Higgs Basis:
Proposal for an EFT basis choice for LHC HXSWG

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Abstract

We review the effective field theory where the Standard Model is extended by higher-dimensional operators. Various existing operator sets (bases) spanning the space of dimension-6 operators are discussed. For the SILH basis and for the Warsaw basis, we derive the map between the Wilson coefficients of dimension-6 operators and the couplings of mass eigenstates in the Lagrangian. We also propose a new parametrization of the space of dimension-6 operators: the so-called Higgs basis. In that basis, the dimension-6 operators that can be best probed by LHC Higgs searches are explicitly separated from the ones strongly constrained by previous experiments. Therefore, the Higgs basis is particularly convenient for leading order effective field theory analyses of the LHC Higgs data.
1 Introduction

For a large class of models beyond the SM, physics at energies below the mass scale $\Lambda$ of the new particles can be parametrized by an effective field theory (EFT) where the SM Lagrangian is supplemented by new operators with canonical dimensions $D$ larger than 4. The theory has the same field content and the same linearly realized $SU(3) \times SU(2) \times U(1)$ local symmetry as the SM. The higher-dimensional operators are organized in a systematic expansion in $D$, where each consecutive term is suppressed by a larger power of $\Lambda$. For a general introduction to the EFT formalism see e.g. [2, 3, 4, 5, 6]; for recent review articles about EFT in connection with Higgs physics see e.g. [7, 8, 9, 10, 11, 12].

Quite generally, the EFT Lagrangian takes the form:

$$L_{\text{eff}} = L_{\text{SM}} + \sum_{i} c_{i}^{(5)} \frac{1}{\Lambda} O_{i}^{(5)} + \sum_{i} c_{i}^{(6)} \frac{1}{\Lambda^{2}} O_{i}^{(6)} + \sum_{i} c_{i}^{(7)} \frac{1}{\Lambda^{3}} O_{i}^{(7)} + \sum_{i} c_{i}^{(8)} \frac{1}{\Lambda^{4}} O_{i}^{(8)} + \cdots,$$

(1.1)

where each $O_{i}^{(D)}$ is an $SU(3) \times SU(2) \times U(1)$ invariant operator of dimension $D$ and the parameters $c_{i}^{(D)}$ multiplying the operators in the Lagrangian are called the Wilson coefficients. This EFT is intended to parametrize observable effects of a large class of BSM theories where new particles, with mass of order $\Lambda$, are much heavier than the SM ones and much heavier than the energy scale at which the experiment is performed. The main motivation to use this framework is that the constraints on the EFT parameters can be later re-interpreted as constraints on masses and couplings of new particles in many BSM theories. In other words, translation of experimental data into a theoretical framework has to be done only once in the EFT context, rather than for each BSM model separately.

The contribution of each $O_{i}^{(D)}$ to amplitudes of physical processes at the energy scale of order $v$ scales as $(v/\Lambda)^{D-4}$. Since $v/\Lambda < 1$ by construction, the EFT in its validity regime typically describes small deviations from the SM predictions, although, under certain conditions, it may be consistent to use this framework to describe large deviations [14, 13].

A complete and non-redundant set of operators that can be constructed from the SM fields is known for $D=5$ [15], $D=6$ [16], $D=7$ [17, 18], and $D=8$ [19, 18]. All $D=5$ operators violate the lepton number [15], while all $D=7$ operators violate $B-L$ (the latter is true for all odd-$D$ operators [20]). Then, experimental constraints dictate that their Wilson coefficients must be suppressed at a level which makes them unobservable at the LHC [21], and for this reason $D=5$ and 7 operators will not be discussed here. Consequently, the leading new physics effects are expected from operators with $D=6$ [22], whose contributions scale as $(v/\Lambda)^{2}$. Contributions from operators with $D \geq 8$ are suppressed by at least $(v/\Lambda)^{4}$, and in most of the following discussion we will assume that they can be neglected.

In this note, we discuss in detail the $D=6$ operators that can be constructed from the SM fields. We review various possible choices of these operators (the so-called basis) and their phenomenological effects. Only the operators that conserve the baryon and lepton numbers are considered. On the other hand, we do not impose a-priori any flavor symmetry. Also, we include CP violating operators in our discussion. One purpose of this note is to propose a common

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1The latter assumption can be relaxed, leading to an EFT with a non-linearly realized electroweak symmetry. This framework is discussed in Section II.2.4 of [1].

2Apart from the scaling with $\Lambda$, the effects of higher-dimensional operators also scale with appropriate powers of couplings in the UV theory. The latter is important to assess the validity range of the EFT description, as discussed in Ref. [13] and Section II.2.2 of [1].
EFT language and conventions that could be universally used in LHC Higgs analyses and be implemented in numerical tools.

In Section 2 we introduce the SM Lagrangian extended by dimension-6 operators. Two popular bases of dimension-6 operators using the manifestly SU(2) × U(1) invariant formalism are introduced. In Section 3 we discuss the interactions of the SM mass eigenstates that arise in the presence of dimension-6 operators beyond the SM, with the emphasis on the Higgs interactions. We also provide a map between the couplings in that effective Lagrangian and Wilson coefficients of dimension-6 operators introduced in Section 2. In Section 4 we define a new basis of D=6 operators, the so-called Higgs basis, which is spanned by a subset of the independent couplings of the mass eigenstate Lagrangian. This material is a slightly extended version of Section II.2.1 of the Yellow Report 4 [1]; additional technical details not included in [1] are collected in Appendices A, B, C, D.

2 SM EFT with dimension-6 operators

We consider an EFT Lagrangian where the SM is extended by dimension-6 operators:

\[ \mathcal{L}_{\text{EFT}} = \mathcal{L}_{\text{SM}} + \sum_i c_i (6) O_i (6). \]  

(2.1)

In our conventions, the scale \( \Lambda \) has been absorbed in the definition of the Wilson coefficients, \( c_i (6) = c_i (6) v^2 / \Lambda^2 \), and we divided the dimension-6 operators by \( v^2 \), \( O_i (6) = O_i (6) / v^2 \).

To fix our notation and conventions, we first write down the SM Lagrangian:

\[ \mathcal{L}_{\text{SM}} = - \frac{1}{4} G^a_{\mu\nu} G^{a\mu\nu} - \frac{1}{4} W^i_{\mu\nu} W^{i\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} + D_\mu H^\dagger D_\mu H + \mu_H^2 H^\dagger H - \lambda (H^\dagger H)^2 + \sum_{f = q, \ell} i f_L \gamma_\mu D_\mu f_L + \sum_{f = u, d, e} i f_R \gamma_\mu D_\mu f_R - \left[ g_L \bar{H} y_u u_R + \bar{q}_L H y_d d_R + \bar{\ell}_L H y_e \ell_R + \text{h.c.} \right]. \]  

(2.2)

Here, \( G^a_{\mu\nu}, W^i_{\mu\nu}, B_\mu \) denote the gauge fields of the \( SU(3) \times SU(2) \times U(1) \) local symmetry. The corresponding gauge couplings are denoted by \( g_s, g, g' \); we also define the electromagnetic coupling \( e = g g' / \sqrt{g^2 + g'^2} \), and the Weinberg angle \( s_\theta = g' / \sqrt{g^2 + g'^2} \). The field strength tensors are defined as \( G^a_{\mu\nu} = \partial_\mu G^a_\nu - \partial_\nu G^a_\mu + g_s f^{abc} G^b_\mu G^c_\nu, \ W^i_{\mu\nu} = \partial_\mu W^i_\nu - \partial_\nu W^i_\mu + g g' f^{ijk} W^j_\mu W^k_\nu \), \( B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu \). The Higgs doublet is denoted as \( H \), and we also define \( H_i = \epsilon_{ij} H^*_j \). The covariant derivative is defined as \( D_\mu H = \partial_\mu H - i \sigma^i W^i_\mu H - i \frac{g'}{2} B_\mu H \). The field \( H \) acquires the VEV \( \langle H^\dagger H \rangle = v^2 / 2 \). In the unitary gauge we have \( H = (0, (v + h) / \sqrt{2}) \), where \( h \) is the Higgs boson field. After electroweak symmetry breaking, the electroweak gauge boson mass eigenstates are defined as \( W^\pm = (W^1 \pm i W^2) / \sqrt{2}, Z = c_\theta W^3 - s_\theta B, A = s_\theta W^3 + c_\theta B \), where \( c_\theta = \sqrt{1 - s_\theta^2} \).

The tree-level masses of W and Z bosons are given by \( m_W = g v / 2, m_Z = \sqrt{g^2 + g'^2} v / 2 \). The left-handed Dirac fermions \( q_L = (u_L, d_L) \) and \( \ell_L = (\nu_L, e_L) \) are doublets of the SU(2) gauge group, and the right-handed Dirac fermions \( u_R, d_R, e_R \) are SU(2) singlets. All fermions are 3-component vectors in the generation space, and \( y_f \) are 3 × 3 matrices. The 3 electroweak parameters \( g, g', v \) are customarily derived from the Fermi constant \( G_F \) measured in muon decays, Z boson mass \( m_Z \), and the low-energy electromagnetic coupling \( \alpha(0) \). The Higgs quartic couplings \( \lambda \) can then be
fixed from the measured Higgs boson mass. The tree-level relations between the input observables and the electroweak parameters are given by:

\[ G_F = \frac{1}{\sqrt{2}v^2}, \quad \alpha = \frac{g^2 g'^2}{4\pi (g^2 + g'^2)}, \quad m_Z = \frac{\sqrt{g^2 + g'^2}v}{2}, \quad m_W^2 = 2\lambda v^2. \quad (2.3) \]

We demand that the dimension-6 operators \( O_6^{(6)} \) in Eq. (2.1) form a complete, non-redundant set - a so-called basis. Complete means that any dimension-6 operator is either a part of the basis or can be obtained from a combination of operators in the basis using equations of motion, integration by parts, field redefinitions, and Fierz transformations. Non-redundant means it is a minimal such set. Any complete basis leads to the same physical predictions concerning possible new physics effects. Several bases have been proposed in the literature, and they may be convenient for specific applications. Historically, a complete and non-redundant set of \( D=6 \) operators was first identified in Ref. [16], and is usually referred to as the Warsaw basis. Below, we work with another basis choice commonly used in the literature: the so-called SILH basis [23]. Later, in Section., we propose a new basis choice that is particularly convenient for leading-order LHC Higgs analyses in the EFT framework.

