Rare $b$ decays at LHCb

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On behalf of the LHCb Collaboration

Les Rencontres de Physique de la Vallee d’Aoste, La Thuile 2015
Why to study rare B decays?

Rare FCNC b decays are sensitive to quantum corrections from degrees of freedom at or above the electroweak scale.
Why to study rare B decays?

Rare FCNC b decays are sensitive to quantum corrections from degrees of freedom at or above the electroweak scale.

Nothing different from what was done at the dawn of the electroweak theory:

1934: Fermi’s theory of the nuclear beta decay (only S & V currents)
1936: Gamow-Teller transitions (also T and A are possible)
1957: Mme Wu experiment: The parity is violated in weak interactions
1957: Feynman-Gellmann: V-A theory
1968: Glashow, Weinberg and Salam: birth of the Standard Model
Why to study rare B decays?

Rare FCNC $b$ decays are sensitive to quantum corrections from degrees of freedom at or above the electroweak scale.

We describe FCNC processes by an effective Hamiltonian in the form of Operator Product Expansion to identify the types of operators that enter in the transitions:

$$H_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \sum_i \left[ C_i(\mu)O_i(\mu) + C'_i(\mu)O'_i(\mu) \right]$$

- $i = 1,2$: Tree
- $i = 3-6,8$: Gluon penguin
- $i = 7$: Photon penguin
- $i = 9,10$: Electroweak penguin
- $i = S$: Higgs (scalar) penguin
- $i = P$: Pseudoscalar penguin
Why to study rare B decays?

Rare FCNC b decays are sensitive to quantum corrections from degrees of freedom at or above the electroweak scale.

NP can modify the Wilson coefficients ($C_i$) affecting observable quantities as angular distributions and decay rates in $B \rightarrow K^{(*)}\mu\mu$ decays ($C_7, C_9, C_{10}$), decay rates in $B \rightarrow \mu\mu$ decays ($C_s, C_p$) and photon polarization ($C'_7$)

$$H_{\text{eff}} = - \frac{4 G_F}{\sqrt{2}} V_{tb} V_{ts}^* \sum_i \left[ C_i(\mu) O_i(\mu) + C'_i(\mu) O'_i(\mu) \right]$$

$i = 1,2$ Tree
$i = 3, 6, 8$ Gluon penguin
$i = 7$ Photon penguin
$i = 9,10$ Electroweak penguin
$i = S$ Higgs (scalar) penguin
$i = P$ Pseudoscalar penguin
...You will see that, despite the variety of topics, today we will be talking always about (almost) the same diagram (in different flavors)....
Angular analysis of $B^0 \rightarrow K^{*0} \mu^+ \mu^-$ decays

Angular distribution of the $B^0 \rightarrow K^{*0} \mu^+ \mu^-$ decay is sensitive to the virtual photon polarisation and new left- and right-handed (axial) vector currents.

Decay described by three angles ($\theta_l$, $\theta_K$, $\phi$) and the dimuon invariant mass squared $q^2$.

3 angles, 12 different $J_\ell$ coefficients (which contain the information from Wilson coefficients) due to 6 complex numbers that define the $K^{*0}$ spin amplitudes. (see spares slides for complete description)
Puzzling deviations: $P'_5$ in $B^0 \to K^{*0} \mu^+ \mu^-$

In 2013, the observation by LHCb of a tension with the SM in $B \to K^* \mu \mu$ angular observables has received considerable attention from theorists and it was shown that the tension could be softened by assuming the presence of new physics (NP).

-3.7 $\sigma$ discrepancy in the region $4.3 < q^2 < 8.68 \text{ GeV}^2/c^4$

[probability that at least one bin varies by this much is 0.5%]

Can be explained by a negative NP contribution to the Wilson coefficient $C_9$, namely $C_9=C_9(\text{SM})-1.5$

Descotes-Genon, Virto, Matias PRD 88 (2013) 074002
D. Van Dyck, C. Bobeth, F. Beaujean arXiv 1310.2478
Altmannshofer, Straub (arXiv 1308.1501)
In 2014, another tension with the SM has been observed by LHCb, namely a suppression of the ratio $R_K$ of $B^+ \rightarrow K^+ \mu^+ \mu^-$ and $B^+ \rightarrow K^+ e^+ e^-$ branching fractions at low di-lepton invariant mass → test of lepton universality

