damaging agent might affect ULM and USA15 devices with different strength, an additional factor would need to be included in the acceleration model.

In order to distinguish between these two possibilities, a long-running test with a small sample size was established by collaborators in the SR1 building on the surface near ATLAS. Four opto-plugins with 12 channels each were operated in the SR1 test for more than 500 days. The opto-plugins also operated without failures before installation in SR1: two plugins operated for 227 days, the other two plugins for 363 days. The setup of the SR1 tests mimicked the USA15 conditions as far as possible. In particular, since the SR1 building and the USA15 cavern are close to each other, the environmental conditions humidity and temperature were expected to be similar (essentially local room-temperature and humidity) and to follow the same seasonal variation pattern. The VCSELs were operated with the same bias and duty cycle. The only difference is that the SR1 devices were not operated under forced airflow, therefore local heating effects might have played a more important
role. In principle, after a long enough operation of the devices until their end of life period, an MTTF can be calculated for this population and compared to the observed MTTFs of USA15 devices and to predictions from ULM data and the acceleration model. If the observed USA15 and SR1 MTTFs are found to be similar and inconsistent with ULM data and the acceleration model, the validity of the functional dependence of MTTF on \( RH, T \) would be further questioned (interpretation 1 above). If the SR1 and USA15 MTTFs differ in spite of their similar environmental conditions, and the SR1 MTTF is consistent with the prediction from the acceleration model, it could rather be assumed that another factor beyond humidity and temperature affected the VCSEL reliability in USA15 (interpretation 2 above).

The result of the SR1 test was that only 1 out of the 48 channels failed. The failure was recorded on one of the plugins that operated for 363 days before SR1 installation. This result is confronted with the ULM and USA15 MTTFs in section 5.7.

Further four arrays were operated in SR1 in a chamber flushed with nitrogen to ensure a zero-humidity environment, and no failures were observed.

### 5.5 Local heating effects

In section 5.6 I collect the extracted MTTFs from the SCT and the accelerated lifetime tests and fit them to an acceleration model, which has as input parameters the temperature and relative humidity. The temperature and relative humidity used need to be those relevant for the VCSEL damaging electrolytic process, the temperature at the VCSEL junction, \( T_{\text{junction}} \) and the relative humidity at the device surface, \( RH_{\text{surface}} \). However, only the ambiental temperature and \( RH \) are available from the various experiments, and they differ from \( T_{\text{junction}}, RH_{\text{surface}} \). Due to the heat dissipation in the VCSEL, in steady-state temperature gradients are set up between the junction, the surface of the device and the ambient. This local heating can in turn reduce the relative humidity at the surface of the device with respect to the ambient \( RH \). A further complication is the fact that some experiments are cooled by
forced airflow. In the case of optimal airflow, the device surface temperature and humidity is constrained to be the same as the ambiental ones, but this should not necessarily be assumed for real-life forced airflow.

In the current section, I set up two models \(5.5.1\) \(5.5.2\) for local heating and for the effect of forced airflow on the local heating. The two models are calibrated in \(5.5.4\). The purpose of the models is to infer the temperature at device surface and at the junction, as well as the \(RH\) at the device surface from the ambiental temperature and humidity of each experiment. In the next section \(5.6\) I will use the inferred \(T_{\text{junction}}, RH_{\text{surface}}\) to perform a fit of the MTTFs to an acceleration model.

### 5.5.1 Model I: forced airflow does not carry away all heat

In model I, it is assumed that the flow of air is unable to carry away all of the dissipated heat, therefore the temperature of the surface of the device is higher than that of the ambient. Assuming that the absolute humidity is the same throughout the chamber and at the surface of the device, the increased surface temperature leads to a lower relative humidity at the surface. The absolute and relative humidity are interrelated, and for any given temperature there is a unique correspondence between the two, as documented in appendix A. In this model, the decrease in the expected MTTF due to higher temperature is partly compensated by the increase in expected MTTF due to lower surface humidity.

### 5.5.2 Model II: forced airflow carries away all heat

In model II, the airflow at the surface of the devices is assumed to be strong enough to constrain the surface temperature to be that of the environment. This is likely to be true for the environmental chambers of the ULM experiments, while for USA15 the forced airflow does not reach all TX arrays equally.

Since there is no forced airflow in the SR1 setup, the \(T\) and \(RH\) for this test are calibrated separately in each model.
5.5 Local heating effects

5.5.3 Heat diffusion model for $\Delta T_{\text{junction-surface}}$ estimation

The prediction of $\Delta T_{\text{junction-surface}}$ is common to both models and is discussed in this subsection. A simple heat diffusion model is built to this purpose. The L-I-V (light power, current, voltage) curves of the VCSELs were measured by collaborators, from which it was possible to infer the bias voltage at different operating currents\(^2\) and calculate the total input power for each setup. From this, the known optical power at given bias current can be subtracted to give the total power dissipated as heat. In steady state, the junction is not heating up and the dissipated heat is conducted through the VCSEL to its surface. The difference $P_{\text{thermal}} = P_{\text{input}} - P_{\text{optical}}$ sets up a thermal current across the VCSEL’s thermal resistance $R_{\text{th}}$, which was provided by the manufacturer to be 1.75 K/W, and was also measured by collaborators. A temperature gradient $\Delta T(\text{junction-surface}) = P_{\text{thermal}}/R_{\text{thermal}}$ is set up as a consequence. The difference $\Delta T_{\text{junction-surface}}$ does not depend on the efficiency of heat removal through the forced airflow, so is the same in both models of local heating. Depending on the local heating model used, a different surface temperature is expected, to which the calculated $\Delta T_{\text{junction-surface}}$ is added to obtain $T_{\text{junction}}$.

The ratio of dissipated powers for different experiments will also be the ratio of temperature differences $\Delta T_{\text{junction-surface}}$ for these experiments due to

$$R_{\text{VCSEL, thermal}} = \frac{P_{\text{dissipated}}^{\text{exp 1}}}{\Delta T_{\text{surface-ambient}}^{\text{exp 1}}} = \frac{P_{\text{dissipated}}^{\text{exp 2}}}{\Delta T_{\text{surface-ambient}}^{\text{exp 2}}}.$$  \hfill (5.7)

The differences in $P_{\text{thermal}}$ for various experiments stem from differences in duty cycle and current of the setups. For example the difference between crates 0-3/4-7 and ULM tests is due to the fact that the crates VCSELs are operated with 10 mA AC current, whereas the ULM VCSELs are operated with 10 mA DC current (see table 5.2). The heat dissipation is a factor $\sim 2$ higher in the ULM case (whereas the thermal resistance is the same), therefore the higher $\Delta T_{\text{junction-surface}}$. The values of $\Delta T_{\text{junction-surface}}$ are summarised in table 5.3.

The second model of local heating assumes the SCT and ULM airflow is strong enough

---

\(^2\)all tests are operated at fixed input current
5.5 Local heating effects

to constrain $\Delta T_{surface-ambient}$ to zero, therefore the above considerations uniquely give a value for $T_{junction}$ and most of the entries for model II in table 5.3 can therefore be filled in. Notice that the SR1 test is exempt from this constraint, since this is the only experiment with no forced airflow, and its value for $\Delta T_{surface-ambient}$ is discussed in the next subsection.

5.5.4 Calibration of models

While for each experiment values for the ambient temperature and humidity were collected, and a method to calculate the difference $\Delta T_{junction-surface}$ was established, there is still need for a measurement or inference of $T_{surface}$. Several tests with 12-channel VCSEL arrays powered at 10 mA AC current, 50% duty cycle, were carried out by members of the collaboration to assess the level of self-heating, by measuring the shift in wavelength during different conditions of operation (with / without airflow, with only one / with several channels turned on). The shift in wavelength is directly proportional to the shift in junction temperature due to the change in optical and physical thickness of the lasing volume and is specified by the manufacturer to be 0.06 nm/K.

A difference of 12.7K in junction temperature between operating a full array of 12 VCSELs with forced airflow and without forced airflow is inferred from the shift in lasing wavelength. This can be directly translated in a difference in junction temperature between the SR1 setup, which has no forced airflow, and the SCT VCSELs operated under forced airflow. This difference is expected to be valid regardless of the local heating model assumed. With this observation, the remaining entries for the SR1 setup for model II in table 5.3 can be filled in.

A second measurement inferred a difference of 9K in junction temperature between operating only one channel on an array with airflow and operating all 12 channels with airflow. This latter measurement can be interpreted as follows in the context of model I. In the case of one channel on, one might assume that even a weak forced airflow is able to carry

---

The strength of airflow in the SCT crates and the environmental chambers is similar to the one used in the experiments measuring the self-heating effects.
Local heating effects

away all heat dissipated, therefore the surface temperature will be the same as the ambient
temperature. Turning on all 12 channels leads to higher heat dissipation, with the forced
airflow not being able to successfully remove all heat from the array surface. The difference
$\Delta T_{\text{junction-surface}}$ for each operating channel is the same regardless of how many channels
are powered or whether forced airflow is present or not, since each device dissipates the
same amount of heat over the same thermal resistance. Since 12 channels on are measured
to have a 9K higher junction temperature than 1 channel on, and $\Delta T_{\text{junction-surface}}$ is the
same for both, it can be inferred that $T_{\text{surface}}$ is 9K higher in the case of 12 VCSELs vs 1
VCSEL on. The ambient temperature is the same in the two operation modes, therefore
$\Delta T_{\text{surface-ambient}}$ must be 9K higher for 12 devices vs 1 device on. Considering the assumption
that $\Delta T_{\text{surface-ambient}} = 0$ K for one device on only, this implies $\Delta T_{\text{surface-ambient}} = 9K$ for a
12-channel array powered at 10 mA AC current, 50% duty cycle, in model I and with forced
airflow. Therefore, the $\Delta T_{\text{surface-ambient}}$ rows of table 5.3 can now be filled in for Crates 0-3
and 4-7. It is assumed that a high fraction of the heat is transferred from the VCSEL surface
through conduction through the air rather than convection through forced airflow, and that
the thermal resistance of the air is the same regardless of the experiment. Therefore the same
reasoning that lead to equation 5.7 implies that the ratio of dissipated powers for two exper-
iments is equal to the the ratio of their $\Delta T_{\text{surface-ambient}}$, which enables the completion of the
$\Delta T_{\text{surface-ambient}}$ entries for the ULM experiments. Again, the SR1 setup makes an exception,
as no forced airflow is applied. From the first measurement described in this subsection a
difference of 12.7 K in surface temperature between an array operated under forced airflow
versus no airflow was inferred, which led to $\Delta T_{\text{surface-ambient}}^{\text{SR1}} = \Delta T_{\text{surface-ambient}}^{\text{USA15}} + 12.7K$.

Notice the second measurement is not consistent with model II, as operating one or more
VCSELs should have no effect on the surface temperature in the case that the forced airflow
removes all heat. This observation suggests that model I is likely to give a more accurate
description of the environmental conditions.

From these measurements, and the described assumptions, the temperatures at the sur-
face and at the junction and therefore the surface humidity were inferred for each experi-
mental setup. The surface humidity is recalculated for each experiment, assuming that the absolute humidity remains the same throughout the environment, but the relative humidity will change close to the surface of the devices due to the increased local temperature. The conversion between relative and absolute humidities at different temperatures is documented in appendix A. Table 5.3 summarizes the temperatures and relative humidities used under the two models.

<table>
<thead>
<tr>
<th></th>
<th>Crates 0-3</th>
<th>Crates 4-7</th>
<th>SR1</th>
<th>85C/85%</th>
<th>60C/85%</th>
<th>85C/60%</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_{ambient}$ ($^\circ$C)</td>
<td>26.1</td>
<td>23.7</td>
<td>25.0</td>
<td>85</td>
<td>60</td>
<td>85</td>
</tr>
<tr>
<td>$RH_{ambient}$ (%)</td>
<td>40.9</td>
<td>48.0</td>
<td>44.5</td>
<td>85</td>
<td>85</td>
<td>60</td>
</tr>
</tbody>
</table>

Model I (forced airflow does not carry away all heat)

<p>| | | | | | | |</p>
<table>
<thead>
<tr>
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<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta T_{surface-ambient}$ ($^\circ$C)</td>
<td>9.0</td>
<td>9.0</td>
<td>21.7</td>
<td>16.4</td>
<td>16.4</td>
<td>16.4</td>
</tr>
<tr>
<td>$\Delta T_{junction-surface}$ ($^\circ$C)</td>
<td>14.8</td>
<td>14.8</td>
<td>14.8</td>
<td>27.0</td>
<td>27.0</td>
<td>27.0</td>
</tr>
<tr>
<td>$\Delta T_{ambient-junction}$ ($^\circ$C)</td>
<td>23.8</td>
<td>23.8</td>
<td>36.5</td>
<td>43.4</td>
<td>43.4</td>
<td>43.4</td>
</tr>
<tr>
<td>$RH_{surface}$ (%)</td>
<td>25.2</td>
<td>29.3</td>
<td>14.4</td>
<td>47.7</td>
<td>43.1</td>
<td>33.7</td>
</tr>
</tbody>
</table>

Model II (forced airflow carries away all heat)

<p>| | | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
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<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta T_{surface-ambient}$ ($^\circ$C)</td>
<td>0</td>
<td>0</td>
<td>12.7</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$\Delta T_{junction-surface}$ ($^\circ$C)</td>
<td>14.8</td>
<td>14.8</td>
<td>14.8</td>
<td>27.0</td>
<td>27.0</td>
<td>27.0</td>
</tr>
<tr>
<td>$\Delta T_{ambient-junction}$ ($^\circ$C)</td>
<td>14.8</td>
<td>14.8</td>
<td>27.5</td>
<td>27.0</td>
<td>27.0</td>
<td>27.0</td>
</tr>
<tr>
<td>$RH_{surface}$ (%)</td>
<td>40.9</td>
<td>48.0</td>
<td>22.5</td>
<td>85</td>
<td>85</td>
<td>60</td>
</tr>
</tbody>
</table>

Table 5.3: Relevant temperature and relative humidity ($RH$) for the different experiments. Values in boldface are measured, the others are inferred. SR1 has no forced airflow, therefore in model II, $\Delta T_{surface-ambient} \neq 0$.

### 5.6 Fit to MTTF data in the Herrick acceleration model

The Herrick acceleration model [86] predicts that the MTTF of a sample is given by

$$\text{MTTF}(T, RH) = k \times \exp \left( \frac{E_a}{k_B T_{junction}} \right) \times \exp (-a_{RH} RH_{surface}),$$

where $k$ is an overall scaling factor, $E_a$ denotes the activation energy, $k_B$ the Boltzmann constant, $T_{junction}$ is the relevant temperature for the aging process in Kelvin, $RH_{surface}$ the relative humidity at the device surface in % and $a_{RH}$ the corresponding humidity acceleration factor. The temperature dependence is an Arrhenius-type equation, while the $RH$ term is empirical and based on failure data of unprotected oxide VCSELs [86]. Based on experi-
mental data [87], a current dependent acceleration can be postulated, where the extra factor has the form \((I_{\text{reference}}/I_{\text{device}})^2\), with larger currents leading to smaller lifetimes. However, Herrick argues in the same paper [86] that the current-dependent acceleration is relevant in the case of dry environments only, in which the failure mechanism cannot be humidity-related. In experiments with humid environments, he finds that the current-dependence of MTTFs is very weak and that it can be attributed to local heating effects.

The function \(\text{MTTF}(T_{\text{junction}}, RH_{\text{surface}})\) from eq. 5.8 was fitted separately to the MTTFs extracted from the accelerated lifetime tests only, and then to the MTTFs from these tests and from USA15 operation (as shown in table 5.2). The procedure was repeated for the two sets of \(T_{\text{junction}}, RH_{\text{surface}}\) resulting from the different local heating models presented in table 5.3 This gives a set of 4 fits. For each fit, a \(\chi^2\) value is calculated as

\[
\chi^2 = \sum \frac{(\text{MTTF}_{\text{measured}} - \text{MTTF}_{\text{expected}})^2}{\sigma^2_{\text{measured}} + \sigma^2_{\text{expected}}},
\]

(5.9)

where the \(\text{MTTF}_{\text{measured}}\) is extracted from the Weibull fit, the \(\text{MTTF}_{\text{expected}}\) is predicted by using the acceleration lifetime model from eq. 5.8 for the appropriate \(T_{\text{junction}}\) and \(RH_{\text{surface}}\). The sum is performed over all accelerated lifetime tests and SCT data. The number of degrees of freedom is due to 5 data points - 3 fit parameters = 2 d.o.f. The results of the fit are summarized in table 5.4.

It becomes apparent that when fitting to ULM data only, the extrapolation given by equation 5.8 to USA15 environmental conditions predicts a MTTF for USA15 devices which is 4 to 6 times higher than observed, with relatively large \(\chi^2\) values. The observation is true regardless which of the two local heating models is assumed. In the case in which the accelerated lifetime model is fitted to both ULM and USA15 data, the prediction for USA15 is, not surprisingly, much closer to the measured values, but the overall \(\chi^2\) values defined in equation 5.9 are even larger. The \(\chi^2\) value can be converted to a \(p\)-value (which is the right-tail probability value for a \(\chi^2\) test). The highest \(p\)-value is 0.06 when the fit is performed to the ULM data only with the model II, however, this level of consistency is achieved only because of the very large errors in the prediction of the USA15 MTTF values.
5.6 Fit to MTTF data in the Herrick acceleration model

<table>
<thead>
<tr>
<th>Data fitted</th>
<th>ULM I</th>
<th>ULM+USA15 I</th>
<th>ULM II</th>
<th>ULM+USA15 II</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_a$ (eV)</td>
<td>0.475 ± 0.048</td>
<td>0.240 ± 0.016</td>
<td>0.528 ± 0.057</td>
<td>0.269 ± 0.018</td>
</tr>
<tr>
<td>$a_{RH}$ (1/%)</td>
<td>0.042 ± 0.012</td>
<td>0.028 ± 0.006</td>
<td>0.024 ± 0.007</td>
<td>0.009 ± 0.002</td>
</tr>
<tr>
<td>$\chi^2$ / d.o.f.</td>
<td>9.1/2</td>
<td>26.4/2</td>
<td>5.5/2</td>
<td>30.7/2</td>
</tr>
<tr>
<td>$p$-value</td>
<td>0.01</td>
<td>&lt; 10$^{-5}$</td>
<td>0.06</td>
<td>&lt; 10$^{-5}$</td>
</tr>
<tr>
<td>Prediction crates 0-3 (10³h)</td>
<td>48.1 ± 18.2</td>
<td>10.5 ± 0.25</td>
<td>69.1 ± 36.3</td>
<td>10.2 ± 0.23</td>
</tr>
<tr>
<td>Prediction crates 4-7 (10³h)</td>
<td>45.9 ± 16.3</td>
<td>10.04 ± 0.22</td>
<td>67.8 ± 33.8</td>
<td>10.3 ± 0.22</td>
</tr>
</tbody>
</table>

Table 5.4: Result of fits of the lifetime acceleration model to the MTTF data. The fit is performed to ULM data only or ULM+USA15 data, and the relevant $RH_{surface}$ and $T_{junction}$ used in the fit are obtained in two different models of local heating. In model I, the forced airflow is assumed not to be able to carry away all the dissipated heat, whereas model II assumes an efficient airflow which constrains $T_{ambient} = T_{surface}$. The $RH_{surface}$, $T_{junction}$ are calculated according to table 5.3.

The predictive power of this fit result for USA15 conditions is questionable. All other fit results with a higher predictive power for the USA15 conditions have a $p$-value smaller than 0.01.

I conclude that both for the case of a fit to ULM data only, and in the case of a fit to ULM and USA15 data, the hypothesis of the ULM and USA15 datasets being consistent under the Herrick acceleration model is excluded at 90% CL.

### 5.6.1 Alternative model for $AH$ acceleration

Alternative models suggested in the literature correlate failure rates with absolute rather than relative humidity. Another ULM test not documented in this work was performed at 85C/29% $RH$, corresponding to the same $AH$ as the 60C/85% $RH$ test. Due to an unexpected failure pattern in the 85C/29% $RH$ devices, their MTTF was not used in the acceleration model fit, but it can nevertheless be said with certainty that the MTTF is higher than the one extracted from the 60C/85% $RH$ test. If the acceleration only depended on the temperature and the $AH$, the $AH$ being the same, the 85C/29% $RH$ devices would be expected to fail sooner than the 60C/85% $RH$ devices due to the higher temperature, which
is the opposite of what was observed, suggesting that the failure rates correlate with $RH$ rather than $AH$. This might be related to the fact that in an environment with high $RH$, it is harder for any water on the device surface to evaporate.

### 5.6.2 Alternative model for current acceleration

In this subsection, I show that that the MTTFs observed for devices operated at different currents can be attributed to local heating, and do not require the introduction of a further current-dependent term in the acceleration model. In a similar fashion as the values in table 5.3 were obtained, one can determine surface $RH$ and junction temperature values for the two 85C/85% ULM tests with “bare” devices that were not coated with epoxy. The two tests were operated at 5 mA and 10 mA DC current. It is assumed that in equation 5.8, the terms depending on $T_{\text{junction}}$ and $RH_{\text{surface}}$ characterise the corrosion process close to the oxide tip of the VCSEL, which is the same regardless of the coating. Therefore the $a_{RH}$ and $E_a$ derived from the fit to MTTF of populations of coated devices are applicable to bare devices. On the other hand, the constant term $k$ is linked to the thermal conductivity and the permeability to humidity of the various VCSEL layers and will be different for VCSELs with and without epoxy coating. While an absolute prediction for the MTTF of a population of bare devices is not possible due to the undetermined $k$ parameter, the ratio of MTTFs (i.e. the acceleration factor, AF) for the two populations of bare devices only depends on $a_{RH}$ and $E_a$ and can be predicted from the acceleration model and the existing fits. Table 5.5 shows the different acceleration factors predicted by each model of local heating. The predicted AF most consistent with the observed value is obtained from the fit to ULM data in model I.

### 5.6.3 Seasonal variation of environmental conditions

The accelerated lifetime tests performed in environmental chambers are guaranteed to have a constant humidity and temperature throughout the period of the test. For USA15 and
5.6 Fit to MTTF data in the Herrick acceleration model

<table>
<thead>
<tr>
<th></th>
<th>AF (MTTF&lt;sub&gt;5mA, DC&lt;/sub&gt; / MTTF&lt;sub&gt;10mA, DC&lt;/sub&gt;)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ULM fit only, model I</td>
<td>1.14</td>
</tr>
<tr>
<td>ULM+USA15 fit, model I</td>
<td>0.96</td>
</tr>
<tr>
<td>ULM fit only, model II</td>
<td>1.85</td>
</tr>
<tr>
<td>ULM+USA15 fit, model II</td>
<td>1.37</td>
</tr>
</tbody>
</table>

Table 5.5: Acceleration factors (AF) achievable using the parameters extracted from fits to different models and different data. The 5mA ULM test with bare VCSELs had a MTTF of 346±16 hours, the 10mA test a MTTF of 297±15.5, giving an AF of 1.165±0.095.

SR1 devices, this is not the case. The relative humidity in USA15 as measured by devices close to the rack fans in which the VCSELs are mounted recorded a variation of up to 10% depending on season. A rescaling procedure was applied to the USA15 data to assess the effect on the MTTF. The extracted MTTF is corrected to the MTTF that would have been obtained, had the environmental conditions been the same throughout. Correcting simply to the mean RH throughout the period would not be correct, because days at high humidity produce exponentially more damage than days with low humidity (this follows from the model of humidity acceleration whereby the acceleration factor is proportional to exp(a<sub>RH</sub> × RH)). A weighted mean (effective RH) can be calculated instead:

\[
RH_{\text{effective}} = \sum_{\text{day } i} \frac{RH_i \times \exp(a_{RH}RH_i) \times N_{\text{modules}}}{\exp(a_{RH}RH_i) \times N_{\text{modules}}}.
\] (5.10)

That is, a weighted average of the daily humidities is calculated by weighing each value by the humidity acceleration factor and the number of modules N<sub>modules</sub> still operating on that day. The daily values are themselves weighted means of more fine measurements. The failure or survival time of each USA15 device is then corrected, by scaling each day by a factor exp(a<sub>RH</sub>(RH<sub>i</sub> − RH<sub>effective</sub>)). Operating at higher RH than the reference RH makes a day count more.

One can use an iterative procedure, starting with an assumed a<sub>RH</sub> = 0, from which the effective RH is found. The failure and survival times in USA15 are rescaled to RH<sub>effective</sub>, and the Weibull fit is performed to extract corrected MTTFs for the USA15 samples, which now depend on the assumed a<sub>RH</sub>. Using the extracted MTTFs, the fit to the acceleration model is performed to obtain a new value for a<sub>RH</sub>. The procedure can be repeated until
the values of $a_{RH}$ and MTTF stabilise. In practice, one can just sample different values for $a_{RH}$ and evaluate the effect on the MTTF. For an extreme value $a_{RH} = 0.5$, the extracted MTTFs are about 20% lower. A 20% effect does not remedy the discrepancy observed between the measured USA15 MTTF and the MTTF predicted from the ULM fit, and in fact the effect enhances the discrepancy. The typical values for $a_{RH}$ extracted from the fit to the acceleration model are one order of magnitude smaller, for which the effect on MTTF is negligible.

5.7 Consistency of SR1 results with other experiments

5.7.1 SR1 - USA15 comparison

From the 4 SR1 VCSEL arrays, 2 of them operated for 227 days and the other 2 for 363 days before installation in SR1. The only recorded death occurred on one of the older plugins, between the 198th and 215th day of operation (the days are counted starting from the day of installation in SR1). In the first part of this subsection, these facts are expressed in mathematical notation. Two categories of channels exist, “old” and “new”, with the “new” channels having $\Delta_2 = 136$ days of operation less than “old” devices. Each category has 2 plugins with a total of $N_{1,2} = 24$ channels. For the “old” category, $n_1 = 1$ death was recorded, for the “new” category, $n_2 = 0$ deaths were recorded. With such low sample size and only one death, it is impossible to constrain meaningfully the MTTF of these devices in a log-likelihood fit.

The null hypothesis in this context is that the SR1 devices are described by the same Weibull distribution as the USA15 devices. Under this hypothesis, given the long running time of the SR1 setup, a considerable number of failures should have been observed, therefore it is an “extreme” result that only one device has failed. One can evaluate the p-value, i.e. the probability of obtaining an experimental result at least as “extreme” as the one actually observed, assuming the null hypothesis is true. Concretely for the situation at hand, it is
the probability \( p_1 \) of any one channel in the “old” category to die at a time equal to or later than the one observed and no deaths in the “new” category, plus the probability \( p_0 \) of no deaths at all. The probability of exactly \( n_1 \) and \( n_2 \) deaths is given by

\[
p(n_1,n_2) = \prod_{\alpha=1,2} \left[ \frac{N_\alpha!}{(N_\alpha - n_\alpha)!} R(T_{\alpha,\text{end}})^{N_\alpha-n_\alpha} \prod_{\text{device failures}} \left( R(T_{\alpha,\text{end}}) - R(t_{\alpha,j}) \right) \right].
\]

(5.11)

The times at which the reliability function \( R(t) \) are evaluated need to take into account the additional time the devices operated for before SR1 installation. \( T_{\alpha,\text{end}} \) is the total time of operation of devices with no failures in category \( \alpha = 1, 2 \) (“new”, “old”) and is given by \( T_{\alpha,\text{end}} = T_{\text{SR1 test length}} + T_{\alpha,\text{previous operation}} \). The times \( t_{\alpha,j} \) are the times when failures were detected and depend on the category \( \alpha \) as they also include the pre-SR1 operation time. The failure could have occurred any time between \( t_{\alpha,j-1} \) and \( t_{\alpha,j} \). From equation (5.11) the special cases of \( p_1, p_0 \) can be calculated. In general, once the experimental facts have been recorded (i.e. the number and time of failed channels, number of total operating channels, length of test) the \( p \)-value will depend on the parameters \( \beta, \eta, \gamma \). Using a conservative approach, the values for \( \beta, \eta, \gamma \) are taken from the Weibull fit in crates 0-3 (which have a longer lifetime) and the \( +1\sigma \) values extracted from the fit are used, giving a \( p \)-value of \( 8.3 \times 10^{-6} \). More devices should have failed in SR1 if their failure times were described by the USA15 parameters.

One might assume that the conditions in SR1 and USA15 are similar, but slightly different, in that there is some constant temperature and humidity difference between the two (as table 5.3 suggests) which will lead to a small \( (O(1)) \) acceleration factor \( AF = \frac{MTTF_{\text{SR1}}}{MTTF_{\text{USA15}}} \). The AF might make the failure rates of the two experiments mutually consistent. Even if it is difficult to assess the AF accurately, a statement can be made about the (in)compatibility of the USA15 and SR1 results, regardless of the exact value of the AF, as long as the AF is of order \( O(1) \). The acceleration model is agnostic to what parameter of the Weibull distribution it is in fact inflating. The fits to the different ULM experiments show that all three Weibull parameters can change, and the acceleration model makes no prediction to which one it affects, but only about the MTTF as a whole. In the following study, the AF is assumed to act linearly on the \( \gamma \) and \( \eta \) parameters, while leaving \( \beta \) constant. The effect
of this is to inflate equally the useful lifetime, as well as the end-of-life period. Alternatively one can inflate the MTTF by the AF by only increasing $\gamma$ or $\eta$ at a time by an appropriately larger factor, while leaving the other component untouched. Repeating the calculation of the $p$-value in eq. 5.11 while varying the AF that affects both $\eta, \gamma$, a $p$-value of 0.05 can be achieved with an AF of 1.36. Table 5.6 reports the maximal AF obtained for different local heating models (yielding different $\Delta T$ and $\Delta RH$ between experiments, as can be extracted from table 5.3) and data fitted, allowing for $E_a$ and $a_{RH}$ to vary within their fit uncertainties. Only if $E_a$ and $a_{RH}$ both deviate by $2\sigma$ in opposite directions from their nominal values can such a high AF be reached. In fact this is still a conservative answer, since errors on $E_a$ and $a_{RH}$ are taken to be independent of each other, even though the fit to the acceleration model constrains them to be correlated.

Table 5.6: Maximum AF achievable between USA15 and SR1 if one is willing to use values for $E_a$ and $a_{RH}$ which are 1, 2 or 3 $\sigma$'s away from the best fit values. The two parameters are varied assuming no correlation between them.

<table>
<thead>
<tr>
<th>Model</th>
<th>Data fitted</th>
<th>nominal AF</th>
<th>1$\sigma$</th>
<th>2$\sigma$</th>
<th>3$\sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>ULM</td>
<td>0.87</td>
<td>1.12</td>
<td>1.36</td>
<td>1.64</td>
</tr>
<tr>
<td>1</td>
<td>ULM+USA15</td>
<td>1.00</td>
<td>1.09</td>
<td>1.19</td>
<td>1.29</td>
</tr>
<tr>
<td>2</td>
<td>ULM</td>
<td>0.65</td>
<td>0.75</td>
<td>0.87</td>
<td>1.02</td>
</tr>
<tr>
<td>2</td>
<td>ULM+USA15</td>
<td>0.77</td>
<td>0.81</td>
<td>0.85</td>
<td>0.88</td>
</tr>
</tbody>
</table>

If correlations between the errors are taken into account, $E_a$ and $a_{RH}$ cannot vary independently anymore. The value of 1.36 is met first time (from all models and all combinations of samples fitted to) on the contour which is 3.1 $\sigma$ away from the best fit values in the fit to ULM data only, assuming model I of local heating. The situation is depicted in figure 5.8. The study was repeated assuming the AF only inflates $\eta$ or $\beta$, with similar conclusions.

5.7.2 ULM - SR1 comparison

USA15 failure rates were found to be 4-6 times higher than expected from the Herrick acceleration model and ULM failure data. Such a predicted long lifetime from the acceleration model is found to be consistent with the time of operation and the one death of SR1 devices. ULM data predicts a lifetime for SR1 devices for operation at 50% duty cycle of $12.8 \pm 2.8$
5.8 Further investigations

Figure 5.8: Contours of equal likelihood in the $a_{RH}, E_a$ plane obtained from a fit of the model in equation 5.8 to MTTFs recorded in ULM experimental setups run at different $T_{junction}$ and $RH_{surface}$ values. The best fit point, as well as 1, 2 and 3 σ confidence level contours are shown. The acceleration factor (AF) of 1.36 necessary to explain the difference in MTTF between the SR1 and USA15 devices is incompatible with the best fit values of $E_a$ and $a_{RH}$.

years under model I and 4.7 ± 2.0 years under model II of local heating. The devices in SR1 were operated between 1.7 (the two newer TXs) and 2.05 years (the two older TXs). Therefore the one failure from 48 channels operating in SR1 is consistent with the predicted lifetimes under all models.