<table>
<thead>
<tr>
<th>Bosonic CP-even</th>
<th>Bosonic CP-odd</th>
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<tr>
<td>( O_H )</td>
<td>( \frac{1}{2\pi^2} [\partial_H (H^\dagger H)]^2 )</td>
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<tr>
<td>( O_T )</td>
<td>( \frac{1}{2\pi^2} (H^\dagger \partial_H H)^2 )</td>
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<td>( O_6 )</td>
<td>( -\frac{1}{2\pi^2} (H^\dagger H)^3 )</td>
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<td>( O_9 )</td>
<td>( \frac{g^2}{m_W^2} H^\dagger H G^{a\mu} G^a_{\mu\nu} )</td>
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<td>( O_{10} )</td>
<td>( \frac{g^2}{m_W^2} H^\dagger H B_{\mu\nu} B_{\mu\nu} )</td>
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<tr>
<td>( O_W )</td>
<td>( \frac{ig}{2m_W^2} (H^\dagger \sigma^i \partial_H H) D_\nu W^{i\mu} )</td>
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<tr>
<td>( O_B )</td>
<td>( \frac{ig}{2m_W^2} (H^\dagger \partial_H H) \partial_\nu B_{\mu\nu} )</td>
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<tr>
<td>( O_{HW} )</td>
<td>( \frac{ig}{m_W} (D_\mu H^\dagger \sigma^i D_\nu H) W^{i\mu} )</td>
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<td>( O_{HB} )</td>
<td>( \frac{ig}{m_W} (D_\mu H^\dagger D_\nu H) B_{\mu\nu} )</td>
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<td>( O_{2W} )</td>
<td>( \frac{1}{m_W^2} D_\mu W^{i\mu} D_\rho W^{i\rho} )</td>
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<td>( O_{2B} )</td>
<td>( \frac{1}{m_W^2} \partial_\mu B_{\mu\nu} \partial_\nu B_{\mu\nu} )</td>
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<td>( O_{2G} )</td>
<td>( \frac{1}{m_W^2} D_\mu G^a_{\mu\nu} D_\rho G^a_{\rho\nu} )</td>
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<tr>
<td>( O_{3W} )</td>
<td>( \frac{g^2}{m_W^2} \epsilon^{ijk} W^{i\mu}<em>{\nu\mu} W^{j\nu}</em>{\rho\nu} W^{k\rho}_{\mu\rho} )</td>
</tr>
<tr>
<td>( O_{3G} )</td>
<td>( \frac{g^2}{m_W^2} \epsilon^{ijk} \epsilon^{abc} G^{a\mu}<em>{\nu\mu} G^{b\nu}</em>{\rho\nu} G^{c\rho}_{\mu\rho} )</td>
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Table 1: Bosonic \( D=6 \) operators in the SILH basis.

The full set of operators in the SILH basis is given in Tables 1, 2, and 3. We use the normalization and conventions of Ref. [23].

\(^3\)In Ref. [23] it was assumed that the flavor indices of fermionic \( D=6 \) operators are proportional to the unit matrix. Generalizing this to an arbitrary flavor structure, one needs to specify flavor indices of the operators \([O_{H1}],[O_{H2}],[O_{\ell\ell}]\) and \([O_{\ell e}]\) which are absent in the SILH basis to avoid redundancy. Here, for concreteness, we made a particular though somewhat arbitrary choice of these indices.
Table 2: Two-fermion dimension-6 operators in the SILH basis. They are the same as in the Warsaw basis, except that the operators $[O_{H\ell}]_{11}$, $[O'_{H\ell}]_{11}$ are absent by definition. We define $\sigma_{\mu\nu} = i[\gamma_\mu, \gamma_\nu]/2$. In this table, $e, u, d$ are always right-handed fermions, while $\ell$ and $q$ are left-handed. For complex operators the complex conjugate operator is implicit.

3 Effective Lagrangian of mass eigenstates

In Section 2 we introduced an EFT with the SM supplemented by $D=6$ operators, using a manifestly $SU(2) \times U(1)$ invariant notation. At that point, the connection between the new operators and phenomenology is not obvious. To relate to high-energy collider observables, it is more transparent to express the EFT Lagrangian in terms of the mass eigenstates after electroweak symmetry breaking (Higgs boson, $W$, $Z$, photon, etc.). Once this step is made, only the unbroken $SU(3)_c \times U(1)_{em}$ local symmetry is manifest in the Lagrangian. Moreover, to simplify the interaction vertices, we will make further field transformations that respect only $SU(3)_c \times U(1)_{em}$. Since field redefinitions do not affect physical predictions, the gauge invariance of the EFT we started with ensures that observables calculated using this mass eigenstate Lagrangian are also gauge invariant. This is possible because the full $SU(2) \times U(1)$ electroweak symmetry is still present, albeit in a non-manifest way, in the form of non-trivial relations between different couplings of mass eigenstates. Finally, for the sake of calculating observables beyond the tree-level one needs to specify the gauge fixing terms. Again, the gauge invariance of the starting point ensures that physical observables are independent of the gauge fixing procedure. Below we only present the Lagrangian in the unitary gauge when the Goldstone bosons eaten by $W$ and $Z$ are set to zero, which is completely sufficient to calculate LHC Higgs observables at tree level; see Appendix C for a generalization to the $R_\xi$ gauge.

In this section we relate the Wilson coefficients of dimension-6 operators in the SILH basis to the parameters of the tree-level effective Lagrangian describing the interactions of the mass eigenstates. The analogous relations can be derived for any other basis; see Appendix A for the
Table 3: Four-fermion operators in the SILH basis. They are the same as in the Warsaw basis \cite{16}, except that the operators \([\tilde{O}_{\ell\ell}]_{1221}, [\tilde{O}_{u\ell}]_{1221}, [\tilde{O}_{au}]_{3333}\) are absent by definition. In this table, \(e, u, d\) are always right-handed fermions, while \(\ell\) and \(q\) are left-handed. A flavor index is implicit for each fermion field. For complex operators the complex conjugate operator is implicit.

map from the Warsaw basis. The form of the mass eigenstate Lagrangian obtained directly by inserting the Higgs VEV and eigenstates into Eq. (2.1) is not convenient for practical applications. For example, applied to the \(\mathcal{H}\) self-interaction term in the SM Lagrangian, it generates \(O_{\mu\nu}^{a}\) interactions at \(O(\Lambda^{-2})\) in the effective Lagrangian that cannot be generated by \(D=6\) operators\footnote{For example, applied to the \(h^4\) self-interaction term in the SM Lagrangian, it generates \(h^5\) and \(h^6\) self-interactions at \(O(\Lambda^{-2})\), which are also generated by the \(O_8\) operator in the SILH basis. Rather than applying the non-linear transformation, one can equivalently use the equations of motion for the Higgs boson field.}. In addition, one is free to add to the Lagrangian a total derivative and/or interactions terms that vanish by equations of motion. These redefinitions of course do not change the physical predictions or symmetries of the theory. However, they allow one to bring the theory to a more convenient form to perform practical calculations. We will use this freedom to demand that the mass eigenstate Lagrangian has the following features:

\#1 All kinetic and mass terms are diagonal and canonically normalized. In particular, higher-derivative kinetic terms are absent.

\#2 The non-derivative photon and gluon interactions with fermions are the same as in the SM.
#3 Tree-level relations between the electroweak parameters and input observables are the same as the SM ones in Eq. (2.3).

#4 Two-derivative self-interactions of the Higgs boson (e.g. $h\partial_\mu h\partial_\mu h$) are absent.

#5 In the Higgs boson interactions with gauge bosons, the derivative does not act on the Higgs (e.g., there is no $\partial_\mu h V_\nu V^\mu_\nu$ terms).

#6 For each fermion pair, the coefficient of the vertex-like Higgs interaction terms $\left(2\frac{h}{v} + \frac{h^2}{v^2}\right)V_\mu f\bar{f}\gamma_\mu f$ is equal to the vertex correction to the respective $V_\mu f\bar{f}\gamma_\mu f$ interaction.

These conditions are a choice of conventions (one among many possible ones) how to represent interactions in the mass eigenstate Lagrangian. It is always possible to implement this choice starting from any D=6 basis: SILH, Warsaw, or any other. The condition #1 simplifies extracting physical predictions of the EFT, and is essential to implement the theory in existing Monte Carlo simulators. The conditions #2-#3 simplify the interpretation of the SM parameters $g$, $g'$ and $v$. If the $[G_F, \alpha, m_Z]$ input is used to determine them (as assumed here), their numerical values should be the same as in the SM, and the input observables are not affected by D=6 operators at the leading order. The conditions #4-#6 are conventions commonly used in the literature that allow one to fix the remaining freedom of fields and couplings redefinitions. These particular conventions match the ones used e.g. in the Higgs characterization framework of Ref. [24]. See Appendix D for physical examples showing these redefinitions do not change the S-matrix. Other convention choices can be made, leading to the same predictions for observables. For example, the features #3, #4, and #6 are not enforced in the alternative approach proposed in Section II.2.3 of [1].

In general, dimension-6 operators do induce interaction terms that do not respect the features #1-#6. However, these features can always be achieved, without any loss of generality, by using equations of motion, integrating by parts, and redefining the fields and couplings. Starting from

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5If other input observables are used, for example $[G_F, m_W, m_Z]$ or $[\alpha, m_W m_Z]$, the shift of input observables due to the presence of D=6 operators must be taken into account to correctly derive physical predictions of the theory. Much as in the SM, the input observables $[G_F, \alpha, m_Z]$ are affected by loop corrections, and this has to be taken into account if the framework is used beyond tree level.
the SILH basis, the conditions #1-#6 fix the free parameters in Eq. (3.1) as

\[ \delta G = \frac{4g_s^2}{g^2} \tilde{c}_g, \]
\[ \delta W = \tilde{c}_W, \]
\[ \delta Z = \tilde{c}_W + \frac{g'^2}{g^2} \tilde{c}_B + \frac{4g'^4}{g^2(g^2 + g'^2)} \tilde{c}_\gamma, \]
\[ \delta_{AZ} = \frac{g'}{g} (\tilde{c}_W - \tilde{c}_B) - \frac{8g'^3}{g(g^2 + g'^2)} \tilde{c}_\gamma, \]
\[ \delta A = \frac{4g'^2}{g^2 + g'^2} \tilde{c}_\gamma, \]
\[ \delta v = \frac{H}{2}, \]
\[ \delta g_s = -\frac{4g_s^2}{g^2} \tilde{c}_g, \]
\[ \delta g = -\frac{g^2}{g^2 - g'^2} \left( \tilde{c}_W + \tilde{c}_2W + \frac{g'^2}{g^2} \tilde{c}_B + \frac{g^2}{g^2} \tilde{c}_{2B} - \frac{1}{2} \tilde{c}_T + \frac{1}{2} [\tilde{c}_H]_{22} \right), \]
\[ \delta g' = \frac{g'^2}{g^2 - g'^2} \left( \tilde{c}_W + \tilde{c}_2W + \frac{g'^2}{g^2} \tilde{c}_B + \frac{g^2}{g^2} \tilde{c}_{2B} - \frac{1}{2} \tilde{c}_T + \frac{1}{2} [\tilde{c}_H']_{22} - 4 \frac{g^2 - g'^2}{g^2} \tilde{c}_\gamma \right), \]
\[ \delta \lambda = \tilde{c}_H - \frac{3}{2} \tilde{c}_0 - [\tilde{c}_H']_{22}, \]
\[ \delta_1 = -\frac{\tilde{c}_H}{2}, \quad \delta_2 = -\frac{\tilde{c}_H}{2}, \quad \delta_3 = -\frac{\tilde{c}_H}{6}. \quad (3.2) \]

Finally, the Higgs mass term in the SM Lagrangian is related by vacuum equations to the other parameters by \( \mu_H^2 = \lambda v^2 (1 - \delta \lambda + 2 \delta v + 3/4 \tilde{c}_0) \). One can repeat this procedure starting from any other basis than SILH, and find a unique solution to the conditions #1-#6 in terms of the Wilson coefficients in that basis.