In SM this ratio is expected to differ from unity only due to tiny Higgs penguin contributions and difference of phase space:

$$R_K (SM) = 1.0003 \pm 0.0001$$

**Bobeth et al., JHEP 12 (2007) 040**

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Puzzling deviations: \( R_K = \frac{\text{BR}(B^+ \rightarrow K^+\mu^+\mu^-)}{\text{BR}(B^+ \rightarrow K^+e^+e^-)} \)

In 2014, another tension with the SM has been observed by LHCb, namely a suppression of the ratio \( R_K \) of \( B^+ \rightarrow K^+\mu^+\mu^- \) and \( B^+ \rightarrow K^+e^+e^- \) branching fractions at low di-lepton invariant mass → test of lepton universality

With 3 fb\(^{-1}\) LHCb measures:

\[
R_K = 0.745^{+0.090}_{-0.074} \text{(stat)} ^{+0.036}_{-0.036} \text{(syst)}
\]

which is (in)consistent with SM at 2.6 \( \sigma \)

LHCb, PRL 113 (2014) 151601
Belle, PRL 103 (2009) 171801
Babar, PRD 86 (2012) 032012
Finally, also branching ratio measurements of \( B^0 \rightarrow K^* \mu^+ \mu^- \), \( B^0 \rightarrow K_S \mu^+ \mu^- \) and \( B^+ \rightarrow K^{*+} \mu^+ \mu^- \) decays published recently seem to be too low compared to the SM predictions when using state-of-the-art form factors from lattice QCD or light-cone sum rules (LCSR).


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Puzzling deviations: $\text{BR}(B^0 \rightarrow K^{*0} \mu^+ \mu^-)$, $\text{BR}(B^0 \rightarrow K_S \mu^+ \mu^-)$, $\text{BR}(B^+ \rightarrow K^{*+} \mu^+ \mu^-)$

Average of the $B^0 \rightarrow K^{*} \mu\mu$ and $B^0_s \rightarrow \phi \mu\mu$ decay rates measured by LHCb, CMS, ATLAS and CDF in the high-$q^2$ range:

Zwicky et al, Phys. Rev. D71 (2005) 014029,

Average from LHCb, CDF, CMS and ATLAS

Theory predictions with $C_9 = C_{(SM)} - 1.5$


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Assuming new physics in $B \rightarrow K^{(*)}\mu\mu$ only, a consistent description of these anomalies seems possible:

- G. Hiller and M. Schmaltz, PRD90 (2014) 054014
- S. L. Glashow et al., arXiv:1411.0565 [hep-ph].

Difficult to explain data in SUSY scenarios or using partial compositeness (why only $C_9$?)
Data can be described using $Z'$ with flavour violating couplings, but mass must be $\mathcal{O}(7\text{ TeV})$
to avoid direct limits and limits from mixing ($\Delta m_s$).

PS: NA62 will probe the same underlying physics with $K \rightarrow \pi\nu\nu$ decays

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However, while $R_K$ is theoretically extremely clean, predicted to be 1 to an excellent accuracy in the SM, the other observables are plagued by sizable hadronic uncertainties. [different treatments of (factorisable/non-factorisable) corrections can give large variation of $P'_5$]

A lot to be done still both on experimental and theoretical sides

Descotes-Genon, Virto, Matias PRD 88 (2013) 074002
D. Van Dyk, C. Bobeth, F. Beaujean EPJC 74 (2014) 2893
Altmannshofer, Straub, EPJC 73 (2013) 2646, JHEP 01 (2014) 069
Jaeger, Camalich JHEP 05 (2013) 043
Study of the rare $B^0_{(s)} \rightarrow \pi^+ \pi^- \mu^+ \mu^-$ decays


$B^0_s \rightarrow f_0 \mu^+ \mu^-$: dominated by “penguin” and “box” $b \rightarrow s$ transition in SM.
Potentially sensitive to non-SM contributions, access similar physics of $B^0 \rightarrow K^{*0} \mu^+ \mu^-$ and $B^0_s \rightarrow \phi \mu^+ \mu^-$

$B^0 \rightarrow \rho \mu^+ \mu^-$: dominated by “penguin” and “box” $b \rightarrow d$ transition in SM.
Potentially sensitive to non-SM contributions, complementary w.r.t. $B^0_s \rightarrow f_0 \mu^+ \mu^-$

Gaia Lanfranchi (LHCb Collaboration) ---- 4 March 2015
LHCb sees first evidence for $B^0_{(s)} \to \pi^+ \pi^- \mu^+ \mu^-$ (7.3 $\sigma$) and $B^0 \to \pi^+ \pi^- \mu^+ \mu^-$ (4.8 $\sigma$)