5.8 Further investigations

5.8.1 Microscopic cause of failures

Further investigations were carried out by collaborators to assess the failure reasons of the TrueLight VCSELs. In particular, research was performed to find the microscopic cause of the failures. References [7] and [8] include some of the results presented in the thesis, and further show other studies concluding that the failure mode for the SCT VCSELs is corrosion failure. Transmission electron microscopy performed on the failed VCSELs showed a dense dislocation network in the active layers in the quantum wells. This type of failure has also been observed much earlier, in 2002, for example in [88]. An important hint to the cause of failure of VCSELs is an observation made in [88], namely that the dislocation networks are not observed in failed VCSELs fabricated with a different type of technology, whereby
Further investigations

the optical index guiding and the current confinement is achieved with ion implants rather than an oxide aperture. The authors of the same article already hypothesized in 2002 that the porous oxide might be subject to a corrosive reaction of electrolytic nature, which would be induced by the simultaneous presence of a voltage bias on the VCSEL and high humidity. This is supported by the observation made by ATLAS collaborators that solely storing devices in a high humidity and temperatures environment, without applying a bias voltage, does not damage them. The corrosive reaction at the oxide aperture is a source of point defects, which may form clusters and loops that grow through the DBR mirror towards the active region. The humidity ingress, the corrosive reaction and the growth of dislocations through to the active region are processes that occur on the timescale of months to years. Thermo-mechanical stress might accelerate the defect propagation. Once the dislocations reach the active region, they grow at a much faster rate due to the much higher density of dopants, building dislocation networks, also called “dark line defects”. They absorb photons at a high rate and reduce the gain, with the VCSEL loosing its ability to lase. The hypothesis that the dislocation network originates at the oxide aperture and is initiated through an electrolytic corrosion process had no conclusive proof for a long time.

The difficulty is that typically, test devices are operated under aging conditions and only checked upon every few hundred hours for failures (e.g. the ULM tests). Especially at high temperature and humidity, the failure of the device is catastrophic with the growth of the dark line defects developing within tens of minutes. If the damage is too extensive, the origin of the dislocation network cannot be tracked down conclusively in electron microscope pictures. Therefore devices need to be caught at the very start of the damage process, which most accelerated lifetime tests are not suited for. ATLAS has worked together with one of the authors of [88] to pin down exactly the microscopic failure mechanism of VCSELs. Following this collaboration, the difficulties of identifying irrefutably the oxide layer as the source of defects described above have recently been overcome in [8] by aging devices operated with a very low current and with more continuous monitoring of the output optical power. A semi-deteriorated, still lasing device was identified, which showed clearly in scanning and transmission electron microscope pictures that defects originating at the tip of the oxide
5.8 Further investigations

aperature gives rise to line dislocations propagating through to the GaAs active region.

A VCSEL diagnostic was established based on the SR1 observations. VCSELs operated at room temperature and humidity were observed to exhibit spectral narrowing within a year of operation. Having established above that humidity operation induces cracking and corrosion at the oxide tip, it is believed that these create larger index discontinuities inside the VCSEL, and therefore a stronger index guide than in the original design. Together with losses from both scattering and absorption around the aperture, this causes the higher order modes to be lost. Spectral narrowing can be used as a device diagnostic before it fails. No spectral narrowing was observed in nitrogen flushed devices in SR1, supporting the claim that these devices are not damaged during operation.

5.8.2 USA15 versus SR1 failure rates

It is still not entirely understood why the USA15 failure rates are inconsistent with the ULM data or the SR1 tests. A possible explanation for the discrepancy between ULM data and USA15 data is that the protective epoxy is very hot and in liquid form when it is applied on the opto-board. It then cools down and transitions to a glass-like state, putting the devices under mechanical pressure. The different expansion coefficients of the two layers might not induce significant stress at the high ULM operating temperatures, which are closer to the temperature of the epoxy liquid state, but could do so at the lower USA15 temperature. From this one might expect MTTFs for samples operating at high temperatures to be consistent between each other and with the acceleration model, and samples operating at lower temperatures to have lower MTTFs than predicted by the acceleration model. Another difference between the setups which might have played a role was the power cycling of the devices in USA15. In both ULM tests and SR1 tests, the devices were powered continuously. In USA15, the devices are typically powered on and off. Shortly after their installation, this happened every few days, later only every few months. This would give rise to additional thermo-mechanical stress, which might have yielded the USA15 VCSELs more susceptible to humidity. Experimentally, having reached the conclusion that the SR1
5.8 Further investigations

VCSELs had a longer lifetime than the USA15 devices, their mode of operation was changed such that the power was turned off once per hour for 5 minutes, with the aim of understanding the effect of repeated self-heating and cooling of the units. One additional channel failed within 3 days, with no further failures in the next few weeks. The tests were suspended, as no conclusive statement could be made from this result.

5.8.3 Mitigation strategies

Commercial alternatives to TrueLight VCSELs

The many failures in USA15 prompted the collaboration to look for commercial alternatives for the TrueLight devices. A large replacement campaign at the end of 2011 and beginning of 2012 has led the TrueLight devices to be substituted with devices manufactured by “Advanced Optical Components” (AOC). The new devices are also oxide VCSELs, but coated with a dielectric layer, which provides better protection against humidity than the epoxy coating of the TrueLight VCSELs. Like the TrueLight arrays, the AOC arrays were also sealed with an epoxy coating. Several infant and random failures were observed in the AOC population, at a much lower rate than for TrueLight devices. A small sub-population of AOC devices might have had pinholes in the dielectric, or incomplete coverage. Thermo-mechanical stress is again believed to have played a role. The different layers expand and contract at different rates as the VCSEL heats up and cools down, with calculations performed by collaborators showing significant mechanical stress being induced in the process. The stress might cause micro-cracks that can grow in time, leading to device failure. Most stress is expected to occur in the middle channel of the array, which is indeed found to be the one that fails most often in the case of AOC VCSELs.

USA15 humidity reduction

Suspecting humidity to be the cause of failures, humidity extractors were installed in USA15, reducing the relative humidity from 40-50% to less than 20%.
5.9 Summary

5.8.4 Importance for other detectors

It is interesting to note that devices of the same type, from the same manufacturer, also operate in the pixel detector and transmit data to the counting room. The on-detector environment is quite different in that the pixel detector volume is continuously flushed with dry nitrogen to prevent condensation. Throughout operation, only a very small number of random failures were recorded, which is consistent with the hypothesis of humidity being the main culprit for USA15 failures. TrueLight VCSELs also operated for the pixel system off-detector in USA15 and experienced a similar failure rate as the SCT VCSELs.

More generally, understanding vulnerabilities of oxide aperture VCSELs is important, since this type of devices from other manufacturers are employed in several other sub-detectors of ATLAS, CMS and LHCb.

5.9 Summary

This chapter developed and validated the statistical tools needed to investigate failures of SCT VCSEL populations. These tools are then applied to the set of failing VCSELs operating off-detector to send the TTC signal to the SCT modules, as well as to analyze results of accelerated lifetime and environmental tests. The SCT VCSELs are found to have a failure rate several times higher than that predicted by the Herrick model and the data from accelerated lifetime tests. In a long-running test under what is believed to be the same environmental conditions as USA15, VCSELs show a much longer lifetime with almost no failures. The results of this test are inconsistent with the USA15 failure rate. While the mechanism of failure of VCSELs has been clearly identified in further studies by collaborators to be corrosion-based due to the presence of atmospheric water vapour, it is still unclear which were the facilitating environmental or operational conditions in USA15 that sped up the intake of humidity and / or the corrosion process. The SCT operability was improved by replacing the entire population of VCSELs driving the TTC links with dielectric-coated devices, which promise to be more robust against humidity, and by reducing the relative humidity in USA15 through the installation of a humidity extractor.
Chapter 6

Single Event Upsets in the p-i-n diode of the SCT front-end modules

For the ATLAS physics programme, the reliable operation of the various detector sub-components under the harsh radiation environment is essential. The SCT optical links make no exception from this requirement. Quantitatively, the optical data links are required to have a BER of less than $10^{-9}$, such that transmission errors should account for far less data loss compared to the inefficiency of the silicon detectors and electronics, which is of the order of 1%.

Intense particle flux from beam collisions can interact with the detector to produce secondary, highly-energetic hadronic particles. The latter can deposit enough energy in the front-end electronics of the SCT to interfere with its normal operation, potentially leading to a non-negligible bit error rate and affecting the data quality. In particular, the $p-i-n$ diode of the modules is the optical component which is most likely to be affected. This chapter reports the first measurement of such single event upset (SEU) events performed in a collider environment and compares it with predictions from beam tests. The SEU rate measurement and understanding the SEU mechanism are important for implementing optimal SEU mitigation strategies and ensuring the high quality of data.

The chapter is organised as follows. Section 6.1 discusses the mechanism of SEU in
6.1 Mechanism in SCT front-end electronics

The optical links of the SemiConductor Tracker [79] are based on GaAs Vertical Cavity Surface Emitting Lasers (VCSEL), and epitaxial silicon $p-i-n$ diodes. Each of the 4088 detector modules is endowed with one opto-package containing two VCSELs for the data links and one $p-i-n$ diode to receive data from the Timing, Trigger and Control (TTC) optical link, as described in more detail in section 4.2. The latter uses BiPhase Mark (BPM) encoding to convolve the trigger and control sequences with the 40 MHz bunch crossing (BC) clock signal. The BPM encoding is exemplified in figure 6.1. The on-detector $p-i-n$ diode converts the BPM encoded optical signal to an electrical signal, which is in turn amplified and deconvolved by the DORIC4A ASIC [89] back to the BC clock and the control signals. The SEU effect investigated in this study pertains to the situation in which high energy particles deposit enough energy in the active region of the $p-i-n$ diode, faking a rising/falling edge structure. Since the $p-i-n$ diode has the largest active region among the opto-electronics of the front-end modules (350 µm diameter and 15 µm height), it is most likely to be affected. It is also the case that the $p-i-n$ diode and the DORIC4A are the sub-system most sensitive to small signals and an energy deposition at a later stage of the signal processing is unlikely to reach the amplitude of the amplified signal. A SEU can occur in the case in which a hadronic particle deposits extra energy in the $p-i-n$ diode, concomitant with the arrival of a logical 0 at the diode, BPM-encoded to 00. If the energy deposition in the diode gives rise to a current larger than the threshold of the DORIC4A, the chip will interpret the signal as an edge and therefore as a logical 1. It is far less likely for the energy deposition to interfere with the correct interpretation of a bit which is BPM-encoded as 01, 10 or 11. In the first two cases, an edge transition is already present and the
6.2 Beam tests

Prior to their installation in the detector, the SCT optical links were tested extensively for SEU effects in beam experiments, a report of which can be found in [82]. Minimally ionizing particles with kinetic energy less than 0.55 MeV, neutrons of a few MeV and protons and pions between 300 and 405 MeV were used to irradiate the components during operation. The setup was such that the particle flux was perpendicular to the $p-i-n$ diode. SEUs

![Figure 6.1: Principle of BiPhase mark (BPM) encoding. The $y$-axis is, for example, intensity of light received by the $p-i-n$ diode, versus time on the $x$-axis. In the BPM-encoded signal (bottom row), no transition in a 25 ns window (BPM-encoded signal is 00 or 11) is decoded by the DORIC4A chip to a data bit with value 0, while a transition (BPM-encoded signal is 01 or 10) is decoded to a data bit with value 1. As a transition occurs at least every 25 ns in the BPM-encoded signal, the DORIC4A chip is able to recover both the 40 MHz clock (shown in the top row) and the original data (middle row).](image-url)
were not observed when no particle flux was present or when the setup was irradiated with minimally ionising particles. All SEUs registered were due to the proton and pion flux.

Pseudo-random number sequences, 32k deep, were sent to the $p-i-n$ diode, received, decoded by the DORIC4A, encoded and sent back. The received signal was compared to a suitably delayed input signal and any bit difference was counted as an error. The bit error rate was converted in a SEU cross-section $\sigma_{SEU}$,

$$\sigma_{SEU} = \frac{N_{\text{ERROR}}}{\text{flux} \times t}. \quad (6.1)$$

The cross-section was measured for various settings of the mean $I_{PIN}$ values, and the expected sharp decrease in cross-section with decreasing $I_{PIN}$ was observed. The measured values are used later in this chapter to make predictions for the SEU rate in the operation of the SCT. The $\sigma_{SEU}$ measured in beam tests does not cover the full range of $I_{PIN}$ values present in SCT operation. Figure 6.2 shows the measured $\sigma_{SEU}$ and the interpolations used in the prediction. $\sigma_{SEU}$ is interpolated using the measured cross-sections for fluxes of protons and pions with kinetic energies of 350, 405, and 465 MeV. In the first approach, $\sigma_{SEU}$ is estimated by piecewise linear interpolation and the $\sigma_{SEU}$ for any $I_{PIN}$ value larger than the highest measured $I_{PIN,\text{max}}$ is taken as $\sigma_{SEU}(I_{PIN,\text{max}})$. In the second approach, a functional fit is performed, where the function is chosen on an empirical basis. In the beam tests, a set of measurements was also performed using 300 MeV pions, which was observed to give a factor $\sim 5$ higher SEU cross-section. This is understood to be due to the formation of a $\Delta$ resonance. This set of measurements is ignored, based on the assumption that the LHC charged pion fluence at this energy is a small fraction of the overall fluence.

### 6.3 SEU effects in SCT operation

One possible measurable effect of SEU in SCT operation is when a bit error in the TTC link causes a SCT module to fail to recognise a first level trigger accept (L1A) signal. Normally on receipt of the L1A signal, the module sends to the counting room the data corresponding
6.3 SEU effects in SCT operation

Figure 6.2: Dependence of $\sigma_{SEU}$ on $I_{PIN}$. Data shown is taken from [82] and refers to $\sigma_{SEU}$ measured with fluxes of pions and protons at kinetic energies of 350, 405 and 465 MeV. The functional form of $\sigma_{SEU}(I_{PIN})$ is empirical.

The module will fail to retrieve the correct event and the BCID and LVL1ID counters will be out of sync with the rest of the detector. In the RODs, the counters from each module are compared with the full LVL1ID and BCID on an event-by-event basis, and an error flag is raised for the affected module if its counters are out of sync with the rest of the SCT. The module will remain desynchronised and the error flag will be present for all events after the SEU occurrence, until a reset command is issued which resets the BCID and LVL1ID counters and the event pipelines of all modules. The outcome is thus a burst of errors for the affected module, i.e. a particular error flag being set for several time-consecutive events. Several error flags are expected to be raised, in particular the LVL1ID and the BCID error flags. The error flag TIMEOUT might also be raised.

Because of the SEU mechanism detailed above, the SCT is only sensitive to the luminosity recorded by ATLAS. The recorded luminosity reflects the DAQ inefficiency when the central
trigger processor sends no L1A to the SCT, as well as the inefficiency of the so called “warm start”: when the stable beam flag is raised, the tracking detectors undergo a ramp of the high-voltage.

Mitigation strategies in 2011 and 2012 running have included using VCSELs in the RODs which gave a fibre-coupled power of greater than 700 $\mu$W, which resulted in large values of $I_{PIN}$ ($> 100\mu$A). These values are much larger than required for error free operation of the links in the absence of beam, but reduced the SEU rates because of the steeply falling $\sigma_{\text{SEU}}$ as a function of $I_{PIN}$. The second mitigation strategy was to automatically reset the pipelines and all counters in the affected front-end chips if a desynchronisation was detected by the DAQ. The procedure typically took 20 to 50 seconds. In order to assess the fraction of data lost without a mitigation strategy, a no-mitigation scenario is defined as one in which the front-end chip counters are only reset at the start of every run, which is required even in the absence of SEUs. In this alternative scenario, and given the observed rate of SEUs, 1.3% of the data would be lost. By comparison, a negligible fraction of data was actually lost due to the mitigation strategy. Note that this does not reflect the mitigation strategy of using larger values of $I_{PIN}$ than would have been required for a low BER link in the absence of SEUs. As the SEU cross section is a rapidly falling function of $I_{PIN}$, this strategy was also very important in reducing the loss of data from SEUs.

Further mitigation strategies for the high luminosity LHC are planned. The TTC link for the high luminosity LHC will be high-bandwidth and will operate at 4.8 GBit/s instead of the current 40 MBit/s. This will allow for forward error correction to be employed [91], and in particular, the use of the Reed-Solomon algorithm is intended [92]. Forward error correction is a technique used for controlling errors in data transmission by encoding the data in a redundant way. The redundant bits are usually the output of a complex function of many original information bits. This allows the receiver to detect a limited number of errors that may occur anywhere in the transmitted sequence and correct the errors without retransmission.
6.4 SCT SEU measurement

6.4.1 Data sample, prerequisites

Runs recorded in 2012, with run numbers between 200498 and 207532, are used in the analysis. The data were recorded during the period from 30th of March 2012 to 27th July 2012 and amount to a total recorded luminosity of 7.8 fb$^{-1}$. In 2012, the peak luminosity was $\sim 7.5 \times 10^{33}\text{cm}^{-2}\text{s}^{-1}$. For this analysis, the ATLAS SCT NTUPles are used, processed from the express stream. The express stream has a mixed trigger menu, overlapping with all physics streams and samples data at a rate of 10-20 Hz. Its purpose is the ability of quickly detecting any problems in data quality. The events are written to the NTUPles in the order in which they come out of the event builder, which is however not the order of increasing BCID or LVL1ID. In the current analysis, the events need to be ordered by their time stamp first before one can search for error bursts.

6.4.2 Analysis cuts

The following cuts are applied for SEU detection. Error bursts of type BCID are scanned for, regardless of the concomitant presence of any other type of error. Each side of a module has its data link, which in principle can lead to independent error bursts. The two sides share the TTC link in which the SEU may occur. In this analysis, both sides of the module need to present the error flag. Modules from manufacturer CiS have encountered unrelated issues in operation, experiencing high leakage currents and voltage trips. This lead to error bursts unrelated to SEU, therefore the 496 CiS modules situated in the endcaps were excluded from the analysis. Since SEU is expected to be rare, at most one error burst per module is allowed in a whole run. A run corresponds typically to a few hours up to a day of running. A list of “noisy” modules with more than 1 error burst per run is kept for each run. The fact that only one error per run and module is allowed might bias the result slightly if there are indeed modules that are susceptible to SEU to a larger extent than this. The bias due to this cut
6.4 SCT SEU measurement

is investigated in a later section (6.5). The BCID and LVL1ID counters and therefore the related errors are believed to be reset by an automatic module recovery procedure which takes place about 30 seconds after the module starts reporting errors. Therefore error bursts longer than 60 seconds are discarded, which leads to 13% of the error bursts to be rejected. A redundancy system exists, such that in the case that the TTC optical link of one module is not operational, an electrical path is set up to a neighboring module and its TTC signal is used instead. If SEU occurs in the neighboring module, the module using redundancy will also report the error flag at the same time. To avoid double counting, any error burst coming from a module with a dead TTC link, where the $I_{\text{PIN}} < 0.04$ mA, is ignored. Concomitant errors in different modules can also be due to a whole ROD going busy and being recovered. The cause of the ROD going busy is unrelated to SEU. To avoid such situations, any pair of two error bursts in different modules starting closer than 10 seconds apart is discarded. From an operational point of view, such ROD busy cases typically last longer than one minute, so not many are expected to pass the time-length cut already discussed. In total 157 such pairs are identified and excluded (with some SEU candidates contributing to more than one pair).

6.4.3 Analysis results for barrel SEUs

The SEU analysis was performed on the barrel and endcap modules separately. The barrel results are easier to interpret due to the simpler geometry and they are presented first. After the above cuts, 2504 SEU events are identified in the barrel modules. The number of error bursts in a run is expected to scale linearly with the fluence and therefore with the integrated luminosity in the run. For the set of SEU bursts that pass the cleaning cuts, this is indeed the case, as can be observed in figure 6.3. The same is true for the SEU rate versus the instantaneous luminosity. In particular, several runs with no beams, but with a non-zero L1A count (for example from cosmic triggers), show no SEU. Also, runs in figure 6.3 with the same integrated luminosity can have different time lengths. The number of SEU in a run is found to scale with the integrated luminosity rather than the time length of the run.
This observation is compatible with SEU, but not necessarily conclusive.

![Figure 6.3: Number of SEU candidates in each run versus the integrated luminosity of the run (fb\(^{-1}\)). Each point represents an analysed run. The total integrated recorded luminosity for all analysed runs is \( \int L\,dt = 7.8 \text{ fb}\(^{-1}\) \), in which 2504 SEU candidates were observed.]

More convincing evidence comes from the observation that the number of SEU in each module scales linearly with the cluster occupancy of that module. Each cluster on an SCT module is formed due to the passage of a charged particle, therefore the cluster occupancy is a good proxy for particle flux, as the rate of noise hits is negligible. Figure 6.4 shows the average number of SEU per module, versus event cluster occupancy. It is obtained in the following way. First, a histogram is filled for each SEU with the average event cluster occupancy of the affected module for the run in which the SEU occurred. Thus each SEU contributes to the first histogram once. Then, a second histogram is filled for each module and each run with the average event cluster occupancy of that module in that run and the entry is assigned a weight equal to the total number of L1As in the run. Thirdly, figure 6.4 is obtained by dividing bin-by-bin the first histogram by the second histogram, thus averaging the number of SEU in a particular occupancy bin by all instances in which an SEU could have occurred for modules in that occupancy bin. This plot effectively uses the spread in occupancy over the different barrel layers as well as the variations in occupancy that arises from different runs having different instantaneous luminosities. The plot shows the expected linear correlation between SEU rate and particle flux. It was also checked that in the absence
of beam there were no SEU candidates when reading out the detector at high rate.

Figure 6.4: SEU rate per module versus module event cluster occupancy. The line shows the result of a linear fit through the origin with a gradient of $2.95 \times 10^{-8} \text{ SEU \cdot mm}^2 \text{ module} \cdot \text{L1A} \cdot \text{cluster}$.

One of the main findings of [82] is that as the $I_{PIN}$ of the module decreases, $\sigma_{SEU}$ rises. In figure 6.5 it can be seen that the SCT modules with SEU (black curve) indeed have a lower $I_{PIN}$ compared to the whole sample (blue curve). The same figure shows how reweighting the $I_{PIN}$ distribution of all modules by $\sigma_{SEU}(I_{PIN})$ is in good agreement with the distribution of $I_{PIN}$ of modules with SEU candidates. These findings are also summarised in table 6.1.

<table>
<thead>
<tr>
<th>Observable</th>
<th>Measured</th>
<th>Pred. A</th>
<th>Pred. B</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\langle I_{PIN} \rangle$, modules with SEU [mA]</td>
<td>0.238</td>
<td>0.223</td>
<td>0.213</td>
</tr>
<tr>
<td>$\langle I_{PIN} \rangle$, all modules [mA]</td>
<td>0.278</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 6.1: Total number of observed and predicted barrel SEUs. Measurements and predictions correspond to 7.8 fb$^{-1}$ of 8 TeV data. Predictions A/B employ a functional fit / piecewise linear approximation to $\sigma_{SEU}$ measured in beam tests and are described in detail in section 6.5.

Had the SEU been produced by continuous ionisation, a dependence $\sigma_{SEU} \propto 1/\cos \phi$ of the SEU cross-section with incidence angle $\phi$ would have been expected. However, the proposed mechanism for SEU in the $p-i-n$ diode is that a hadronic interaction may occur in the vicinity of the diode and give rise to a hadronic shower.\footnote{The interaction may occur in the $p-i-n$ diode itself, though the probability for this is negligible due to the small volume of the diode.} The $p-i-n$ diode
acts as a micro-calorimeter, collecting part of the energy of the shower. In this case, the energy deposited in the diode and thus the SEU cross-section is proportional to its volume, which is independent of the incidence angle. The SEU cross-section $\sigma_{SEU}$ is indeed observed to be independent of the incidence angle $\phi$ (fig. 6.6), which reinforces the proposed SEU mechanism in the $p-i-n$ diode. The result holds when the analysis is performed for the whole barrel or by barrel layer.

A missed L1A can possibly also give rise to a time-out error. This possibility has been investigated by identifying TIMEOUT error bursts with the same algorithm and cleaning cuts in data corresponding to $\int L dt = 5.35 \text{ fb}^{-1}$ at $\sqrt{s} = 7 \text{ TeV}$ data. 750 SEU candidates were found. It was observed that TIMEOUT error bursts correlate neither with occupancy, nor integrated luminosity. It can be concluded that the main cause of TIMEOUT error bursts is not SEU.
Figure 6.6: SEU rates normalized to module occupancy as a function of incidence angle for the four barrel layers.
6.4.4 Analysis results for endcaps SEUs

The results presented in the previous subsection pertaining to barrel SEUs were published in [6]. The analysis was repeated for the endcap modules and further 1517 SEU candidates were observed. Contrary to expectations, the distribution equivalent to figure 6.4 of the SEU rate versus occupancy for all endcap modules did not show the expected linear dependence. In order to investigate this further, the endcap modules were divided in three classes, corresponding to the “outer”, “middle” and “inner” module rings, depending on their distance from the beampipe (each endcap disk consists of up to three module rings). There are 936 outer, 640 middle and 400 inner modules in total in the two endcaps. The $\sigma_{SEU}$-weighted $I_{PIN}$ distribution for each class of modules agrees well with the $I_{PIN}$ distribution of the modules with SEUs in that class, as can be seen on the left hand side of figure 6.7.

The observed SEU rate versus module occupancy follows a linear trend in the outer, middle and inner modules individually, but with different slopes (RHS of fig. 6.7). The interpretation of this finding is that the energy spectra of the particles might be different in different detector regions. It was found in the beam tests that the SEU cross-section can vary with energy (for example the 300 MeV pions have a much higher $\sigma_{SEU}$ compared to 350 MeV pions due to the formation of a $\Delta$ resonance), therefore different detector regions can experience different SEU rates for the same occupancy.

6.5 SEU prediction for the SCT

In order to gain confidence in the SCT SEU measurements, $\sigma_{SEU}$ data from beam tests measurements was used to predict the number of SEU expected in the SCT for the runs investigated. An error burst can be observed in operation whenever the L1A trigger signal is misinterpreted due to SEU. Therefore the predicted number of SEUs in a run will depend on the probability to miss a L1A word and the number of events passing the level 1 trigger in that run. The probability to miss the L1A word will depend on the flux through the module, on $\sigma_{SEU}$ and therefore on the $I_{PIN}$, and on the way the L1A word is encoded.
Figure 6.7: From top to bottom: distributions for endcap outer, middle and inner modules. 
Left: Number of BCID SEU candidates in the endcaps versus module $I_{PIN}$ (black). The $I_{PIN}$ distribution for all modules (blue) is weighted by $\sigma_{SEU}(I_{PIN})$ from figure 6.3 giving rise to the red (functional fit to $\sigma_{SEU}$) and orange (piecewise linear interpolation) curves. All histograms are normalised to the total number of SEU candidates observed. The weighted histograms agree well with the distribution of $I_{PIN}$ for modules with SEU (black). Right: SEU rate per module versus module event cluster occupancy. The linear fits through the origin have gradients $1.55 \times 10^{-8}$, $0.82 \times 10^{-8}$, $1.37 \times 10^{-8}$ SEU·mm$^{-2}$ module·L1A·cluster$^{-1}$. 
A possible mechanism by which a module can miss a L1A signal has been described in section 6.1. It is worth investigating if other bit flips occurring at other times and induced by the same mechanism can also create SEU and lead to BCID errors. For example, most of the time, the ABCD3TA is listening to logical 0s. If only one bit flips in this sequence, it will not look like L1A or a control sequence and will be ignored by the ASIC. In principle SEUs can incur multiple bit flips, but at the large values of \( \langle I_{PIN} \rangle \) used, this is a negligible effect. A valid concern however is the bit structure of control commands sent to the ABCD3TA, which have the pre-amble 101, preceded by 0s. If the immediately preceding 0 flips, the sequence will look like the L1A signal. The control commands are however not very frequent compared to the level 1 trigger rate, and this effect is negligible. SEU occurring when other types of TTC data are transmitted might compromise the control commands, but are unlikely to give rise to LVL1ID and BCID errors.

Therefore it can safely be assumed that the number of BCID error bursts satisfying the cleaning cuts is equal to the number of missed L1A. The predicted number of missed L1A signals for any given module in any given run is given by:

\[
\text{predicted \# missed L1A} = Pr_{\text{miss L1A}} \times \#\text{L1A (run)}
\]

\[
= k \times \frac{\text{Average Event Cluster Occupancy (module, run)}}{A_{\text{module}}} \times \sigma_{\text{SEU}}^{\text{predicted}}(I_{PIN, \text{module}}, \text{run}) \times \#\text{L1A (run)}
\]

(6.2)

where \( Pr_{\text{miss L1A}} \) is the probability to miss a L1A; “\#L1A” is the number of L1A counts in a run; \( k \) is a factor correcting for the overlap between the time windows in which the particle flux is received by the module and the time windows in which the DORIC4A ASIC is sensitive to SEU effects in the diode, assumed to be \( = 1 \) for the prediction and discussed in detail in subsection 6.5.2. \( A_{\text{module}} \) is the active area of the module; \( \sigma_{\text{SEU}}^{\text{predicted}}(I_{PIN, \text{module}}, \text{run}) \) is the SEU cross-section predicted by either of the fits in figure 6.2. The average fluence
per event for any module is taken from the average event cluster occupancy of that module and is computed as follows. For each run, events which are recorded during stable beams and have passed event filter triggers are identified in the SCT NTUPles. A subset of these events were recorded because they have passed calibration triggers, even though no collisions were recorded at that time. In order not to bias the cluster occupancy calculation, events with very low cluster occupancy consistent with coming from noise are excluded from the calculation. For the remaining events, the total number of clusters collected in a module are divided by the known area of each SCT module (e.g. 8003 mm$^2$ for a barrel module) and by the number of events to obtain the average occupancy. The number of L1A signals is then retrieved for each run. $\sigma_{SEU}^{predicted}$ is evaluated at the $I_{PIN}$ of the module for which the prediction is made, either by using a piecewise linear interpolation to the measured values in beam tests, or by using the fitted function. The $I_{PIN}$ used is that measured for that module on the day of the run for which the prediction is made.

Predictions are made based on equation 6.2 for each module in each run. For each run, modules identified as “noisy” in data, CiS modules or modules with non-operational TTC link ($I_{PIN} < 0.04$ mA) are excluded from the prediction, since they were also excluded from the SEU measurement (see section 6.4.2). The predictions are first summed over all runs, and then averaged over all modules in a barrel layer or endcap disk. The total SEU prediction for each barrel layer or endcap disk is then compared to measurements in figures 6.8. For the barrel, good agreement is obtained both in the overall number of SEU (table 6.1) as well as the distribution of SEU by barrel layer. In the endcaps, the agreement is reasonable.

### 6.5.1 Systematic uncertainties

Several factors can affect the SEU rate prediction. The SCT radiation environment and its operation are rather different from those in the beam tests, given that the bunch structure of the beams, the particle flux, and the composition and energy spectra of the radiation affecting the modules are different. For example, complications arise from the fact that the $\sigma_{SEU}$ measurements as a function of $I_{PIN}$ in beam tests is rather coarse in certain ranges of
Figure 6.8: Average number of BCID SEU candidates per module versus barrel layer or endcap disk. The average is performed over all modules in a layer or disk. The prediction has an absolute normalization obtained by setting $k = 1$ in equation 6.2.

$I_{PIN}$ compared to the values of $I_{PIN}$ used in the SCT, giving rise to systematic uncertainties in the prediction. As an indication of the systematic uncertainties related to this, the prediction was performed using a linear interpolation of the cross-section data, as well as a functional fit (table 6.1).