We move to discussing the interactions in the mass eigenstate Lagrangian once conditions #1-#6 are satisfied. We will focus on interaction terms that are most relevant for LHC phenomenology. To organize the presentation, we split the Lagrangian into the following parts,

\[ L_{\text{EFT}} = L_{\text{kinetic}} + L_{\text{aff}} + L_{\text{vertex}} + L_{\text{dipole}} + L_{\text{tgc}} + L_{\text{qgc},0} + L_{\text{qgc},2} + L_{\text{hff}} + L_{\text{hv},0} + L_{\text{hv},2} + L_{\text{h},0} + L_{\text{h},2} + L_{\text{other}}. \quad (3.3) \]

Below we define each term in order of appearance. We also express the corrections to the SM interactions in \( L_{\text{EFT}} \) in terms of linear combinations of Wilson coefficients of \( D=6 \) operators in the SILH basis (the analogous formulas for the Warsaw basis are given in Appendix [A]). These corrections start at \( O(1/\Lambda^2) \) in the EFT expansion, and we will ignore all \( O(1/\Lambda^4) \) and higher contributions.
Kinetic Terms

By construction, the kinetic terms of the mass eigenstates are diagonal and canonically normalized:

\[
\mathcal{L}_{\text{kinetic}} = -\frac{1}{2} W^\mu W_{\mu} - \frac{1}{4} Z_{\mu\nu} Z^{\mu\nu} - \frac{1}{4} A_{\mu\nu} A^{\mu\nu} - \frac{1}{4} G^{\mu\nu}_a G_a^{\mu\nu} \\
+ \frac{g^2 v^2}{4} (1 + \delta m)^2 W^\mu W_{\mu} + \frac{(g^2 + g'^2)v^2}{8} Z_{\mu} Z_{\mu} \\
+ \frac{1}{2} \partial_\mu h \partial_\mu h - \nu^2 h^2 + \sum_{f=q,\ell,u,d,e} \bar{f} (i \gamma_\mu \partial_\mu - m_f) f.
\]

Above, the parameter \( \lambda \) is defined by the tree-level relation \( m_h^2 = \frac{2}{\lambda} v^2 \). There is no correction to the Z boson mass terms, in accordance with the condition #3. With this convention, the corrections to the W boson mass cannot be in general redefined away, and are parametrized by \( \delta m \). The relation between \( \delta m \) and the Wilson coefficients in the SILH basis is given by

\[
\delta m = -\frac{g'^2}{g^2 - g'^2} \left( c_W + c_B + c_{2W} + c_{2B} - \frac{g^2}{2g'^2} \delta g + \frac{1}{2} |\delta g_{H\ell}|^2 \right). \quad (3.5)
\]

Gauge boson interactions with fermions

By construction (condition #2), the non-derivative photon and gluon interactions with fermions are the same as in the SM:

\[
\mathcal{L}_{\text{aff}} = e A_\mu \sum_{f=u,d,e} \bar{f} \gamma_\mu Q_f f + g_s G^a_\mu \sum_{f=u,d} \bar{f} \gamma_\mu T^a_f.
\]

The analogous interactions of the W and Z boson may in general be affected by dimension-6 operators:

\[
\mathcal{L}_{\text{vertex}} = \frac{g}{\sqrt{2}} \left( W^\mu_\mu \bar{u}_L \gamma_\mu \left( I_3 + \delta g_{W}^W \right) e_L + W^\mu_\mu \bar{u}_L \gamma_\mu \left( I_3 + \delta g_{W}^W \right) d_L + W^\mu_\mu \bar{u}_R \gamma_\mu \delta g_{W}^R d_R + \text{h.c.} \right) \\
+ \sqrt{g^2 + g'^2} Z_\mu \left[ \sum_{f=u,d,e,\nu} \bar{f}_L \gamma_\mu \left( T^f_3 - s^f_\nu Q_f + \delta g^Z_f \right) f_L + \sum_{f=u,d,e} \bar{f}_R \gamma_\mu \left( -s^f_\nu Q_f + \delta g^Z_f \right) f_R \right].
\]

Here, \( I_3 \) is the \( 3 \times 3 \) identity matrix, and the vertex corrections \( \delta g \) are \( 3 \times 3 \) Hermitian matrices in the generation space, except for \( \delta g_{W}^R \) which is a general \( 3 \times 3 \) complex matrix. The vertex corrections to W and Z boson couplings to fermions are expressed by the Wilson coefficients in
Another type of gauge boson interactions with fermions are the so-called dipole interactions. These do not occur in the tree-level SM Lagrangian, but they in general may appear in the EFT and it is implicit that $\bar{\sigma}$, where strength tensors are defined as

$$\delta g^Z_L = \frac{1}{2} \varepsilon'_{H\ell} - \frac{1}{2} \varepsilon_{H\ell} + \hat{f}(1/2,0),$$

$$\delta g^Z_e = -\frac{1}{2} \varepsilon'_{H\ell} - \frac{1}{2} \varepsilon_{H\ell} + \hat{f}(-1/2,-1),$$

$$\delta g^Z_R = -\frac{1}{2} \varepsilon_{H\ell} + \hat{f}(0,-1),$$

$$\delta g^Z_u = \frac{1}{2} \varepsilon_{Hq} - \frac{1}{2} \varepsilon_{Hq} + \hat{f}(1/2,2/3),$$

$$\delta g^Z_d = -\frac{1}{2} \varepsilon'_{Hq} V_{\text{CKM}} \bar{c} V_{\text{CKM}} - \frac{1}{2} \varepsilon_{Hq} V_{\text{CKM}} + \hat{f}(-1/2,-1/3),$$

$$\delta g^Z_R = -\frac{1}{2} \varepsilon_{Hd} + \hat{f}(0,2/3),$$

$$\delta g^Z_R = -\frac{1}{2} \varepsilon_{Hd} + \hat{f}(0,-1/3),$$

$$\delta g^W_L = \varepsilon_{H\ell} + \hat{f}(1/2,0) - \hat{f}(-1/2,-1),$$

$$\delta g^W_R = \varepsilon'_{Hq} + \hat{f}(1/2,2/3) - \hat{f}(-1/2,-1/3) \right) V_{\text{CKM}},$$

$$\delta g^W_R = -\frac{1}{2} \varepsilon_{Hwd},$$

where

$$\hat{f}(T_f^3, Q_f) = \left[ \bar{c}_{2W} + \frac{g'^2}{g^2} \bar{c}_{2B} + \frac{1}{2} \bar{c}_T - \frac{1}{2} \bar{c}'_{H\ell} \right] T_f^3$$

$$- \frac{g'^2}{(g^2-g'^2)} \left[ (2g^2-g'^2) \bar{c}_{2B} + \bar{c}_{2W} + \bar{c}_W + \bar{c}_B - \frac{1}{2} \bar{c}_T + \frac{1}{2} \bar{c}'_{H\ell} \right] Q_f,$$

and it is implicit that $\bar{c}'_{H\ell}^{11} = [\bar{c}_{H\ell}]^{11} = 0$.

Another type of gauge boson interactions with fermions are the so-called dipole interactions. These do not occur in the tree-level SM Lagrangian, but they in general may appear in the EFT with $D=6$ operators. We parametrize them as follows:

$$\mathcal{L}_{\text{dipole}} = -\frac{1}{4v} \sum_{f \in u,d} \sqrt{m_f m_{f'}} \bar{f}_{L,i} \sigma_{\mu \nu} T^a \left[ d_{Gf} \right]_{ij} f_{R,j} \sigma_{\mu \nu} + e \sum_{f \in u,d,e} \sqrt{m_{f'} m_{f}} \bar{f}_{L,i} \sigma_{\mu \nu} [d_{Af}]_{ij} f_{R,j} A_{\mu \nu}$$

$$+ \sqrt{g^2 + g'^2} \sum_{f \in u,d,e} \sqrt{m_{f'} m_{f}} \bar{f}_{L,i} \sigma_{\mu \nu} [d_{Zf}]_{ij} f_{R,j} Z_{\mu \nu}$$

$$+ \sqrt{2g} \sqrt{m_{u_{f_i}} m_{u_{f_j}}} \bar{d}_{L,i} \sigma_{\mu \nu} [d_{W_{u_{f_i}}}]_{ij} u_{R,j} W_{\mu \nu}^+ + \sqrt{2g} \sqrt{m_{d_{f_i}} m_{d_{f_j}}} \bar{u}_{L,i} \sigma_{\mu \nu} [d_{W_{d_{f_i}}}]_{ij} d_{R,j} W_{\mu \nu}^+$$

$$+ \sqrt{2g} \sqrt{m_{e_{f_i}} m_{e_{f_j}}} \bar{e}_{L,i} \sigma_{\mu \nu} [d_{W_{e_{f_i}}}]_{ij} e_{R,j} W_{\mu \nu}^+$$

\[+ \text{h.c.}]$$

(3.10)

where $\sigma_{\mu \nu} = i[\gamma_\mu, \gamma_\nu]/2$, and $g_{Gf}$, $d_{Af}$, $d_{Zf}$, and $d_{Wf}$ are complex $3 \times 3$ matrices. The field strength tensors are defined as $X_{\mu \nu} = \partial_\mu X_\nu - \partial_\nu X_\mu$, and $\tilde{X}_{\mu \nu} = \epsilon_{\mu \nu \rho \sigma} \partial_\rho X_\sigma$. The coefficients $d_{\nu f}$
are related to the Wilson coefficients in the SILH basis as
\[
\begin{align*}
    d_{Gf} &= -\frac{16}{g^2} \hat{c}_{fG}, \\
    d_{Af} &= -\frac{16}{g^2} (\eta_f \hat{c}_{fW} + \hat{c}_{fB}), \\
    d_{Zf} &= -\frac{16}{g^2} (\eta_f c_{fW}^2 - s_{fB}^2 \hat{c}_{fB}), \\
    d_{Wf} &= -\frac{16}{g^2} \hat{c}_{fW},
\end{align*}
\] (3.11)

where \(\eta_u = +1, \eta_{d,e} = -1\).