NB: $b \to d$ transitions are suppressed by $|V_{td}/V_{ts}|^2$ w.r.t. $b \to s$ in SM;

Gaia Lanfranchi (LHCb Collaboration) ---- 4 March 2015
Study of the rare $B^0_{(s)} \to \pi^+ \pi^- \mu^+ \mu^-$ decays


The results for the decay rates:

$$
B(B^0_s \to \pi^+ \pi^- \mu^+ \mu^-) = (8.6 \pm 1.5 \text{ (stat)} \pm 0.7 \text{ (syst)} \pm 0.7 \text{ (norm)}) \times 10^{-8}
$$

$$
B(B^0 \to \pi^+ \pi^- \mu^+ \mu^-) = (2.11 \pm 0.51 \text{ (stat)} \pm 0.15 \text{ (syst)} \pm 0.16 \text{ (norm)}) \times 10^{-8}
$$

are compatible with SM expectations..

We need input from the theory community.

<table>
<thead>
<tr>
<th>$\text{Br}(B^0_s \to f_0(980)\mu^+\mu^-)$</th>
<th>$\text{Br}(B^0 \to \rho(770)^0\mu^+\mu^-)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(5.21 \pm 3.23) \times 10^{-7}$</td>
<td>$(5.0 \pm 2.1) \times 10^{-8}$</td>
</tr>
<tr>
<td>$(9.5 \pm 3.1) \times 10^{-8}$</td>
<td>$(8.6 \pm 3.4) \times 10^{-8}$</td>
</tr>
<tr>
<td>$(1.67 \pm 0.61) \times 10^{-7}$</td>
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</tr>
<tr>
<td>$(0.81 \pm 2.02) \times 10^{-8}$</td>
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</tr>
<tr>
<td>$(0.63 \pm 3.37) \times 10^{-9}$</td>
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</tbody>
</table>

.. but SM predictions differ up to two orders of magnitude among each other!!

Gaia Lanfranchi (LHCb Collaboration) ---- 4 March 2015
Differential branching fraction and angular analysis of $\Lambda_b \rightarrow \Lambda \mu^+\mu^-$ decays
[to be submitted to JHEP]

Same $b \rightarrow s$ quark level transition as for $B^0 \rightarrow K^{*0}\mu^+\mu^-$

Unique features:
1) $\Lambda_b$ baryon has non-zero spin:
→ potential to improve the limited understanding of the helicity structure of the underlying Hamiltonian, which cannot be extracted from mesonic decays.

2) composition of the $\Lambda_b$ baryon may be considered as the combination of a heavy quark with a light di-quark system:
→ the hadronic physics differs significantly from that of the B meson decay.

3) $\Lambda$ baryon decays weakly: (vs $K^{*0}$ that decays strongly)
→ complementary information to that available from meson decays

D. Van Dyk et al., arXiv.1410.2115
Gaia Lanfranchi (LHCb Collaboration) ---- 4 March 2015
Differential branching fraction and angular analysis of $\Lambda_b \rightarrow \Lambda \mu^+ \mu^-$ decays
[to be submitted to JHEP]

Signal seen for the first time with a significance $> 3 \sigma$ between $0.1 < q^2 < 2 \text{ GeV}^2/c^4$ and between charmonium resonances;
→ no significant signal observed in the $1.1 < q^2 < 6.0 \text{ GeV}^2/c^4$ range.
→ uncertainty in the decay rate within $15 < q^2 < 20 \text{ GeV}^2/c^4$ is reduced by a factor of $\sim 3$ w.r.t previous LHCb measurement.

3 fb$^{-1}$ data analyzed.

Inner error: stat + syst
Outer error: including normalization

Inner error: $\text{stat + syst}$
Outer error: including normalization

SM predictions: PRD 87 (2013) 074502

LHCb preliminary

$q^2 [\text{GeV}^2/c^4]$
Differential branching fraction and angular analysis of $\Lambda_b \rightarrow \Lambda \mu^+ \mu^-$ decays [to be submitted to JHEP]

Measurement of the forward-backward asymmetry for the lepton $A_{FB}^l$ (same as $B^0 \rightarrow K^{*0} \mu^+ \mu^-$) and for the hadron side $A_{FB}^h$.