The effect of only allowing at most 1 SEU error burst per run can be investigated by slightly altering the prediction algorithm. It is observed that a few modules do have a prediction of more than 1 SEU for very long runs with many L1A signals, so understanding the bias of this cut is important. The average expected number $\lambda$ of SEU bursts for a given run and module can be split as

$$\lambda = \sum_{j=0}^{\infty} j \times \frac{\lambda^j \exp(-\lambda)}{j!},$$

(6.3)

i.e. the prediction $\lambda$ can be understood as a weighted average of the prediction being 0, 1, 2, ..., SEU bursts for the given run, where the different outcomes are weighted by their Poisson probabilities. If one enforces the constraint of at most 1 SEU/run, only the term $1 \times \lambda \exp(-\lambda)$ contributes to the prediction. When summed over the whole SCT, the cut-off prediction is less than 6% lower than the full prediction, and the difference does not materially affect the distributions shown in this section. The prediction of total number of SEUs in table 6.1 and the distributions in figure 6.8 are cut-off predictions.
The next subsection deals with a further systematic effect which impacts the overall normalisation of the prediction.

### 6.5.2 Overall normalization of the prediction

The overall normalisation of the prediction can be affected by differences in the beam structure of the beam tests and the LHC, which lead to differences in the overlap between the time of arrival of the optical signal and the time of arrival of the particles. Consequently, the $p - i - n$ diode might be sensitive to only a small fraction of the received particle flux. Only logical 0’s BPM-encoded to 00 can be affected by SEU, if the fluence is received concomitantly with the signal. Some of the bits received cannot be affected by SEU, since a bit is transmitted every 25 ns, but the LHC bunch crossing is 50 ns, while the beam tests bunch crossing is 20 ns. Both the measurement in the SCT as well as that in beam test measurements are subject to this effect, and the factor $k$ in equation [6.5.2] is meant to correct for this. For the prediction the value $k = 1$ was used, and I present an argument in this section that this is the correct order of magnitude. I discuss in turn when the particle flux is received at the $p - i - n$ diode relative to the optical signal in the case of the beam tests and the SCT.

**SEU in the SCT**

The L1A trigger signal (in red) has the logical form ..00110.. and is preceded by logical 0s. A logical bit is received every 25 ns. The LHC had in 2011 and 2012 a 50 ns bunch structure, which means that only every second bit of received data can in principle be affected.

In general,

$$Pr_{\text{miss \, L1A}} = \sum_{(\text{pre-})\text{L1A word bits}} Pr_{0 \rightarrow 1} = k \rho,$$

where $p$ is defined as $p = \sigma_{\text{SEU, true}} \times \text{fluence}$ to be the probability of a bit flip $0 \rightarrow 1$, if there is particle flux in the time bin in which the bit is received and if the bit has the BPM-resolved structure 00. It will depend on $\langle I_{PIN} \rangle$ and on the particle flux. The sum is over the three
Table 6.2: The L1A bit structure and the LHC bunch structure. Each “x” depicts the flux from the primary interaction. Given the typical length of LHC bunches of $\gtrsim 7$ cm, it is expected the fluence to be concentrated in a 0.25 ns window within the larger 25 ns window in which a bit is received. Only the transmission of bits depicted in red can be affected by SEU effects as they are the only instances of bits BPM-encoded to 00 that are transmitted at the same time as the particle flux is received at the module. As discussed in section 6.1, $01 \rightarrow 11$, $10 \rightarrow 11$ and $11 \rightarrow$ anything transitions are not possible from SEU. On average, the $P_{\text{miss L1A}}$ is $3/4p$.

bits of the L1A and the two preceding ones. The probability of a bit flip is 0 if there is no fluence or if the BPM-resolved structure is any other than 00. The constant $\kappa$ is obtained as an average over the two distinct phases of the BPM encoding, and the relative phases of the collisions versus optical signal data. The different cases of overlap between the L1A signal and the LHC bunch structure are presented in table 6.2. It is not clear whether only the first two, the last two, or all rows represent configurations actually present in data taking and therefore contribute to the calculation of $\kappa$. If it is assumed that all do, $\kappa$ is 3/4, but values as low as 1/4 can be obtained, if only the last two rows contribute.

The working assumption of equation 6.5 is that a bit flip occurs due to the fluence accumulated in a 25ns time window, and that the DORIC4A is sensitive to a deposition in energy in the $p-i-n$ diode anytime within that time window. In reality, in order to minimise clock jitter effects, the DORIC4A was designed to only be sensitive to edges that occur within a smaller time-window of 12.5 ns, in the middle of the 25 ns bin. In the SCT case, the fluence of particles is only present for the time it takes an LHC bunch to pass the interaction point, i.e. $< 0.25$ ns, and must coincide with the sensitivity time-window. If the two do not coincide in one clock cycle, they will not coincide in any following clock cycles, since the bunch spacing is an exact multiple of the clock period with which TTC data is
sent, and no SEU at all would be observed.

### SEU in beam tests

The probability of a SEU-induced bit flip is trivially 0 if there is no particle flux. It is also 0 if the BPM-resolved bits that are transmitted are not 00. Therefore the measured $\sigma_{\text{SEU}}$ in the random data in beam tests is only an effective cross-section, since in fact only a quarter of the time are logical bits sent which are BPM-encoded to 00. Based on this alone, $\sigma_{\text{SEU, true}} = 4 \times \sigma_{\text{SEU}}$, where $\sigma_{\text{SEU}}$ is the measurement result quoted in [82] and shown in figure 6.2. Since $p = \sigma_{\text{SEU, true}} \times \text{fluence}$ it must hold that $p = 4 \times \sigma_{\text{SEU}} \times \text{fluence}$. The fluence refers to that received within the 25 ns bin in which the bit is received by the $p_{\text{in}}$ diode. Similar to the discussion leading to table 6.2, the overlap between the particle flux and the DORIC4A sensitivity window is essential. In the case of the beam tests, the bunch spacing was 20 ns, and the bunches were very short compared to the bunch spacing, such that the particle flux was delivered within less than 1 ns. Even if the particle flux and the sensitivity window coincide in one clock cycle, they will not necessarily be synchronised in the following ones. The overall effect is that the effective fluence is only a fraction of the delivered fluence, which will depend on the exact phase difference between the particle flux and the data clock. It can be shown that this fraction is $\approx 1/2$. The true $\sigma_{\text{SEU, true}}$ picks up another factor of 2 it can be concluded that

$$p = 8 \times \sigma_{\text{SEU}} \times \text{fluence}. \quad (6.5)$$

### Conclusion on the normalization of the prediction

From equations 6.4, 6.5 and the above considerations, the probability to miss a L1A signal is

$$Pr_{\text{miss L1A}} = \frac{3}{4}p = 6 \times \sigma_{\text{SEU}} \times \text{fluence}. \quad (6.6)$$

The overall factor 6 can be identified as the constant $k$ in equation 6.2. Still, the value
of this normalisation is not entirely specified, since is not clear which rows in table 5.2 contribute, and because certain aspects in the above discussion might have been modeled crudely. For example, little attention was given to the differences in the energy profiles of the fluences, and what fraction of the occupancy is due to pions and protons. However, it is several factors of order $\mathcal{O}(1)$ that contribute towards the factor $k$, which overall does not exceed the value of 10. Due to these uncertainties, a value of $k = 1$ was chosen for the predictions, but we acknowledge that the overall normalisation of the prediction is difficult to control.

### 6.6 Summary

A first measurement of SEU in the $p - i - n$ diodes of the SCT modules under operation conditions is reported. The SEU rate is found to be proportional to the instantaneous luminosity and the module occupancy. Modules with lower $I_{PIN}$ are affected more often. Predictions based on beam test measurements are found to be in good agreement with data, describing well the ratio of average SEU rates in different barrel layers and endcap disks. The absolute normalisation of the prediction is difficult to control. It is found that SEUs are broadly in line with predictions and fortunately give rise to a small error rate and negligible data loss for the SCT.
Chapter 7

Search for direct sbottom and stop pair production

Scalar top and bottom partners are searched for in 20.1 fb$^{-1}$ of $pp$ collisions at a centre-of-mass energy of 8 TeV using the ATLAS experiment. The study focuses on final states with large missing transverse momentum, no leptons (electrons or muons) and two jets, each identified as originating from a $b$-quark. This final state can be produced in a $R$-parity conserving minimal supersymmetric scenario, when the scalar bottom decays into a bottom quark and a neutralino or the scalar top decays into a bottom quark and a chargino, with a small mass difference with the neutralino.

The 2 $b$-jets + $E_T^{\text{miss}}$ analysis has been initially performed with the 2011 dataset [57], and was later repeated with 12.8 fb$^{-1}$ of data recorded at $\sqrt{s} = 8$ TeV [2, 3]. It is the final version analysing the full 2012 dataset and published in reference [5] that is described in this chapter. For the purpose of this thesis, the analysis was rerun using the latest available object definitions and calibrations at the time of writing, but no kinematic selections were altered. Differences with respect to the published analysis are mentioned.

The current chapter is organised as follows. A general overview of the search, signal targeted and main backgrounds is given in section 7.1. Section 7.2 gives an account of the triggers and analysed data samples. The Monte Carlo (MC) samples used are summarised
7.1 Signal, signature and backgrounds

Events with two $b$-tagged jets, large missing transverse momentum and no leptons arise in the first of the investigated simplified model scenarios, in which sbottom quarks $\tilde{b}_1$ are pair produced and each decays to the Standard Model $b$-quark and the lightest supersymmetric particle (LSP), the neutralino $\tilde{\chi}_0^0$, with 100% branching ratio. All other SUSY particles are assumed to be at a higher, inaccessible scale. In the context of $R$-parity conservation, the LSP is stable and non-interacting, giving rise to large transverse momentum imbalance ($E_T^{\text{miss}}$). Hard initial state radiation (ISR) may boost the sbottom pair system.

A different simplified model in which the results of this search are interpreted is that of stop $\tilde{t}_1$ pair production, with exclusive decay to $b$ and lightest chargino $\tilde{\chi}_1^\pm$. For small mass difference between the chargino and the LSP ($\Delta m = 5$ or 20 GeV are considered in this analysis), the chargino is assumed to decay to a virtual $W$ and the LSP, $\tilde{\chi}_1^0$. Given the small mass difference, the off-shell $W$ decay products may have low $p_T$ and therefore be below the thresholds required for efficient object reconstruction. The signature of such events reduces to a 2 $b$-jets + $E_T^{\text{miss}}$ final state. The Feynman diagrams corresponding to sbottom and stop decays are shown in figure 7.1. The simplified signal models described above have only two free parameters: the mass of the sbottom or stop squark and the mass of the LSP. These define 2D signal planes, of which there is one for the sbottom pair production case and two for the stop pair production case for $\Delta m(\tilde{\chi}_1^\pm - \tilde{\chi}_1^0) = 5$ GeV or 20 GeV.
Figure 7.1: Feynman diagrams for sbottom (left) and stop (right) pair production and decay.

The cross-section for such models ranges from 560 pb (±16% theory uncertainty) for sbottom or stop mass of 100 GeV, down to 0.014 pb (±18% theory uncertainty) for sbottom or stop mass of 650 GeV. The cross-sections as a function of stop / sbottom mass and the theoretical uncertainty on the cross-section are summarised in figure 7.2. The uncertainties are obtained as the envelope of cross-section predictions using different PDF sets and varying the factorisation and renormalisation scales as described in reference [93]. The cross-sections and their uncertainties are provided centrally within the SUSY working group. They are calculated to next-to-leading order in the strong coupling constant, and the soft gluon emission is resummed at next-to-leading-logarithmic accuracy (NLO+NLL) [94-96].

The main backgrounds for this signature are semi-leptonic $t\bar{t}$ and single top production and $Z/W$ produced in association with heavy flavour (HF) jets. These are estimated using semi-data driven methods, whereby the shape of various distributions is simulated using dedicated Monte Carlo simulations, but the overall normalisation is determined in control regions from data. Minor backgrounds include QCD multijets, $t\bar{t}$ produced in association with a vector boson and diboson production.

### 7.2 Triggers and Data Sample

This analysis uses data collected during 2012 corresponding to a total integrated luminosity of $20.1 \pm 0.6 \text{ fb}^{-1}$. The trigger employed for the signal selection uses a requirement on the
7.2 Triggers and Data Sample

Figure 7.2: Pair production cross-section and theoretical uncertainty thereon for top squarks, squarks of the first two generations and gluinos, as a function of sparticle mass. The sbottom pair production cross-section is found to be equal to the stop pair production cross-section. The diagram was produced by collaborators [97] following prescriptions as indicated on the top left.

missing transverse momentum only and is found to provide the best acceptance among other possibilities studied (e.g. $b$-jet or jet + $E_T^{\text{miss}}$ triggers). In order to populate control regions, single lepton and dilepton triggers are used.

7.2.1 $E_T^{\text{miss}}$-only trigger

Two triggers are employed depending on their availability in different periods, as shown in table 7.1. These are the lowest threshold, unprescaled triggers available for the specified runs. Both triggers were found to reach their efficiency plateaus after requiring 150 GeV of $E_T^{\text{miss}}$ and a leading jet with 90 GeV of transverse momentum in the offline analysis. The EF_xe80T_tclcw_loose has the peculiarity that due to the high luminosity in the first bunch.

<table>
<thead>
<tr>
<th>Run</th>
<th>Trigger chain</th>
<th>L1 Seed</th>
<th>L2 Chain</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt; 203719</td>
<td>EF_xe80T_tclcw_loose</td>
<td>L1_XE40_BGRP7</td>
<td>L2_xe45T</td>
</tr>
<tr>
<td>≥ 203719</td>
<td>EF_xe80_tclcw_loose</td>
<td>L1_XE40</td>
<td>L2_xe45</td>
</tr>
</tbody>
</table>
crossings of each train, the trigger rate would have been too high for all events to be recorded. Because of this, the trigger was inactive during the first three bunch crossings of the train, leading to the efficiency on the plateau to be at 90% only. This has been taken into account by applying a 0.9 weight to MC events using a random number generator procedure. The use of $\text{EF}\_xe80T\_tclcw\_loose$ effectively reduces the luminosity by $0.2 \text{ fb}^{-1}$ to a total luminosity of $20.1 \text{ fb}^{-1}$.

### 7.2.2 Lepton triggers

In order to populate the different control regions used in the analyses, other triggers were used as described in Table 7.2. The performance of the single and 2-lepton triggers was characterised in other ATLAS SUSY analyses exploring strong production with one and two leptons in the final state. Events with two leptons of different flavours might pass both the single $\mu$ and single $e$ triggers and thus be recorded in both the Muons and Egamma streams. In order to remove the overlap between the streams, events which pass both the single $\mu$ and single $e$ triggers are removed from the Muons stream and taken from the Egamma stream. The data collected in any of the leptonic channels corresponds to an integrated luminosity of $20.3 \text{ fb}^{-1}$.

<table>
<thead>
<tr>
<th>Channel</th>
<th>Trigger chain</th>
<th>Threshold used</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-electron</td>
<td>EF_e24vhi_medium1 OR EF_e60_medium1</td>
<td>$p_T(e) &gt; 25 \text{ GeV}$</td>
</tr>
<tr>
<td>1-muon</td>
<td>EF_mu24_tight OR EF_mu36_tight</td>
<td>$p_T(\mu) &gt; 25 \text{ GeV}$</td>
</tr>
<tr>
<td>2-electron</td>
<td>EF_2e12Tv_loose1</td>
<td>$p_T(\mu, e) &gt; (20, 20) \text{ GeV}$</td>
</tr>
<tr>
<td>2-muon</td>
<td>EF_mu18_tight_mu8_EFFS</td>
<td>$p_T(\mu, \mu) &gt; (20, 20) \text{ GeV}$</td>
</tr>
<tr>
<td>1-e + 1-\mu</td>
<td>same as single $e/\mu$ channel (stream overlap removed)</td>
<td>$p_T(e/\mu) &gt; 25 \text{ GeV}$</td>
</tr>
</tbody>
</table>

Table 7.2: Triggers used for the different control regions and lepton $p_T$ thresholds needed for $> 99\%$ efficiency.
The simulated event samples used in this thesis were produced centrally by and for the ATLAS collaboration, and most samples are common to all of the SUSY group. In a first step, an MC generator produces events containing the desired hard process, followed in a second step by the parton shower and hadronisation, which might be performed by the same MC generator or a different one. The third step is the simulation of the underlying event, consisting of soft spectator interactions other than the hard scatter and the remnant of the colliding protons. The ATLAS underlying event tune 2B [98] was used to correct the default underlying event simulations. Additional minimum bias events simulated with PYTHIA 6 can be overlaid to model the effect of pile-up interactions. Finally, the interaction of the outgoing particles with the detector is simulated according to [99] using the GEANT4 [100] toolkit or the ATLFASTII (AFII) fast simulation framework [101]. The detector electronics response is simulated in the digitization phase. From here onwards, the simulated events are further fed through the same reconstruction algorithms as the real data. Table 7.3 summarises the different models used to simulate different physics processes, as well as the corresponding parton shower / hadronisation model, parton distribution functions and type of detector simulation. The background predictions are normalised to the best available theoretical calculations of the cross-sections.

Since signal samples are more specific to particular analyses, I have worked on producing the sbottom pair signal samples in the ATLAS production system. When simulating sbottom pair production, with each sbottom decaying to a bottom quark and the LSP, one expects 2 $b$-jets and missing transverse momentum in the final state. Further jets can, however,
be created from quark or gluon splitting, either in the final state (final state radiation) or in the evolution of the initial partons from the proton to the scale of the hard scattering (initial state radiation). For the 7 TeV sbottom analysis, the signal samples were already available and had been produced with HERWIG++ \cite{63,64}, a leading order, $2 \rightarrow 2$ particles generator. In HERWIG++, ISR is modelled by convolving the parton distribution functions with Sudakov factors, which give the probability of (non-)splitting. This limits the ISR to occur at a smaller scale than the scale of the hard process and therefore HERWIG++ underestimated the amount of initial state radiation, which is an essential feature targeted by one of this analysis’ signal regions.

Using MADGRAPH 5 interfaced with PYTHIA, one can overcome this limitation. MADGRAPH 5 is a $2 \rightarrow n$ leading order generator and therefore is expected to model ISR better by explicitly producing additional partons in the matrix element thus making high $p_T$ ISR jets part of the hard scattering process. The four-vectors produced by MADGRAPH are used as input to PYTHIA for the parton shower, hadronisation and underlying event. For the 8 TeV iterations of the analysis, I have produces such MADGRAPH samples, in which the production of $\tilde{b}_1 \tilde{b}_1^*$ and $\tilde{b}_1 \tilde{b}_1^* + 1$ additional jet with decays $\tilde{b}_1 \rightarrow b + \tilde{\chi}_1^0$ were simulated.

### 7.4 Objects and Variable Definitions

#### 7.4.1 Objects

Section [3.3] describes the object reconstruction in ATLAS. The objects provided by those definitions typically have high object identification efficiencies, but also high fake-rates. After the object overlap removal, further requirements might be imposed on the objects in order to reduce fake rates, at the cost of lowering identification efficiencies. It is these requirements that are described in this section.

**Electrons** “Baseline” electron candidates are required to have $p_T > 7$ GeV and $|\eta| < 2.47$ and to satisfy the “mediumPP” selection criteria \cite{75}. In order to ensure efficient
triggering in the leptonic control regions, “signal” electrons used in these regions have $p_T > 20$ GeV or $> 25$ GeV (see table 7.2) and are selected using the “tightPP” criteria. Additional isolation requirements are applied, which were updated with respect to the published result to follow an improved recipe yielding lower rates of jets faking electrons. The isolation requires the scalar sum of the tracks’ $p_T$ other than the electron track itself, in a cone of $\Delta R = 0.3$ around the candidate to be less than 16% of the reconstructed electron $p_T$, and the total energy of clusters in the EM calorimeter in a cone of $\Delta R = 0.3$ around the candidate to be less than 18% of the reconstructed electron $p_T$. Furthermore, the transverse and longitudinal impact parameters must be within 5 standard deviations and 0.4 mm, respectively, of the primary vertex.

**Muons** “Baseline” muon candidates are required to have $p_T > 6$ GeV and $|\eta| < 2.4$. In the control regions, “signal” muons are required to have $p_T > 20$ GeV or $> 25$ GeV for efficient triggering (c.f. 7.2) and to be isolated: the scalar sum of the transverse momenta of the tracks, excluding the muon track itself, within a cone of $\Delta R = 0.3$ around the muon candidate must not exceed 12% of the muon’s $p_T$ and the scalar sum of calorimeter transverse energy in the same cone, excluding that from the muon, must not exceed 12% of the muon’s $p_T$. Muon candidates must have transverse and longitudinal impact parameters within 5 standard deviations and 0.4 mm, respectively, of the primary vertex. As in the case of electrons, these requirements were also updated with respect to the published result.

The effect of the modified isolation compared to the approach used in the published result [5] is to reduce the data and MC yields in the different control regions by a maximum of 12%. The benefit of the new isolation is a larger fake lepton rejection, at the cost of lower prompt lepton identification efficiency. The impact on the extracted background normalisations was found to be negligible, with the highest difference being 3%. However, the higher fake lepton rejection increases the confidence that no multijet events enter the control regions by faking a lepton, and therefore that the background estimation is robust.

**Jets** are required to have $p_T > 20$ GeV and are reconstructed in the range $|\eta| < 4.9$. For the
7.4 Objects and Variable Definitions

analysis, jets within $|\eta| < 2.8$ are selected and ordered according to their $p_T$. Electromagnetic calorimeter energy depositions will be reconstructed by the ATLAS software as jets and electrons at the same time. These overlaps are resolved by discarding any jet candidate within $\Delta R = 0.2$ of an electron candidate. Subsequently, any electron or muon candidate found within $\Delta R = 0.4$ of any surviving jet is discarded, as in most cases these electrons or muons originate from a $b$-jet semi-leptonic decay.

**$b$-jets** Jets with $p_T > 30$ GeV and within $|\eta| < 2.5$, i.e. within the acceptance of the ID, are selected by the MV1 algorithm as originating from a $b$-quark. The MV1 selection requires an MV1 weight above 0.9827, which was found to give in a $t\bar{t}$ dominated sample a maximum $b$-tagging efficiency of 60% and rejection factors of 580, 8 and 23 against jets originating from light quarks, $c$-quarks and $\tau$-leptons, respectively.

$E_T^{\text{miss}}$ The missing transverse momentum 2-vector $p_T^{\text{miss}}$ is constructed as the negative vector sum of all muons, electrons and jets selected according to the above definitions, that have been calibrated to give the most accurate energy, together with any calibrated calorimeter energy clusters within $|\eta| < 4.9$ that have not been associated with any objects ("CellOut" term) [78]. The magnitude of $p_T^{\text{miss}}$ is denoted by $E_T^{\text{miss}}$.

### 7.4.2 Variables

The following variables are used in the analysis:

**$m_{CT}$** The contransverse mass is a kinematic variable that was designed to measure the masses of pair-produced heavy particles decaying semi-invisibly. For two identically decaying heavy particles $\delta_{1,2}$ with visible decay products $v_{1,2}$ and invisible decay products $\alpha_{1,2}$, the $m_{CT}$ variable is defined as [110]

$$m^2_{CT}(v_1, v_2) = [E_T(v_1) + E_T(v_2)]^2 - [p_T(v_1) - p_T(v_2)]^2,$$

(7.1)

where $E_T = \sqrt{p_T^2 + m^2}$. In this analysis, the $m_{CT}$ variable is built using as inputs
the two $b$-jets expected from the sbottom decays. $m_{CT}$ is an invariant quantity under equal and opposite boosts of the decaying particles in the transverse plane. If the visible decay products are massless, and the system of two decaying particles does not experience itself a transverse boost, it can be shown that $m_{CT}$ has an upper bound given by

$$m_{CT}^{\text{max}} = \frac{m^2(\delta) - m^2(\alpha)}{m(\delta)}. \quad (7.2)$$

The maximal value is achieved when the visible objects are collinear. An interesting observation is that for top pair production, the following must hold

$$m_{CT}^{\text{max}} = \frac{m^2(t) - m^2(W)}{m(t)} = 135 \text{ GeV}. \quad (7.3)$$

The $t\bar{t}$ background can effectively be reduced by using a high $m_{CT}$ requirement above the expected kinematic end-point, since in the case of sbottom pair production, the maximal value of $m_{CT}$ is expected at higher values given by

$$m_{CT}^{\text{max}} = \frac{m^2(\tilde{b}_1) - m^2(\tilde{\chi}_1^0)}{m(\tilde{b}_1)}. \quad (7.4)$$

In the case in which the pair of heavy decaying particles is boosted itself in the transverse plane by initial state radiation, the invariance of the quantity is broken and the above result does not hold exactly. The definition of $m_{CT}$ can be altered to a boost-corrected contransverse mass [111] to conservatively correct for such transverse boosts, and it is the “corrected” $m_{CT}$ that is used in this analysis. The effect of the correction is to preserve the kinematic endpoint of the distribution.

$m_{bb}$ is the invariant mass of the two $b$-tagged jets.

$\Delta \phi_{\text{min}}(n)$ is defined as the minimum azimuthal distance between any of the leading $n$ jets and the $p_T^{\text{miss}}$:

$$\Delta \phi_{\text{min}}(n) = \min(|\phi_1 - \phi_{p_T^{\text{miss}}}|, ..., |\phi_n - \phi_{p_T^{\text{miss}}}|). \quad (7.5)$$

Selecting events with large $\Delta \phi_{\text{min}}$ is efficient at rejecting the multijet background.
\( m_{\text{eff}} (n) \) is defined as the scalar sum of the \( p_T \) of the first leading \( n \) jets with \( p_T > 20 \text{ GeV} \) and \( |\eta| < 2.8 \) and the \( E_{\text{T}}^{\text{miss}} \):

\[
m_{\text{eff}} = \sum_{i \leq n} (p_{\text{jet}}^T)_i + E_{\text{T}}^{\text{miss}},
\]

(7.6)

where the index refers to the \( p_T \)-ordered list of jets. For the multijet background, the \( E_{\text{T}}^{\text{miss}} \) is expected to be a small contribution of the total \( m_{\text{eff}} \) in the event. The ratio of the two quantities is used to reject this background.

\( H_{T,x} \) is the scalar sum of the \( p_T \) of all jets in the event with \( p_T > 20 \text{ GeV} \) and \( |\eta| < 2.8 \), but excluding the leading \( x \) jets:

\[
H_{T,x} = \sum_{i=x+1}^{n} (p_{\text{jet}}^T)_i.
\]

(7.7)

An \( H_{T,3} \) upper limit requirement is useful to suppress the jet activity beyond that of the leading 3 jets. For example in the case of a pair of sbottom quarks recoiling off an ISR jet, one only expects the ISR jet, and two \( b \)-jets from the sbottom decays, with small further jet activity. \( H_{T,3} \) is found to be an effective discriminating variable against the \( t\bar{t} \) background.

For jets below 50 GeV, there is a chance that they originate from an interaction other than the primary proton-proton collision. For jets that are well within the tracker acceptance (\( |\eta| < 2.4 \)), the jet vertex fraction (JVF) can be evaluated, which is defined as

\[
\text{JVF} = \frac{\sum_{i \in \text{jet tracks associated w. primary vertex}} p_T^i}{\sum_{i \in \text{jet tracks associated w. any vertex}} p_T^i}.
\]

(7.8)

The primary vertex is defined to be the vertex with the highest \( \sum p_T^2 \) of all tracks associated with it. A selection \( |\text{JVF}| > 0 \) is equivalent to requiring that at least one jet track is associated with the primary vertex. For this analysis, only such jets enter the calculation of \( H_{T,3} \) and it was found that consequently the \( H_{T,3} \) distribution is independent of the number of primary vertices and therefore insensitive to pile-up.
$m_T$ can be defined in the case of an object decaying to two massless particles with transverse momenta $p_T^{1,2}$ as:

$$m_T = \sqrt{2p_T^{1}\cdot p_T^{2} - 2p_T^{1} \cdot p_T^{2}}. \quad (7.9)$$

The quantity is particularly useful when studying the $W$ leptonic decay, for which one decay product is the lepton and the other is a neutrino giving rise to missing transverse momentum. In such events, the above definition becomes

$$m_T = \sqrt{2p_{lep}^{1}\cdot E_{miss}^{T} - 2p_{lep}^{1} \cdot p_{miss}^{T}}. \quad (7.10)$$

It can be shown that the $m_T$ distribution has an endpoint at the true mass of the decaying particle, $m_T \lesssim m_W$.

$m_{T2}$ The $m_{T2}$ variable is a generalization of the transverse mass to the case of decays of pairs of particles [112, 113]. It can be shown that the $m_{T2}$ distribution also has an endpoint at the true mass of the decaying particle. While $m_{T2}$ is not used in this analysis, it was one of the discriminating variables tested in the optimisation procedure as an alternative to $m_{CT}$.

$\alpha_T$ is a variable proposed by [114] and used by the CMS experiment in their searches for bottom quark superpartners [115], [116]. The $\alpha_T$ variable was found to be sub-optimal in the optimisation procedure when compared with $m_{CT}$ or $m_{T2}$. Also, in case of the discovery of a supersymmetric particle, $m_{CT}$ or $m_{T2}$ distributions would naturally provide the mass scale of new physics, whereas $\alpha_T$ is a scaleless variable.

### 7.5 Event cleaning

Several “cleaning” requirements are employed to reduce non-collision backgrounds. The first type of events that need to be rejected are because of hardware malfunctions. Events are required to have been recorded in “good” luminosity blocks listed in the “Good Runs List” (GRL). These are events for which most of the detector was operational and data quality
is expected to be at a high standard. However, in some events included in the GRL, there might still be essential sub-detectors that were only partially operational. For example, events are required not to have any reported LAr or tile calorimeter errors, and the trigger system is expected to have read the full information from the detector. Some events have objects falling within non-operational cells in the tile and the hadronic end-cap calorimeters, which will give rise to fake $E_T^{\text{miss}}$, and such events are removed from the dataset.

Another type of events that are rejected are those in which physics objects like jets or muons are correctly measured, but these do not originate from proton-proton collisions. For example, “bad jets” are jets which can stem from coherent detector noise, or be part of cosmic-ray showers, or they can be due to LHC beam conditions. Several quality criteria described in [117] are applied to jets, and events are rejected if they contain jets failing these criteria. For example, requirements are placed on the charged $p_T$ fraction $f_{\text{ch}}$ of jets, which refers to the sum of $p_T$ of all tracks associated with the jet divided by the jet $p_T$, and on the fraction $f_{\text{em}}$ of the jet energy contained in the electromagnetic calorimeter. If any of the two leading jets in an event with $p_T > 100 \text{ GeV}$ and within $|\eta| < 2.0$ are characterised by either $f_{\text{ch}} < 0.02$ or both $f_{\text{ch}} < 0.05$ and $f_{\text{em}} > 0.9$, the event is rejected.

Muons with a longitudinal impact parameter larger than 1 mm or transverse impact parameter larger than 0.2 mm with respect to the primary vertex are believed to be of cosmic ray origin and the event is vetoed. Events with poorly measured muon momenta are vetoed in order to protect the $E_T^{\text{miss}}$ calculation. The primary vertex must be associated with at least five tracks in order to ensure that it is well reconstructed.

7.6 Signal Regions

After objects are selected, calibrated, overlap-removed and events are cleaned, the signal regions selections can be applied to the dataset.

Given the studied sbottom signal, exactly two $b$-jets, large $E_T^{\text{miss}}$ and no leptons are expected. Two signal regions (SRs), SRA and SRB, are defined, and in both events with
electrons or muons are vetoed, and exactly two \( b \)-tagged jets are required. In order to ensure fully efficient triggers, the leading jet \( p_T \) is required to be above 130 GeV, while the \( E_T^{\text{miss}} \) lower threshold is placed at 150 GeV.