### Gauge boson self-interactions

Gauge boson self-interactions are not directly relevant for LHC Higgs searches, however we include them in this presentation because of the important synergy between the triple gauge couplings and Higgs couplings measurements \([25, 26, 27, 8, 28, 29, 30]\). The triple gauge interactions in the effective Lagrangian are parameterized by

\[
\mathcal{L}_{\text{gc}} = \frac{i e}{g} \left( W^+_\mu W^- - W^- W^+_\mu \right) A_\nu + i e \left[ (1 + \delta \gamma) A_{\mu\nu} W^+ W^- + \bar{\kappa}_\gamma \bar{A}_\mu A_\nu + \bar{\kappa}_z \bar{A}_\mu A_\nu \right]
\]

\[
\begin{align*}
    &+ i e c \left[ (1 + \delta g_{1z}) (W^+ W^- - W^- W^+) Z_\nu + (1 + \delta \kappa) Z_{\mu\nu} W^+ W^- + \bar{\kappa}_z \bar{Z}_{\mu\nu} W^+ W^- \right] \\
    &+ \frac{g}{m_W^2} \left[ \lambda g W^+ W^- A_{\rho\mu} + \bar{\lambda} g W^+ W^- A_{\rho\mu} \right] + i \frac{g}{m_W^2} \left[ \lambda g W^+ W^- Z_{\rho\mu} + \bar{\lambda} g W^+ W^- \bar{Z}_{\rho\mu} \right] \\
    &- g_s f^{abc} \partial_\mu G_\mu^a G_\mu^b G_\mu^c + \frac{c_{3g}}{v^2} g_s f^{abc} G_\mu^a G_\nu^b G_\rho^c + \frac{\tilde{c}_{3g}}{v^2} g_s f^{abc} \bar{G}_\mu^a \bar{G}_\nu^b G_\rho^c.
\end{align*}
\] (3.12)

The anomalous triple gauge couplings of electroweak gauge bosons are related to the Wilson coefficients in the effective Lagrangian as

\[
\begin{align*}
    \delta g_{1z} &= -\frac{g^2 + g'^2}{g^2 - g'^2} \left[ \frac{g^2}{g^2} \hat{c}_{HW} + \hat{c}\tilde{W} + \hat{c}_3W + \frac{g'^2}{g^2} \hat{c}_B + \frac{g^2}{g^2} \hat{c}_2B - \frac{1}{2} \hat{c}_T + \frac{1}{2} |\hat{c}'_{H\ell}|_2 \right], \\
    \delta \kappa &= -\hat{c}_{HW} - \hat{c}_{HB}, \\
    \delta \kappa &= -\hat{c}_{HW} - \hat{c}_{HB} - \frac{g^2 + g'^2}{g^2 - g'^2} \left[ \hat{c}_W + \hat{c}_2W + \frac{g^2}{g^2} \hat{c}_B + \frac{g^2}{g^2} \hat{c}_2B - \frac{1}{2} \hat{c}_T + \frac{1}{2} |\hat{c}'_{H\ell}|_2 \right], \\
    \lambda &= -\frac{6g^2}{g^2} \hat{c}_3W, \\
    \delta \kappa &= -\hat{c}_{HW} - \hat{c}_{HB}, \\
    \delta \kappa &= \frac{g^2}{g^2} \left[ \hat{c}_{HW} + \hat{c}_{HB} \right], \\
    \tilde{\lambda} &= -\frac{6g^2}{g^2} \hat{c}_3W, \\
    \bar{\lambda} &= \frac{4}{g^2} \hat{c}_3G, \\
    \tilde{\lambda} &= \frac{4}{g^2} \hat{c}_3G.
\end{align*}
\] (3.13)

The tilded Wilson coefficients refer to the tilded (CP-odd) operators in Table 1.
Quartic gauge boson self-interactions may also receive corrections from $D=6$ operators. Those with zero derivatives take the form

\[\mathcal{L}_{\text{qgc,0}} = e^2 (W^+_{\mu} A_\mu W^-_{\nu} A_\nu - W^+_{\mu} W^-_{\nu} A_\mu A_\nu)\]
\[+ \frac{g^2}{2} \left(1 + 2c_\theta^2 \delta g_{1,2}\right) \left(W^+_{\mu} W^-_{\nu} W^+_{\nu} - W^+_{\mu} W^-_{\nu} W^+_{\nu}\right)\]
\[+ g^2 c_\theta (1 + \delta g_{1,2}) \left(W^+_{\mu} Z_\mu W^-_{\nu} Z_\nu - W^+_{\mu} W^-_{\nu} Z_\mu Z_\nu\right)\]
\[+ egc_\theta (1 + \delta g_{1,2}) \left(W^+_{\mu} Z_\mu W^-_{\nu} A_\nu + W^+_{\mu} A_\mu W^-_{\nu} Z_\nu - 2W^+_{\mu} W^-_{\nu} Z_\nu A_\nu\right).\] (3.14)

In this case, the deformations from the SM are controlled by the anomalous triple gauge couplings $\delta g_{1,2}$. On top of that, two-derivative quartic gauge couplings appear with the coefficients related to $\lambda_z$:

\[\mathcal{L}_{\text{qgc,2}} = -\frac{g^2}{2} \frac{\lambda_z}{m_W^2} \left(W^+_{\mu \nu} W^-_{\nu \rho} - W^+_{\mu \nu} W^+_{\nu \rho}\right) \left(W^+_{\mu} W^-_{\rho} - W^+_{\mu} W^-_{\rho}\right)\]
\[+ g^2 c_\theta \frac{\lambda_z}{m_W^2} \left[W^+_{\mu} \left(W^-_{\mu} Z_{\nu \rho} - Z_{\mu \nu} W^-_{\nu \rho}\right) Z_\rho + W^-_{\mu} \left(W^+_{\mu} Z_{\nu \rho} - Z_{\mu \nu} W^+_{\nu \rho}\right) Z_\rho\right]\]
\[+ e^2 \frac{\lambda_z}{m_W^2} \left[W^+_{\mu} \left(W^-_{\mu} A_{\nu \rho} - A_{\mu \nu} W^-_{\rho}\right) A_\rho + W^-_{\mu} \left(W^+_{\mu} A_{\nu \rho} - A_{\mu \nu} W^+_{\rho}\right) A_\rho\right]\]
\[+ egc_\theta \frac{\lambda_z}{m_W^2} \left[W^+_{\mu} \left(W^-_{\mu} Z_{\nu \rho} - Z_{\mu \nu} W^-_{\rho}\right) A_\rho + W^-_{\mu} \left(W^+_{\mu} Z_{\nu \rho} - Z_{\mu \nu} W^+_{\rho}\right) A_\rho\right],\] (3.15)

where CP odd stands for analogous terms with $\lambda_z \to \tilde{\lambda}_z$, and one of the field strength tensor replaced by the dual one.

**Single Higgs couplings**

In this subsection we discuss the terms in the effective Lagrangian that involve a single Higgs boson field $h$. This part is the most relevant one from the point of view of the LHC Higgs phenomenology.

We first define the Higgs boson couplings to a pair of fermions:

\[\mathcal{L}_{\text{hff}} = -\frac{h}{v} \sum_{f \in u,d,e} \sum_{ij} \sqrt{m_f m_f'} \left(\delta_{ij} + [\delta y_f]_{ij} e^{i[\phi_f]_{ij}}\right) \bar{f}_{R,i} f_{L,j} + \text{h.c.},\] (3.16)

where $[\delta y_f]_{ij}$ and $[\phi_f]_{ij}$ are general $3 \times 3$ matrices with real elements. The corrections to the SM Yukawa interactions are related to the Wilson coefficients in the SILH basis by

\[[\delta y_f]_{ij} e^{i[\phi_f]_{ij}} = -[\bar{c}_f]_{ij} - \delta_{ij} \frac{1}{2} [\bar{c}_H + [\bar{c}_H^c]_{22}].\] (3.17)
Next, we define the following single Higgs boson couplings to a pair of the SM gauge fields:

\[ L_{hVV} = \frac{h}{v} \left[ (1 + \delta c_w) \frac{g^2 v^2}{2} W_{\mu}^+ W_{\mu}^- + \frac{(g^2 + g'^2) v^2}{4} Z_{\mu} Z_{\mu} \right. \\
+ c_{ww} \frac{g^2}{2} W_{\mu\nu}^- W_{\mu\nu}^- + \tilde{c}_{ww} \frac{g^2}{2} W_{\mu\nu}^+ \tilde{W}_{\mu\nu}^- + c_{w\Box} g^2 \left( W_{\mu\nu} \partial_\nu W_{\mu\nu}^+ + \text{h.c.} \right) \\
+ c_{gg} \frac{g^2}{4} G_{\mu\nu}^a G^{a\mu\nu} + \tilde{c}_{gg} \frac{g^2}{4} A_{\mu\nu} A^{\mu\nu} + c_{\gamma\gamma} \frac{e \sqrt{g^2 + g'^2}}{2} Z_{\mu\nu} A_{\mu\nu} + c_{zz} \frac{g^2 + g'^2}{4} Z_{\mu\nu} Z_{\mu\nu} \\
+ c_{z\gamma} \frac{g^2}{2} Z_{\mu} \partial_\nu Z_{\mu\nu} + c_{z\gamma} g g' Z_{\mu\nu} \partial_\mu A_{\nu} \\
+ \tilde{c}_{gg} \frac{g^2}{4} G^{a\mu\nu} G^{a\mu\nu} + \tilde{c}_{\gamma\gamma} \frac{e \sqrt{g^2 + g'^2}}{2} Z_{\mu\nu} \tilde{A}_{\mu\nu} + \tilde{c}_{zz} \frac{g^2 + g'^2}{4} Z_{\mu\nu} \tilde{Z}_{\mu\nu} \right], \\
\]

(3.18)

where all the couplings above are real. The terms in the first two lines describe corrections to the SM Higgs couplings to W and Z, while the remaining terms introduce Higgs couplings to gauge bosons with a tensor structure that is absent in the SM Lagrangian. Note that, using equations of motion, we could get rid of certain 2-derivative interactions between the Higgs and gauge bosons: \( hZ_{\mu} \partial_\nu Z_{\nu\mu} \), \( hZ_{\mu} \partial_\nu A_{\nu\mu} \), and \( hW_{\mu\nu}^+ \partial_\nu W_{\nu\mu}^- \). These interactions would then be traded for contact interactions of the Higgs, gauge bosons and fermions in Eq. (3.7). However, one of the defining features of our effective Lagrangian is that the coefficients of the latter couplings are equal to the corresponding vertex correction in Eq. (3.7). This form can be always obtained, without any loss of generality, starting from an arbitrary dimension-6 Lagrangian provided the 2-derivative \( hV_{\mu} \partial_\nu V_{\nu\mu} \) are kept in the Lagrangian. Note that we work in the limit where the neutrinos are massless and the Higgs boson does not couple to the neutrinos. In the EFT context, the couplings to neutrinos induced by dimension-5 operators are proportional to neutrino masses, therefore they are far too small to have any relevance for LHC phenomenology.