$A_{FB}^l$ is compatible with SM predictions at 2 $\sigma$ level
$A_{FB}^h$ is fully compatible with SM predictions

S. Meinel, arXiv 1401.2685, Proceedings of Lattice2013
Computed at the leading order of HQET (accurate up to corrections $o(m_b/\Lambda)$).
Differential branching fraction and angular analysis of $\Lambda_b \rightarrow \Lambda \mu^+\mu^-$ decays
[to be submitted to JHEP]

Measurement of the forward-backward asymmetry for the lepton $A^l_{FB}$ (same as $B^0 \rightarrow K^{*0} \mu^+\mu^-$) and for the hadron side $A^h_{FB}$

\[
A^l_{FB} = -0.05 \pm 0.09 \text{ (stat)} \pm 0.03 \text{ (sys)}
\]
\[
A^h_{FB} = -0.29 \pm 0.07 \text{ (stat)} \pm 0.03 \text{ (sys)}
\]

for $15 < q^2 < 20 \text{ GeV}^2/c^4$

Gaia Lanfranchi (LHCb Collaboration) ---- 4 March 2015
The DNA of the Wilson coefficients

Straub, Altmannshofer arXiv:1308.1501

Hints that $C_9(NP)$ is negative

$$H_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \sum_i \left[ \frac{C_i(\mu) O_i(\mu)}{\text{left-handed part}} + \frac{C_i'(\mu) O_i'(\mu)}{\text{right-handed part}} \right] + \sum \frac{c}{\Lambda_{NP}^2} O_{NP}$$
The DNA of the Wilson coefficients

\[ H_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \sum_i \left[ \frac{C_{i}(\mu) O_{i}(\mu)}{4} + \frac{C'_{i}(\mu) O'_{i}(\mu)}{4} \right] + \sum \frac{c}{\Lambda_{NP}^2} O_{NP} \]

C\text{7}^* compatible with zero (so far) [in the assumption that \text{Im}(C\text{7}^*) =0]

\[ C_9(\text{NP}) \text{ is negative} \]

Straub, Altmannshofer arXiv:1308.1501
In the SM, photons from $b \rightarrow s \gamma$ decays are predominantly left-handed ($C_7/C_7' \sim m_b/m_s$), due to the charged current interaction.

Several models beyond the SM predict the photon to acquire a significant right-handed component due to the exchange of a heavy fermions in the electroweak penguin loop.

A 3D angular analysis (like $B^0 \rightarrow K^{*0}\mu^+\mu^-$) at the photon pole ($q^2 = [0.0004,1]$ GeV$^2$/c$^4$) allows to assess the photon polarization in $b \rightarrow s \gamma$ transition.
Angular analysis of $B^0 \rightarrow K^{*0} e^+e^-$ decays

Angular analysis as $B^0 \rightarrow K^{*0} \mu^+\mu^-$

Measured observables:

\[
\begin{align*}
F_L &= 0.16 \pm 0.06 \pm 0.03 \\
A_T^{(2)} &= -0.23 \pm 0.23 \pm 0.05 \\
A_T^{Im} &= 0.14 \pm 0.22 \pm 0.05
\end{align*}
\]

Quantities related to photon polarization

Kruger et al. PRD71 (2005) 094009
Becimeric et al. NPB854 (2012) 321

Consistent with SM expectations

[adapted from Jager et al. JHEP 05 (2013) 043]

Gaia Lanfranchi (LHCb Collaboration)
Angular analysis of $B^0 \to K^{*0} e^+ e^-$ decays

Theory predictions on $C_7/C_7'$ based on the B-factories results

Assumptions: $C_7 = C_7'(SM)$, $C_7' = \text{free to vary}$

$F_L = 0.16 \pm 0.06 \pm 0.03$

$A_T^{Re} = +0.10 \pm 0.18 \pm 0.05$

$A_T^{(2)} = -0.23 \pm 0.23 \pm 0.05$

$A_T^{Im} = +0.14 \pm 0.22 \pm 0.05$

$3\sigma$ contours based on B factories results


(Unfortunately) everything is consistent with the SM predictions.