Signal region A (SRA) targets scenarios with large mass splittings between the \( \tilde{b}_1 \) and \( \tilde{\chi}^0_1 \), therefore the events are required to have exactly two \( b \)-tagged jets with high \( p_T > 130, 50 \) GeV, which should be the leading two jets. Given this signature, several standard model backgrounds can be expected. In the case of \( t\bar{t} \) and \( W + \) heavy flavour production, events with \( W \) leptonic decays will give rise to \( E_T^{\text{miss}} \) and might pass the lepton veto requirement. This can happen if the lepton is either out of detector geometric acceptance, or is not reconstructed or identified, or if the lepton is a hadronically-decaying \( \tau \). In the case of \( t\bar{t} \), more jets can be expected beyond the two \( b \)-jets, and a selection can be applied to limit further jet activity beyond the two jets expected from decays of pair produced sbottom quarks. Therefore an event is vetoed if a 3rd jet with \( p_T > 50 \) GeV is found. In order to further suppress \( t\bar{t} \), the \( m_{\text{CT}} \) variable is calculated using as inputs the two \( b \)-tagged jets. Events with high \( m_{\text{CT}} \) values are selected, as \( t\bar{t} \) events are expected to exhibit a kinematic endpoint at 135 GeV. However, some \( t\bar{t} \) events can still by-pass this requirement when a \( c \)-quark from a hadronic \( W \) decay is mistagged as a \( b \)-jet. For a \( bc \)-quark pair, equation 7.3 does not hold, as incorrect kinematic assignments have been made for the particles \( v_{1,2}, \alpha_{1,2} \) and such events can extend beyond the kinematic endpoint. Therefore several sub-regions are defined, requiring \( m_{\text{CT}} > 150, 200, 250, 300 \) and 350 GeV, with higher threshold values targeting higher sbottom or stop masses. A further important background process is \( Z \) produced in association with heavy flavour jets, where the \( Z \) decays \( Z \to \nu \nu \) giving rise to \( E_T^{\text{miss}} \). This background is tackled by applying an additional requirement on the invariant mass of the two \( b \)-jets > 200 GeV, since \( b \)-jets produced in association with a \( Z \)-boson tend to come from near-collinear gluon splitting and have low \( \Delta \phi(b, b) \) and \( m_{bb} \). The importance of the \( m_{bb} \) and \( m_{\text{CT}} \) requirements is illustrated in figure 7.3, where the correlations of \( m_{bb} \) with \( m_{\text{CT}} \) and \( \Delta \phi(b, b) \) are shown. It can be observed that the signal is distributed at higher \( m_{bb} \) and \( m_{\text{CT}} \) values compared to the backgrounds, and that the \( m_{bb} \) and \( m_{\text{CT}} \) are not correlated for the signal. The \( Z+\text{HF} \) background is concentrated at lower \( m_{bb} \) and \( \Delta \phi(b, b) \) values due
to the two $b$-jets originating from gluon splitting.

Signal region B (SRB) targets scenarios with small mass splittings between the $\tilde{b}_1$ and $\tilde{\chi}_1^0$. Signal events for such models are expected to have low $E_{T}^{\text{miss}}$ and low $b$-jet $p_T$’s. However, SRB exploits the possibility of the $\tilde{b}_1 \tilde{b}_1$ system recoiling against a high $p_T$ initial state radiation (ISR) jet. A requirement is made that the leading jet is not $b$-tagged and has $p_T > 150$ GeV (the ISR jet). The second and third leading jets are expected to come from
the sbottom decays and are therefore required to be $b$-tagged and to have $p_T > 30 \text{ GeV}$. Due to the ISR jet, the $\chi^0_1$'s can experience significant boost, and the $E_T^{\text{miss}}$ is required to be above $250 \text{ GeV}$. The leading jet and $p_T^{\text{miss}}$ are expected to be back-to-back, therefore their angular separation in the transverse plane must be $\Delta \phi > 2.5$. The dominant background in this region is $t\bar{t}$ production, with $W/Z+\text{HF}$ production having sub-leading contributions. The high $E_T^{\text{miss}}$ requirement is effective at rejecting $t\bar{t}$ events. Similar to SRA, the jet activity beyond the leading three jets is restricted by placing an upper limit at $50 \text{ GeV}$ on the $H_{T,3}$ variable, i.e. on the sum of all jet $p_T$'s after the expected ISR jet and 2 $b$-tagged jets, which also rejects $t\bar{t}$ events.

For both signal regions, requirements on the ratio of $E_T^{\text{miss}}$ to the sum of $E_T^{\text{miss}}$ and $p_T$'s of the first two (SRA) or three jets (SRB) and on the smallest angle between $E_T^{\text{miss}}$ and the leading three jets are used to suppress the multijet background. The full set of requirements is summarised in table 7.4.

<table>
<thead>
<tr>
<th>Description</th>
<th>SRA</th>
<th>SRB</th>
</tr>
</thead>
<tbody>
<tr>
<td>Event cleaning</td>
<td>Common to both SRs</td>
<td></td>
</tr>
<tr>
<td>Lepton veto</td>
<td>No baseline $e/\mu$ after overlap removal with $p_T &gt; 7(6) \text{ GeV}$ for $e(\mu)$</td>
<td></td>
</tr>
<tr>
<td>$E_T^{\text{miss}}$</td>
<td>$&gt; 150 \text{ GeV}$</td>
<td>$&gt; 250 \text{ GeV}$</td>
</tr>
<tr>
<td>Leading jet $p_T(j_1)$</td>
<td>$&gt; 130 \text{ GeV}$</td>
<td>$&gt; 150 \text{ GeV}$</td>
</tr>
<tr>
<td>Second jet $p_T(j_2)$</td>
<td>$&gt; 50 \text{ GeV}$</td>
<td>$&gt; 30 \text{ GeV}$</td>
</tr>
<tr>
<td>Third jet $p_T(j_3)$</td>
<td>veto if $&gt; 50 \text{ GeV}$</td>
<td>$&gt; 30 \text{ GeV}$</td>
</tr>
<tr>
<td>$\Delta \phi(p_T^{\text{miss}}, j_1)$</td>
<td></td>
<td>$&gt; 2.5$</td>
</tr>
<tr>
<td>$b$-tagging</td>
<td>leading 2 jets</td>
<td>2nd- and 3rd-leading jets</td>
</tr>
<tr>
<td></td>
<td>($p_T &gt; 50 \text{ GeV},</td>
<td>\eta</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$n_{b\text{-jets}} = 2$</td>
</tr>
<tr>
<td>$\Delta \phi_{\text{min}}(3)$</td>
<td>$&gt; 0.4$</td>
<td>$&gt; 0.4$</td>
</tr>
<tr>
<td>$E_T^{\text{miss}}/m_{\text{eff}}(k)$</td>
<td>$E_T^{\text{miss}}/m_{\text{eff}}(2) &gt; 0.25$</td>
<td>$E_T^{\text{miss}}/m_{\text{eff}}(3) &gt; 0.25$</td>
</tr>
<tr>
<td>$m_{CT}$</td>
<td>$&gt; 150, 200, 250, 300, 350 \text{ GeV}$</td>
<td>-</td>
</tr>
<tr>
<td>$H_{T,3}$</td>
<td>-</td>
<td>$&lt; 50 \text{ GeV}$</td>
</tr>
<tr>
<td>$m_{bb}$</td>
<td>$&gt; 200 \text{ GeV}$</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 7.4: Event selection in SRA and SRB.
These requirements are the result of a 1-D optimisation procedure performed by the author. The procedure applied the whole analysis selection to the background and signal, but the requirement on the variable that was being optimised. A total integrated luminosity of 21 fb$^{-1}$ was assumed, as the optimisation was performed before data collection has finished. The requirement on the variable to be optimised was then scanned in suitably chosen steps and the sensitivity of the analysis in the sbottom signal plane was assessed by estimating the expected 95% confidence level exclusion limit. The analysis is performed using the candidate set of requirements on the MC samples, and an expectation $B(S)$ for the number of background (signal) events is obtained. The systematic uncertainty on the background prediction is assumed to be $\sigma_B = 20\%$ throughout, and the true, unknown number of background events $\mu_B$ is modelled as being sampled from a Gaussian probability distribution, $p(\mu_B)$, centered at $B$ and with width $\sigma_B$. The $Z_N$ procedure [118] is employed to calculate the $p$-value of the null hypothesis that physics is described by the Standard Model only (which would predict $\mu_B$ events), when $S+B$ events are observed, by marginalising over the assumed distribution $p(\mu_B)$ of the true number of background events:

$$p = \int d\mu_B \left( \sum_{N_{tot}=S+B} \frac{\mu_B}{N_{tot}} p(\mu_B) \right).$$

(7.11)

The $p$-value can also be converted to a significance value $Z_N$ which corresponds to the number of standard deviations in a one-tailed test of a Gaussian probability distribution:

$$Z_N = \sqrt{2} \text{erf}^{-1} (1 - 2p).$$

(7.12)

The $Z_N$ value is required to be above 1.64 in order to exclude the background-only hypothesis at 95% confidence level (CL). The values of most requirements from the 7 TeV iteration of the analysis were checked this way in the optimisation procedure, and new variables were tested. The results of the optimisation are as follows:

- the requirements on most variables like $E_T^{\text{miss}}$ or $p_T$ of jets were found to already give optimal sensitivity, even though the values were inherited from the 7 TeV analysis.
some requirements were dropped. In SRB, an upper limit on the sub-leading jet $p_T$ was previously placed at 110 GeV, motivated by the fact that in the case of low mass splitting $m_{b_1} - m_{\tilde{\chi}^0_1}$, the leading jet is believed to be an ISR jet, and the sub-leading jet is believed to come from the sbottom decay. Due to the small mass splitting, the sub-leading jet can only have a limited amount of energy available, and might therefore be lower in $p_T$ than the $t\bar{t}$ background. In the re-optimisation of the sub-leading jet $p_T$ upper threshold, it was found that this requirement was no longer providing any sensitivity in the 8 TeV case. Another finding of the optimisation procedure was that a previously used hybrid signal region, having features of both SRA and SRB and aiming at bridging the gap at intermediate mass splittings between the sbottom and the neutralino mass, was found to be obsolete. As the luminosity increased and the previous versions of SRA and SRB were optimised, the region of the simplified model parameter space covered by the hybrid region was now covered by either SRA or SRB.

some new requirements were introduced. The $m_{bb} > 200$ GeV requirement in SRA was newly introduced, which for a sbottom mass between 500 and 700 GeV increased the exclusion limit by 30-40 GeV in neutralino mass. The SRA signal region consists of several sub-regions which all have a common selection, apart from the final $m_{CT}$ requirement, which starts at $m_{CT} > 150$ GeV and increases in steps of 50 GeV. This led to introducing the SRA sub-region with $m_{CT} > 350$ GeV selection, and it was found that any more stringent requirement would have added no further sensitivity.

a range of new requirements were tested, but found not to provide any extra sensitivity. The possibility of placing a $\tau$ veto was investigated, of restricting the $|\eta|$ of the leading jets, the $\Delta R$ separation between the jets, or using $m_{T2}$ or $\alpha_T$ as discriminating variables instead of $m_{CT}$.

Figures 7.4, 7.5 comprise a selection of intermediate optimisation results showing the effect that varying the requirements on various variables has on the exclusion limits. Most of the optimisation was performed on the sbottom pair production model, but some ideas were also tested in the context of the $\tilde{t} \rightarrow b\tilde{\chi}^\pm_1$ model. Particular attention was given to
Figure 7.4: Exclusion limits in the $\tilde{b}_1 - \tilde{\chi}_0^0$ mass plane for SRA (left) and SRB (right). Top left: $m_{CT}$ requirement is varied (no $m_{bb}$ requirement applied). Top right: the leading jet $\eta$ requirement is varied. Bottom left: $b$-tagging operating point is varied. The $b$-tagging uncertainties related to the particular operating point are used. Bottom right: leading $b$-jet requirement is varied.

the third jet veto in SRA and the $H_{T,3}$ requirement in SRB, which reduce the stop signal acceptance due to the chargino 3-body decay. It was attempted to loosen or remove these requirements and complement them with others, for example with a $\tau$ veto. In most tests performed, it was found that the $t\bar{t}$ acceptance increases more than the signal acceptance and is detrimental to the exclusion limit. Any new requirements other than the 3rd jet veto are unable to compensate for the increase in background acceptance. Only in the case of the $\tilde{t}_1 - \tilde{\chi}_1^0$ mass plane with $\Delta m(\tilde{\chi}_1^\pm - \tilde{\chi}_1^0) = 20$ GeV, removing the $H_{T,3}$ requirement altogether would have excluded further scenarios with low stop masses between 150 and 280 GeV, but the additional sensitivity was considered not to be commensurate with the complication of introducing additional SRs, especially since a dedicated analysis was also expected to target this scenario. The optimal requirements are presented in table 7.4. Combined exclusion
7.7 Background Estimation

In order to estimate the background contributions to the signal regions, a partially data-driven approach is taken. MC predictions of dominant backgrounds are normalised in control regions (CR). Several CRs are designed to be similar in kinematics, but orthogonal to the SRs and enhanced in the particular background of interest. If \( n \) control regions are defined for a signal region, and each CR has mixed contributions from several background processes, \( n \) normalisation parameters \( \mu_{i=1,\ldots,n} \) for \( n \) background processes can in principle be extracted by performing a simultaneous fit of the MC predicted contributions to the observed data. However, if the purity of a background process in the control regions is small, its normalisation cannot be estimated precisely. For SRA, three control regions are defined. They have large enough contributions from the top, \( Z \) and \( W \) production processes, which makes it possible to extract three normalisation parameters \((\mu_{t\bar{t}}, \mu_Z, \mu_W)\) from the fit. For SRB, while \( W \) production is an important background, no control region can be defined with a large enough \( W \) production purity. Therefore only two control regions are defined, with
Figure 7.6: Combined exclusion limits and best expected signal region in the sbottom - neutralino (top) and stop - neutralino (bottom) (left: $\Delta m = 5$ GeV, right: $\Delta m = 20$ GeV) mass planes. The numbers indicate the best expected signal region for the given point; short descriptions of the signal regions are shown in the legend.

Figure 7.7: Acceptance (left) and acceptance $\times$ efficiency (right) for the sbottom signal plane for the best expected signal region.
Z and top production dominating, and only their normalisations are extracted from the fit. The fit procedure is described in section 7.9. Other electroweak backgrounds are negligible and their contribution is taken from MC, normalising to luminosity and the best available theoretical calculation of the inclusive cross-section. The multijet background is estimated using a dedicated, data-driven method described in section 7.7.2.

### 7.7.1 Electroweak backgrounds

**Control regions for SRA**

For signal region SRA, the following control regions have been defined:

- **CRA.1L**: The $t\bar{t}$, single top and $W + HF$ processes enter the signal region because of a $W$ leptonic decay, where the lepton is a hadronic $\tau$ or out of acceptance and the $E_T^{\text{miss}}$ is due to the escaping $\nu$ or lepton. The $t\bar{t}$, single top and $W + HF$ jets CR therefore has an explicit one-lepton requirement. The transverse mass is calculated using the lepton momentum and the $p_T^{\text{miss}}$ and a requirement is made $40 \text{ GeV} < m_T < 100 \text{ GeV}$; the upper limit has the purpose of reducing generic signal contamination, as an endpoint in the $m_T$ distribution is expected at $m_W$, while the lower limit ensures there is only a negligible contribution from multijet production where a jet is mis-identified as a lepton. The rest of the selection mirrors the SRA selection by applying jet multiplicity and $p_T$ requirements, $b$-tag selections and an $m_{CT}$ requirement. The $t\bar{t}$ and single top contributions are treated together as one process with the same normalisation, as their $m_{CT}$ shapes are observed to be similar, and the single top contribution is much smaller than the $t\bar{t}$ one.

- **CRA_SF**: $Z$ produced in association with heavy flavour jets enters SRA when $Z$ decays to a pair of neutrinos. The $Z$ background normalisation is extracted from CRA_SF, where the $Z \rightarrow \nu\nu$ decay is replaced with the visible $Z \rightarrow ee$ or $\mu\mu$ decay. Two “signal” electrons or two “signal” muons with opposite sign are required. The invariant mass of the two leptons is constrained to be consistent with the mass of the $Z$ boson, $75 \text{ GeV} < m_{\ell\ell} < 105 \text{ GeV}$. In order to mimic the kinematics of the $Z \rightarrow \nu\nu$ decay, the $p_T$'s of the two leptons in CRA_SF are added vectorially to the $p_T^{\text{miss}}$. The leading lepton is required to have $p_T > 90 \text{ GeV}$ in
order to increase the purity of the Z sample and to improve the MC / data agreement. For lower leading lepton $p_T$, $t\bar{t}$ production dominates, but the $t\bar{t}$ MC underestimates the data in this region of phase-space. Jet multiplicity and $p_T$ requirements as well as $b$-tag selections are made.

**CRA_DF:** An additional very pure $t\bar{t}$ CR is defined for SRA only by requiring two different-flavour signal leptons ($e$ or $\mu$).

In general the CRA selections mirror the kinematics imposed in SRA ($p_T$ of jets, jet multiplicity, $b$-jet multiplicity, $m_{CT}$, $m_{bb}$). Depending on the control region, several requirements need to be relaxed to ensure enough events pass the selections and to reduce the statistical uncertainty of the extracted normalizations. The exact definitions of the various control regions are summarised in **Table 7.5**. Three control regions were defined, with sufficient contributions from the top, $Z+HF$ and $W+HF$ backgrounds, therefore the normalisations $\mu_{t\bar{t}}, \mu_{Z}, \mu_{W}$ of these backgrounds are extracted from the simultaneous fit of the MC predicted backgrounds in the CRs to the observed data, as detailed in section **7.9**. Good agreement is found between the Monte Carlo prediction and data in all control regions. Several distributions are shown for the control regions defined for signal region SRA: CRA_1L (fig. **7.8**), CRA_SF (fig. **7.9**), CRA_DF (fig. **7.10**). In all figures shown, the SM prediction is normalised.
malised according to the MC expectations before the fit is performed in the control regions, unless otherwise stated. The shaded band includes both detector and theoretical systematic uncertainties, as well as statistical uncertainties. The last bin in each histogram contains the integral of all events with values greater than the upper axis bound.

Control regions for SRB

A 1-lepton region, CRB_1L, dominated by $t\bar{t}$ and single top, and a 2-lepton region, CRB_SF, dominated by $Z+HF$ production are defined. The kinematic requirements mirror the SRB selection, unless small numbers of events dictate relaxing them. Table 7.6 gives the complete kinematic selections. In contrast with the CRs defined for SRA, in this case the $W+HF$ is normalised according to theoretical calculations, as it was not possible to find a suitable CR with a large enough $W$ contribution to perform the normalisation. Therefore in the case of SRB control regions, only two normalisations $\mu_{t\bar{t}}$ and $\mu_Z$ are extracted. Distributions are shown for CRB_1L in fig. 7.11 and for CRB_SF in fig. 7.12.

<table>
<thead>
<tr>
<th>CRB_1L</th>
<th>CRB_SF</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 signal $e$ or $\mu$</td>
<td>$e^+e^-$ or $\mu^+\mu^-$</td>
</tr>
<tr>
<td>Veto additional baseline leptons with $p_T(e)&gt;7$ GeV or $p_T(\mu)&gt;6$ GeV</td>
<td></td>
</tr>
<tr>
<td>At least three reconstructed jets ($p_T&gt;30$ GeV)</td>
<td></td>
</tr>
<tr>
<td>$p_T(j_1)&gt;130$ GeV</td>
<td>$p_T(j_1)&gt;50$ GeV</td>
</tr>
<tr>
<td>$E_T^{\text{miss}} &gt; 120$ GeV</td>
<td>$E_T^{\text{miss (lepton-corrected)}} &gt; 100$ GeV</td>
</tr>
<tr>
<td>$j_1$ anti-$b$-tagged; $j_2$ and $j_3$ $b$-tagged</td>
<td></td>
</tr>
<tr>
<td>$40$ GeV &lt; $m_T &lt; 100$ GeV</td>
<td>$75$ GeV &lt; $m_{t\bar{t}} &lt; 105$ GeV</td>
</tr>
<tr>
<td>lepton $p_T &gt; 90$ GeV</td>
<td></td>
</tr>
<tr>
<td>$H_{T,3} &lt; 50$ GeV</td>
<td></td>
</tr>
</tbody>
</table>

Table 7.6: Definition of the two SRB control regions.

The combined profile likelihood fit of the MC expectation to data in the control regions is described in section 7.9. From the fit, normalizations for $t\bar{t} + $ single top, $Z + HF$ in both SRA and SRB and $W + HF$ in SRA are extracted, and these are applied to the MC
Figure 7.8: CRA_{1L} distributions before the fit. Top: $m_{T_{CT}}$ distribution, $m_{T_{lep}}$, $E_{T_{miss}}$. Middle: lepton $p_T$, $E_{T_{miss}}$. Bottom: leading and sub-leading jet $p_T$. On the top row, distributions are shown with all selections applied, omitting the requirement on the shown variable, which is indicated by the red arrows. The shaded band shown includes both detector and theoretical systematic uncertainties, as well as statistical uncertainties on the standard model prediction. The last bin in each histogram contains the integral of all events with values greater than the upper axis bound.
Figure 7.9: CRA_SF distributions; from left to right: $E_T^{\text{miss}}$, $m_{bb}$ (top row); leading lepton $p_T$, $m_{\ell\ell}$ before $m_{bb}$ and $m_{\ell\ell}$ requirement (bottom row). The red arrows indicate the requirement on the variable shown used in the definition of the control region.
Figure 7.10: CRA_{DF} distributions; from left to right: $m_{CT}$ distribution, $E_{T}^{miss}$ (top row); leading lepton and leading jet $p_T$ (bottom row). The red arrow indicates the requirement on the variable shown used in the definition of the control region.
Figure 7.11: CRB_1L distributions; from left to right: $m_T$ distribution, $E_T^{\text{miss}}$ (top row); leading jet and leading lepton $p_T$ (bottom row). Distributions are shown with all selections applied, omitting the requirement on the shown variable, which is indicated by the red arrows.
Figure 7.12: CRB_SF distributions; from left to right: $m_{\ell\ell}$ distribution, leading jet $p_T$ (top row); original $E_T^{\text{miss}}$, $E_T^{\text{miss}}$ with leading two lepton $p_T$’s re-added vectorially. Distributions are shown with all selections applied, apart from the $m_{\ell\ell}$ distribution, where the requirement on the shown variable is indicated by the red arrows.
expectation in the signal region. Other, less significant backgrounds are estimated directly from MC. Goodness of fit is assessed in multiple, orthogonal validation regions:

- **VRA,Mct** - as SRA, but invert the $m_{CT}$ requirement such that $m_{CT} < 100$ GeV. Relevant distributions shown in top row of figure 7.13.

- **VRA,Mbb** - as SRA, but invert the $m_{bb}$ requirement such that $m_{bb} < 200$ GeV. Relevant distributions shown in bottom row of figure 7.13.

- **VRB,HT3** - as SRB, but invert the $H_{T,3}$ requirement such that $H_{T,3} > 50$ GeV. Relevant distributions shown in top row of figure 7.14.

- **VRB,emu** - defined in an analogous way to the 2 lepton, different flavour CRADF control region. Relevant distributions shown in bottom row of figure 7.14.

Validation regions have also been defined for the 0, 1, and 2 lepton same flavour control regions, but relaxing the $b$-tagging requirement to only one $b$-tag. These also showed good MC to data agreement.
Figure 7.13: Validation regions distributions for SRA; from left to right: $E_T^{\text{miss}}$ and $m_{bb}$ distributions in VRA:Mct (top row); $E_T^{\text{miss}}$ and $m_{CT}$ distributions in VRA:Mbb (bottom row).
Figure 7.14: Validation regions distributions for SRB; from left to right: $E_T^{\text{miss}}$ and $H_{T,3}$ distribution in VRB-HT3 (top row); $E_T^{\text{miss}}$ and $H_{T,3}$ distribution in VRBemu (bottom row).
7.7 Background Estimation

7.7.2 Multijet Estimate from Jet Smearing Method

In the case of QCD multi-jet events, the energy measurement of one or more jets may fluctuate such that they give rise to an overall transverse momentum imbalance. This background is difficult to estimate since it involves a high multijet production cross-section and a very small probability of multijet events to pass the analysis selection, leading to the necessity to simulate a very large multijets sample. The jet smearing method [119] is a data-driven method that overcomes this difficulty and is widely used in SUSY analyses. It uses events with well measured jets, which are smeared using the jet resolution functions to produce large $E_T^{\text{miss}}$. The smearing can be repeated an arbitrary number of times, eventually leading to enough events to populate the signal regions. The pseudo-data can be normalised in a dedicated control region. The method comprises the following steps:

In a first step, the smearing function is derived from simulated dijet events by comparing the truth jet $p_T$ with the reconstructed jet $p_T$. The smearing function is derived in bins of truth jet $p_T$, for jets originating from light and $b$-jets. The latter are expected to have an underestimation of their measured energy due to the escaping $\nu$ from the $b$-quark decay. The smearing functions can be validated in data, from which corrections to the MC-only smearing functions can be derived, as well as a set of smearing functions for systematic uncertainties variations. This first step is performed centrally in the ATLAS SUSY working group.

Further steps are analysis specific and were performed by the author. The second step involves selecting well-measured multi-jet events in data by requiring low values of $E_T^{\text{miss}}$ significance, $E_T^{\text{miss}}/\sqrt{H_T} < 0.6 \sqrt{\text{GeV}}$. Due to the low $E_T^{\text{miss}}$, the $E_T^{\text{miss}}$ trigger cannot be employed, and is replaced by jet triggers. Their prescales need to be taken into account by assigning appropriate weights to the events. In a third step, the transverse momenta of all jets in these seed events are smeared using the smearing functions. Each seed event is used multiple times to obtain 20,000 different jet configurations, some of which would have acquired a high enough $E_T^{\text{miss}}$ in the process to pass the analysis selection. With a large sample of pseudo-events, the signal regions will be sufficiently populated such that the smeared events can be used to give the multijet contribution of any variable of interest.
The normalisation is obtained separately for each SR from a dedicated QCD CR with the same selection as the analysis (table 7.4), but with an inverted $\Delta\phi_{\text{min}}$ requirement, such that $\Delta\phi_{\text{min}} < 0.4$. In order to reduce statistical uncertainties, the $m_{bb}$, $m_{CT}$ and $H_{T,3}$ requirements are not applied. In this CR, the QCD is normalised to the difference between the data and the the electroweak backgrounds, and the normalisation is applied to the pseudo-data contribution in the SR. The process is repeated for both SRA and SRB, and good agreement of relevant distributions is found between the multijet pseudo-data and the data in the control regions (fig. 7.15).

![Figure 7.15: Distribution of $E_{\text{T}}^{\text{miss}}$ in CRA_QCD (left) and CRB_QCD (right). The uncertainty band shows statistical uncertainties only. While the author has produced the smeared multijet events and has calculated the normalisations in the CRs, the histograms shown were produced by collaborators [120].](image)

While the multijet contribution in the dedicated CRs is high, the estimated contribution to the SRs is negligible, with less than 1 event expected in any SR (table 7.12). Repeating the procedure with the set of smearing functions dedicated for the assessment of systematic uncertainties, the yields are found to be similar, and a conservative uncertainty of 100% on this background is considered.
7.8 Systematic uncertainties

Systematic uncertainties on the backgrounds or signal can arise from two sources. On the one hand, there are uncertainties related to the experimental measurement of objects and how well these are described by the simulations. On the other hand, uncertainties may arise from the different theoretical approaches and approximations used to generate the events.

7.8.1 Experimental uncertainties due to object identification and calibrations

The calibration of objects and object identification efficiencies are discussed first. Systematic uncertainties arise due to the electron identification efficiency, energy scale and resolution, the muon energy resolution, the jet energy scale and resolution, the MET CellOut term scale and resolution and the $b$-tagging efficiency for $b$, $c$ and light quarks.

Lepton identification, energy scale and resolution Scale factors and related uncertainties that correct the lepton reconstruction and trigger efficiencies in MC to those in data can be obtained from $Z \rightarrow \ell^{+}\ell^{-}$ events using the tag-and-probe method [75,76,121]. The lepton energy scale and resolution are derived from $Z \rightarrow \ell^{+}\ell^{-}$ or $J/\Psi \rightarrow \ell^{+}\ell^{-}$ events and the lepton momenta are corrected such that their invariant mass corresponds to the known mass of these resonances. Uncertainties on the correction come from a wide variety of sources and depends on the lepton type. The electron and muon energy resolution is also measured in data, and used the smear the momenta of all simulated electrons and muons. The uncertainties due to the lepton reconstruction, trigger efficiencies, lepton energy scale and resolution have a very small effect on the event yields in the signal and control regions, on the order of 1%. These uncertainties are therefore neglected.

Jet energy scale (JES) The jet calibration scheme of ATLAS consists of several steps. Jets are first reconstructed from calorimeter topological clusters which have been cali-
brated using the local cluster weighting method. In a first instance, the energy of jets is corrected for the energy offset introduced by pile-up. The correction is derived as a function of the number of primary vertices and the average number of interactions per bunch crossing. In a second step, the jet direction is altered such that the jet points to the primary vertex, without changing the jet energy. Further, a multiplicative factor is applied to jet 4-vectors which aims to correct for the difference between true jet energy (obtained from MC) and jet energy measured in the calorimeter. A residual calibration is derived using in-situ techniques and applied to data only. These exploit the transverse momentum balance between a jet and a well-measured reference object such as a photon or a $Z$ boson for jets with $p_T < 1000$ GeV. For jets with $p_T > 1$ TeV, an alternative technique uses in situ single hadron response measurements \cite{122} and test beam results. The calibration of forward jets ($|\eta| > 1.2$) is derived from dijet $p_T$ balance measurements. The different steps are described in detail in \cite{117}.

The derived calibration has uncertainties related to it due to physics and detector modeling effects. The MC simulation might use an inaccurate underlying event model, and different MC programs lead to different predictions. The detector simulation might use an imprecise detector geometry model. The in-situ techniques also rely on approximations that are only partially fulfilled, for example the reference object might not balance completely the jet in the event in the case where there are additional high $p_T$ particles present. Uncertainties are also propagated from the calibration of the reference objects. Many of the in situ calibration techniques use jets that are due to a high $p_T$ quark, whereas physics analyses make use of jets stemming from both gluons, light quarks and heavy flavour quarks, for which the detector response is different. The jet calibration can in principle be affected by the presence of close-by jets. The JES uncertainty is a measure of all these effects.

**Jet energy resolution (JER)** The JES corrects the jet energies on average to the true jet energy. However, for events containing jets of a given true energy, there will still be a spread of reconstructed jet energies around the mean, characterised by the JER. It can be measured using two different in-situ methods, using either di-jet $p_T$ balance or the
bisector method, both in data and MC [123]. It is found that the JER in data and MC agree to within 10%, with the resolution in data being larger. To assess the uncertainty due to this difference, each jet in MC is smeared by a random factor sampled from a Gaussian with mean 1 and a suitably chosen \( \sigma \) which reflects the difference between the JER in data and MC.

\[ E_{\text{miss}} \] The corrections described above and their uncertainties are propagated to the \( E_{\text{miss}} \) calculation. Additionally, the scale and resolution uncertainty of the calorimeter clusters that were not associated with any physics object (the CellOut term) has to be taken into account.

**b-tagging calibration** For a given operating point representing a requirement on the MV1 output distribution, efficiencies for various jet flavours to pass the requirement can be evaluated. The efficiencies can be parametrized as a function of their relevant kinematic variables, in particular the jet \( p_T \) and \( \eta \). The \( b \)-tagging efficiency from simulation can be confirmed in measurements using a variety of techniques, and scale factors can be derived which correct the MC efficiency to the one observed in data.

The technique used to obtain scale factors for the published version of this analysis is based on isolating a pure sample of \( t\bar{t} \) in data using a dedicated *dileptonic kinematic selection* [124]. The fraction of \( b \), \( c \) and light jets expected in such a sample are evaluated from simulations and the \( b \)-tagging efficiency can be obtained by measuring the fraction of \( b \)-tagged jets in this data sample.