The shifts of the Higgs couplings to W and Z bosons are related to the Wilson coefficients in the SILH basis by

\[ \delta c_w = -\frac{1}{2} \bar{c}_H - \frac{1}{g^2} \left[ 4g'^2 \bar{c}_W + \bar{c}_B + \bar{c}_2 B + c_2 W \right] - 2g^2 \bar{c}_T + \frac{3}{2} \frac{g^2 + g'^2}{2} [\bar{c}'_{H\ell}]_{22}, \]

\[ \delta c_z = -\frac{1}{2} \frac{3}{2} [\bar{c}'_{H\ell}]_{22}. \]

(3.19)

The two-derivative Higgs couplings to gauge bosons are related to the Wilson coefficients in the
SILH basis by

\[ c_{gg} = \frac{16}{g^2} \tilde{c}_g, \]
\[ c_{\gamma\gamma} = \frac{16}{g^2} \tilde{c}_\gamma, \]
\[ c_{zz} = -\frac{4}{g^2 + g'^2} \left[ \tilde{c}_{HW} + \frac{g'^2}{g^2} \tilde{c}_{HB} - \frac{4g'^2}{g^2} s_\theta^2 \tilde{c}_\gamma \right], \]
\[ c_{z\phi} = \frac{2}{g^2} \left[ \tilde{c}_W + \frac{g'^2}{g^2} (\tilde{c}_B + \tilde{c}_{HB} + \tilde{c}_{2B}) - \frac{1}{2} \tilde{c}_T + \frac{1}{2} [\tilde{c}'_{H\ell}]_{22} \right], \]
\[ c_{z\gamma} = \frac{2}{g^2} (\tilde{c}_{HB} - \tilde{c}_{HW}) + \frac{4}{g^3 - g'^2} \left[ \tilde{c}_W + \frac{g'^2}{g^2} (\tilde{c}_B + \tilde{c}_{2B}) - \frac{1}{2} \tilde{c}_T + \frac{1}{2} [\tilde{c}'_{H\ell}]_{22} \right], \]
\[ c_{ww} = -\frac{4}{g^2} \tilde{c}_{HW}, \]
\[ c_{w\phi} = \frac{2\tilde{c}_{HW}}{g^2} + \frac{2}{g^2 - g'^2} \left[ \tilde{c}_W + \tilde{c}_{2W} + \frac{g'^2}{g^2} (\tilde{c}_B + \tilde{c}_{2B}) - \frac{1}{2} \tilde{c}_T + \frac{1}{2} [\tilde{c}'_{H\ell}]_{22} \right]. \] (3.20)

Next, couplings of the Higgs boson to a gauge field and two fermions (which are not present in the SM Lagrangian) can be generated by dimension-6 operators. The vertex-like contact interactions between the Higgs, electroweak gauge bosons, and fermions are parametrized as:

\[ \mathcal{L}_{hvff} = \sqrt{2} g \frac{h}{v} W_\mu^+ \left( \bar{u}_L \gamma_\mu \delta g_L^{hWq} d_L + \bar{u}_R \gamma_\mu \delta g_R^{hWq} d_R + \bar{\nu}_L \gamma_\mu \delta g_L^{hW\ell} e_L \right) + h.c. \]
\[ + \frac{2h}{v} \sqrt{g^2 + g'^2} Z_\mu \left[ \sum_{f=u,d,e,\nu} \bar{f}_L \gamma_\mu \delta g_L^{hZf} f_L + \sum_{f=u,d,e} \bar{f}_R \gamma_\mu \delta g_R^{hZf} f_R \right]. \] (3.22)

By construction (condition #6), the coefficients of these interaction are equal to the corresponding vertex correction in Eq. (3.7):

\[ \delta g^{hZf} = \delta g^{Zf}, \quad \delta g^{hWf} = \delta g^{Wf}. \] (3.23)
The dipole-type contact interactions of the Higgs boson are parametrized as:

\[
\mathcal{L}_{h_{\text{dip}}f} = -\frac{h}{4v^2} \left[ g_s \sum_{f=u,d} \sqrt{m_f m_j} \bar{f} L, i \sigma_{\mu\nu} T^a [d_{hGf}]_{ij} f_{R,j} G^a_{\mu\nu} + e \sum_{f=u,d,e} \sqrt{m_f m_j} \bar{f} L, i \sigma_{\mu\nu} [d_{hAf}]_{ij} f_{R,j} A_{\mu\nu} \right. \\
+ \sqrt{g^2 + g'^2} \sum_{f=u,d,e} \sqrt{m_f m_j} \bar{f} L, i \sigma_{\mu\nu} [d_{hZf}]_{ij} f_{R,j} Z_{\mu\nu} \\
+ \sqrt{2} g \sqrt{\frac{m_u m_w}{v}} \bar{d} L, i \sigma_{\mu\nu} [d_{hvW}]_{ij} u_{R,j} W^+_{\mu\nu} + \sqrt{2} g \sqrt{\frac{m_d m_w}{v}} \bar{u} L, i \sigma_{\mu\nu} [d_{hWd}]_{ij} d_{R,j} W^+_{\mu\nu} \\
\left. + \sqrt{2} g \sqrt{\frac{m_{e_i} m_{\ell_j}}{v}} \bar{\nu} L, i \sigma_{\mu\nu} [d_{hWf}]_{ij} e_{R,j} W^+_{\mu\nu} \right] + \text{h.c.},
\]

(3.24)

where \(d_{hGf}, d_{hAf}, d_{hZf}, \) and \(d_{hvWf}\) are general complex \(3 \times 3\) matrices. The coefficients are simply related to the corresponding dipole interactions in Eq. (3.10):

\[
d_{hVf} = d_{Vf}.
\]

Finally, the CP-conserving single Higgs couplings to 3 gauge bosons take the form

\[
\mathcal{L}_{hv\nu\nu} = e g^2 \frac{h}{v} \left\{ i c_{\nu\mu} W^+_{\mu} W^-_{\nu} A_{\nu} + 2 i c_{\nu\mu} \partial_{\nu} W^+_\mu W^-_{\nu} A_{\nu} - i c_{\nu\mu} \partial_{\mu} W^+_\nu W^-_{\nu} A_{\nu} \right\} - i e g^2 \frac{h}{v} A_{\mu\nu} W^+_{\mu} W^-_{\nu} \left( 3 c_{\nu\mu} + c_{z\gamma} + s^2_{\gamma} c_{\gamma\gamma} \right) \\
+ \sqrt{g^2 + g'^2} g^2 \frac{h}{v} \left\{ i c_{\nu\mu} c^2_{\nu} W^+_{\mu} W^-_{\nu} Z_{\nu} + i c_{\nu\mu} \left( 1 + 2 c_{\nu\mu} \right) \partial_{\mu} W^+_{\mu} W^-_{\nu} Z_{\nu} \right\} \\
- i e g^2 \frac{h}{v} \left( 2 + c^2_{\nu\mu} \right) \partial_{\mu} W^+_{\nu} W^-_{\nu} Z_{\nu} + i c_{\nu\mu} s^2_{\gamma} \partial_{\mu} W^+_{\mu} W^-_{\nu} Z_{\nu} + \text{h.c.} \} \\
- i \sqrt{g^2 + g'^2} g^2 \frac{h}{v} Z_{\mu\nu} W^+_{\mu} W^-_{\nu} \left( 3 c_{\nu\mu} c^2_{\nu} + c_{\nu\mu} - s^2_{\gamma} c_{z\gamma} - s^4_{\gamma} c_{\gamma\gamma} \right).
\]

(3.26)

There are also analogous CP-violating couplings which can be obtained from Eq. (3.26) by setting \(c_{\nu\mu} = 0\) and, in the remaining terms, replacing \(c_i \rightarrow \tilde{c}_i, V_{\mu\nu} \rightarrow \tilde{V}_{\mu\nu}\).

**Higgs boson self-couplings and double Higgs couplings**

The cubic Higgs boson self-coupling and couplings of two Higgs boson fields to matter play a role in the EFT description of double Higgs production [32, 33]. Self-interactions of the Higgs boson are parametrized as

\[
\mathcal{L}_{h,\text{self}} = - (\lambda + \delta \lambda_3) v h^3 - \frac{1}{4} (\lambda + \delta \lambda_4) h^4 - \frac{\delta \lambda_5}{v^2} h^5 - \frac{\delta \lambda_6}{v^4} h^6.
\]

(3.27)
The relation between the Higgs self-coupling corrections and the Wilson coefficients in the SILH basis is given by

\[
\delta \lambda_3 = \lambda \left( \tilde{c}_6 - \frac{3}{2} \tilde{c}_H - \frac{1}{2} [\tilde{c}'_{H\ell}]_{22} \right), \\
\delta \lambda_4 = \lambda \left( 6 \tilde{c}_6 - \frac{25}{3} \tilde{c}_H - [\tilde{c}'_{H\ell}]_{22} \right), \\
\delta \lambda_5 = \lambda \left( \frac{3}{4} \tilde{c}_6 - \tilde{c}_H \right), \\
\delta \lambda_6 = \lambda \left( \frac{1}{8} \tilde{c}_6 - \frac{1}{6} \tilde{c}_H \right). \tag{3.28}
\]