Gaia Lanfranchi (LHCb Collaboration) ---- 4 March 2015
Rare Decays and the Higgs
(or how rare processes can test non-SM Higgs sectors)
Rare Decays and the Higgs
(or how rare processes can test non-SM Higgs sector)

You can recognize here the main diagrams that drive the $B^0_{(s)} \rightarrow \mu^+\mu^-$ decays…
B⁰ → μ⁺μ⁻ and B⁰ₛ → μ⁺μ⁻: a story 30 years long

Gaia Lanfranchi (LHCb Collaboration) ---- 4 March 2015
B⁰ → μ⁺μ⁻ and B⁰ₛ → μ⁺μ⁻: a story 30 years long

La Thuile 2011: LHCb presents its first results based on 37 pb⁻¹ (competitive with CDF with 3.7 fb⁻¹)
B⁰→μ⁺μ⁻ and B⁰ₛ→μ⁺μ⁻: a story 30 years long

November 2012: LHCb publishes the first evidence for the B⁰ₛ→μ⁺μ⁻
[PRL 110 (2013) 021801]
B⁰ → μ⁺μ⁻ and Bˢ₀ → μ⁺μ⁻: a story 30 years long

Another puzzling deviation: $\text{BR}(B_d \rightarrow \mu^+\mu^-)$

**Results:**

\[
\text{BR}(B^0) = (3.94^{+1.58}_{-1.41}^{+0.31}_{-0.24}) \times 10^{-10}
\]

3.2σ observed (0.8σ expected)

\[
\text{BR}(B^0_s) = (2.79^{+0.66}_{-0.60}^{+0.26}_{-0.19}) \times 10^{-9}
\]

6.2σ observed (7.6σ expected)

**Theory predictions:**

\[
\text{BR}(B^0 \rightarrow \mu^+\mu^-) = (1.06 \pm 0.09) \times 10^{-10}
\]

\[
\text{BR}(B^0_s \rightarrow \mu^+\mu^-) = (3.66 \pm 0.23) \times 10^{-9}
\]

Bobeth et al, PRL 112 (2014) 101801

Compatibility with the SM predictions: 2.2 σ for $B^0$ and 1.2 σ for $B_s$
BR($B^0 \rightarrow \mu^+\mu^-$) and BR($B^0 \rightarrow \mu^+\mu^-$) in a model independent approach:

LHCb+CMS $B^0 \rightarrow \mu^+\mu^-$ result out of scale

D. Straub, CKM 2014
Conclusions

• LHCb with three years of data taking has performed a major step forward in constraining Wilson coefficients related the b rare decays realm.
• Only hints of discrepancy so far, to be confirmed with more data.
• New results still based on Run I dataset coming soon:
  - Full angular analysis of $B^0 \rightarrow K^{*0} \mu^+ \mu^-$ with full dataset
  - Measurement of the photon polarization in $B_s^0 \rightarrow \phi \gamma$ decays
  - Differential BR of $B^+ \rightarrow \pi^+ \mu^+ \mu^-$ decays
  - Differential BR of $B_s^0 \rightarrow \phi \mu^+ \mu^-$
  - $R_{K^*}$ and $R_{\phi}$
  - ....

.. And many more during Run II!
SPARES
Angular analysis of $B^0 \rightarrow K^{*0} \mu^+ \mu^-$ decays

\[ \frac{d^4 \Gamma}{dq^2 \, d \cos \theta_K \, d \cos \theta_l \, d \phi} = \frac{9}{32 \pi} \left[ J_{1s} \sin^2 \theta_K + J_{1c} \cos^2 \theta_K + (J_{2s} \sin^2 \theta_K + J_{2c} \cos^2 \theta_K) \cos 2\theta_l \\ + J_3 \sin^2 \theta_K \sin^2 \theta_l \cos 2\phi + J_4 \sin 2\theta_K \sin 2\theta_l \cos \phi + J_5 \sin 2\theta_K \sin \theta_l \cos \phi \\ + (J_{6s} \sin^2 \theta_K + J_{6c} \cos^2 \theta_K) \cos \theta_l + J_7 \sin 2\theta_K \sin \theta_l \sin \phi + J_8 \sin 2\theta_K \sin 2\theta_l \sin \phi \\ + J_9 \sin^2 \theta_K \sin^2 \theta_l \sin 2\phi \right], \quad (1) \]