By the time of writing of the thesis, a better calibration with smaller uncertainties had been derived, combining the results of the dileptonic \( t\bar{t} \) kinematic selection and of the System8 method, described in [125]. The System8 method uses \( b \)-jets involving semileptonic decays. The method is designed to minimize the dependence on simulation by applying three independent selection criteria to a data sample containing a muon associated with a jet. Only two out of the three selection criteria are applied at a time, resulting in 8 regions of phase space for which comparisons can be made between observed and expected event counts. These eight observables are sufficient to solve a
system of equations with eight unknowns: the efficiencies of $b$ and non-$b$ jets to pass each of the three selection criteria, and the initial number of $b$ and non-$b$ jets present in the sample. The three selection requirements are the tagging weight under study, the momentum of the muon relative to the jet+muon axis, $p_T^{rel}$, and a requirement of a $b$-tagged jet on the opposite side of the detector with respect to the jet under study. The $b$-tagging efficiencies can thus be obtained for simulated samples and in data, and a calibration can be extracted.

It is this latter, combined calibration that was used for the results presented in this chapter, which has the advantage of smaller systematic uncertainties. However, the System8 and dileptonic kinematic selection calibrations agree within their uncertainties. Similar scale factors that correct for the efficiency in data of tagging $c$ and light jets as $b$-jets are derived using methods described in [126, 127].

The method by which the $b$-tagging efficiency in MC is corrected to that observed in data is as follows. Scale factors (SF) for each flavour are defined as

$$SF_{\text{flavour}}(p_T) = \frac{\epsilon_{\text{data}}}{\epsilon_{\text{MC}}}$$

where $\epsilon_{\text{data}} / \epsilon_{\text{MC}}$ is the efficiency of $b$-tagging a jet of a certain flavour in data / MC. Calibration scale factors are used to determine a per-jet weight. If a jet is $b$-tagged, the weight is taken to be the scale factor itself. If the jet is not $b$-tagged, the weight is given by

$$w_{\text{jet}} = 1 - \frac{\epsilon_{\text{data}}}{1 - \epsilon_{\text{MC}}}.$$  

An event weight is obtained by the product of all weights of the individual jets in the event and is included in all MC estimations of event yields. Since scale factors are determined independently for $b$-jets, $c$-jets and light jets, their uncertainties are uncorrelated.

Scale factors correcting the identification or trigger efficiencies in MC to those observed in data are applied to all events in the simulated samples.
7.8 Systematic uncertainties

7.8.2 Further experimental uncertainties

Further detector-related uncertainties need to be taken into account:

Luminosity The uncertainty on the integrated luminosity is ±2.8%. It is derived following the methodology detailed in [128], from a preliminary calibration of the luminosity scale derived from beam-separation scans performed in November 2012.

Pileup reweighting In order to simulate pileup, several minimum bias events are overlaid on the nominal interaction. However, at the time of Monte Carlo simulation, the pileup conditions and therefore the $\mu$ distribution of the number of interactions per bunch crossings in data are unknown and the MC is produced with a guessed distribution. The simulated events are reweighted at a later time such that the $\mu$ distribution of the MC matches the data. An uncertainty is associated with the pileup scaling factor due to the fact that at given $\mu$, the minimum bias vertex multiplicity does not agree, and the best description of this distribution is obtained when the MC $\mu$ is scaled up by 9%. An associated uncertainty is determined by changing the average number of interactions per bunch crossing by 10% up and down in MC. The impact of this uncertainty on the SR and CR yields was found to be lower than 1% and was neglected.

JVF The effect of the JVF distribution not being accurately described in the simulation is taken into account by varying the JVF requirement from 0.0 to 0.1. This only has an effect in SRB, where a JVF requirement is made for the jets entering the $H_{T,3}$ calculation.

7.8.3 Background theoretical uncertainties

While the author has evaluated these uncertainties in previous iterations of the analysis, these have been re-evaluated by collaborators for the re-optimised selection. For completeness, this subsection gives an account of the general strategy. The theoretical uncertainties on the Monte Carlo are evaluated by comparing different samples at truth level. The uncertainties
7.8 Systematic uncertainties

are propagated to the histograms used to estimate the backgrounds in the fit.

For each background, the different uncertainties are assumed to be independent of each
other and therefore they are added in quadrature to give an overall systematic uncertainty
for that particular process. The uncertainties are derived for each SR, VR and CR and are
treated in the statistical fit as fully correlated between regions. For the SRs with high $m_{CT}$
requirements, most samples only have very few events that are in the required region of phase
space. Therefore large statistical uncertainties might be associated with this procedure, and
the theoretical uncertainties cannot be trusted. In such cases, the theoretical uncertainties
derived for lower $m_{CT}$ requirements are used at high $m_{CT}$ values as well. Requirements on
$E_T^{\text{miss}}/m_{\text{eff}}$ and $\Delta\phi_{\text{min}}$ have been removed from all selections.

Top pair production

Different theoretical uncertainties were evaluated as follows:

Monte Carlo Generator uncertainty: Generators with the same parton shower model
are compared, POWHEG BOX + HERWIG and ALPGEN + HERWIG.

Parton Shower uncertainty: Samples produced with the same generator interfaced with
different parton shower models are compared: POWHEG BOX + PYTHIA 6 vs.
POWHEG BOX + HERWIG.

Scale uncertainties: Dedicated POWHEG BOX + PYTHIA 6 samples exist for the evalu-
ation of scale uncertainties, with the renormalisation and factorisation scales changed
by a factor of two up and down.

ISR/FSR uncertainty: Samples generated with AcerMC exist, interfaced with PYTHIA
6. In the parton shower model, two tunes with varied parton shower parameters exist,
“morePS” and “lessPS”.

PDF uncertainties: The baseline POWHEG BOX + PYTHIA 6 sample was produced
using the NLO PDF set CT10. To evaluate the PDF uncertainty, the events were
The total $t\bar{t}$ uncertainty varies between 9% and 41% depending on region. The dominant uncertainty comes from differences in the predictions of different Monte Carlo generators, followed by parton shower and scale uncertainties. ISR/FSR and PDF uncertainties are mostly negligible. The single top process is added to the $t\bar{t}$ contribution and the same uncertainties are considered in a fully correlated manner. The alternative approach of using dedicated single top samples with varied theoretical parameters led to too large statistical fluctuations, while results were compatible with $t\bar{t}$ uncertainties.

**$Z/W+HF$**

Dedicated samples which include heavy flavour jets are used as follows:

**Monte Carlo Generator uncertainty** The uncertainty is evaluated at truth level by comparing ALPGEN +PYTHIA 6 with the nominal SHERPA sample.

**Scale uncertainties** No dedicated SHERPA samples exist to evaluate scale uncertainties. However, it is possible to have a very rough estimate of these uncertainties using the alternative ALPGEN samples with varied renormalisation and factorisation scales. Four samples are investigated, in which each of the scales is varied individually up and down by a factor of 2.

**PDF uncertainties** Same procedure as in the top theory uncertainty case.

**Heavy flavour component** In the case of $W+HF$, an additional 24% is added in quadrature, which is the full (systematic + statistical) uncertainty on the measured fiducial $W + b$ cross-section in the 2 jets bin [129].

Depending on region, the total uncertainties vary between between 51% and 74% for $Z+HF$ and between 56% and 86% for $W+HF$. The $Z$ uncertainties are dominated by scale uncer-
Systematic uncertainties, while for $W$ production, both the generator and the scale uncertainties contribute significantly to the total uncertainty.

Other: $tt + W/Z$ and di-boson uncertainties

These backgrounds are grouped under the “Other” category. The theoretical uncertainties on the cross-sections are 30% for $ttW$ and $ttZ$ ([130, 131]) and 7% for di-bosons. Given the minor impact of these backgrounds in the final event yield and the fact that there could be some shape uncertainties not accounted for, which are difficult to estimate in the regions proposed by this analysis, an overall 30% conservative uncertainty has been assigned to all of them.

7.8.4 Signal uncertainties

In the statistical treatment, the experimental uncertainties on the signal are assumed to be fully correlated with those of the backgrounds. The most important experimental uncertainties are the $b$-tagging uncertainty (effect between 20% and 30% in SRA, and 15% to 30% in SRB before the fit in the CRs) and the JES (3-30% in SRA, 20-40% in SRB before the fit in the CRs).

The theoretical uncertainties on the cross-section due to the PDF and the renormalisation and factorisation scales have been already discussed in 7.1. For SRB, which relies on the presence of an ISR jet, further uncertainties due to the modeling of ISR/FSR processes are taken into consideration.

ISR / FSR related uncertainties

The SRB signal region has been optimised for the cases of sbottom pair production with small mass differences $\Delta m(\tilde{b}_1,\tilde{\chi}^0_1)$, and relies on the presence of initial state radiation. The smaller $\Delta m$, the more the signal events need to rely on ISR in order to enter the signal region,
therefore models with low $\Delta m$ are expected to be particularly sensitive to systematic uncertainties in the description of ISR. Both SRA and SRB selections rely on rejecting hadronic activity in the event in order to suppress the $t\bar{t}$ background, which makes the selections sensitive to final state radiation in case of the signal. Uncertainties in the $\alpha_s$, the factorisation and renormalisation scale, the strength of the parton shower and the matching scale used for merging the matrix element and the parton shower calculations are all expected to affect the amount of ISR and FSR in the event and their evaluation is crucial to a robust prediction of the expected signal. In order to assess the theoretical uncertainties related to the ISR/FSR, the author has simulated dedicated samples. For 8 signal points in the sbottom-neutralino mass plane ($m_{\tilde{b}_1} = 350$ GeV, 250 GeV, $\Delta m(\tilde{b}_1, \tilde{\chi}^0_1) = 10, 30, 50, 100$ GeV), “up” and “down” variations for four types of variations were generated:

- **MadGraph** samples are produced using a variable factorization and renormalization scale, set to the central $m_T^2$ scale after $k_T$-clustering of the event. For a single heavy particle, this corresponds to the $M^2 + p_T^2$, for a pair of heavy particles like two sbottoms it is the geometric mean of $M^2 + p_T^2$ for each particle, for massless particles it is the $p_T^2$ of the last pair after clustering. The `scalefact` parameter multiplies this scale. The default signal plane uses `scalefact = 1.0`, the `scaleup`, `scaledown` samples use 2.0 and 0.5.

- The signal samples are produced using the MLM matching procedure (see [59]). One important parameter in this algorithm is the matching scale `qcut`, which is the maximal distance between a parton and a jet to be matched with each other. The parameter uses the $k_T$ measure as distance definition, where $k_T = \min(p_{T,\text{jet}}^2, p_{T,\text{parton}}^2) \frac{\Delta R(\text{parton, jet})^2}{R^2}$ with $R$ being the jet-radius parameter, $R = 0.4$. The `qcut` is set by default to $1/4 \times m_{\tilde{b}_1}$. The variation samples `qcutup`, `qcutdown` multiply the matching scale by 2.0 and 0.5.

- **alpsfact** - scale factor for QCD emission vertex. The evolution scale at which $\alpha_S$ is evaluated in MadGraph is multiplied by `alpsfact`. The equivalent parameter in Pythia is PARP(64), which controls the scale at which $\alpha_S$ is evaluated in ISR branchings. PARP(64) is varied consistently at the same time with `alpsfact`. The variations for `alpsfact` are 0.5 and 2.0, corresponding to PARP(64) variations of 0.25.
and 4, respectively. The lower the value of alpafact, the higher $\alpha_S$ and therefore the amount of ISR.

- **FSR tunes** - they change the strength of the parton shower by modifying Pythia parameters PARP(72) and PARJ(82). PARP(72) controls the lambda value for timelike showers (FSR showers), and is varied between 0.2635 and 0.7905. PARJ(82) sets the FSR parton-shower cut-off scale and is modified between 0.5 and 1.66.

All variation samples are normalised to the NLO+NLL cross-section, such that the above variations measure the uncertainty due to the difference in shape of various distributions rather than the uncertainty in the overall normalisation, which is already accounted for in the cross-section uncertainty. For each of the 4 sets of variations above, the following ratio is computed:

$$ r = \frac{A^\uparrow - A^\downarrow}{A^\uparrow + A^\downarrow}, \quad (7.15) $$

where $A^\uparrow$ ($A^\downarrow$) indicates the acceptance in SRB for the variation up (down) sample. For each variation binomial errors are assumed, $\sigma = \sqrt{N_{SRB}}$, where $N_{SRB}$ is the number of events SRB. The statistical uncertainties for each variation are propagated to give a statistical uncertainty on the ratio. Results for individual sets of variations are shown in figure 7.16.

The uncertainties due to individual sources are added in quadrature to give the total systematic uncertainty shown in table 7.7 and figure 7.17. Given the compatible results for both sbottom masses considered, the final uncertainty is taken as the mean value of the two results per mass difference. The total uncertainties are between 11% at large mass splittings $\Delta m(\tilde{b}_1, \tilde{\chi}_1^0) \geq 100$ GeV and increase up to 29% at low mass splittings $\Delta m = 10$ GeV. At the lowest mass splitting for which the uncertainties are calculated, the statistical uncertainty on these is large, at the level of 18%.

Since the samples used to derive the uncertainties are distributed in $\Delta m(\tilde{b}_1, \tilde{\chi}_1^0)$ in the same way as the points in the signal plane, no interpolation for intermediate values of $\Delta m$ is necessary. For $\Delta m \geq 100$ GeV, the total uncertainty is taken as the same value as for the $\Delta m = 100$ GeV point. A conservative approach was taken for SRA and the same
7.8 Systematic uncertainties

Figure 7.16: Magnitude of various signal theory systematic uncertainties calculated as (Yield from variation up - Yield from variation down)/(Sum of yields) versus the mass difference $\Delta m(\tilde{b}_1, \tilde{\chi}_0^1)$. The uncertainties shown are, clockwise from top left: alpsfact, matching scale qscale, scalefact, FSR Pythia tunes. The purple (blue) squares indicate the magnitude of the variation for signal models with $m_{\tilde{b}_1} = 250$ GeV (350 GeV).

<table>
<thead>
<tr>
<th>$\Delta M$ (GeV)</th>
<th>Average Uncertainty (%)</th>
<th>Statistical error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>28.7</td>
<td>18.1</td>
</tr>
<tr>
<td>30</td>
<td>7.6</td>
<td>6.1</td>
</tr>
<tr>
<td>50</td>
<td>8.0</td>
<td>5.1</td>
</tr>
<tr>
<td>100</td>
<td>11.2</td>
<td>6.5</td>
</tr>
</tbody>
</table>

Table 7.7: Total signal ISR/FSR systematic uncertainty obtained by averaging the systematic uncertainties derived for the $m_{\tilde{b}_1} = 250$ GeV and $m_{\tilde{b}_1} = 350$ GeV points.

Contrary to expectations, no strong dependence is observed on $\Delta m(\tilde{b}_1, \tilde{\chi}_1^0)$ or on $m_{\tilde{b}_1}$ for any of the studied uncertainties, within statistical uncertainties. This can be further
7.8 Systematic uncertainties

Figure 7.17: Signal total systematic uncertainty, obtained by adding in quadrature the alpsfact, matching scale, final state radiation and scalefact uncertainties. Left: uncertainties derived using the yields in SRB after the full selection including the $H_{T,3}$ requirement, separately for the $m_{\tilde{b}_1} = 350$ GeV and $m_{\tilde{b}_1} = 250$ GeV points; right: signal total systematic uncertainty as a function of $\Delta m$, obtained by averaging the uncertainty for the $m_{\tilde{b}_1} = 350$ GeV and 250 GeV points. The statistical uncertainties for the points with the same $\Delta m$ are added in quadrature.

understood by inspecting table 7.8, which contains the acceptances for two samples with varied alpsfact parameter after each selection requirement. The difference between the two samples increases significantly for example when requiring 3 jets with $p_T > 20$ GeV, which is consistent with the effect of alpsfact of varying the strength of ISR. Increasing the amount of ISR leads to higher acceptance after this requirement. The last requirement on $H_{T,3}$ has the opposite effect of reducing the acceptance for the high ISR variation. The increased radiation helped initially to provide additional jets in the event and to increase acceptance, the veto on the jet activity beyond the 3rd leading jet now cancels this effect. Figure 7.18 shows how the uncertainty due to the alpsfact variation changes throughout the selection for all simulated points. Similar cancelling effects are at play for the other uncertainties, giving overall the almost flat distribution in figure 7.17.
Table 7.8: Signal acceptances for samples in which the \textit{alpsfact} parameter is varied, for the point with $m_{\tilde{b}_1} = 250$ GeV, $m_{\tilde{\chi}^0_1} = 240$ GeV. \textit{alpsfact} $\uparrow$ corresponds to less ISR, therefore a negative uncertainty implies that less ISR reduces the acceptance. The samples were produced with a truth-$E_T^{\text{miss}}$ filter with a requirement at 80 GeV, which has an acceptance of about 30% for the points shown.

<table>
<thead>
<tr>
<th>Requirement</th>
<th>Acceptance \textit{alpsfact} $\uparrow$ (%)</th>
<th>Acceptance \textit{alpsfact} $\downarrow$ (%)</th>
<th>Derived uncertainty \textit{alpsfact} $\uparrow$−\textit{alpsfact} $\downarrow$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Electron Veto</td>
<td>28.0</td>
<td>33.4</td>
<td>-8.8</td>
</tr>
<tr>
<td>Muon Veto</td>
<td>27.4</td>
<td>32.7</td>
<td>-8.8</td>
</tr>
<tr>
<td>$E_T^{\text{miss}} &gt; 250$ GeV</td>
<td>3.10</td>
<td>3.85</td>
<td>-10.9</td>
</tr>
<tr>
<td>at least 3 jets with $p_T &gt; 20$ GeV</td>
<td>1.71</td>
<td>2.46</td>
<td>-17.8</td>
</tr>
<tr>
<td>leading jet $p_T &gt; 150$ GeV, 2nd, 3rd jet $p_T &gt; 30$ GeV</td>
<td>1.02</td>
<td>1.52</td>
<td>-19.5</td>
</tr>
<tr>
<td>$\Delta \phi($leading jet, $p_T^{\text{miss}})$</td>
<td>0.93</td>
<td>1.37</td>
<td>-19.2</td>
</tr>
<tr>
<td>at least one $b$-jet</td>
<td>0.32</td>
<td>0.47</td>
<td>-18.7</td>
</tr>
<tr>
<td>exactly 2 $b$-jets</td>
<td>0.058</td>
<td>0.089</td>
<td>-20.7</td>
</tr>
<tr>
<td>leading $b$-jet veto</td>
<td>0.009</td>
<td>0.012</td>
<td>-13.2</td>
</tr>
<tr>
<td>second,third jets are $b$-jets</td>
<td>0.009</td>
<td>0.012</td>
<td>-13.2</td>
</tr>
<tr>
<td>$\Delta \phi_{\text{min}}(p_T^{\text{miss}}, j1, 2, 3)$</td>
<td>0.007</td>
<td>0.008</td>
<td>-6.8</td>
</tr>
<tr>
<td>$E_T^{\text{miss}}/M_{\text{eff}}$</td>
<td>0.007</td>
<td>0.008</td>
<td>-6.8</td>
</tr>
<tr>
<td>$H_{T,3} &lt; 50$ GeV</td>
<td>0.006</td>
<td>0.006</td>
<td>1.3</td>
</tr>
</tbody>
</table>

Figure 7.18: Signal systematic uncertainties for the \textit{alpsfact} variation at different stages in the selection. A negative uncertainty corresponds to the acceptance being larger for the sample with more ISR (\textit{alpsfact} $\downarrow$). Left: after the requirement of 3 jets with $p_T > 20$ GeV. Center: before the $H_{T,3}$ requirement. Right: after the $H_{T,3}$ requirement. Table 7.8 provides the full selection.
7.9 Statistical treatment with HistFitter

The statistical model used to fit the MC contributions to data in the control regions, and to assess the sensitivity of the analysis to generic BSM models and to SUSY models, was built using the ATLAS-internal HistFitter framework. HistFitter uses the HistFactory tool [132], which is part of the RooStats package [133]. While the HistFitter treatment was initially performed by collaborators for the paper result in reference [5], the results were reproduced by the author for the purpose of this thesis and a potential improvement in the statistical treatment was tested.

The “background-only fit” is described first. The HistFitter framework sets up the statistical model in form of a likelihood function. By maximising the likelihood, the background normalisations that best describe the data in the control regions can be extracted. The different terms that enter the likelihood function are as follows:

- Poisson probability to observe $n_c$ events when $\nu_c$ events are expected in channel $c$ (i.e. control region), given by

$$\text{Pois}(n_c|\nu_c) = \frac{\nu_c^n}{n_c!} \exp(-\nu_c).$$

(7.16)

The expectation $\nu_c$ is summed over all background samples $s$ and is given by

$$\nu_c = \sum_s \lambda_c \mu_s \eta_s,\nu_c(\alpha).$$

(7.17)

$\lambda_c$ is the luminosity accumulated in channel $c$. $\mu_s$ is a per-sample normalisation factor and the $\eta_s(\alpha)$ corresponds to an overall shift up or down in $\nu_c$ as an effect from shifting up or down the source of systematic uncertainty like JES, JER, $b$-tagging, etc. by an amount $\alpha$. They are explained in detail below.

- the parameter $\lambda_c$ is constrained by a Gaussian probability function $G(\lambda_c|L_0, \Delta_L)$ centered at the measured luminosity value $L_0$ and with width $\Delta_L$ representative of the uncertainty in the luminosity measurement.
within ATLAS, performance groups provide a nominal calibration, as well as “±1σ” values of the calibration as constrained by data. These calibrations are mapped to the parameters $\alpha_p$, where $p$ is an index counting the sources of systematic uncertainties. $\alpha_p = 0$ corresponds to the nominal calibration as measured by the performance group, and $\alpha_p = \pm 1$ correspond to the “±1σ” variations. The values $\eta_s(-1)$ and $\eta_s(+1)$ are the relative values of the event yields with respect to the $\alpha_p = 0$ case for which $\eta_s(0) = 1$, and can be obtained by running the analysis with the nominal calibration, and then repeating with the ±1σ JES, $b$-tagging, etc. values. For intermediate values of $\alpha_p$, $\eta_s$ is interpolated. There are many interpolation methods (described in [132]), and they are all arbitrary to a certain extent. The selected option is a recipe with a piecewise exponential interpolation strategy which guarantees that $\eta(\alpha_p) > 0$. The $\alpha_p$ nuisance parameters are profiled over in the fit. However, auxiliary measurements, for example that of the JES, have already constrained the extent by which the JES can vary and correspondingly in the likelihood function, any deviation of the $\alpha_p$ parameter too far from the nominal calibration value should be penalised. This is obtained by introducing Gaussian constraints $P_p(a_p|\alpha_p)$ with $a_p$ being the value of the calibration measured in the auxiliary measurement of the source. Because of the mapping used, $a_p = 0$ and the constraint term on the true value of the calibration, $\alpha_p$, is a Gaussian centered at 0 and with width 1. In the case in which a background process has a floating normalisation, and a source of systematic uncertainty (like the JES) is fully correlated between the SR and CRs, the uncertainty can be factorised in an overall normalisation factor and a residual term. The overall normalisation factor is absorbed in the floating $\mu_{bkg}$ parameter, thus effectively reducing the impact of the uncertainty source.

Using these terms, the probability of the data given the model parameters can be written as

$$P(n_c, a_p|\mu_s, \alpha_p) = \prod_c \text{Pois}(n_c|\nu_c) \cdot G(L_0|\lambda, \Delta_L) \cdot \prod_p P_p(a_p|\alpha_p).$$  \hspace{1cm} (7.18)$$

The probability distribution can be reinterpreted as a likelihood $L(\mu_s, \alpha_p|n_c, a_p)$. By max-
7.9 Statistical treatment with HistFitter

<table>
<thead>
<tr>
<th>Parameter</th>
<th>SRA</th>
<th>SRB</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu_{t\bar{t}}$</td>
<td>1.04 ± 0.12</td>
<td>1.06 ± 0.08</td>
</tr>
<tr>
<td>$\mu_Z$</td>
<td>1.20 ± 0.17</td>
<td>0.82 ± 0.19</td>
</tr>
<tr>
<td>$\mu_W$</td>
<td>1.06 ± 0.52</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 7.9: Dominant background normalisations from fit.

imising the likelihood function, $\mu_{t\bar{t}}$, $\mu_Z$, and in SRA $\mu_W$ can be extracted. This is called a “background-only” fit.

In the case when exclusion of a SUSY model is attempted, an additional Poisson term for the observed and expected number of events in the signal region is added and the $\nu_c$ expected number of events in the signal and control regions have an additional contribution from the signal, characterised by a signal strength $\mu_{\text{sig}}$. By construction, the fit takes into account correlations due to common systematic uncertainties in different CRs, the fact that a given background can contribute to several CRs, and potential contamination from the SUSY signal to the CRs when the sensitivity to a given model is assessed.

7.9.1 Background-only fit results

Appendix B shows the results of the background-only fit for the nuisance parameters and the parameters of interest, as well as the corresponding correlation matrix. Table 7.9 shows the fit results for the parameters of interest only. The $t\bar{t}$ normalisation is very similar for both SRA and SRB and consistent with 1.0, but the contribution of $Z$ produced in association with heavy flavour jets is scaled differently in SRA and SRB.

The effect of the fit is to improve the MC to data agreement in the control regions by adjusting the normalisation of the dominant backgrounds. Tables 7.10 7.11 report the number of data events observed in each control region and the standard model background expectation before and after the fit.

The following section projects the fit results obtained from the CRs to the signal regions.
### Channel CRA

<table>
<thead>
<tr>
<th>Event Type</th>
<th>Total</th>
<th>CRASF</th>
<th>CRADF</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Observed events</strong></td>
<td>138</td>
<td>71</td>
<td>72</td>
</tr>
<tr>
<td><strong>Fitted background events</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Total SM</strong></td>
<td>$138 \pm 12$</td>
<td>$71 \pm 8$</td>
<td>$72 \pm 8$</td>
</tr>
<tr>
<td>Top-quark production</td>
<td>$89 \pm 16$</td>
<td>$11 \pm 2$</td>
<td>$71 \pm 9$</td>
</tr>
<tr>
<td>W production</td>
<td>$39 \pm 19$</td>
<td>$&lt; 0.1$</td>
<td>$&lt; 0.1$</td>
</tr>
<tr>
<td>Z production</td>
<td>$0.41 \pm 0.10$</td>
<td>$60 \pm 9$</td>
<td>$&lt; 0.1$</td>
</tr>
<tr>
<td>Others</td>
<td>$9.2 \pm 3.6$</td>
<td>$0.29 \pm 0.13$</td>
<td>$0.84 \pm 0.28$</td>
</tr>
</tbody>
</table>

### Table 7.10: Results of the fit for the control regions adopted for SRA. Expected yields derived from MC simulations using theoretical cross-sections are given for comparison. The uncertainties shown include the statistical and detector systematic uncertainties. The central values of the fitted sum of backgrounds in the control regions agree with the observations by construction. The uncertainty on the total background estimate can be smaller than the sum of the individual uncertainties due to correlations.

### Channel CRB

<table>
<thead>
<tr>
<th>Event Type</th>
<th>Total</th>
<th>CRBSF</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Observed events</strong></td>
<td>433</td>
<td>46</td>
</tr>
<tr>
<td><strong>Fitted background events</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Total SM</strong></td>
<td>$433 \pm 21$</td>
<td>$46 \pm 7$</td>
</tr>
<tr>
<td>Top-quark production</td>
<td>$394 \pm 31$</td>
<td>$15 \pm 2$</td>
</tr>
<tr>
<td>W production</td>
<td>$31 \pm 22$</td>
<td>$&lt; 0.1$</td>
</tr>
<tr>
<td>Z production</td>
<td>$0.25 \pm 0.14$</td>
<td>$30 \pm 7$</td>
</tr>
<tr>
<td>Others</td>
<td>$8.1 \pm 2.8$</td>
<td>$0.90 \pm 0.44$</td>
</tr>
</tbody>
</table>

### Table 7.11: Results of the fit for the control regions adopted for SRB. Expected yields derived from MC simulations using theoretical cross-sections are given for comparison. The uncertainties shown include the statistical and detector systematic uncertainties. The central values of the fitted sum of backgrounds in the control regions agree with the observations by construction. The uncertainty on the total background estimate can be smaller than the sum of the individual uncertainties due to correlations.
Table 7.12: Results of the fit for the signal regions. The uncertainties shown include the statistical and detector systematic uncertainties. The uncertainty on the total background estimate can be smaller than the sum of the individual uncertainties due to correlations.

### 7.10 Results and interpretation

#### 7.10.1 Projection of background-only fit results to the SRs

Table 7.12 reports the number of data events observed in each signal region and the standard model background expectation after the fit. Figures 7.19, 7.20 show the distribution of the observed data for various kinematic distributions in SRA and SRB, and the comparison with the SM prediction. No significant excess is observed in any of the signal regions.

Tables 7.13, 7.14 show the systematic uncertainties on the SRA and SRB background yields. The dominant uncertainties are those on the normalisation of the $Z$ and $W$, arising from low number of events in the $Z$ control region and the small contribution of $W$ events in the 1-lepton CR.