In accordance with the condition #4, the 2-derivative Higgs boson self-couplings have been traded for other equivalent interactions and do not occur in the mass eigenstate Lagrangian. The interactions between two Higgs bosons and two other SM fields are parametrized as follows:

\[
\mathcal{L}_{h^2} = h^2 \left( 1 + 2 \delta c_z^{(2)} \right) \frac{g^2 + g'^2}{4} Z_{\mu} Z_{\mu} + h^2 \left( 1 + 2 \delta c_w^{(2)} \right) \frac{g^2}{2} W_{\mu}^+ W_{\mu}^- - \frac{h^2}{2 v^2} \sum_{f,i,j} \sqrt{m_f m_j} \left[ \bar{f}_{i,R} y_f^{(2)} | ij \bar{f}_{j,L} + h.c. \right]
\]

\[
+ \frac{h^2}{8 v^2} \left( c_{gg}^{(2)} g_s^2 G_{\mu\nu}^a G^a_{\mu\nu} + 2 c_{ww}^{(2)} g^2 W_{\mu\nu}^+ W_{\mu\nu}^- + c_{zz}^{(2)} (g^2 + g'^2) Z_{\mu\nu} Z_{\mu\nu} + 2 c_{zz}^{(2)} g' g Z_{\mu\nu} A_{\mu\nu} + c_{zz}^{(2)} e^2 A_{\mu\nu} \bar{A}_{\mu\nu} \right)
\]

\[
+ \frac{h^2}{8 v^2} \left( c_{gg}^{(2)} g_s^2 G_{\mu\nu}^a \tilde{G}^a_{\mu\nu} + 2 c_{ww}^{(2)} g^2 W_{\mu\nu}^+ \tilde{W}_{\mu\nu}^- + c_{zz}^{(2)} (g^2 + g'^2) Z_{\mu\nu} \tilde{Z}_{\mu\nu} + 2 c_{zz}^{(2)} g' g Z_{\mu\nu} \tilde{A}_{\mu\nu} + c_{zz}^{(2)} e^2 A_{\mu\nu} \bar{\tilde{A}}_{\mu\nu} \right)
\]

\[
- \frac{h^2}{2 v^2} \left( g^2 c_{ww}^{(2)} (W_{\mu}^+ \partial_\mu \bar{W}_{\nu\mu}^- + W_{\mu}^- \partial_\mu \bar{W}_{\nu\mu}^+) + g^2 c_{zz}^{(2)} Z_{\mu} \partial_\mu Z_{\nu\mu} + g' c_{zz}^{(2)} Z_{\mu} \partial_\nu A_{\nu\mu} \right). \tag{3.29}
\]

All double Higgs couplings arising from D=6 operators can be expressed by the single Higgs couplings:

\[
\delta c_z^{(2)} = \delta c_z, \quad \delta c_w^{(2)} = \delta c_z + 3 \delta m, \\
[y_f^{(2)}]_{ij} = 3 [\delta y_f]_{ij} e^{i \phi_{ij}} - \delta c_z \delta_{ij}, \\
c_{VV}^{(2)} = c_{VV}, \quad \tilde{c}_{VV}^{(2)} = c_{VV}, \quad \forall \in \{g, w, z, \gamma \},
\]

\[
\begin{align*}
\delta m = 3 \bar{c}_{VV} = \bar{c}_{V}, & \quad \delta c_{vv}^{(2)} = c_{vv}, \quad \forall \in \{w, z, \gamma \}. \tag{3.30}
\end{align*}
\]

Other interaction terms with two Higgs bosons involve at least 5 fields: e.g the $h^2 V^3$ or $h^2 f f V$ contact interactions, and are not displayed here.

**Other terms**

In this section we have written down the interaction terms of mass eigenstates in the dimension-6 EFT Lagrangian which are most relevant for LHC Higgs phenomenology. They either enter the single and double Higgs production at tree level, or they affect electroweak precision observables that are complementary to Higgs couplings measurements. The remaining terms in the mass eigenstate Lagrangian, which are not explicitly displayed in this chapter, are contained in $\mathcal{L}_{\text{other}}$ in Eq. [3.3]. They include 4-fermion terms, dipole-like interactions of two gauge bosons and two
fermions, and interaction terms with 5 or more fields. For a future reference, we only comment on two 4-lepton terms involving left-handed electrons and muons and the corresponding neutrinos:

\[ \mathcal{L}_{4\ell} \supset \frac{1}{v^2} \left[ [c_{\ell\ell}]_{1122}(\bar{\ell}_1 \gamma \mu \ell_1)(\bar{\ell}_2 \gamma \mu \ell_2) + [c_{\ell\ell}]_{1221}(\bar{\ell}_1 \gamma \mu \ell_2)(\bar{\ell}_2 \gamma \mu \ell_1) \right]. \]  

(3.31)

The coefficients of these 4-lepton terms are related to the Wilson coefficients in the SILH basis by

\[
\begin{align*}
[c_{\ell\ell}]_{1122} &= \frac{2g'^2}{g^2} \bar{c}_{2B} - 2 \bar{c}_{2W}, \\
[c_{\ell\ell}]_{1221} &= 4\bar{c}_{2W}.
\end{align*}
\]  

(3.32)

Note that the corresponding 4-fermion operators are absent in the SILH basis. However, in the mass eigenstate Lagrangian, these operators do appear, once the SILH operators \(O_{2W}\) and \(O_{2B}\) are traded for other interactions terms by using equations of motion. By the same token, the 4-top term \([O_{uu}]_{3333}\) does appear in the mass eigenstate Lagrangian, with the coefficient proportional to \(\bar{c}_{2G}\).

\section{Higgs basis}

In the previous section we related the Wilson coefficients in the SILH bases of \(D=6\) operators to the couplings of mass eigenstates in the Lagrangian. With this information at hand, one can proceed to calculating observables at a given order in the EFT as a function of the Wilson coefficients. The information provided above is enough to calculate the leading order EFT corrections to SM predictions for single and double Higgs production and decays in all phenomenologically relevant channels.

There is no theoretical obstacle to present the results of LHC Higgs analyses as constraints on the Wilson coefficients in the SILH, Warsaw, or any other basis. However, this procedure may not be the most efficient one from the experimental point of view. The reason is that the relation between the Wilson coefficients in the SILH basis and the relevant couplings of the Higgs boson in the mass eigenstate Lagrangian is somewhat complicated, c.f. Eqs (3.8), (3.17), (3.19), (3.20). The situation is similar for the Warsaw basis, see Appendix A. In this section we propose another, equivalent parametrization of the EFT with \(D=6\) operators. The idea, put forward in Ref. [34], is to parametrize the space of \(D=6\) operators using a subset of couplings in a mass eigenstate Lagrangian, such as the one defined in Eq. (3.3) of Section 3. The parametrization described in this section, which differs slightly from that in Ref. [34], is referred to as the Higgs basis.

The salient features of the Higgs basis are the following. The goal is to parametrize the space of \(D=6\) operators in a way that can be more directly connected to observable quantities in Higgs physics. The variables spanning the Higgs basis correspond to a subset of the couplings parametrizing interaction terms in the mass eigenstate Lagrangian in Eq. (3.3). Since these couplings have been expressed as linear combinations of the SILH basis Wilson coefficients, technically the Higgs basis is defined as a linear transformation from the SILH basis. All couplings

\[\text{[3.3]}\]

\[\text{Section 4.3}\]

\[\text{[34]}\]

\[\text{[34]}\]
in the subset have to be independent, in the sense that none can be expressed by the remaining ones at the level of a general $D = 6$ EFT Lagrangian. It is also a maximal such subset, which implies that their number is the same as the number of independent operators in the Warsaw or SILH basis. We will refer to this set as the independent couplings. They parametrize all possible deformations of the SM Lagrangian in the presence of $D=6$ operators. Therefore, they can be used on par with any other basis to describe the effects of dimension-6 operators on any physical observables (also those unrelated to Higgs physics). By definition of the Higgs basis, the independent couplings will include single Higgs boson couplings to gauge bosons and fermions. Thanks to that, the parameters of the Higgs basis can be connected in a more intuitive way to LHC Higgs observables calculated at leading order in the EFT. Furthermore, the vertex corrections to the $Z$ boson interactions with fermions are chosen to be among the independent couplings. As a consequence, combining experimental information from Higgs and electroweak precision observables is more transparent in the Higgs basis.

4.1 Independent couplings

We now describe the choice of independent couplings which defines the Higgs basis.

The first group of independent couplings parametrizes the interactions of the Higgs boson with itself and with the SM gauge bosons and fermions:

\[
\begin{align*}
&c_{gg}, \delta c_z, c_{\gamma\gamma}, c_{zz}, c_{Z\gamma}, \bar{c}_{gg}, \bar{c}_{\gamma\gamma}, \bar{c}_{zz}, \delta \lambda_3, \\
&[\delta y_u]_{ij}, [\delta y_d]_{ij}, [\delta y_e]_{ij}, [\phi_u]_{ij}, [\phi_d]_{ij}, [\phi_e]_{ij}.
\end{align*}
\]

(4.1)

The parameters in the first line are defined by Eq. (3.18) and Eq. (3.29), and in the second line by Eq. (3.16). Overall, there is 65 independent parameters in Eq. (4.1), and they all affect Higgs boson production and/or decay at the leading order in the EFT expansion. Therefore they are of crucial importance for LHC Higgs phenomenology. Moreover, at the leading order, they are not constrained at all by LEP-1 electroweak precision tests or low-energy precision observables.

The second group of independent couplings parametrizes the W boson mass and the Z and W boson couplings to fermions:

\[
\begin{align*}
&\delta m, [\delta g_{L}^{\ell}]_{ij}, [\delta g_{R}^{\ell}]_{ij}, [\delta g_{L}^{W\ell}]_{ij}, [\delta g_{R}^{W\ell}]_{ij}, [\delta g_{L}^{Z\ell}]_{ij}, [\delta g_{R}^{Z\ell}]_{ij}, [\delta g_{L}^{Zd}]_{ij}, [\delta g_{R}^{Zd}]_{ij}, [\delta g_{L}^{Wq}]_{ij}, \\
&[d_{Gu}]_{ij}, [d_{Gd}]_{ij}, [d_{Au}]_{ij}, [d_{Ad}]_{ij}, [d_{Ze}]_{ij}, [d_{ Zu}]_{ij}, [d_{Zd}]_{ij}.
\end{align*}
\]

(4.2)

Here the mass correction $\delta m$ is defined in Eq. (3.4), the vertex corrections $\delta g^i$ are defined in Eq. (3.7), and the dipole moments $d_i$ are defined in Eq. (3.10). All these parameters also affect the Higgs boson production and/or decay at the leading order in the EFT. However, as opposed to the ones in Eq. (4.1), they affect at the same order electroweak and/or low-energy precision observables.

The third group of independent couplings parametrizes the self-couplings of gauge bosons:

\[
\begin{align*}
&\lambda_z, \tilde{\lambda}_z, c_{3G}, \tilde{c}_{3G}.
\end{align*}
\]

(4.3)

They are defined in Eq. (3.12). These couplings do not affect Higgs production and decay at the leading order in EFT.