- Large number of terms simplified by angular folding, e.g. $\phi \rightarrow \phi + \pi$
  - if $\phi < 0$ to cancel terms in $\cos \phi$ and $\sin \phi$, or integration.
Angular analysis of $B^0 \rightarrow K^{*0} \mu^+ \mu^-$ decays

\[
\frac{d^4 \Gamma}{dq^2 d\cos \theta_K d\cos \theta_\ell d\phi} = \frac{9}{32\pi} \left[ J_{1s}\sin^2 \theta_K + J_{1c}\cos^2 \theta_K + (J_{2s}\sin^2 \theta_K + J_{2c}\cos^2 \theta_K) \cos 2\theta_\ell \\
+ J_3\sin^2 \theta_K \sin^2 \theta_\ell \cos 2\phi + J_4\sin 2\theta_K \sin 2\theta_\ell \cos \phi + J_5\sin 2\theta_K \sin \theta_\ell \cos \phi \\
+ (J_{6s}\sin^2 \theta_K + J_{6c}\cos^2 \theta_K) \cos \theta_\ell + J_7\sin 2\theta_K \sin \theta_\ell \sin \phi + J_8\sin 2\theta_K \sin \theta_\ell \sin \phi \\
+ J_9\sin^2 \theta_K \sin^2 \theta_\ell \sin 2\phi \right],
\]

(1)

$J_i$ terms depend on the complex spin amplitudes $A_{0,L,R}^{L,R}, A_{\parallel,L,R}^{L,R}, A_{\perp,L,R}$

\[
J_3 = \frac{1}{2} \beta^2 \epsilon \left[ |A_{\perp}^L|^2 - |A_{\parallel}^L|^2 + |A_{\perp}^R|^2 - |A_{\parallel}^R|^2 \right]
\]
Angular analysis of $B^0 \rightarrow K^{*0} \mu^+\mu^-$ decays

\[
\frac{d^4\Gamma}{dq^2 \, d\cos \theta_K \, d\cos \theta_l \, d\phi} = \frac{9}{32\pi} \left[ J_{1s} \sin^2 \theta_K + J_{1e} \cos^2 \theta_K + (J_{2s} \sin^2 \theta_K + J_{2e} \cos^2 \theta_K) \cos 2\theta_l \\
+ J_3 \sin^2 \theta_K \sin^2 \theta_l \cos 2\phi + J_4 \sin 2\theta_K \sin 2\theta_l \cos \phi + J_5 \sin 2\theta_K \sin \theta_l \cos \phi \\
+ (J_{6s} \sin^2 \theta_K + J_{6e} \cos^2 \theta_K) \cos \theta_l + J_7 \sin 2\theta_K \sin \theta_l \sin \phi + J_8 \sin 2\theta_K \sin 2\theta_l \sin \phi \\
+ J_9 \sin^2 \theta_K \sin^2 \theta_l \sin 2\phi \right],
\]

(1)

$J_i$ terms depend on the complex spin amplitudes $A^{L,R}_0, A^{L,R}_||, A^{L,R}_\perp$

\[
J_3 = \frac{1}{2} \beta^2 \left( |A^{L}_\perp|^2 - |A^{R}_|||^2 + |A^{R}_\perp|^2 - |A^{L}_|||^2 \right)
\]

The spin amplitudes depend on the Wilson coefficients via form factors $q^2$ dependent;

\[
A^{L(R)}_{\perp} = N \sqrt{2} \chi \left\{ \left( (C_6^{\text{eff}} + C_9^{\text{eff}}) \mp (C_{10}^{\text{eff}} + C_{10}^{\text{eff}}) \right) \frac{V(q^2)}{m_B + m_{K^*}} + \frac{2m_b}{q^2} (C_7^{\text{eff}} + C_7^{\text{eff}}) T_1(q^2) \right\}
\]
Angular analysis of $B^0 \rightarrow K^{*0} \mu^+ \mu^-$ decays

\[
\frac{d^4 \Gamma}{dq^2 \, d\cos \theta_K \, d\cos \theta_l \, d\phi} = \frac{9}{32 \pi} \left[ J_{1s} \sin^2 \theta_K + J_{1c} \cos^2 \theta_K + (J_{2s} \sin^2 \theta_K + J_{2c} \cos^2 \theta_K) \cos 2\theta_l \\
+ J_3 \sin^2 \theta_K \sin^2 \theta_l \cos 2\phi + J_4 \sin 2\theta_K \sin 2\theta_l \cos \phi + J_5 \sin 2\theta_K \sin \theta_l \cos \phi \\\n+ (J_{6s} \sin^2 \theta_K + J_{6c} \cos^2 \theta_K) \cos \theta_l + J_7 \sin 2\theta_K \sin \theta_l \sin \phi + J_8 \sin 2\theta_K \sin 2\theta_l \sin \phi \\\n+ J_9 \sin^2 \theta_K \sin^2 \theta_l \sin 2\phi \right],
\]