No significant excess is observed in any of the signal regions. The level of agreement between the standard model prediction and the number of events observed in the signal regions is quantified in the next section, in which a test statistic that characterises the relative goodness of fit for the background-only and the background plus signal hypotheses is introduced. Corresponding $p$-values for the background-only hypothesis will then be derived.
Table 7.13: Breakdown of the dominant systematic uncertainties on background estimates in the various signal regions. Note that the individual uncertainties can be correlated, and do not necessarily add up quadratically to the total background uncertainty. The percentages show the size of the uncertainty relative to the total expected background. Sources of uncertainties which contribute more than 5% are emboldened.
## 7.10 Results and interpretation

### Uncertainty of channel

<table>
<thead>
<tr>
<th>Source of Uncertainty</th>
<th>SRB</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total background expectation</td>
<td>62</td>
</tr>
<tr>
<td>Total statistical ($\sqrt{N_{\text{exp}}}$)</td>
<td>7.9</td>
</tr>
<tr>
<td>Total background systematic</td>
<td>7.2 [11.6%]</td>
</tr>
<tr>
<td>Top-quark normalisation from fit</td>
<td>3.1 [4.9%]</td>
</tr>
<tr>
<td>Top quark production theory</td>
<td>2.5 [4.0%]</td>
</tr>
<tr>
<td><strong>Z normalisation from fit</strong></td>
<td>3.1 [5.0%]</td>
</tr>
<tr>
<td>Z production theory</td>
<td>2.0 [3.2%]</td>
</tr>
<tr>
<td><strong>W production theory</strong></td>
<td>5.6 [8.9%]</td>
</tr>
<tr>
<td>“Other” theory</td>
<td>0.96 [1.5%]</td>
</tr>
<tr>
<td>QCD</td>
<td>0.16 [0.26%]</td>
</tr>
<tr>
<td>CellOut energy scale</td>
<td>0.86 [1.4%]</td>
</tr>
<tr>
<td>CellOut energy resolution</td>
<td>0.19 [0.30%]</td>
</tr>
<tr>
<td>Jet energy scale</td>
<td>1.9 [3.0%]</td>
</tr>
<tr>
<td>Jet energy resolution</td>
<td>2.9 [4.7%]</td>
</tr>
<tr>
<td>b-tagging</td>
<td>0.98 [1.6%]</td>
</tr>
<tr>
<td>c-tagging</td>
<td>0.87 [1.4%]</td>
</tr>
<tr>
<td>light-tagging</td>
<td>0.08 [0.13%]</td>
</tr>
<tr>
<td>JVF</td>
<td>0.95 [1.5%]</td>
</tr>
</tbody>
</table>

Table 7.14: Breakdown of the dominant systematic uncertainties on background estimates in the various signal regions. Note that the individual uncertainties can be correlated, and do not necessarily add up quadratically to the total background uncertainty. The percentages show the size of the uncertainty relative to the total expected background. Sources of uncertainties which contribute more than 5% are emboldened.
Figure 7.19: SRA distributions; from left to right: $m_{CT}$ distribution, $m_{bb}$ distribution (top row). In the top row only, distributions are shown with all selections applied, omitting the requirement on the shown variable, which is indicated by the red arrows. In the middle (bottom) row, the $E_{T}^{miss}$ and leading jet $p_T$ (second and third leading jet $p_T$) distributions are shown in SRA with an $m_{CT}$ requirement at 150 GeV. Distributions in the middle and bottom rows are shown with all selections applied. The first bin in the third leading jet $p_T$ distribution indicates the number of events which have no third jet with $p_T > 20$ GeV, which is the reconstruction threshold used in this analysis. The $t\bar{t}$, $Z$ and $W$ contributions have been normalised in all plots using the values obtained from the fit in the control regions.
Figure 7.20: SRB distributions; from left to right: $E_T^{\text{miss}}$ distribution; $H_{T,3}$ distribution (top row), leading and sub-leading jet $p_T$ (middle row) and third jet $p_T$ (bottom row). For the top row, distributions are shown with all selections applied, omitting the requirement on the shown variable, which is indicated by the red arrows. The first bin in the $H_{T,3}$ distribution indicates the number of events which have no more than three jets with $p_T > 20$ GeV, which is the reconstruction threshold used in this analysis. Because of this threshold, if any jets enter the $H_{T,3}$ calculation, the lowest value it can take is 20 GeV. The $t\bar{t}$ and $Z$ contributions have been normalised in all plots using the values obtained from the fit in the control regions.
7.10.2 Model-independent limits

As no significant excess is observed in any of the signal regions, the results are used to derive upper limits on the number of beyond the Standard Model (BSM) events for each signal region. This is done assuming no systematic uncertainties on the (unknown type of) signal events and assuming no signal contamination in the control regions. The model in eq. 7.18 is extended by the addition of only one new parameter $\mu_{\text{sig}}$, which quantifies the signal strength. The new parameter contributes to the expected number of events in the SRs, $\nu_{\text{SR}}$. From now onwards $\mu_{\text{sig}}$ is referred to as $\mu$, while the background normalisations are considered nuisance parameters. The following test statistic, called the profile log-likelihood ratio, is defined:

$$
q_\mu = \begin{cases} 
\lambda(\mu) = -2 \ln \frac{L(\mu, \hat{\theta}(\mu))}{L(\hat{\mu}, \hat{\theta})}, & \mu > \hat{\mu} \\
0, & \mu < \hat{\mu} 
\end{cases}
\tag{7.19}
$$

$\hat{\mu}, \hat{\theta}$ are the maximum likelihood estimators, i.e. the values of $\mu, \theta$ that maximise the likelihood, and $\hat{\theta}(\mu)$ is the value of $\theta$ that maximises the likelihood for a given value of $\mu$. Notice only values of $\mu$ larger than the best fit result $\hat{\mu}$ are allowed. The test statistic distribution for the profile log-likelihood ratio, $f(q_\mu|\mu, \hat{\theta}(\mu))$, can be found analytically in the limit of large number of events ($[134]$). Alternatively, if the number of events is small, more accurate results are obtained by using a Monte Carlo technique, as has been performed in this analysis. Many ($\mathcal{O}(10^3)$) pseudo-datasets are sampled from the model probability distribution function in 7.18 and the value of the test statistic is calculated for each one to obtain a simulated $f(q_\mu)$. The $p$-value for a signal strength $\mu$ is given by

$$
p_{s+b}(\mu) = \int_{q_{\mu,\text{obs}}}^{\infty} f(q_\mu|\mu) dq_\mu. 
\tag{7.20}
$$

For 95% confidence level exclusion of the signal strength $\mu$, the $p$-value of the signal plus background hypothesis is required to be $p_{s+b} < 0.05$. The $\mu$ parameter is scanned over and for each step, the test statistic distribution $f(q_\mu)$ is simulated and the integral above is evaluated, until the necessary $p_{s+b}$-value is reached. The model independent limits on the
Results and interpretation

The number of BSM signal events in the SRs are found using this procedure and the results are given in table 7.15. These limits can be interpreted as an upper limit on the visible BSM cross-section $\sigma_{\text{vis}} = \sigma \cdot A \cdot \epsilon$, where $\sigma$ is the BSM production cross-section, $A$ is the detector and analysis acceptance and $\epsilon$ is the detector efficiency.

Using a similar approach, the agreement of the background-only hypothesis with the data can be quantified in order to conclusively state that no excess is observed. The test statistic 7.19 is modified to

$$q_{\mu=0} = \begin{cases} -2 \ln \frac{\mathcal{L}(\mu = 0, \hat{\theta}(\mu = 0))}{\mathcal{L}(\hat{\mu}, \hat{\theta})} & , \hat{\mu} > 0 \\ 0 & , \hat{\mu} < 0 \end{cases}$$

(7.21)

Notice the difference in definition of this test statistic compared to $q_{\mu}$ in eq. 7.19 as now the best fitted signal strength is allowed to take up any non-negative value. The test statistic $q_0$ compares the background-only hypothesis (signal strength $\mu = 0$) with the hypothesis with $\mu = \hat{\mu}$ that best fits the data. One can determine the probability $p_b$ (also denoted CL$_b$) for the $q_0$ test statistic to be at least as incompatible with the data as the observed value. Mathematically, $p_b$ is given by eq. 7.20 with $\mu = 0$ and the appropriate distribution for the $q_0$ statistic. A value $p_b < 0.05$ excludes the background-only hypothesis at 95% confidence level. Table 7.15 contains the $p_b$ values for all SRs.

The $p_{s+b}$ value has the disadvantage that in the case of data underfluctuation in the signal region, signals will be excluded even if the analysis is not expected to have any sensitivity to them. This can be seen as follows. Let us assume the number of expected signal events is much lower than the number of expected background events, $\nu_{\text{sig}} \ll \nu_{\text{bckg}}$ and $\nu_{\text{sig+bckg}} \approx \nu_{\text{bckg}}$. If the data shows an underfluctuation in the signal regions, both the $p_{s+b}$ and $p_b$ would be low, i.e. both the background-only and the signal plus background hypothesis would be excluded, even if there is no true sensitivity to the signal model. The exclusion power for the signal plus background hypothesis needs to be penalised if the data is inconsistent with the background-only hypothesis. Therefore the $p_{s+b}$ value is corrected
Results and interpretation

conservatively by the $p_b$ value of the background-only hypothesis to give the CL$_s$ value

$$\text{CL}_s = \frac{p_{s+b}}{p_b}. \tag{7.22}$$

Notice that while CL$_s$ is not a true $p$-value, it is always larger than the CL$_{s+b}$ value. This is a widely adopted prescription in ATLAS and CMS and is further described in [135]. Both the model-independent limits in table 7.15 and the model-dependent limits presented in the following section are calculated using $p_s$ values.

The observed model-independent and model-dependent limits are calculated as above using the observed yields in the signal and control regions to calculate the likelihood $\mathcal{L}$. For the expected limits, a large number, $O(10^3)$ MC simulations are performed to obtain pseudo-data. The pseudo-data yields in each CR and SR are obtained by sampling from the probability distribution in eq. 7.18 with $\mu = 0$ and the set of nuisance parameters $\hat{\Theta}(\mu = 0, \text{observed data})$ obtained from the background-only fit to the real data. For each pseudo-dataset, the parameters $\hat{\Theta}(\mu, \text{pseudo-data})$ and the distribution of the test statistic, $f(q_\mu | \mu, \hat{\Theta}(\mu, \text{pseudo-data}))$ are evaluated. A CL$_s$ value is calculated for each pseudo-experiment, and these values are histogrammed. The median value of this ensemble is taken to be the median expected CL$_s$ value. The $\pm 1\sigma$ CL$_s$ values are calculated such that the integral of the CL$_s$ distribution between $-\infty$ and CL$_s^{-1\sigma}$ and between CL$_s^{+1\sigma}$ and $\infty$ each gives an area equal to $\Theta(1)$, where $\Theta(x)$ is the cumulative distribution function of the standard normal distribution with mean 0 and variance 1.

### 7.10.3 Model-dependent limits

The results are also interpreted in the context of simplified SUSY models, in which the sbottom or the stop squarks are the only strongly interacting particles that are kinematically accessible. The assumed mass hierarchy is such that the lightest third-generation squark decays exclusively via $\tilde{b}_1 \rightarrow \tilde{\chi}^0_1$ or $\tilde{t}_1 \rightarrow \tilde{\chi}^\pm_1$. In order for the stop pair production events to have a signature similar to that of the sbottom pair production scenario, the $\Delta m$ between
Table 7.15: Expected and observed event yields with the corresponding upper limits on BSM signal yields and $\sigma_{\text{vis}} = \sigma \cdot A \cdot \epsilon$ for all the signal regions defined.

<table>
<thead>
<tr>
<th>Signal Regions</th>
<th>Background estimate</th>
<th>Observed data</th>
<th>$p_b$</th>
<th>95% CL upper limit on BSM event yield</th>
<th>$\sigma_{\text{vis}}$ (fb)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>exp.</td>
<td>obs.</td>
</tr>
<tr>
<td>SRA ($m_{\text{CT}} &gt; 150$ GeV)</td>
<td>97 ± 12</td>
<td>104</td>
<td>0.66</td>
<td>32$^{+12}_{-8.7}$</td>
<td>36</td>
</tr>
<tr>
<td>SRA ($m_{\text{CT}} &gt; 200$ GeV)</td>
<td>40 ± 6</td>
<td>49</td>
<td>0.83</td>
<td>18$^{+7.4}_{-5.0}$</td>
<td>25</td>
</tr>
<tr>
<td>SRA ($m_{\text{CT}} &gt; 250$ GeV)</td>
<td>16 ± 2</td>
<td>14</td>
<td>0.33</td>
<td>10$^{+4.5}_{-3.0}$</td>
<td>8.5</td>
</tr>
<tr>
<td>SRA ($m_{\text{CT}} &gt; 300$ GeV)</td>
<td>5.9 ± 1.0</td>
<td>7</td>
<td>0.65</td>
<td>6.5$^{+3.3}_{-2.1}$</td>
<td>7.4</td>
</tr>
<tr>
<td>SRA ($m_{\text{CT}} &gt; 350$ GeV)</td>
<td>2.4 ± 0.5</td>
<td>3</td>
<td>0.61</td>
<td>4.6$^{+2.6}_{-1.5}$</td>
<td>5.1</td>
</tr>
<tr>
<td>SRB</td>
<td>62 ± 7</td>
<td>65</td>
<td>0.59</td>
<td>22$^{+8.9}_{-6.1}$</td>
<td>24</td>
</tr>
</tbody>
</table>

The lightest chargino and the neutralino is assumed to be 5 or 20 GeV. A similar statistical formalism as already discussed is adopted for the model-dependent limits, but the potential signal contamination to the CRs is also taken into account, and the $\nu_{\text{signal}}$ variables now include the effect of the systematic uncertainties. The same statistical formalism as in eq. 7.19 is used to obtain a test statistic, and the $p_{s+b}$ value, i.e. the $p$-value of the signal plus background hypothesis, is computed for each signal point, testing a signal strength $\mu = 1$.

**Sbottom simplified model exclusion limits**

The 95% confidence level (CL$_s$) limits are computed, and are shown in the sbottom signal plane for each SR in fig. 7.21 together with the expected CL$_s$ values. For the combined limit, for each signal point, the SR with the best expected exclusion limit was taken. Figure 7.22 shows the signal region with the highest expected CL$_s$ value, as well as the expected and observed CL$_s$ value for the best signal region. Figure 7.22 also shows the exclusion limit published in [5], which was obtained using the same procedures, but older calibrations and object definitions as described. The two limits agree very well within their systematic uncertainties, and the exclusion presented here has thinner uncertainty bands due to the smaller systematic uncertainties of the new $b$-tagging calibration. For example for massless LSP, the uncertainty band has decreased from a width of 140 GeV to 100 GeV.
The exclusion in the sbottom - neutralino plane agrees well with the expectation from the optimisation (fig. 7.6). Some differences are observed, for example the expected limit in the optimisation reaches about 740 GeV, whereas the final result reaches 680 GeV, and the highest neutralino mass excluded was at around 380 GeV, whereas the final limit extends only to 320 GeV. The differences can be attributed to the fact that the $Z_N$ procedure utilises as null hypothesis the background only hypothesis, and is therefore a discovery limit, whereas the HistFitter framework has a more sophisticated calculation for the exclusion limit. Another difference is that $Z_N$ marginalises over the probability distribution of the true background yield, whereas HistFitter profiles the uncertainties. Furthermore, no signal systematic uncertainties were taken into account in the $Z_N$ calculation.

Figure 7.23 shows the expected and observed $p_{s+b}$ and the observed $p_s, p_b$ values versus the signal strength $\mu_{\text{sig}}$ for two particular signal models. The excluded cross-section at 95% CL is obtained by multiplying the nominal signal model cross-section with the $\mu_{\text{sig}}$ excluded at 95% CL. This is shown for the whole sbottom plane on the left hand side of figure 7.24. The impact of reducing the branching ratios (BR) of the sbottom decay into $b + \tilde{\chi}_1^0$, assuming no sensitivity to the other decay possibilities, is shown on the right hand side of figure 7.24. The excluded BR can be obtained as

$$\text{BR} = \sqrt{\frac{\text{excluded model } \sigma}{\text{nominal model } \sigma}}. \quad (7.23)$$

The green lines show the exclusion under the assumption of only 50% BR to $b + \tilde{\chi}_1^0$ and no sensitivity to other decay modes. For example, the point with $m_{\tilde{b}_1} = 600$ GeV and $m_{\tilde{\chi}_1^0} = 60$ GeV is no longer excluded if the BR is 50%. In chapter 8 I show for the first time an otherwise unpublished analysis with which sensitivity is gained for the first time to such a scenario.
Figure 7.21: Expected and observed exclusion limits at 95% CL for the sbottom simplified model for each SR individually. The dashed (solid) lines show the expected (observed) limits, including all uncertainties except for the theoretical signal cross-section uncertainty (PDF and scale). The yellow bands around the expected limits show the ±1σ experimental uncertainties. The dotted lines around the observed limits represent the results obtained when moving the nominal signal cross-section up or down by the ±1σ theoretical uncertainty. The excluded regions are those below the curves. The numbers give expected CLs values for the signal points. Points with CLs < 0.05 are excluded at 95% CL.
Figure 7.22: Top left: combined exclusion and best expected signal region resulting from the analysis presented in this thesis; top right: combined exclusion published in [5]; bottom left: expected CL$_s$ values; bottom right: observed CL$_s$ values.
7.10 Results and interpretation

Figure 7.23: Left (right): 95% CL limit on $\mu_{\text{sig}}$ for the signal model with $m_{b_1} = 300$ GeV and $m_{\tilde{\chi}_0^0} = 200$ GeV ($m_{b_1} = 500$ GeV and $m_{\tilde{\chi}_0^0} = 1$ GeV), using SRB (SRA250), the best expected signal region for this model.

Figure 7.24: Expected and observed exclusion limits at 95% CL for the sbottom-neutralino mass plane. Numbers represent the observed, excluded cross-sections at 95% CL (left) and the observed, excluded maximum branching ratio of the model $b_1 \rightarrow b + \tilde{\chi}_0^0$ at 95% CL (right), assuming no sensitivity to other decay modes. The region between the green lines shows the observed exclusion at 95% CL under the assumption of only 50% branching ratio.
Stop simplified models exclusion limits

Limits in various other signal planes are shown in figure 7.25. As expected from the optimisation studies, the exclusion limit is weaker with increasing $\Delta m(\tilde{\chi}_1^\pm - \tilde{\chi}_1^0)$, as the decay products from the $\tilde{\chi}_1^\pm$ reduce the acceptance due to either the lepton veto or the jet veto / the $H_{T,3}$ selection.

The published analysis [5] includes additional interpretations for simplified models. These are based on the same type of stop simplified models already discussed, with three SUSY particles accessible at LHC energy: $\tilde{t}_1$, $\tilde{\chi}_1^\pm$ and $\tilde{\chi}_1^0$. In these additional models, either the $\tilde{t}_1$ mass is fixed $m_{\tilde{t}_1} = 300$ GeV or the $\tilde{\chi}_1^\pm$ mass is fixed $m_{\tilde{\chi}_1^\pm} = 150$ GeV, and a scan of the other two mass parameters is performed. The resulting exclusion limits are shown in fig. 7.26 and were obtained by collaborators.

Figure 7.25: Expected and observed exclusion limits at 95% CL for the different simplified model scenarios considered. Left: $(m_{\tilde{b}_1}, m_{\tilde{\chi}_1^0})$ mass plane. Middle (right): $(m_{\tilde{t}_1}, m_{\tilde{\chi}_1^0})$, $\Delta m(\tilde{\chi}_1^\pm - \tilde{\chi}_1^0) = 5(20)$ GeV. The signal region providing best expected CL$_s$ background-only hypothesis exclusion limit is chosen at each point. The excluded regions are those below the curves. For the stop mass planes, the lower bound of the neutralino mass axis corresponds to the LEP limit of the lightest chargino mass, 103.5 GeV [49].

7.10.4 Improvements in the statistical treatment

In the previous section which described the methodology used to obtain the published result [5], the combined limit of all signal regions was obtained by picking for each point the signal region which gives the best expected exclusion and using its CL$_s$ value to set the limit.
7.10 Results and interpretation

Figure 7.26: Expected and observed exclusion limits at 95% CL for the different simplified model scenarios considered. Left: \((m_{\tilde{t}^\pm}, m_{\tilde{\chi}_1^0})\) mass plane for \(m_{\tilde{t}_1} = 300\) GeV. Right: \((m_{\tilde{t}_1}, m_{\tilde{\chi}_1^0})\), \(m_{\tilde{\chi}_1^\pm} = 150\) GeV. The signal region providing best expected CLs exclusion limit is chosen at each point. The excluded regions are those above the curves and below the grey lines. The plots shown are from [5] and were made by collaborators.

Figure 7.27: Left: Nominal exclusion in the sbottom plane using SRA signal regions only. Right: Exclusion limit obtained from a shape fit to the \(m_{\text{CT}}\) distribution.
Alternatively, one might expect to achieve a better exclusion through a shape fit to the $m_{CT}$ distribution. Instead of having 5 sub-regions of SRA with increasing requirement of $m_{CT}$ from 150 to 350 GeV, and performing the signal hypothesis test in each such signal region, a shape fit can be attempted. Figure 7.27 shows the comparison of the nominal fit results as was performed for the paper and the resulting exclusion limit from a shape fit to a histogram of $m_{CT}$ with 5 bins of 50 GeV between 150 and 400 GeV and an additional overflow bin. The normalisation of the backgrounds was performed as in the case of the nominal statistical setup in 1-bin control regions, while the $m_{CT}$ shape was used in the signal region only.

The resulting limit from the shape fit ranges further out towards higher sbottom and neutralino masses by about 5-10 GeV and is smoother. Since the default SRA already has multiple sub-regions with increasing $m_{CT}$ requirement, it was expected that most of the exclusion power has already been used in the nominal statistical setup.

\section*{7.11 Summary}

Over the different iterations of the ATLAS sbottom search, I was involved in most of its aspects. In particular for the last iteration, I optimised the signal regions, evaluated the standard model and signal contributions to the various signal and control regions, performed checks in cases of MC/data disagreement and assessed the signal uncertainties. The statistical treatment with HistFitter and the limit setting was performed by another member of the analysis team, which I have repeated independently for the purpose of this thesis.

In this chapter, the results of a search for third-generation squark pair production in events with large $E_T^{\text{miss}}$ and two $b$-tagged jets were reported. The full 2012 dataset of ATLAS $pp$ collisions collected at a center of mass energy of $\sqrt{s} = 8$ TeV and amounting to a total integrated luminosity of 20.3 fb$^{-1}$ has been analysed. No significant excess above the standard model expectation is observed, which translate into 95\% CL$_{s}$ upper limits on the sbottom / stop and neutralino masses in particular simplified model scenarios which assume exclusive decays $\tilde{b}_1 \to b\tilde{\chi}_1^0$ ($\tilde{t}_1 \to b\tilde{\chi}_1^\pm$). ATLAS quotes conservative exclusion limits
using the observed exclusion $-1 \sigma_{\text{theory}}$ on the signal cross-section. Using this convention, sbottom masses up to 640 GeV are excluded at 95% CL for neutralino masses of up to 150 GeV. Differences in mass between $\tilde{b}_1$ and $\tilde{\chi}_1^0$ larger than 50 GeV are excluded up to sbottom masses of 300 GeV. For sbottom masses higher than 450 GeV, the exclusion limit can be extended by 5-10 GeV to higher $\tilde{b}_1$ and $\tilde{\chi}_1^0$ masses if a shape fit to the $m_{\text{CT}}$ distribution is employed. For stop pair production, the sensitivity of the analysis is reduced due to the presence of additional jets and leptons in the signal events. In the case of $\Delta m = m_{\tilde{\chi}_1^\pm} - m_{\tilde{\chi}_1^0} = 5$ GeV (20 GeV), stop masses up to 580 GeV (440 GeV) are excluded for $m_{\tilde{\chi}_1^0} = 100$ GeV. Neutralino masses up to 280 GeV (230 GeV) are excluded for $m_{\tilde{t}_1} = 420$ GeV for $\Delta m = 5$ GeV (20 GeV).

These are world leading results, and this analysis is the only one sensitive to models with highly compressed scenarios in the regime $\Delta m(\tilde{t}_1/\tilde{b}_1, \tilde{\chi}_1^0) < 50$ GeV.
Chapter 8

Search for the Higgs boson in sbottom decays

The current chapter reports on a search for pair production of the scalar partners of bottom quarks cascade-decaying to at least one Higgs boson, using neural network (NN) based selections. The search is performed in 20.1 fb$^{-1}$ of $pp$ collisions at a centre-of-mass energy of 8 TeV using the ATLAS experiment. The final states targeted have large $E_T^{\text{miss}}$, no electrons or muons and multiple jets, of which at least 3 are identified to originate from $b$-quarks. These final states can be produced in a $R$-parity conserving minimal supersymmetric scenario, when scalar bottoms are pair produced and decay in one or both legs as $\tilde{b}_1 \rightarrow b \tilde{\chi}_2^0$ with subsequent decay $\tilde{\chi}_2^0 \rightarrow H + \tilde{\chi}_1^0$ and $H \rightarrow bb$. The $H \rightarrow bb$ final state is chosen because of the high branching ratio (BR) of 57.7% [136] for the Standard Model Higgs boson.

The search was initiated by the author, who performed all aspects of the analysis. The analysis follows a similar structure to that presented in the previous chapter; the same trigger and object definitions are used and there is a large overlap between the Monte Carlo samples analysed. The background estimation strategy is similar to section 7.7 and the normalisation of the dominant $t\bar{t}$ background is extracted from a profile likelihood fit of MC yields to data in dedicated control regions. The limit-setting employs the statistical algorithms already described in section 7.9.
### 8.1 Signal, signature and backgrounds

The current chapter is organised as follows. A general overview of the search, signal targeted and main backgrounds is given in section 8.1. Objects and variables are defined in section 8.2 followed in 8.3 by an account of the triggers and data samples used. The Monte Carlo samples used are summarised in 8.4. Section 8.5 lists the pre-selection and cleaning used in the analysis. The signal regions and the optimisation procedure are described in section 8.6 followed by an account of the background estimation strategy in 8.7. Experimental and theoretical uncertainties are discussed in section 8.8. Fit results of the background normalisations and kinematic distributions in the control and validation regions are presented in section 8.9. Results of the search are presented in 8.10.

**8.1 Signal, signature and backgrounds**

In the previous chapter exclusion limits have been derived in the context of a sbottom simplified model for the case that both sbottoms decay \( \tilde{b}_1 \rightarrow b + \tilde{\chi}_1^0 \). Figure 7.24 showed the consequence of reducing the BR of this decay under the assumption that the sbottom analysis has no sensitivity to the other decay modes. The assumption is justified by considering alternatives to the \( \tilde{b}_1 \rightarrow b + \tilde{\chi}_1^0 \) decay. For example, \( \tilde{b}_1 \rightarrow t + \tilde{\chi}_1^\pm \) or \( \tilde{b}_1 \rightarrow b + \tilde{\chi}_2^0 \) are possible. In the first case, events are very likely to fail the sbottom analysis selection because of either the lepton veto or the requirements suppressing hadronic activity. In the latter decay channel, depending on the neutralino mixing matrix, the \( \tilde{\chi}_2^0 \) can decay through a \( Z \) or a \( H \) and give additional leptons or jets. The analysis presented in this chapter targets the scenario in which the sbottom BR to \( b + \tilde{\chi}_2^0 \) is 100%, and the mixed decay scenario when the BR to \( b + \tilde{\chi}_2^0 \) and \( b + \tilde{\chi}_1^0 \) is 50% each. In particular SUSY models, for example in the case of the next to minimal supersymmetric standard model (NMSSM), the neutralino mixings can be easily chosen such that the decay \( \tilde{\chi}_2^0 \rightarrow \tilde{\chi}_1^0 + H \) is enhanced (see 2.3.8).

The model under consideration has the following free parameters: \( m_{\tilde{b}_1}, m_{\tilde{\chi}_2^0}, m_{\tilde{\chi}_1^0} \) and 

\[
\text{BR} \equiv \text{BR}(\tilde{b}_1 \rightarrow b + \tilde{\chi}_2^0) = 1 - \text{BR}(\tilde{b}_1 \rightarrow b + \tilde{\chi}_1^0)
\]

Signal points are abbreviated using the convention \( Bxxx.Cyyyy_Nzzz.BRff \), where \( xxx \) stands for \( m_{\tilde{b}_1} \), \( yyy \) for \( m_{\tilde{\chi}_2^0} \), \( zzz \) for \( m_{\tilde{\chi}_1^0} \).
and $ff$ is “1” if the branching ratio of $\tilde{b}_1 \rightarrow b + \tilde{\chi}_2^0 = 1$ and “05” if the branching ratio is 0.5.

The following signal planes are defined:

- fix $m_{\tilde{\chi}_1^0} = 60$ GeV, scan $m_{\tilde{b}_1}, m_{\tilde{\chi}_2^0}$
- fix $m_{\tilde{\chi}_2^0} = 2 \times m_{\tilde{\chi}_1^0}$, scan $m_{\tilde{b}_1}, m_{\tilde{\chi}_2^0}$.

The above is repeated for BR = 1 and BR = 0.5, effectively giving four signal planes. While samples generated using HERWIG++ existed for the BR = 1 case, dedicated signal samples with mixed sbottom decays were generated by the author using MADGRAPH 5 + PYTHIA 6. The processes simulated at matrix element level are $\tilde{b}_1 \tilde{b}_1^*$ and $\tilde{b}_1 \tilde{b}_1^* + 1$ additional parton, with additional partons being produced by the parton shower.

An existing analysis requiring events with no electrons or muons, high $E_T^{\text{miss}}$ and at least 3 $b$-tagged jets has already shown some sensitivity in one of the two planes with BR = 1 [137]. Figure 8.1 shows the exclusion for the case $m_{\tilde{\chi}_1^0} = 60$ GeV, however the analysis had no sensitivity for the signal plane with $m_{\tilde{\chi}_2^0} = 2 \times m_{\tilde{\chi}_1^0}$ due to the lower $p_T$ of jets, the lower $E_T^{\text{miss}}$ and lower $m_{\text{eff}}$ specific to such signal events. In this chapter, I show that better sensitivity can be obtained for the models with BR = 1 than was found in [137], and show for the first time sensitivity for the signal models with sbottom mixed decays. The dominant background for this analysis is by far $t\bar{t}$ production, where a light or $c$-jet from a $W$ decay

![Figure 8.1: Exclusion from [137] in the $\tilde{b}_1 - \tilde{\chi}_2^0$ mass plane.](image-url)
is mis-tagged as a $b$-jet, and from $t\bar{t} + b\bar{b}$ production. Higgs (and in particular $t\bar{t}H$) events, even though they contain a true Higgs and therefore potentially a large number of $b$-jets, have a very small production cross-section and give only a small contribution to the signal, control and validation regions. In particular, $t\bar{t}H$ has a production cross-section of 0.075 pb. The background is characterised in more detail after the pre-selection stage in section 8.5.2.

8.2 Object and Variable Definitions

Most of the variables already defined in section 7.4 are also used in this analysis. In addition to these, the following variables are defined:

- $m_{T\mathrm{Gen}}$ is a variable designed [138] to measure the mass scales associated with pair-produced particles. On an event-by-event basis, $m_{T\mathrm{Gen}}$ supplies a lower bound for the mass of either of the two particles which were pair produced and whose decay products were observed. $m_{T\mathrm{Gen}}$ is defined to be the smallest value of $m_{T2}$ obtained over all possible partitions of momenta in two disjoint sets. Each set contains the decay products from one side of the event.

- $\sigma_{mb}(b_i, b_j)$ is defined as the difference between the invariant mass of the two input $b$-jets, $m_{b_i, b_j}$, and the mass of the Higgs boson, $m_H = 125$ GeV in units of the jet energy resolution uncertainty of $m_{b_i, b_j}$:

$$\sigma_{mb}(b_i, b_j) = \frac{m_{b_i, b_j} - m_H}{\mathrm{JER}_{m_{b_i, b_j}}}.$$  \hspace{1cm} (8.1)

The smaller the absolute value of this quantity, the more likely it is that the two $b$-jets stem from a Higgs decay. The JER uncertainty on the invariant mass is obtained by propagating the individual JER of each of the two $b$-jets to the invariant mass,

$$\frac{\mathrm{JER}_{mb_{b_i, b_j}}}{m_{b_i, b_j}} = \sqrt{\left(\frac{\mathrm{JER}_{pT,b_i}}{p_{T,b_i}}\right)^2 + \left(\frac{\mathrm{JER}_{pT,b_j}}{p_{T,b_j}}\right)^2}.$$  \hspace{1cm} (8.2)
8.3 Triggers and Data Sample

The trigger strategy followed by this analysis and the analysed data samples are the same as those for the sbottom analysis, as presented in section 7.2.

8.4 Monte Carlo samples

The Monte Carlo samples used in this analysis have a high overlap with those already presented in section 7.3. Some samples have been replaced due to higher event numbers, and samples involving Higgs production have been added as shown in table 8.1. The $t\bar{t}$ PowHEG Box sample employs the “P2011C” tune for the $p_T$-ordered shower and the underlying event. All other samples employ the AUET2B tune.

<table>
<thead>
<tr>
<th>Process</th>
<th>Generator</th>
<th>Order</th>
<th>Hadronisation</th>
<th>Underlying event</th>
<th>PDF</th>
<th>Simulation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t\bar{t}$</td>
<td>PowHEG Box</td>
<td>NLO</td>
<td>Pythia 6</td>
<td>Pythia 6</td>
<td>CTEQ6L1</td>
<td>AFII</td>
</tr>
<tr>
<td>$t\bar{t} + WW$</td>
<td>MadGRAPH 5</td>
<td>LO</td>
<td>Pythia 6</td>
<td>Pythia 6</td>
<td>CTEQ6L1</td>
<td>full sim.</td>
</tr>
<tr>
<td>$W(\rightarrow \nu\nu) + H(\rightarrow bb)$</td>
<td>Pythia 8 [139]</td>
<td>LO</td>
<td>Pythia 8</td>
<td>Pythia 8</td>
<td>CTEQ6L1</td>
<td>full sim.</td>
</tr>
<tr>
<td>$Z(\rightarrow \ell^+\ell^-) + H(\rightarrow bb)$</td>
<td>Pythia 8</td>
<td>LO</td>
<td>Pythia 8</td>
<td>Pythia 8</td>
<td>CTEQ6L1</td>
<td>full sim.</td>
</tr>
<tr>
<td>$Z(\rightarrow \nu\nu) + H(\rightarrow bb)$</td>
<td>Pythia 8</td>
<td>LO</td>
<td>Pythia 8</td>
<td>Pythia 8</td>
<td>CTEQ6L1</td>
<td>full sim.</td>
</tr>
<tr>
<td>$H + t\bar{t}$ (dileptonic)</td>
<td>Pythia 8</td>
<td>LO</td>
<td>Pythia 8</td>
<td>Pythia 8</td>
<td>CTEQ6L1</td>
<td>AFII</td>
</tr>
<tr>
<td>$H + t\bar{t}$ (semileptonic)</td>
<td>Pythia 8</td>
<td>LO</td>
<td>Pythia 8</td>
<td>Pythia 8</td>
<td>CTEQ6L1</td>
<td>full sim.</td>
</tr>
<tr>
<td>$H + t\bar{t}$ (hadronic)</td>
<td>Pythia 8</td>
<td>LO</td>
<td>Pythia 8</td>
<td>Pythia 8</td>
<td>CTEQ6L1</td>
<td>full sim.</td>
</tr>
</tbody>
</table>

Table 8.1:MC samples and generators complementing or replacing the samples listed in section 7.3

8.5 Pre-selection

In order to ensure the triggers operate at > 99% efficiency, the requirements $E_T^{miss} > 150$ GeV and $p_T^{j1} > 90$ GeV are made. Given the expected signal, at least 4 jets with $p_T > 30$ GeV are required. Jets are $b$-tagged using the MV1 algorithm configured at the 60% operating point, and at least three $b$-jets with $p_T > 30$ GeV are required. To reduce the effects of pile-up, all jets with $p_T$ less than 50 GeV and within $|\eta| < 2.4$ are required to have $|JVF| > 0$. Further requirements designed to reject the multi-jet background are made as follows. The
8.5 Pre-selection

$\Delta \phi_{\text{min}}$ between the $p_T^{\text{miss}}$ and each of the leading four jets is required to be greater than 0.4, and the $E_T^{\text{miss}}/m_{\text{eff}}$(leading 4 jets) is required to be greater than 0.2. The requirements are summarised in table 8.2.