To complete the definition of the Higgs basis, one has to select the independent couplings corresponding to 4-fermion operators. We choose to parametrize them by the same set of Wilson
coefficients as in the SILH basis, c.f. Table[3]

\[ \begin{aligned}
&c_{\ell\ell}, \ c_{qq}, \ c'_{qq}, \ c'_{\ell q}, \ c'_{quqd}, \ c'_{\ell\ell e}, \ c'_{\ell\ell d}, \ c_{\ell q}, \ c'_{\ell q}, \ c_{quqd}, \ c'_{quqd}, \ c_{\ell\ell e}, \ c'_{\ell\ell d}, \ c_{\ell\ell quqd}, \ c'_{\ell\ell quqd}.
\end{aligned} \]  \hspace{1cm} (4.4)

Each parameter \( c_{f f} \) has 4 flavor indices, which are not displayed here. The non-trivial question of which combination of flavor indices constitutes an independent set was worked out in Ref. [35]. In the Higgs basis we take the same choice of independent 4-fermion couplings as in that reference, with one exception. As explained in the next subsection, in a dimension-6 EFT Lagrangian, the coupling \([c_{\ell\ell}]_{1221}\) multiplying a particular 4-lepton operator can be expressed by \( \delta m \) and \( \delta g^i \). Therefore \([c_{\ell\ell}]_{1221}\) is not among the independent couplings defining the Higgs basis.

### 4.2 Dependent couplings

In the mass eigenstate Lagrangian in Eq. (3.3), all deviations from the SM Lagrangian originate from \( D=6 \) operators. However, the number of interaction terms characterizing these deviations is larger than the number of Wilson coefficients multiplying the \( D=6 \) operators in Eq. (2.1). Therefore, there must be relations among the couplings in the mass eigenstate Lagrangians. Working in the Higgs basis, some of these couplings can be expressed by the independent couplings defining the Higgs basis; we call them the dependent couplings. The relations between dependent and independent couplings can be inferred from the matching between the effective Lagrangian and the SILH basis in Section 3. These relations hold at the level of the dimension-6 Lagrangian, and they are in general not respected in the presence of dimension-8 and higher operators.

We start with the dependent couplings in Eq. (3.18) parametrizing the single Higgs boson interactions with gauge bosons. They can be expressed in terms of the independent couplings as:

\[ \begin{aligned}
\delta c_{w} &= \delta c_z + 4\delta m, \\
c_{ww} &= c_{zz} + 2s_b^2c_{z\gamma} + s_b^4c_{\gamma\gamma}, \\
\tilde{c}_{ww} &= \tilde{c}_{zz} + 2s_b^2\tilde{c}_{z\gamma} + s_b^4\tilde{c}_{\gamma\gamma}, \\
c_{w\Box} &= \frac{1}{g^2 - g'^2} \left[ g^2c_{z\Box} + g'^2c_{zz} - e^2s_b^2c_{\gamma\gamma} - (g^2 - g'^2)s_b^4c_{z\gamma} \right], \\
c_{\gamma\Box} &= \frac{1}{g^2 - g'^2} \left[ 2g^2c_{z\Box} + (g^2 + g'^2)c_{zz} - e^2c_{\gamma\gamma} - (g^2 - g'^2)c_{z\gamma} \right].
\end{aligned} \]  \hspace{1cm} (4.5)

The coefficients of W-boson dipole interactions in Eq. (3.10) are related to those of the Z and the photon as

\[ \eta_f d_{Wf} = d_{Zf} + s_b^2d_{Af}, \]  \hspace{1cm} (4.6)

where \( \eta_u = 1 \) and \( \eta_d,e = -1 \). The coefficients of the dipole-like Higgs couplings in Eq. (3.24) are simply related to the corresponding dipole moments:

\[ \begin{aligned}
d_{hVf} &= d_{Vf}, \quad \tilde{d}_{hVf} = \tilde{d}_{Vf}, \quad V \in \{G, W, Z, A\}.
\end{aligned} \]  \hspace{1cm} (4.7)

The coefficients of quartic and higher self-interaction terms of the Higgs bosons in Eq. (3.27)

\[ \text{The relation between } c_{ww}, \tilde{c}_{ww} \text{, and other parameters can also be viewed as a consequence of the accidental custodial symmetry at the level of the dimension-6 operators [23].} \]
are dependent couplings. They can be expressed by the Higgs basis parameters as
\[
\delta \lambda_4 = 6\delta \lambda_3 - \frac{4\lambda}{3} \delta c_z, \\
\delta \lambda_5 = \frac{3}{4} \delta \lambda_3 - \frac{\lambda}{4} \delta c_z, \\
\delta \lambda_6 = \frac{1}{8} \delta \lambda_3 - \frac{\lambda}{24} \delta c_z. 
\] (4.8)

The coefficients of all interaction terms with two Higgs bosons in Eq. (3.29) are dependent couplings. They can be expressed in terms of other Lagrangian parameters as:
\[
\delta c^{(2)}_z = \delta c_z, \quad \delta c^{(2)}_w = \delta c_z + 3\delta m, \\
[g^f_i]_{ij}^{(2)} = 3[\delta g^f_i]_{ij}^{(2)} - \delta c_z \delta_{ij}, \\
c^{(2)}_{\nu
u} = c_{\nu
u}, \quad \delta c^{(2)}_{\nu
u} = \delta c_{\nu
u}, \quad v \in \{g, w, z, \gamma\}, \\
c^{(2)}_{\nu e} = c_{\nu e}, \quad v \in \{w, z, \gamma\}. 
\] (4.9)

The dependent vertex corrections are expressed in terms of the independent couplings as
\[
\delta g^Z_L = \delta g^Z_L + \delta g^W_L, \quad \delta g^W_L = \delta g^Z_L V_{\text{CKM}} - V_{\text{CKM}}^\dagger \delta g^Z_L. 
\] (4.10)

All but four triple gauge couplings in Eq. (3.12) are dependent couplings expressed in terms of the Higgs basis parameters as
\[
\delta g_{1,z} = \frac{1}{2(g^2 - g'^2)} [c_{\gamma\gamma} e^2 g^2 + c_{z\gamma}(g^2 - g'^2)g'^2 - c_{zz}(g^2 + g'^2)g'^2 - c_{z\gamma}(g^2 + g'^2)g^2], \\
\delta \kappa_{\gamma} = -\frac{g^2}{2} \left( c_{\gamma\gamma} \frac{e^2}{g^2 + g'^2} + c_{z\gamma} \frac{g^2 - g'^2}{g^2 + g'^2} - c_{zz} \right), \\
\tilde{\kappa}_{\gamma} = -\frac{g^2}{2} \left( c_{\gamma\gamma} \frac{e^2}{g^2 + g'^2} + c_{z\gamma} \frac{g^2 - g'^2}{g^2 + g'^2} - c_{zz} \right), \\
\delta \kappa_z = \delta g_{1,\nu} - t_{\nu}^2 \delta \kappa_{\gamma}, \quad \tilde{\kappa}_z = -t_{\nu}^2 \tilde{\kappa}_z, \\
\lambda_{\gamma} = \lambda_z, \quad \tilde{\lambda}_z = \tilde{\lambda}_z. 
\] (4.11)

Finally, we discuss how the Wilson coefficient \([c_{\ell \ell}]_{1221}\) is expressed by the independent couplings. One defining feature of the mass eigenstate Lagrangian Eq. (3.3) is that the tree-level relations between the SM electroweak parameters and input observables are not affected by \(D=6\) operators (condition \# 3). On the other hand, one of the four-fermion couplings in the Lagrangian,
\[
\mathcal{L}_{4f}^{D=6} \ni [c_{\ell \ell}]_{1221}(\bar{e}_1 \gamma_{\rho} e_{2,1,2}(\bar{e}_2 \gamma_{\rho} e_{1,1,1}), 
\] (4.12)

does affect the relation between the parameter \(v\) and the muon decay width from which \(v = (\sqrt{2}G_F)^{-2}\) is determined:
\[
\frac{\Gamma(\mu \to e\nu\nu)}{\Gamma(\mu \to e\nu\nu)_{\text{SM}}} \approx 1 + 2[\delta g^W_L]_{11} + 2[\delta g^W_L]_{22} - 4\delta m - [c_{\ell \ell}]_{1221}. 
\] (4.13)

Therefore, the muon decay width is unchanged with respect to the SM when \([c_{\ell \ell}]_{1221}\) is related to \(\delta m\) and \(\delta g\) as
\[
[c_{\ell \ell}]_{1221} = 2[\delta g^W_L]_{11} + 2[\delta g^W_L]_{22} - 4\delta m. 
\] (4.14)
This relation can be verified using the expressions of these parameters in terms of the SILH Wilson coefficients in Eqs. (3.5), (3.8), and (3.32). In other words, due to the fact that we selected $\delta m$ and $6g$ as independent couplings in the Higgs basis, $[c_{\ell r}]_{121}$ has to be a dependent coupling. Of course, one could equivalently choose $[c_{\ell r}]_{121}$ to define a basis, and remove e.g. $\delta m$ from the list of independent couplings. The remaining 4-fermion parameters in Eq. (4.4) are independent couplings.

### 4.3 Gauge invariant definition

In summary, in the Higgs basis the parameters spanning the space of $D=$6 EFT operators are the independent couplings in Eqs. (4.1), (4.2), (4.3), and (4.4). In the EFT expansion, the independent couplings are formally of order $O(\Lambda^{-2})$. These parameters are directly linked to deviations from the SM interactions in the mass eigenstate Lagrangian in Eq. (3.3). All other deviations in the mass eigenstate Lagrangian can be expressed by the independent couplings.