(1)

$J_i$ terms depend on the complex spin amplitudes $A_0^{L,R}, A_\parallel^{L,R}, A_\perp^{L,R}$

\[
J_3 = \frac{1}{2} \beta_\ell^2 \left[ |A_\perp|^2 - |A_\parallel|^2 + |A_\perp^R|^2 - |A_\parallel^R|^2 \right]
\]

The spin amplitudes depend on the Wilson coefficients via form factors $q^2$ dependent;

\[
A_\perp^{L,R} = 2\sqrt{2} \lambda \left\{ \left[ (C_9^{\text{eff}} + C_9^{\text{eff}}) + (C_{10}^{\text{eff}} + C_{10}^{\text{eff}}) \right] \frac{V(q^2)}{m_B + m_{K^*}} + \frac{2m_b}{q^2} (C_{7}^{\text{eff}} + C_{7}^{\text{eff}}) T_1(q^2) \right\}
\]

Build CP averaged observables:

\[
S_i = \frac{(J_i + J_i)}{(\Gamma + \Gamma)}
\]
Angular analysis of $B^0 \rightarrow K^{*0} \mu^+ \mu^-$ decays

\[
\frac{d^4 \Gamma}{dq^2 \, d\cos \theta_K \, d\cos \theta_l \, d\phi} = \frac{9}{32\pi} \left[ J_{1s} \sin^2 \theta_K + J_{1e} \cos^2 \theta_K + (J_{2s} \sin^2 \theta_K + J_{2e} \cos^2 \theta_K) \cos 2\theta_l \\
+ J_3 \sin^2 \theta_K \sin^2 \theta_l \cos 2\phi + J_4 \sin 2\theta_K \sin 2\theta_l \cos \phi + J_5 \sin 2\theta_K \sin \theta_l \cos \phi \\
+ (J_{6s} \sin^2 \theta_K + J_{6e} \cos^2 \theta_K) \cos \theta_l + J_7 \sin 2\theta_K \sin \theta_l \sin \phi + J_8 \sin 2\theta_K \sin 2\theta_l \sin \phi \\
+ J_9 \sin^2 \theta_K \sin^2 \theta_l \sin 2\phi \right],
\]

(1)

$J_i$ terms depend on the complex spin amplitudes $A_0^{L,R}, A_\|^{L,R}, A_\perp^{L,R}$

$J_3 = \frac{1}{2} \beta^2 \delta [ |A_\perp|^2 - |A_\|^2 + |A_\|^2 - |A_\perp|^2 ]$

The spin amplitudes depend on the Wilson coefficients via form factors $q^2$ dependent:

$A_\perp^{(R)} = \sqrt{2} \lambda \left\{ \left[ (C_6^{eff} + C_9^{eff}) + (C_{10}^{eff} + C_{10}^{eff}) \right] \frac{V(q^2)}{m_B + m_{K^*}} + \frac{2m_b}{q^2} \left( C_7^{eff} + C_7^{eff} \right) \right\}$

Build CP averaged observables:

$S_i = (J_i + \bar{J}_i) / (\Gamma + \bar{\Gamma})$

Build observables where form factors uncertainties cancel at leading order:

$P_i' = S_i / \sqrt{(F_L (1-F_L))}$
BR($B_s^0 \rightarrow \mu^+\mu^-$) and BR($B^0 \rightarrow \mu^+\mu^-$): constraints on Wilson coefficients

\[
BR(B_s \rightarrow \mu^+\mu^-) \propto m_\mu^2 \left( \left| (C_{10}^{SM} + C_{10}^{NP} - C_{10}') - \frac{m_{B_s}}{2m_\mu} (C_{SP} + C_{SP}') \right|^2 + \left| \frac{m_{B_s}}{2m_\mu} (C_{SP} - C_{SP}') \right|^2 \right)
\]

If we neglect NP in $C_{(10)}$, BRs are proportional to the squared sum/difference of $C_S$ and $C_P$.