8.5.1 Pile-up dependence

Since the selection requires a large number of jets with $p_T > 30$ GeV, the analysis might be susceptible to pile-up, as some of the required jets might originate from an interaction other than the hard scattering, even when the JVF requirement is applied. In order to check if this is the case, the selection efficiency at different stages of the pre-selection is shown versus the number of reconstructed vertices in figure 8.2 for the JetTauEtmiss stream, for the nominal $t\bar{t}$ sample, and for the signal point with $m_{b_1} = 800$ GeV, $m_{\tilde{\chi}_2^0} = 300$ GeV, $m_{\tilde{\chi}_1^0} = 60$ GeV.

<table>
<thead>
<tr>
<th>Requirement number</th>
<th>Variable</th>
<th>Cut value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Cleaning variables</td>
<td>see section 7.5</td>
</tr>
<tr>
<td>2</td>
<td>baseline $e/\mu$ veto</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>$E_T^{\text{miss}}$</td>
<td>$&gt; 150$ GeV</td>
</tr>
<tr>
<td>4</td>
<td>$p_T^{4j}$</td>
<td>$&gt; 90$ GeV</td>
</tr>
<tr>
<td>5</td>
<td>$N_{\text{jets}}$ ($p_T &gt; 30$ GeV)</td>
<td>$\geq 4$</td>
</tr>
<tr>
<td>6</td>
<td>$</td>
<td>\text{JVF}</td>
</tr>
<tr>
<td>7</td>
<td>$\Delta \phi_{\text{min}}$(4jets)</td>
<td>$&gt; 0.4$</td>
</tr>
<tr>
<td>8</td>
<td>$E_T^{\text{miss}}/m_{\text{eff}}$(4jets)</td>
<td>$&gt; 0.2$</td>
</tr>
<tr>
<td>9.a</td>
<td>$N_{b-\text{jets}}$ (MV1, 60% OP)</td>
<td>$\geq 1$</td>
</tr>
<tr>
<td>9.b</td>
<td>$N_{b-\text{jets}}$ (MV1, 60% OP)</td>
<td>$\geq 2$</td>
</tr>
<tr>
<td>9.c</td>
<td>$N_{b-\text{jets}}$ (MV1, 60% OP)</td>
<td>$\geq 3$</td>
</tr>
</tbody>
</table>

Table 8.2: Pre-selection requirements

No dependence on the number of vertices is observed for the efficiency of selecting four or more low $p_T$ jets. However, the $b$-tagging efficiency is observed to depend on the number of vertices. This is not an issue, as long as the dependence in data is well described in the simulation. The difference in $b$-tagging efficiency in data and simulation is already corrected for by the $b$-tagging scale factors, which are derived with an uncertainty associated as described in 7.8.1. However, these correct the $b$-tagging efficiency in the simulation on average, regardless of $N_{\text{vtx}}$. An additional pileup uncertainty is applied in this analysis to all
8.5 Pre-selection

Figure 8.2: Selection efficiency versus number of reconstructed vertices ($N_{\text{vtx}}$) after the 0-lepton pre-selection. Left top: selection efficiency before $b$-tagging requirements, $\epsilon = N_8/N_4$, where $N_i$ refers to the number of events after the $i$-th requirement in table 8.2. Top right, bottom left, bottom right: efficiency of tagging at least 1, 2, or 3 $b$-jets, $\epsilon = N_9/N_8$. The efficiencies have been scaled by a sample-specific factor indicated in the legend. The distribution of the number of vertices, $N_{\text{vtx}}$ is shown after requirement 4 in table 8.2.

yields, which is obtained by changing the average number of interactions per bunch crossing in the simulation by 10% up and down. In addition, the total number of data events with $N_{\text{vtx}} < 6$, for which the selection efficiency after $b$-tagging is significantly different compared to the average, is a small fraction of the total dataset, as can be seen from the distribution of $N_{\text{vtx}}$ in figure 8.2.

8.5.2 Background composition

After the pre-selection, the background is characterised in some detail. Of particular interest are the origin of the $E_{\text{T}}^{\text{miss}}$ and the flavour composition of the three $b$-tagged jets.

For the $E_{\text{T}}^{\text{miss}}$ origin, I explore the cases of whether an electron or muon is out of acceptance, or whether there is a hadronic tau in the event, in which case the $E_{\text{T}}^{\text{miss}}$ is often a
result of the $\tau$ neutrino. Establishing the source of the $E_T^{\text{miss}}$ is important for the definition of the control regions. Figure 8.3 shows the source of $E_T^{\text{miss}}$ in $t\bar{t}$ events in a MC sample. In less than 30% of the pre-selected events, hadronic $\tau$ jets exist, while in the remaining over 70% of the events, the $E_T^{\text{miss}}$ is associated with a lepton ($e$, $\mu$ or $\tau$) out of acceptance or not identified. In the events in which a prompt $\mu$ exists at truth level, the $\mu$ is likely to be out of kinematic acceptance, for example by escaping through the gap at $\eta = 0$ in the muon systems. These muons (together with the $\nu_\mu$) will give rise to $E_T^{\text{miss}}$. In the events in which a prompt electron exists, the electron is likely to have been mis-identified as a jet instead, which will however enter the $E_T^{\text{miss}}$ calculation at the wrong scale, again giving rise to some $E_T^{\text{miss}}$. Therefore in the 1-lepton control region, the lepton $p_T$ is added to the $E_T^{\text{miss}}$ vectorially.

Figure 8.4 shows the flavour composition of the 3 $b$-tagged jets in the 0-lepton pre-selection region and in the equivalent 1-lepton region. It is found that in both the 0 and 1-lepton regions the $t\bar{t}$ process most commonly enters the region because one $c$-jet is mistagged as a $b$-jet. The pair of $b$-tagged jets with the smallest $|\sigma_{mbs}|$, and therefore the pair which is most consistent with originating from a $H \rightarrow b\bar{b}$ decay, is also investigated for its flavour composition in the same figure.

### 8.6 Signal Regions

Several of the proposed variables are found to provide signal versus background discrimination. Multivariate techniques are applied to combine these weak discriminators to a stronger one. Using the Toolkit for Multivariate Data Analysis (TMVA [140]), the performance of several multivariate (MVA) techniques is examined. The following ten variables were chosen as initial inputs to the optimisation procedure:

- each of the leading 4 jets’ $p_T$
- missing transverse momentum, $E_T^{\text{miss}}$
- $m_{T\text{Gen}}$
Figure 8.3: Source of $E_T^{\text{miss}}$. Left top: number of missed electrons or muons versus number of $\tau$ leptons. For the bin with exactly one missed $e$ or $\mu$ and no $\tau$, the pseudo-rapidity $\eta$ of the missed lepton is shown in the bottom row, on the left for muons and on the right for electrons. Detector features such as the gap in the muon chambers at $\eta = 0$ are visible in this plot. Right top: number of hadronic $\tau$ jets versus number of $\tau$ leptons. Overall, only 28.3% of the events have one or more hadronic $\tau$ jets, while the rest of the events are likely to contain a lepton ($e$, $\mu$ or leptonic $\tau$) which was out of detector acceptance or has not been reconstructed.
8.6 Signal Regions

Figure 8.4: Number of MC events after the pre-selection in table 8.2 versus truth flavour of the b-tagged jets. Top left (right): events with 0 (1) leptons and at least 3 b-tagged jets. The dominant contribution after the pre-selection is by far due to $t\bar{t}$ production. Bottom: Number of $t\bar{t}$ MC events after the pre-selection (no leptons, 3 b-tagged jets) versus the truth flavour of the two b-tagged jets most consistent with a $H \to b\bar{b}$ decay.
\section*{8.6 Signal Regions}

- $N_{\text{jets}}$ - number of jets with $p_T > 30$ GeV
- $N_{b\text{-jets}}$ - number of $b$-jets with $p_T > 30$ GeV
- $\sigma_{m_{bb}}(b_i, b_j)$ - is calculated for all combinations $i, j$ of $b$-jets. The value that is used as input is the one closest to zero, i.e. the $b$-jet pair most consistent with having originated from a Higgs boson is picked for the $\sigma_{m_{bb}}$ calculation
- $m_{\text{eff}}$ - built with all jets in the event with $p_T > 20$ GeV.

Multiple multivariate techniques were trained and their performance was compared, as detailed in appendix C.1. The neural network technique was found to have overall the best discriminating power and exhibit very little overtraining. Signal regions were therefore defined based on this technique and the results of the other multivariate methods were not pursued.

The type of neural network studied in this context can be thought of as a non-linear function which maps from a space of input variables $x_i, i = 1, \ldots, n$ to a one-dimensional space consisting of one output variable $y_{\text{NN}}$, also called the output score. An output score close to one (zero) signifies the event is signal (background)-like. The expression for the non-linear function is

$$y_{\text{NN}} = \sum_{j=1}^{n_{\text{hidden}}} \tanh \left( \sum_{i=1}^{n_{\text{var}}} x_i w_{ij}^{(1)} \right) \cdot w_{j1}^{(2)}, \quad (8.3)$$

where $n_{\text{var}}$ and $n_{\text{hidden}}$ are the number of neurons in the input layer and in the hidden layer, respectively. $w_{ij}^{(1)}$ is the weight between input-layer neuron $i$ and hidden layer neuron $j$, and $w_{j1}^{(2)}$ is the weight between the hidden-layer neuron $j$ and the output neuron. The weights are function parameters which are being fitted during the training process. In the training process, the weights vector $\mathbf{w}$ is calculated by minimising the cost function

$$E(\mathbf{w}) = \sum_{\text{events}} (y_{\text{NN}} - y_{\text{true}})^2,$$

where $y_{\text{NN}}$ is the NN output score and $y_{\text{true}}$ is equal to one (zero) if the event is a signal (background) event. The Broyden-Fletcher-Goldfarb-Shannon (BFGS) method [141–144] is used for the minimisation.

Neural networks were trained on the $t\bar{t}$ background and several signal points representative of the different kinematic configurations achievable in the different signal planes, as
Signal Regions

shown in table 8.3. Once the NN method has been trained, a final requirement on the NN output score needs to be made in order to fully define a signal region. This requirement is dependent on the background and signal cross-sections, the luminosity and the systematic uncertainty on the event yields. For the purpose of the optimisation only, the significance $Z$ for exclusion is estimated assuming a flat background uncertainty of 30% and is calculated as

$$Z = \frac{S}{\sqrt{S + B + (0.3 \times B)^2}},$$

(8.4)

with $S$ ($B$) being the number of expected signal (background $t\bar{t}$) events. In equation 8.4, the null hypothesis tested is that the data is described by the background and signal model, i.e. $N_{\text{expected, total}} = S + B$. In the limit of large number of events, the probability distribution for the number of observed events is described by a Gaussian centered at $N_{\text{expected, total}} = S + B$ with standard deviation given by the statistical error on $N_{\text{expected, total}}$, i.e. $\sqrt{S + B}$, added in quadrature with the systematic uncertainty, $(0.3 \times B)$. However, if the data is described by the standard model only, $B$ events will be observed, i.e. the deviation from the expectation $S + B$ of the null hypothesis is $S$ events. The exclusion significance $Z$ is given by the observed deviation, $S$, measured in units of standard deviations $\sigma = \sqrt{S + B + (0.3 \times B)^2}$ of the distribution of expected number of events given the null hypothesis of background plus signal. This approach is the default used by TMVA in the optimisation procedure.

The signal regions are defined by choosing the requirement on the NN output score that maximises the $Z$ significance. As an example, figure 8.5 shows the background and signal efficiencies as a function of the requirement on the neural network (NN) output score in the case of a NN optimised on the signal point B800_C300_N60_BR1. The other quantities shown are the signal purity defined as $S/B$ and the significance $Z$. It should be emphasised that the statistical treatment used in the final limit setting procedure is the same as described in section 7.9 which correctly takes into accounts cases with small numbers of events, as well as systematic uncertainties.

The procedure for finding the maximal significance is repeated for each signal sample trained on, for a NN and for a selection based on rectangular cuts, with the results being
Figure 8.5: Left: signal and background selection efficiencies, signal purity and discovery significance as a function of the requirement on the NN output score for the NN trained on B800_C300_N60_BR05. In 20.1 fb$^{-1}$ of data, a total of 4 signal and 337 $t\bar{t}$ background events are predicted to pass the pre-selection. The maximum significance $Z = 1.28$ is obtained when making a requirement on the NN output score at 0.8812. Right: NN score output distributions for training and test sub-samples for the B800_C300_N60_BR1 signal point, for signal and $t\bar{t}$ background. This is the NN which exhibits the lowest $p_{KS}$ values amongst the four signal NNs used for the exclusion limits.

summarised in table 8.3. For all points used in the exclusion limits presented in this chapter, the significance according to eq. 8.4 obtained from the neural network selection is superior to that obtained from the optimal “cut & count” analysis. The expected exclusion significances are only indicative, as equation 8.4 is a naive approximation to the more sophisticated statistical treatment used to calculate the exclusion limits. More importantly, it uses guessed values for the systematic uncertainties, the background $B$ estimate used in the optimisation procedure only contains $t\bar{t}$ event counts and no systematic uncertainties on the signal event yield are considered.

Amongst all MVA techniques trained, the neural network exhibits an acceptable amount of overtraining, while providing the best overall significances “out-of-the-box”. The concept of overtraining is explained briefly in the paragraph below.

In the case when a machine learning algorithm has too many model parameters and too few data points to train on, overtraining may occur. The symptoms of overtraining are an apparent increase in the classification performance above that of the objectively achievable one when the technique is applied on the training sample, and the factual decrease
8.6 Signal Regions

<p>| Signal region | Point               | Significance from selection based on |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th>Neural network</th>
<th>Rectangular cuts</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>B600_C200_N60_BR05</td>
<td>2.60</td>
<td>1.96</td>
</tr>
<tr>
<td>2</td>
<td>B600_C575_N60_BR05</td>
<td>2.56</td>
<td>2.29</td>
</tr>
<tr>
<td>3</td>
<td>B800_C300_N60_BR05</td>
<td>1.02</td>
<td>0.92</td>
</tr>
<tr>
<td>4</td>
<td>B800_C300_N60_BR1</td>
<td>1.05</td>
<td>0.92</td>
</tr>
<tr>
<td>Σ</td>
<td></td>
<td>7.23</td>
<td>6.09</td>
</tr>
</tbody>
</table>

Table 8.3: Summary of signal regions and discovery significances achievable with a neural network selection versus a selection based on rectangular cuts. The significance is obtained according to eq. (8.4). The last row shows a sum of significances for all SRs. While this value has no physical meaning, it is an indicator that overall, the neural network technique performs better than rectangular cuts.

of discrimination power when the technique is applied on an independent test dataset. In order to assess the level of overtraining, in this analysis the signal and background samples are each split in two equal sub-samples, a training and a test sub-sample. The training of the NN is performed on the training sample only, and the resulting NN is applied on the test sample. For each trained NN, distributions of the output score for the training and test samples are compared using the Kolmogorov-Smirnov (KS) test statistic, from which the probability $p_{KS}$ that two samples are sampled from the same underlying distribution can be calculated. An overtrained discriminator produces discrepant distributions for the training and test sub-samples. The smallest $p_{KS}$ value for the four NNs trained for this analysis was found to be 0.14, for which the NN output distributions for the training and test sub-samples are shown in figure 8.5. Values of $p_{KS}$ as low as $O(10^{-2})$ are shown in appendix C.2 not to degrade the performance of the discriminator.

In order to reduce overtraining and increase the $p_{KS}$ values, an optimisation procedure (see app. C.2) was devised to remove weak variables from the set of input variables and to reduce the NN complexity. The remaining input variables for each NN are summarised in table 8.4. The variables are ranked according to the decrease in sensitivity suffered if the variable under consideration is removed from the training. It can be observed that most of the input variables pre-selected for training are indeed necessary and carry useful information about the signal and background events, as removing any further variables from the training...
degrades the sensitivity of the discriminant. All NNs used in this analysis are robust against large model complexity, exhibiting no signs of overtraining.

Table 8.4: Variable ranking for the NNs used in this analysis. The variables are removed one at a time from the set of input variables and the largest reduction in the signal sensitivity is interpreted to be due to the removal of the most important variable from the training.

### 8.7 Background Estimation

#### 8.7.1 Control and validation regions

For each point optimised on, a set consisting of a signal region (SR), a 1-lepton control region (CR1L) and 0-lepton validation region (VR0L), is defined. They all share the same pre-selection, apart from the lepton multiplicity requirement. The NNs employed are those optimised in section 8.6 and summarised in table 8.4. For the SR, the requirement on the NN output score is chosen such that it gives the highest \(Z\) discovery significance for the point the NN was trained on. The requirements on the NN output scores differ in CR1L and VR0L as indicated in table 8.5 in order to ensure that there is no overlap with the SR, that the control and validation regions include a large enough sample of events and that the statistical uncertainties on the event yields in these regions are below 15%.

Since most of the background is due to cases in which the \(E_T^{\text{miss}}\) comes from an escaping lepton or a hadronically decaying \(\tau\), and their associated neutrinos, it is useful to construct a 1-lepton control region, CR1L, for each SR. The same pre-selection is required, as well as exactly one signal lepton. Events with any further baseline leptons with \(p_T > 10\) GeV are vetoed. Since in the signal region the \(E_T^{\text{miss}}\) is most often due to an escaping lepton, it is
8.8 Systematic uncertainties

added to the $E_T^{\text{miss}}$ in the control region, resulting in corrected $E_T^{\text{miss}}$. The $m_{T\text{Gen}}$ and $m_{\text{eff}}$ variables for CR1L are calculated from the jets in the event and the corrected $E_T^{\text{miss}}$. All other variables serving as input to the neural network are calculated as in the SR. In order to populate the control region, the requirement on the NN output score is relaxed such that there are at least 50 expected background events. The 1 lepton CR is dominated by $t\bar{t}$, and is used to fit the overall normalisation of this background.

The goodness of fit is evaluated for each SR/CR1L pair using a 0 lepton region, VR0L, which has the same pre-selection as the SR but a disjoint requirement on the NN output score. The VR0L has an upper cut on the NN score equal to the one defining the corresponding SR, and a lower cut such that there are about 100 expected background events in each validation region. The VR0L region is also dominated by $t\bar{t}$ and is used to derive a closure test systematic uncertainty by comparing the data event yield with the expected $t\bar{t}$ event yield after the normalisation of the background in the CR1L region.

The requirements on the NN output scores are summarised in table 8.5.

<table>
<thead>
<tr>
<th>Point optimised</th>
<th>SR</th>
<th>CR1L</th>
<th>VR0L</th>
</tr>
</thead>
<tbody>
<tr>
<td>B600_C200_N60_BR05</td>
<td>(0.69413, +\infty)</td>
<td>(0.5185, +\infty)</td>
<td>(0.0535, 0.69413)</td>
</tr>
<tr>
<td>B600_C575_N60_BR05</td>
<td>(0.67269, +\infty)</td>
<td>(0.5235, +\infty)</td>
<td>(0.0905, 0.67269)</td>
</tr>
<tr>
<td>B800_C300_N60_BR05</td>
<td>(0.85164, +\infty)</td>
<td>(0.4745, +\infty)</td>
<td>(0.0225, 0.85164)</td>
</tr>
<tr>
<td>B800_C300_N60_BR1</td>
<td>(0.74847, +\infty)</td>
<td>(0.3105, +\infty)</td>
<td>(0.0125, 0.74847)</td>
</tr>
</tbody>
</table>

Table 8.5: Requirement on the neural network output score for each signal, control and validation region.

8.8 Systematic uncertainties

The same detector systematic uncertainties are considered as in the case of the sbottom pair analysis (c.f. section 7.8). In particular, systematic uncertainties that were previously found to be negligible and therefore were not included in the statistical model are now explicitly checked and included in the fit model. These include the pileup uncertainty and the electron and muon momentum and energy scale, resolution and identification efficiency uncertainties.
The theory systematic uncertainties are most important for the $t\bar{t}$ sample, as this is the dominant background in the inclusive 0-lepton region. An overall shift in the yield in the SR and CR can be absorbed by the $t\bar{t}$ normalisation when the fit is performed. Therefore the relative uncertainty in the SR is calculated, after it has been normalised in the CR. Several sources of theoretical uncertainties have been considered:

- **MC generator** - a sample simulated with **PowHEG Box** interfaced with **HERWIG +Jimmy** is compared to a sample produced with **ALPGEN**, also interfaced with **HERWIG +Jimmy** for the parton shower and underlying event.

- **Parton Shower and Underlying Event** - $t\bar{t}$ final state 4-vectors from **PowHEG Box** are showered with **PYTHIA 6**. The sample is compared with a sample in which the same 4-vectors are showered with **HERWIG**.

- **ISR/FSR** - several samples produced with the LO generator **AcerMC**, interfaced with **PYTHIA 6** are compared. One sample uses the **PYTHIA 6** “more parton shower” tune, the other sample “less parton shower” tune.

In the case of the uncertainty due to the factorisation and renormalisation scale variations, the required samples only exist at truth level and have not been reconstructed, which presents a challenge for the current analysis. In particular, even in the inclusive phase space, the analysis requires at least 3 $b$-tagged jets, and in $t\bar{t}$ events one of these jets is usually a $c$ or a light jet. Therefore a good understanding of the rates for misidentifying jets as $b$-jets is needed in order to perform this analysis at truth level. The approach taken for these preliminary results is to assume the scale and PDF uncertainties are of the same size as the total theoretical uncertainties from other sources and to add these in quadrature. This effectively inflates the total theoretical uncertainty calculated from known sources by 41%.

The final total theory uncertainty after normalisation in the CR1L is shown in table 8.6.
8.9 Fit results for control and validation regions

8.8.1 Systematic uncertainty from 1L versus 0L modeling

The $t\bar{t}$ normalisation is extracted from the corresponding CR1L of each signal region. After the background fit, the data yield and the Standard Model expectation are compared in the 0-lepton validation VR0L. Differences between data and the MC expectation which has been normalised in CR1L are evaluated and used to derive an additional systematic uncertainty on the $t\bar{t}$ yield in the SR. The uncertainty is defined as

$$VR_{sys} = \frac{N_{observed} - N_{MC, non-t\bar{t}}}{N_{fitted, t\bar{t}}} \quad (8.5)$$

where all numbers refer to those relevant for VR0L. Table 8.6 shows the values obtained for the VR0L uncertainty.

<table>
<thead>
<tr>
<th>SR name</th>
<th>Total theory uncertainty</th>
<th>VR0L closure uncertainty</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 B600_C200_N60_BRO5</td>
<td>14%</td>
<td>7.0%</td>
</tr>
<tr>
<td>2 B600_C575_N60_BRO5</td>
<td>4%</td>
<td>12.3%</td>
</tr>
<tr>
<td>3 B800_C300_N60_BRO5</td>
<td>24%</td>
<td>18.5%</td>
</tr>
<tr>
<td>4 B800_C300_N60_BR1</td>
<td>16%</td>
<td>9.3%</td>
</tr>
</tbody>
</table>

Table 8.6: Summary of $t\bar{t}$ theory uncertainties in the signal regions after normalisation in the CR. Last column shows the $t\bar{t}$ closure systematic uncertainty derived in the 0-lepton validation region VR0L.

8.9 Fit results for control and validation regions

Table 8.7 shows the number of expected and measured event yields in the various control and validation regions. The derived values of the $t\bar{t}$ normalisations are consistent with 1 within their uncertainties. After normalisation of the $t\bar{t}$ process in the CR1L regions, the agreement in the VR0L validation region is very good, which is reflected in the small closure systematic uncertainties in table 8.6. Both CR1L and VR0L regions are dominantly populated by $t\bar{t}$ events, with small contributions from $W/Z$ production and production of $t\bar{t}$ in association with a $W$, $Z$ or Higgs boson.

Figures 8.6-8.9 show the most important three kinematic variables (according to table 8.4) and NN output scores in the CR1L and VR0L regions for each of the four sets of SR/CR1L/VR0L regions. The $t\bar{t}$ normalisations derived from the fit are applied to the
Fit results for control and validation regions

MC $t\bar{t}$ expectation in all plots. The systematic uncertainty bands include all detector and theoretical uncertainties detailed in section 8.8 but not the VR0L closure uncertainty. Generally, good MC to data agreement is observed in all regions for all kinematic distributions. The agreement for the distributions of the highest ranked input variables is particularly important, as these will influence the distribution of the NN output score the most.
### 8.9 Fit results for control and validation regions

Table 8.7: Results of the fit for the control and validation regions adopted for the SRs. Expected yields derived from MC simulations using theoretical cross-sections are given for comparison. The uncertainties shown include the statistical and detector systematic uncertainties. The central values of the fitted sum of backgrounds in the control regions agree with the observations by construction. The uncertainty on the total background estimate can be smaller than the sum of the individual uncertainties due to correlations.
Figure 8.6: Distributions for regions optimised on signal point B600_C200_N60_BR05; Left: CR1L; right VR0L. Top row: NN output score (the lower value of the requirement on the NN score defining the region is indicated with the red arrow; in the case of VR0L, the SR starts at the high end of the NN score shown; in the case of CR1L, there is no upper requirement on the NN score). Further rows, from top to bottom: $N_{b-jets}$, $m_{\text{eff}}$, $p_T$. 
Figure 8.7: Distributions for regions optimised on signal point B600_C575_N60_BR05; Left: CR1L; right VR0L. Top row: NN output score (the lower value of the requirement on the NN score defining the region is indicated with the red arrow; in the case of VR0L, the SR starts at the high end of the NN score shown; in the case of CR1L, there is no upper requirement on the NN score). Further rows, from top to bottom: \( N_{\text{jets}} \), \( E_{\text{T}}^{\text{miss}} \), \( m_{\text{T,gen}} \).
Figure 8.8: Distributions for regions optimised on signal point B800_C300_N60_BR05; Left: CR1L; right VR0L. Top row: NN output score (the lower value of the requirement on the NN score defining the region is indicated with the red arrow; in the case of VR0L, the SR starts at the high end of the NN score shown; in the case of CR1L, there is no upper requirement on the NN score). Further rows, from top to bottom: NN output score, $m_{T,\text{Gen}}$, $E_{T}^{\text{miss}}$, $p_{T}^{\text{2nd jet}}$. 
Figure 8.9: Distributions for regions optimised on signal point B800_C300_N60_BR1; Left: CR1L; right VR0L. Top row: NN output score (the lower value of the requirement on the NN score defining the region is indicated with the red arrow; in the case of VR0L, the SR starts at the high end of the NN score shown; in the case of CR1L, there is no upper requirement on the NN score). Further rows, from top to bottom: $N_{b-jets}$, $p_T^{1st}$, $\sigma_{bb}$. 

8.9 Fit results for control and validation regions
8.10 Results and Interpretation

Table 8.8 shows the number of expected and observed events in each signal region. Production of \( Z \) in association with heavy flavor jets accounts for 69%-84% of the \( W/Z \) component, depending on the signal region. The dominant process among the "Others" component is \( \bar{t}tZ \) production (30%-54% of "Others" depending on the signal region), followed by \( \bar{t}tH \) (14%-41%) and \( \bar{t}tW \) (7%-18%).

Table 8.9 shows the breakdown of the systematic uncertainties on the signal region yields. No excess is observed in data above the Standard Model expectation and in the next section I proceed to set 95% exclusion limits on SUSY signal models.
### Table 8.9: Breakdown of the dominant systematic uncertainties on background estimates in the various signal regions.

Note that the individual uncertainties can be correlated, and do not necessarily add up quadratically to the total background uncertainty. The percentages show the size of the uncertainty relative to the total expected background. Sources of uncertainties larger than 5% are emboldened. Negligible sources of uncertainty smaller than 1% are not shown.

<table>
<thead>
<tr>
<th>Uncertainty of channel</th>
<th>$m_{b\bar{b}}$</th>
<th>$m_{\chi_0^0}$</th>
<th>$m_{\chi_0^0}$</th>
<th>BR</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>600</td>
<td>200</td>
<td>60</td>
<td>0.5</td>
</tr>
<tr>
<td></td>
<td>600</td>
<td>575</td>
<td>60</td>
<td>0.5</td>
</tr>
<tr>
<td></td>
<td>800</td>
<td>300</td>
<td>60</td>
<td>0.5</td>
</tr>
<tr>
<td></td>
<td>800</td>
<td>300</td>
<td>60</td>
<td>1</td>
</tr>
<tr>
<td>Total background expectation</td>
<td>7.98</td>
<td>10.09</td>
<td>3.23</td>
<td>3.27</td>
</tr>
<tr>
<td>Total statistical ($\sqrt{N_{\text{exp}}}$)</td>
<td>±2.8</td>
<td>±3.2</td>
<td>±1.8</td>
<td>±1.8</td>
</tr>
<tr>
<td>Total background systematic</td>
<td>±1.47 [18.4%]</td>
<td>±1.49 [14.8%]</td>
<td>±0.86 [26.6%]</td>
<td>±0.70 [21.5%]</td>
</tr>
<tr>
<td>$tt$ fit normalisation</td>
<td>±1.36 [17.1%]</td>
<td>±1.63 [16.1%]</td>
<td>±0.58 [18.0%]</td>
<td>±0.63 [19.3%]</td>
</tr>
<tr>
<td>$t\bar{t}$ production theory</td>
<td>±0.78 [9.8%]</td>
<td>±0.28 [2.8%]</td>
<td>±0.51 [15.9%]</td>
<td>±0.40 [12.4%]</td>
</tr>
<tr>
<td>$W/Z$ production theory</td>
<td>±0.45 [5.6%]</td>
<td>±0.57 [5.7%]</td>
<td>±0.19 [5.8%]</td>
<td>±0.08 [2.4%]</td>
</tr>
<tr>
<td>“Other” production theory</td>
<td>±0.22 [2.8%]</td>
<td>±0.43 [4.3%]</td>
<td>±0.09 [2.6%]</td>
<td>±0.11 [3.4%]</td>
</tr>
<tr>
<td>Closure in VR0L</td>
<td>±0.45 [5.6%]</td>
<td>±0.72 [7.1%]</td>
<td>±0.41 [12.7%]</td>
<td>±0.27 [8.3%]</td>
</tr>
<tr>
<td>Jet energy scale</td>
<td>±0.28 [3.5%]</td>
<td>±0.26 [2.5%]</td>
<td>±0.44 [13.8%]</td>
<td>±0.13 [3.9%]</td>
</tr>
<tr>
<td>Jet energy resolution</td>
<td>±0.78 [9.7%]</td>
<td>±0.38 [3.7%]</td>
<td>±0.10 [3.1%]</td>
<td>±0.11 [3.4%]</td>
</tr>
<tr>
<td>$b$-tagging</td>
<td>±1.23 [15.4%]</td>
<td>±1.51 [15.0%]</td>
<td>±0.53 [16.4%]</td>
<td>±0.57 [17.4%]</td>
</tr>
<tr>
<td>$c$-tagging</td>
<td>±0.34 [4.2%]</td>
<td>±0.44 [4.4%]</td>
<td>±0.12 [3.8%]</td>
<td>±0.07 [2.0%]</td>
</tr>
<tr>
<td>light-tagging</td>
<td>±0.20 [2.5%]</td>
<td>±0.13 [1.3%]</td>
<td>±0.05 [1.7%]</td>
<td>±0.03 [1.0%]</td>
</tr>
<tr>
<td>Pileup reweighting</td>
<td>±0.12 [1.5%]</td>
<td>&lt; 1%</td>
<td>±0.05 [1.7%]</td>
<td>±0.09 [2.7%]</td>
</tr>
</tbody>
</table>
Figures 8.10-8.13 show distributions in the signal regions of the NN score, and distributions of the three most important kinematic variables as detailed in table 8.4. The $t\bar{t}$ normalisations derived from the fit are applied to the MC $t\bar{t}$ expectation in all plots. The systematic uncertainty bands include all detector and theoretical uncertainties detailed in section 8.8 and specifically also the VR0L closure uncertainty. Kinematic distributions are also shown for signal points which this analysis is expected to be sensitive to. Generally, good agreement is observed between the number of data events in each SR and the MC expectation, though comparing kinematic distributions is difficult due to the low number of events (between 4 and 11 events are observed in the different signal regions).
Figure 8.11: Distributions for signal region optimised on point B600,C575,N60, BR05. Top row: NN output score, $N_{jets}$. Bottom row: $E_{T}^{miss}$, $m_{T\text{gen}}$. 
Figure 8.12: Distributions for signal region optimised on point B800_C300_N60_BR05. Top row: NN output score, $m_{T_{\text{Gen}}}$. Bottom row: $E_{T_{\text{miss}}}$, $p_{T}^{2}$. 
Figure 8.13: Distributions for signal region optimised on point B800_C300_N60_BR1. Top row: NN output score, $N_{\text{b-jets}}$. Bottom row: $p_{T1}$, $\sigma_{m_{bb}}$. 
Figure 8.14 shows the combined exclusion limit in each of the 4 signal planes, obtained by taking at each signal point the signal region that gives the best expected CLS value.

The neural networks most sensitive in the different regions of the signal planes are as expected from the optimisation. In the signal model plane with $\text{BR}(\tilde{b}_1 \rightarrow b + \tilde{\chi}_0^2) = 100\%$ and $m_{\tilde{\chi}_1^0} = 60 \text{ GeV}$, signal region B800_C300_N60_BR1 provides sensitivity at large mass splittings $\Delta m(\tilde{b}_1, \tilde{\chi}_2^0)$, as the NN was optimised on a model with the same characteristics (large mass splitting, $m_{\tilde{\chi}_1^0} = 60 \text{ GeV}$ and $\text{BR} = 1$). In such signal events, a large number of high $p_T$ $b$-jets are expected from the $\tilde{b}_1 \rightarrow b + \tilde{\chi}_2^0$ decay as well as from the Higgs decays. Therefore one expects $N_{b-jets}$, $p_T^{b-jets}$ and $\sigma_{m_{b\bar{b}}}$ to be some of the most sensitive variables, which were indeed found to be among the highest ranked variables for the neural network trained on B800_C300_N60_BR1 (table 8.4). For models in the same signal plane exhibiting a small mass differences $\Delta m(\tilde{b}_1, \tilde{\chi}_2^0) \lesssim 100 \text{ GeV}$, the neural network optimised for the signal point B600_C575_N60_BR05 is found to be most sensitive, as again it was trained on a model with similar characteristics. For such events, the $p_T$ of the $b$-jets resulting from the decay $\tilde{b}_1 \rightarrow b + \tilde{\chi}_2^0$ is expected to be low and the $b$-jets are potentially out of acceptance. Significant jet activity is still expected from the $\tilde{\chi}_2^0$ decays, so $N_{jets}$, $E_{T\text{miss}}$ and $m_{TGen}$ are the best ranked variables. The same two signal regions are sensitive to models in the signal plane with $m_{\tilde{\chi}_2^0} = 2 \times m_{\tilde{\chi}_1^0}$ and $\text{BR} = 1$ for similar reasons as described.

For the signal planes with $\text{BR} = 0.5$, the NN optimised on B600_C575_N60_BR05 is still sensitive to models with low and moderate $\Delta m(\tilde{b}_1, \tilde{\chi}_2^0)$. In the case $m_{\chi_2^0} = 2 \times m_{\chi_1^0}$, the same signal region is sensitive to most of the plane, due to the fact that it does not rely heavily on the $p_T$ of the jets or on the $b$-jet multiplicity. Models with large mass separation $\Delta m(\tilde{b}_1, \tilde{\chi}_2^0)$ and $m_{\chi_1^0} = 60 \text{ GeV}$ are expected to have high $p_T$ ($b$-)jets and are therefore excluded by either signal region B600_C200_N60_BR05 or B800_C300_N60_BR05, which were optimised on models belonging to this signal plane.

\footnote{Currently, the exclusion limits exhibit artefacts due to the limited number of signal simulations. For example, in the exclusion plots with $m_{\chi_2^0} = 2 \times m_{\chi_1^0}$ and $\text{BR} = 1$ (bottom left of fig. 8.14), the expected exclusion limit has a step at $m_{\chi_2^0} = 420 \text{ GeV}$ and low $m_{\chi_1^0}$. This is an artefact due to the interpolation procedure for the CLS values as well as due to the lack of simulated signal points below (outside) of the exclusion limit. The dents in the exclusion limit at $m_{\chi_2^0} = 500 \text{ GeV}$, $m_{\chi_1^0} = 300 \text{ GeV}$ and $m_{\chi_1^0} = 700 \text{ GeV}$, $m_{\chi_2^0} = 500 \text{ GeV}$ in the same plane are also interpolation artefacts, both points having an expected CLS value below 0.05.}
Figure 8.14: Observed and expected exclusion limits in the $\tilde{b}_1 - \tilde{\chi}_2^0$ mass plane. Top row: planes with $m_{\tilde{\chi}_1^0} = 60$ GeV. Middle row: planes with $m_{\tilde{\chi}_1^0} = 2 \times m_{\tilde{\chi}_1^0}$. Left: $\text{BR}(\tilde{b}_1 \to b + \tilde{\chi}_2^0) = 1$, right: $\text{BR}(\tilde{b}_1 \to b + \tilde{\chi}_2^0) = \text{BR}(\tilde{b}_1 \to b + \tilde{\chi}_1^0) = 0.5$. The numbers shown report the best expected signal region according to the legend:

1 B600_C200_N60_BR05
2 B600_C575_N60_BR05
3 B800_C300_N60_BR05
4 B800_C300_N60_BR1
8.11 Summary and discussion

In this thesis, I have presented a novel, yet unpublished analysis sensitive to the pair production of the scalar partners of bottom quarks cascade-decaying to a final state containing at least one Higgs boson. The search is optimised for 20.1 fb$^{-1}$ of $pp$ collisions at a centre-of-mass energy of 8 TeV.

The sensitivity of this analysis to the signal models investigated surpasses that of other analyses. Following the convention followed by the ATLAS and CMS experiments, exclusion limits are quoted considering the signal cross-section $-1\sigma$ theoretical uncertainty. In the case of $m_{\tilde{\chi}^0_1} = 60$ GeV, $\text{BR}(\tilde{b}_1 \to b + \tilde{\chi}^0_2) = 1$, sbottom masses up to $m_{\tilde{b}_1} = 760$ GeV are excluded for $m_{\tilde{\chi}^0_2}$ between 500 GeV and 600 GeV. The performance of the analysis for this signal plane is similar to that shown in [137]. However, the analysis presented in this chapter has also been optimised for the case of the mixed decay with $\text{BR}(\tilde{b}_1 \to b + \tilde{\chi}^0_2) = \text{BR}(\tilde{b}_1 \to b + \tilde{\chi}^0_1) = 0.5$, for which sbottom masses up to $m_{\tilde{b}_1} = 680$ GeV are excluded for $m_{\tilde{\chi}^0_2} = 500$ GeV. No other analysis has attempted exclusions in this plane.

A more difficult case is posed by the constraint $m_{\tilde{\chi}^0_2} = 2 \times m_{\tilde{\chi}^0_1}$, for which reference [137] has no sensitivity. The current analysis excludes sbottom masses up to 650 GeV for a $\tilde{\chi}^0_2$ mass of 480 GeV for $\text{BR}(\tilde{b}_1 \to b + \tilde{\chi}^0_2) = 1$ and sbottom masses up to 610 GeV for a $\tilde{\chi}^0_2$ mass between 320 GeV and 490 GeV for the mixed decays scenarios.

Prior to publication of this analysis, a more accurate understanding of theoretical uncertainties will be made, in particular the parton distribution functions and the renormalisation and factorisation scale uncertainties. Additional signal samples will be simulated in order to eliminate the interpolation artefacts present in the exclusion limits. The improvements are unlikely to change the conclusions of the analysis.

The analysis can be repeated with run II data, for which several improvements can be made. The definition of the control and validation region can be optimised to reach a better compromise between the cancellation of systematic uncertainties between the SR and CR and the statistical uncertainty on the $tt$ normalisation factor due to the limited size of the CR.
data sample. Alternative background estimation techniques can be explored, for example the data-driven “matrix method” [137]. Furthermore, it may be instructive to split the $t\bar{t}$ background by flavour composition into a $bbb$ and $bbc$ component. An additional control region with 2 different-flavour leptons could be used to control the $bbb$ component, while the $bbc$ contribution would continue to be normalised in the 1-lepton control region or could be determined using the matrix method.

Amongst the SRs chosen for the exclusions in fig. 8.14 the non-$t\bar{t}$ background contributes between 19% in the case of B800_C300_N60_BR1 up to 33% in the case of B600_C575_N60_BR05. A better understanding of the non-$t\bar{t}$ background and dedicated control regions could be introduced in order to gain more confidence in the background prediction.

We have not yet found SUSY. However, if sbottom quarks do exist, it is unlikely they will decay in a single channel as investigated in chapter [7] i.e. $\tilde{b}_1 \rightarrow b + \tilde{\chi}_1^0$. The analysis presented in this chapter is the first analysis at a LHC experiment shown to be sensitive to sbottom mixed decays to $b + \tilde{\chi}_2^0$ and $b + \tilde{\chi}_1^0$. Repeated with the run II data, the search will have leading sensitivity to such complex decay scenarios.
Chapter 9

Conclusions

Since the LHC switched on in March 2010, the operation of both the accelerator and the ATLAS detector have been very successful. The collaboration was able to record data with high efficiency thanks to a concerted effort dedicated to detector operation and maintenance. With the recorded data, the collaboration was able to put the Standard Model to some of the most stringent tests to date and to obtain unprecedented sensitivity to physics beyond the Standard Model.

Within this context, chapters 5 and 6 contributed to the high data taking efficiency. Chapter 5 established the statistical techniques necessary to analyse the failure rate of VCSEL devices in the ATLAS SCT. By comparing this failure rate to results from accelerated lifetime tests, it was confirmed that humidity affects the reliability of the devices. This led to the adoption of several countermeasures by the collaboration, which ensured the operability of the SCT.

This thesis presents in chapter 6 the first evidence of SEU and a measurement of the SEU rate in opto-electronics in a collider environment. The mitigation strategies in place were effective at reducing the impact of SEU on the data taking efficiency to a negligible level.

Later chapters of this thesis use the 2012 ATLAS data set to search for supersymmetry.
Chapter 9. Conclusions

Chapter 7 outlines a search for pair-production of 3rd generation quarks. The search is optimised for scenarios in which the scalar bottom decays exclusively to a bottom quark and a neutralino and the scalar top decays to a bottom quark and a chargino, with a small mass difference with the neutralino. As no signal is observed above Standard Model expectation, competitive exclusion limits on scalar bottom and top production are set, surpassing previously existing limits. Sbottom masses up to 640 GeV are excluded at 95% CLs for neutralino masses of up to 150 GeV. In the case of stop pair production and decay, slightly weaker exclusion limits are obtained.

The assumption of the search reported in chapter 7 is that the sbottom quark decays in one channel only, though nature might not be so kind to realise such a simple theory. If the branching ratio of $\tilde{b}_1 \to b + \tilde{\chi}_1^0$ is reduced, the sensitivity of the search to high sbottom masses is lost. Chapter 8 extends the search by considering further possible decays of the sbottoms, including cascade-decays to the newly discovered Higgs boson, thus recovering sensitivity to high sbottom masses. This is the first ATLAS search to show sensitivity to sbottom mixed decays.

No evidence of SUSY or new physics has been observed in the data collected during 2011 and 2012 at a centre-of-mass energy $\sqrt{s} = 7$ TeV and 8 TeV, respectively. During the shutdown following the end of data-taking in 2012, the machine has been consolidated to be able to operate in 2015 at a centre-of-mass-energy of 13 TeV, with the possibility of future 14 TeV collisions. The techniques developed and tested for the two searches described in this thesis will also be applicable to the new dataset. The searches have shown world-leading sensitivity to 3rd generation squarks when applied on the 2011 and 2012 datasets, and could be amongst the first ones to see the early signs of SUSY in 2015 running. Projections of sensitivity made using the analysis targeting the $b\tilde{\chi}_1^0$ final state, with the only change being higher requirements on the $m_{CT}$ variable up to $m_{CT} > 750$ GeV, show that bottom squark with masses up to 1100 GeV can be discovered with $5\sigma$ significance with 300 fb$^{-1}$ of 14 TeV collisions, for a massless $\tilde{\chi}_1^0$ [145]. This amount of data is expected to be collected between 2015 and 2022, following which the accelerator is foreseen to be upgraded to the High Lumi-
nosity LHC (HL-LHC), which will be able to achieve luminosities of $5 \times 10^{34} \text{cm}^{-2}\text{s}^{-1}$. The HL-LHC is expected to deliver 3000 fb$^{-1}$ of data from 2025 until 2032, which will extend the discovery reach for bottom squarks up to a mass of 1300 GeV. Future improvements in the analysis techniques and in the understanding of experimental and theoretical systematic uncertainties on the SM background could provide additional sensitivity gains at high luminosity.
Appendix A

Absolute versus relative humidity

The relative humidity is defined as the ratio of the partial pressure of water vapour in the air mixture to the saturated vapour pressure of water at a given temperature,

\[ RH = \frac{p_{\text{partial}}}{p_{\text{saturated}}} \]  \hspace{1cm} (A.1)

The saturated vapour pressure (in millibar) is well described by the empirical formula [146]

\[ p_{\text{saturated}} = 6.112 \times \exp \left( \frac{17.67 \times T_{\text{amb}}}{T_{\text{amb}} + 243.5} \right) \]  \hspace{1cm} (A.2)

where \( T_{\text{amb}} \) is the ambient temperature in degree Celsius.

The numerator needed for the calculation of \( RH \) is the partial pressure of the water vapour, which can be calculated from the ideal gas law. It is given by

\[ p_{\text{partial}} = \frac{\nu_{\text{water}} R (T + 273.15 K)}{V} = \frac{m_{\text{water}}}{M_{\text{water}}} \frac{R (T + 273.15 K)}{V} \]  \hspace{1cm} (A.3)

where \( \nu_{\text{water}} \) is the number of moles of water in the volume \( V \), \( m_{\text{water}} \) is the mass of water vapour in the same volume and \( M_{\text{water}} \) is the molar mass of water, 18 g/mol. \( R \) denotes the gas constant. The term \( \frac{m_{\text{water}}}{V} \) is recognised as being the absolute humidity, therefore the above expression can be rewritten as

\[ p_{\text{partial}} = AH \times T \times \frac{R}{M_{\text{water}}} \]  \hspace{1cm} (A.4)

The above equations establish an unique inter-dependency between \( AH, RH, T \). This allows for the calculation of any third environmental property if the other two are known.
Appendix B

Fit setup and results discussion

B.1 Modelling of the systematic uncertainties

When modelling the yield $\eta$ as a function of the uncertainty parameter $\alpha$, a piecewise exponential function is used to interpolate between the $\eta(-1), \eta(0), \eta(+1)$ values ([132]). The exponential interpolation ensures that $\eta(\alpha) > 0$ for any $\alpha$. Uncertainties of the following type have been used in the HistFitter fit setup:

- **overallNormHistoSys**: used for JES. The nominal distribution of a variable, as well as histograms of the same distribution corresponding to the up and down variations of a parameter are saved as a histogram. The up and down variation histograms are renormalized in specific control regions. For example, the $t\bar{t}$ yield is driven by the dedicated $t\bar{t}$ control region. The top histograms with up and down variations are renormalized in various regions based on variations seen in the $t\bar{t}$ control region. The normalization of the systematic uncertainty is absorbed into the floating $\mu_{t\bar{t}}$ parameter.

- **normHistoSys**: used for $b, c$, light-tagging. In the case of 1-bin histograms relevant to the current analysis, **normHistoSys** is equivalent to **overallNormHistoSys**.

- **normHistoSysOneSideSym**: used for JER as above, but there is no up and down variation. The yield as a function of $\alpha_{\text{JER}}$ is parameterised as $\eta(+1) = \text{JER-varied yield}$,
The relative affect of the +/-1 sigma variations of the systematic source on the integral of the histogram is calculated and the systematic variation is applied as an overall scaling to the histogram, correlated across the bins but without using shape information. The values for the up and down variations are not renormalised in a control region.

- **userOverallSys**: used for theory uncertainties. User passes in high- and low-values manually, rather than through input histograms or trees. The values are applied as an overall scaling to the histograms. The theory uncertainties are also normalised “by hand” in the control regions.

### B.2 Background-only fit results

The linear correlation coefficient between two parameters is obtained as the covariance of the two parameters, divided by the square root of the product of the variances:

$$
\rho_{ij} = \frac{\text{cov}(\alpha_i, \alpha_j)}{\sigma(\alpha_i)\sigma(\alpha_j)}.
$$

The quantity $\rho_{ij}$ is in the range $-1 \leq \rho_{ij} \leq 1$. If $|\rho_{ij}| = 1$, $\alpha_i$ and $\alpha_j$ are linearly dependent. The global correlation coefficient is a measure for the strongest correlation between variable $i$ and a linear combination of all other variables, and is defined by

$$
\rho_i = \sqrt{1 - \frac{1}{C_{ii}(C^{-1})_{ii}}},
$$

with $C_{ii}$ and $(C^{-1})_{ii}$ being elements in the diagonal of the covariance matrix and of its inverse, respectively. The **alphaXXX** parameters refer to the $\alpha_p$ nuisance parameters, and their naming is mostly self-explanatory. $\alpha_{JC}$ refers to the JES, $\alpha_{JR}$ to the JER, $\alpha_{MET}$ to the $E_{\text{T}}^{\text{miss}}$ CellOut term energy scale, and $\alpha_{RESOST}$ to its resolution.
### B.2 Background-only fit results

<table>
<thead>
<tr>
<th>Floating Parameter</th>
<th>InitialValue</th>
<th>FinalValue +/- Error</th>
<th>GblCorr.</th>
</tr>
</thead>
<tbody>
<tr>
<td>alpha_JC</td>
<td>0.0000e+00</td>
<td>-3.9328e-03 +/- 9.93e-01</td>
<td>0.300229</td>
</tr>
<tr>
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<td>0.0000e+00</td>
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<td>1.1988e-02 +/- 9.94e-01</td>
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</tr>
<tr>
<td>alpha_lTagging</td>
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<tr>
<td>alpha_otherTheory</td>
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<tr>
<td>alpha_topTheory</td>
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<td>1.0400e+00 +/- 1.24e-01</td>
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<td>mu_W</td>
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<td>1.0555e+00 +/- 5.23e-01</td>
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<td>mu_Z</td>
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<td>1.1995e+00 +/- 1.74e-01</td>
<td>0.214617</td>
</tr>
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</table>

Table 1: SRA fit parameters.

<table>
<thead>
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<th>Floating Parameter</th>
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<th>FinalValue +/- Error</th>
<th>GblCorr.</th>
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<tr>
<td>alpha_RESOST</td>
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<td>3.9976e-04 +/- 9.93e-01</td>
<td>0.004250</td>
</tr>
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<td>alpha_WTheory</td>
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<td>-3.4498e-04 +/- 9.93e-01</td>
<td>0.736514</td>
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<tr>
<td>alpha_ZTheory</td>
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<td>2.3130e-05 +/- 9.93e-01</td>
<td>0.002602</td>
</tr>
<tr>
<td>alpha_bTagging</td>
<td>0.0000e+00</td>
<td>2.3566e-03 +/- 9.93e-01</td>
<td>0.222074</td>
</tr>
<tr>
<td>alpha_cTagging</td>
<td>0.0000e+00</td>
<td>7.7980e-04 +/- 9.93e-01</td>
<td>0.063243</td>
</tr>
<tr>
<td>alpha_lTagging</td>
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<td>3.0085e-04 +/- 9.93e-01</td>
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<tr>
<td>alpha_otherTheory</td>
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<tr>
<td>alpha_topTheory</td>
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<td>0.206173</td>
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<td>mu_Z</td>
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<td>0.289594</td>
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</table>

Table 2: SRB fit parameters.
B.2 Background-only fit results

Figure B.1: SRA fit parameter correlations.

Figure B.2: SRB fit parameter correlations.
Appendix C

Optimisation of the search for the Higgs boson in sbottom decays

The current appendix presents further details on the optimisation procedure used to determine the signal regions used in the search for the Higgs boson in sbottom decays described in chapter 8.

C.1 Multivariate techniques and their sensitivity

With TMVA, the following several multivariate techniques were trained and their performance was compared:

- Rectangular cuts optimised using a genetic algorithm (CutsGA) - essentially a “standard” cut & count analysis
- Multi-layer perceptron backpropagating neural network (NN)
- Boosted decision trees (BDT)
- Support vector machine (SVM)
- Linear discriminant and function discriminant analysis (LD and FDA_GA)
- $k$-nearest neighbors (KNN)
- Rule ensembles (RuleFit)
• “Likelihood” - automatic non-parametric probability density function (PDF) estimation through histogram smoothing and interpolation with various spline functions and quasi-unbinned kernel density estimators.

Details about each of these techniques and further references can be found in the TMVA user guide [147]. The techniques are used with the default settings in TMVA. The neural networks used in this analysis are feed-forward, have a layer of input neurons with linear activation functions, a layer of hidden neurons with tanh activation functions and a final output layer consisting of one neuron with linear activation function. The number of input neurons is equal to the number of input variables, plus an additional bias neuron. The number of neurons in the hidden layer is equal to the number of input neurons plus five.

For one particular signal sample, B600_C200_N60_BR05, figure C.1 shows the background rejection versus signal efficiency curve for each of the trained techniques. The curves are obtained in the case of MVA techniques by scanning through the value of the requirement on the MVA output score and evaluating the background rejection and the signal efficiency. In the case of the rectangular cuts method, the various sets of cuts corresponding to particular values of the signal selection efficiency (operating point) are scanned over. Several MVA techniques exhibit a higher background rejection rate for a given signal efficiency compared to the simple “cut & count” selection, i.e. they exhibit superior performance.

![Background rejection versus Signal efficiency](image)

Figure C.1: Signal efficiency versus background rejection for the MVAs trained on the point B600_C200_N60_BR05.
C.2 Overtraining

For several signal points, a signal region per point and per MVA technique each was optimised. Table C.1 shows the achievable significance as defined in eq. 8.4. Neural networks exhibit overall the best performance. The other MVA techniques were therefore not pursued any further.

<table>
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<tr>
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<th>NN</th>
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<th>FDA_GA</th>
<th>SVM</th>
<th>BDT</th>
<th>RuleFit</th>
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<tr>
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<td>29.47</td>
<td>21.36</td>
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</table>

Table C.1: Best significance achieved for each trained MVA. The last row shows a sum of significances for all SRs. While this value has no physical meaning, it is an indicator that overall, the neural network technique (NN) performs best out-of-the-box. NNs used in the final exclusion limits of chapter 8 are emboldened.

C.2 Overtraining

Identifying overtraining

In order to test for overtraining, the TMVA suite splits the background and signal samples in equal halves; the training is performed on one half of the sample (training sample) and the resulting neural network is applied on the second half of the sample (test sample). The test is repeated for the signal and the background sample.

In the case of the neural network training, the output score for the training and test sub-samples agrees well for both signal and background, as shown in figure C.2. A standard test that compares two distributions (in this case the output score of the training and test sub-samples), is the Kolmogorov-Smirnov (K-S) test. The K-S statistic is given by the
maximum distance between the cumulative probability distributions of the two distributions to be compared, and the probability distribution for the K-S statistic can be calculated. A low \( p_{KS} \) value implies inconsistent NN score distributions for the training and test samples, therefore indicating overtraining.

Overtraining is illustrated on one example. An optimal selection, based on the training samples of the overtrained BDT shown on the bottom row of figure C.2, would be to require the BDT score to be above the value 0.0. When applying this requirement on the test samples, one can observe leakage of the signal to lower BDT values, outside of the signal region. Signal events in real data are likely to have a BDT score distribution similar to that exhibited by the test sample. When applying the same BDT on data events and requiring the BDT score to be above 0.0, potential signal events are therefore likely to be rejected. Conversely, more background events in data than expected from training could enter the signal region. There is no straightforward way though to quantify the correlation between the \( p_{KS} \) value and the degradation in sensitivity (e.g. the reduction in the final \( Z \) discovery significance).

Avoiding overtraining

Neural networks in TMVA are by design more robust against overtraining compared to other MVA techniques implemented in TMVA due to the use of a Bayesian regulator which adaptively regularises the model complexity [148]. A certain amount of overtraining is nevertheless still possible. From all trained SRs, the largest amount of overtraining was observed in the case of the NN trained on the point B700,C300,N150_BR1, giving the lowest \( p_{KS} = 0.03 \) (see left histogram of fig. C.2 for training and test sample score distributions for this NN). The following procedure was devised in order to minimise the overtraining and to confirm the robustness of the adaptive regularisation of the model complexity. A neural network is trained on all \( N \) input variables (this is called the “exhaustive NN”), as well as on the \( N \) combinations of \( N - 1 \) variables, i.e. the combinations obtained from the initial variables by excluding one at a time. The maximum achievable significance, as well as the K-S test \( p_{signal} \) and \( p_{bckg} \) are assessed for all \( N + 1 \) neural networks (1 with \( N \) variables and \( N \) with
Of the $N$ NNs with a variable removed, that with the largest significance is retained. The procedure is repeated until a further variable removal would not lead to any increase in the sensitivity.

Naively, the reduction in the number of input variables is expected to reduce the degree of overtraining (increase the $p_{KS}$ values), though it was observed that this is not always the case. If the $p_{KS}$ values decrease, any reported increase in significance for the training sample might only be a subjective one, while objectively the sensitivity might in fact drop when the classification is applied to the independent test sample.
An alternative approach to the “best significance” (BS) procedure described above is therefore investigated. It adopts a similar strategy: the variable to be removed is chosen such that it maximises significance, but the product of the K-S probabilities $p_{\text{signal}} \times p_{\text{bckg}}$ is not allowed to decrease. I will call this procedure “best significance with barrier on $p_{\text{KS}}$” (BS_BpKS).

It was observed that for 6 out of the 14 signal samples trained on, the two different overtraining optimisation procedures converge to the same result. For the analysis reported in chapter 8, the signal regions obtained with the “best significance with barrier on $p_{\text{KS}}$” (BS_BpKS) procedure were used. Table C.2 reports which variables have been removed from the training as a consequence of this procedure. Most often, the variable removed is the fourth jet $p_T$. Variables removed also include the jet multiplicity $N_{\text{jets}}$, $\sigma_{m_{bb}}$, leading, subleading jet $p_T$ and $E_{\text{miss}}^T$. Notice that the information contained in these variables is not completely lost upon their removal. For example, the sub-leading jet $p_T$ intrinsically carries the information that the leading jet has at least the same $p_T$, and similarly the $N_{b-\text{jets}}$ variable places a lower limit on the $N_{\text{jets}}$ value. It should be noted that the leading jet $p_T$ already has a requirement imposed at 90 GeV in order for the trigger to be in the efficiency plateau. For the optimised signal regions, table C.3 shows the ranking of importance of the remaining variables that enter the training. Table C.4 compares the performance of the initial, “exhaustive” NN compared to that of the final SRs. The study confirms that the Bayesian regulator employed to adaptively reduce the NN model complexity is robust.

**Effect of overtraining on exclusion limits**

The SR which is optimal for a given signal point and which employs a NN trained on that signal point is not guaranteed to be optimal for nearby points in $m_{\tilde{b}_1}$, $m_{\tilde{\chi}_2^0}$, $m_{\tilde{\chi}_1^0}$ space. In this subsection, I compare the exclusion sensitivity in the whole signal plane for the exhaustive NN trained on all variables and the NN giving minimal overtraining, as obtained by the procedures explained above. Large overtraining for the nominal sample can be expected to lead to lower sensitivity for nearby signal points.
The smallest $p_{KS}$ value achieved amongst all SRs when the NN is trained on all variables is 0.032 for the point B700_C300_N150_BR1. Upon removal of the $N_{jets}$ variable from the training, the significance when applied to the test sample increases most, from previously 1.40 to 1.50, while the $p_{KS}$ value increases to 0.83. Any further removal of a variable does not further increase the significance when evaluated on the test sample. Having established these differences in $p_{KS}$, the impact of this amount of overtraining on the final exclusion limit is assessed\(^1\). Running the statistical machinery separately for the signal region defined with a requirement on the exhaustive NN and on the NN trained with the removed variable shows a maximum improvement of 5 GeV towards higher sbottom masses in the expected exclusion limit\(^2\). This check shows that $p_{KS,signal}$ values as low as 0.03 can be tolerated without a significant decrease in sensitivity. This can also be expected from the distribution of the neural network output score for the training and test samples shown in figure C.2 (top

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\(^1\)The same statistical limit setting procedure is used as described in section 7.9.

\(^2\)The improvement is achieved in the plane with $\text{BR}(\tilde{b}_1 \to b \tilde{\chi}_2^0)=100\%$, $m_{\tilde{\chi}_1}=60$ GeV.
### Table C.3: Variable ranking for the optimal SRs obtained with the BS_BpKS procedure.

Each of the remaining variables is removed one at a time and the largest reduction in the signal sensitivity is interpreted to be due to the removal of the most important variable from the training. NNs used in the final exclusion limits of chapter 8 are emboldened.

A $p_{KS}$ value of 0.03 corresponds to a low amount of “leakage” of the signal events from high to low NN score. A more concerning situation is shown on the bottom row of the same figure, in which a BDT technique exhibits a large degree of overtraining, giving a $p_{KS}$ value for the signal $\mathcal{O}(10^{-24})$. BDT techniques can in principle be adapted to be more robust against overtraining, though this analysis only employs signal regions based on neural networks.

### C.3 Control and validation regions

Only four of the above defined signal regions are used in the final exclusion limits presented in chapter 8. The background and signal contributions, and all related uncertainties were evaluated for all SRs and related control and validation regions, and expected limits in all simplified model planes were calculated. For each point, the best expected signal region was chosen. Only seven of the fourteen SRs contributed to the expected combined exclusion limit. Three of the seven signal regions gave the best expected sensitivity for signal points with low $m_{b_1}$. However, the points were also expected to be excluded with slightly weaker...
C.3 Control and validation regions

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<th>$m_{b_1}$</th>
<th>$m_{\tilde{q}}$</th>
<th>$m_{\tilde{g}}$</th>
<th>BR</th>
<th>$\sigma$</th>
<th>$P_{KS,signal}$</th>
<th>$P_{KS,bckg}$</th>
</tr>
</thead>
<tbody>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>initial</td>
<td>final</td>
<td>initial</td>
</tr>
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Table C.4: Effect of overtraining control. The “best significance with barrier on $p_{KS}$” (BS_BpKS) procedure is used to remove weak variables from the NN training. The initial and final significance value, and signal and background $p_{KS}$ probabilities are shown. Emboldened regions are those used in the final exclusion limits.

CLs values by the other four signal regions, and after interpolation there was no visible difference in the expected exclusion when using the full set of seven signal regions or the reduced set of four signal regions. Chapter 8 refers to the reduced set of four signal regions only in order to avoid cluttering the discussion.

The fact that a region has not been chosen in the final exclusion limit for a particular point, despite it being optimised for that point, is due to the contribution of non-$t\bar{t}$ processes to such SRs and a more accurate picture of the systematic uncertainties used in the limit setting. In the optimisation using TMVA, only $t\bar{t}$ events were discriminated against and a flat systematic uncertainty was assumed throughout.
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