Constraints on the $C_{(10)}$ Wilson coeff. from the measured BRs (pre-combination)

Alonso et al. arXiv: 1407.7044

The radius of the rings is proportional to the measured branching fractions, the width of the rings is proportional to the experimental accuracy. → improving the experimental accuracy in these modes reduces the width of the ring, other observables are required to break the degeneracy (effective lifetime)
Rare decays with ew-penguins: prospects

<table>
<thead>
<tr>
<th></th>
<th>$3\text{fb}^{-1} (7+8\text{ TeV})$</th>
<th>2015</th>
<th>Run II</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B^0 \to K^{*0}\mu^+\mu^-$</td>
<td>$2.6k^\dagger$</td>
<td>$1.4 - 2.1k$</td>
<td>$8.5k$</td>
</tr>
<tr>
<td>$B^+ \to K^+\mu^+\mu^-$</td>
<td>$4746 \pm 81$</td>
<td>$2.6 - 3.9k$</td>
<td>$15.4k$</td>
</tr>
<tr>
<td>$B^+ \to \pi^+\mu^+\mu^-$</td>
<td>$100$</td>
<td>$50 - 80$</td>
<td>$320$</td>
</tr>
<tr>
<td>$B^+ \to K^+e^+e^- (1 &lt; q^2 &lt; 6)$</td>
<td>$256^{+25}_{-23}$</td>
<td>$140 - 210$</td>
<td>$830$</td>
</tr>
</tbody>
</table>

$^\dagger$ with enlarged $q^2$ windows for $3\text{fb}^{-1}$ analysis

By 2028 the statistical uncertainty on each point will be reduced by a factor 3-4

Zwicky & Lyon in [arXiv:1406.0566]

Will we be able to control the hadronic uncertainties at the same level?
Expected precision on \( R = \frac{\text{BR}(B^0 \rightarrow \mu^+ \mu^-)}{\text{BR}(B_s^0 \rightarrow \mu^+ \mu^-)} \)

Main limiting factor will be the control of the peaking backgrounds
(pure particle identification problem)
The uncertainty of CKM matrix elements is now larger than the uncertainty on $f_{B_{s,d}}$.  

$BR(B_s^0 \rightarrow \mu^+ \mu^-) = (3.66 \pm 0.23) \times 10^{-9} (6.3\%)$  
$BR(B^0 \rightarrow \mu^+ \mu^-) = (1.06 \pm 0.09) \times 10^{-10} (8.5\%)$

**Theory predictions: error budget**

<table>
<thead>
<tr>
<th>$B_s^0 \rightarrow \mu^+ \mu^-$</th>
<th>$f_{B_s}$</th>
<th>CKM</th>
<th>$\tau_H^s$</th>
<th>$M_t$</th>
<th>$\alpha_s$</th>
<th>other param.</th>
<th>non-param.</th>
<th>$\sum$</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.0%</td>
<td>4.3%</td>
<td>1.3%</td>
<td>1.6%</td>
<td>0.1%</td>
<td>&lt; 0.1%</td>
<td></td>
<td>1.5%</td>
<td>6.4%</td>
</tr>
</tbody>
</table>

<table>
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<tr>
<th>$B^0 \rightarrow \mu^+ \mu^-$</th>
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</thead>
<tbody>
<tr>
<td>4.5%</td>
<td>6.9%</td>
<td>0.5%</td>
<td>1.6%</td>
<td>0.1%</td>
<td>&lt; 0.1%</td>
<td></td>
<td>1.5%</td>
<td>8.5%</td>
</tr>
</tbody>
</table>

- $f_{B_s} = 227.4(4.5) \text{ MeV}$  
  [FLAG '13, arXiv:1310.8555]
- $V_{cb}$ from recent inclusive fit  
- $f_{B_d} = 190.5(4.2) \text{ MeV}$  
  [FLAG '13, arXiv:1310.8555]

The uncertainty of CKM matrix elements is now larger than the uncertainty on $f_{B_{s,d}}$.  

Bobeth et al. ‘13
Theory predictions: error budget

\[ \text{BR}(B_s^0 \rightarrow \mu^+\mu^-) = (3.66 \pm 0.23) \times 10^{-9} \ (6.4\%) \]
\[ \text{BR}(B^0 \rightarrow \mu^+\mu^-) = (1.06 \pm 0.09) \times 10^{-10} \ (8.5\%) \]

\[ R = \frac{\text{BR}(B^0 \rightarrow \mu^+\mu^-)}{\text{BR}(B_s^0 \rightarrow \mu^+\mu^-)} = 0.0295^{+0.0028}_{-0.0025} \ (+8.7\% - 7.7\%) \]

The theoretical uncertainty on R is due:
- 8\% uncertainty from CKM elements ;
- 3.7\% uncertainty from \( f_{B_s}/f_{B_d} \)
- 1.4\% uncertainty on the \( B_s \) lifetime

These uncertainties do not cancel in the ratio.