In this note, the Higgs basis was introduced by choosing a subset of independent couplings in the mass eigenstate Lagrangian defined in Section 3. The latter is not manifestly invariant under the full gauge symmetry of the SM, as the electroweak symmetry $SU(2) \times U(1)$ is broken to $U(1)_{em}$ at the mass eigenstate level. Nevertheless, one can provide an equivalent and manifestly gauge invariant definition of the Higgs basis. To this end, one can introduce the $SU(3) \times SU(2) \times U(1)$ invariant $D=$6 operators as follows:

\[
O_{\delta \lambda_3} = -\frac{1}{v^2}(H^\dagger H)^3,
\]

\[
O_{\delta g g} = \frac{g^2}{4v^2} H^\dagger H G_{\mu\nu} G^{\mu\nu},
\]

\[
O_{\delta \epsilon z} = -\frac{1}{v^2} \left[ \partial_{\mu} (H^\dagger H) \right]^2 + \frac{3\lambda}{v^2} (H^\dagger H)^3 + \left( \sum_f \frac{\sqrt{2m_f}}{v^3} H^\dagger H f_{L,i} H f_{R,i} + \text{h.c.} \right),
\]

\[
O_{c_{e c}} = \frac{ig^3}{v^2(g^2 - g'^2)} \left( H^\dagger \sigma^i D^\mu_{\mu} H \right) D_{\nu} W_{i\mu} - \frac{ig'g^2}{v^2(g^2 - g'^2)} \left( H^\dagger D^\mu_{\mu} H \right) \partial_{\nu} B_{\mu\nu},
\]

\[
O_{c_{z z}} = \frac{ig(g^2 + g'^2)}{2v^2(g^2 - g'^2)} \left( H^\dagger \sigma^i D^\mu_{\mu} H \right) D_{\nu} W_{i\mu} - \frac{ig'(g^2 + g'^2)}{2v^2(g^2 - g'^2)} \left( H^\dagger D^\mu_{\mu} H \right) \partial_{\nu} B_{\mu\nu}
\]

\[
- \frac{ig}{v^2} \left( D_{\mu} H^\dagger \sigma^i D_{\nu} H \right) W_{i\mu} - \frac{ig'}{v^2} \left( D_{\mu} H^\dagger D_{\nu} H \right) B_{\mu\nu},
\]

\[
O_{c_{\gamma \gamma}} = \frac{-2igg'g^2}{v^2(g^2 + g'^2)} \left( D_{\mu} H^\dagger \sigma^i D_{\nu} H \right) W_{i\mu} + \frac{2igg'g^2}{v^2(g^2 + g'^2)} \left( D_{\mu} H^\dagger D_{\nu} H \right) B_{\mu\nu},
\]

\[
O_{c_{\gamma g}} = \frac{-igg'g^2}{2v^2(g^4 - g'^4)} \left( H^\dagger \sigma^i D^\mu_{\mu} H \right) D_{\nu} W_{i\mu} + \frac{igg'g^2}{2v^2(g^4 - g'^4)} \left( H^\dagger D^\mu_{\mu} H \right) \partial_{\nu} B_{\mu\nu}
\]

\[
- \frac{igg'^4}{v^2(g^2 + g'^2)^2} \left( D_{\mu} H^\dagger \sigma^i D_{\nu} H \right) W_{i\mu} + \frac{igg'(2g^2 + g'^2)}{v^2(g^2 + g'^2)^2} \left( D_{\mu} H^\dagger D_{\nu} H \right) B_{\mu\nu} + \frac{g'^2}{4v^2} H^\dagger H B_{\mu\nu} B_{\mu\nu},
\]

\[
\left[ O_{\delta g f} \right]_{ij} = \frac{-\sqrt{2m_f m_j}}{v^3} H^\dagger H f_{L,i} H f_{R,j} + \text{h.c.},
\]

\[
\ldots
\]

(4.15)

The coefficients of the operators on the right-hand side in Eq. (4.15) are determined by the linear map relating the SILH Wilson coefficients to those in the Higgs basis, which can be obtained by
inverting the relations between the Higgs and SILH coefficients derived earlier in this note. By following this algorithm, a complete and non-redundant set of $D=6$ operators $O_{ci}$ defining the Higgs basis can be constructed. Then the Higgs basis Lagrangian can be defined in a manifestly gauge invariant way as $\mathcal{L}_{\text{EFT}} = \mathcal{L}_{\text{SM}} + \sum_i c_i O_{ci}$.

### 4.4 Simplified scenarios

In total, the Higgs basis, as any complete basis at the dimension-6 level, is parametrized by 2499 independent real couplings [35]. One should not, however, be intimidated by this number. The point is that a much smaller subset of the independent couplings is relevant for analyses of Higgs data at leading order in EFT. First of all, the coefficients of 4-fermion interactions in Eq. (4.4) and triple gauge interactions in Eq. (4.3) do not enter Higgs observables at the leading order. At that order, the parameters relevant for LHC Higgs analyses are those in Eqs. (4.1) and (4.2), which already reduces the number of variables significantly. Furthermore, there are several motivated assumptions about the UV theory underlying the EFT which could be used to further reduce the number of parameters:

- **Minimal flavor violation**, in which case the matrices $\delta y_f$, $\phi_f$, $d_{Vf}$, and $\delta g^V_f$, reduce to a single number for each $f$.

- **CP conservation**, in which case all CP-odd couplings vanish: $\tilde{c}_i = \phi_f = \text{Im} d_f = 0$.

- **Custodial symmetry**, in which case $\delta m = 0$.

We stress that independent couplings should not be arbitrarily set to zero without an underlying symmetry assumption. Furthermore, the relations between the dependent and independent couplings in the mass eigenstate Lagrangian should be consistently imposed, so as to preserve the structure of the dimension-6 EFT Lagrangian.

Finally, to reduce the number of free parameters in an analysis, one may take advantage of the fact that, in addition to Higgs observables, other measurements are sensitive to the parameters in Eq. (4.2). In particular, the parameters in the first line of Eq. (4.2) are constrained by electroweak precision tests in LEP-1. These are among the most stringent constraints on EFT parameters, and they have an important impact on possible signals in Higgs searches. Assuming minimal flavor violation, all the vertex corrections in Eq. (4.2) are constrained to be smaller than $O(10^{-3})$ (for the leptonic vertex corrections and $\delta m$), or $O(10^{-2})$ (for the quark vertex corrections) [26, 28, 36]. Even when the assumption of minimal flavor violation is not imposed, all the leptonic, bottom and charm quark vertex corrections are still constrained at the level of $O(10^{-2})$ or better [35]. Similarly, many parameters in the second line of Eq. (4.2) are strongly constrained by measurements of the magnetic and electric dipole moments. In the LHC environment, experimental sensitivity is often not sufficient to probe these parameters with a comparable accuracy. If that is indeed the case, it is well-motivated to neglect the parameters in Eq. (4.2) in LHC Higgs analyses.

---

*Custodial symmetry implies several relations between Higgs couplings to gauge bosons: $\delta c_w = \delta c_z$, $c_{w\phi} = c_{z\phi} = c_{zz} + 2 s^2 \theta c_{z\gamma}$, and $c_{ww} = c_{zz} + 2 s^2 \theta c_{z\gamma} + s^4 \theta c_{\gamma\gamma}$. The last three are satisfied automatically at the level of dimension-6 Lagrangian, while the first one is true for $\delta m = 0$, see Eq. (4.5).*

*These constraints may be relaxed if the leading-order dimension-6 EFT does not provide an adequate description of electroweak precision observables [37]. If that is the case, the vertex-like and dipole-like Higgs boson couplings in Eqs. (3.22) and (3.24) could in principle be sizable enough to be relevant for the LHC searches without conflict with electroweak precision constraints. However, it is not clear whether there exist explicit BSM models where this concern is relevant.*
Once the parameters in Eq. (4.2) are neglected, this leaves the parameters collected in Eq. (4.1) to describe leading order deformations of Higgs observables. This set consists of 11 bosonic and $2 \times 3 \times 3 = 54$ fermionic couplings. While that number is still large, it represents a significant simplification compared to the 2499 Wilson coefficients parametrizing a complete $D=6$ basis. Further simplifications can be introduced by making more specific assumptions about the high-energy theory that generates $D=6$ operators in the EFT. For example, if the high-energy theory respects the minimal flavor violation paradigm, the flavor structure of the fermionic parameters in Eq. (4.1)) is proportional to the unit matrix: $[\delta y_f]_{ij} = \delta_{ij} \delta y_f$ and $[\phi_f]_{ij} = \delta_{ij} \phi_f$. This reduces down to 17 (11 bosonic and 6 fermionic) the number of parameters relevant for LHC Higgs observables. In the Higgs basis, these parameters are:

$$CP\text{-}even : c_{gg}, \delta c_z, c_{\gamma \gamma}, c_{z \gamma}, c_{z2}, c_{\Omega}, \delta y_u, \delta y_d, \delta y_e, \delta \lambda_3;
CP\text{-}odd : \tilde{c}_{gg}, \tilde{c}_{\gamma \gamma}, \tilde{c}_{z \gamma}, \tilde{c}_{z2}, \phi_u, \phi_d, \phi_e.$$  

Assuming in addition CP conservation in the Higgs sector leaves only 10 CP-even parameters to describe leading order EFT corrections to single and double Higgs production and decay.

Providing model-independent constraints on the 17 parameters in Eq. (4.16) or at least the 10 CP-even ones, is a realistic target for run-2 LHC Higgs searches. The CP-even parameters are weakly constrained by prior precision experiments, with $O(0.1)-O(1)$ values allowed by current global fits to Higgs and electroweak data [29]. The CP-odd parameters are even less constrained by Higgs and electroweak data, though they are indirectly constrained by low-energy probes of CP violation [30, 40, 41, 42]. Better constraints on this reduced sets of EFT parameters from the ensemble of LHC Higgs measurements would already be a valuable input for constraining a large class of theories beyond the SM.

4.5 Relation to other frameworks

The Higgs basis can be used in par with any other basis to describe the effects of dimension-6 operators on physical observables. Other popular SM EFT approaches in the literature use the so-called SILH [23], Warsaw [16], or HISZ [31] bases of $D=6$ operators. At the leading order in EFT all these approaches are completely equivalent, as there exists a 1-to-1 correspondence between the parameter of the Higgs basis and Wilson coefficients of any other $D=6$ basis. Therefore, the results of leading order EFT analyses can be always translated from and to the Higgs basis without any loss of generality (see e.g. [29] for the translation of the LHC Higgs and TGC constraints). Formulas necessary for translations between various bases are provided in this note: see Section 3 for the Higgs-SILH basis translation, and Appendix A for the Higgs-Warsaw basis translation. A map between the Higgs basis parameters in Eq. (4.16) and the HISZ basis can be found in Appendix B.2. These maps are used by the Rosetta package [43], which provides automated translation between different bases and an interface to Monte Carlo simulations in the MadGraph 5 framework [44].

Using the Higgs basis for leading order Higgs EFT analysis is then simply a matter of convenience. Its usefulness is in the fact that description of Higgs observables and electroweak precision observables at the leading EFT order (tree-level $O(\Lambda^2)$) is more transparent than in other bases.

10The CP-odd parameters affect inclusive Higgs observables only at the quadratic level, ($O(\Lambda^4)$ in the EFT expansion). Therefore they can be neglected in the leading order approximation, even without assuming CP conservation, if one restricts the analysis to inclusive measurements, such as the Higgs signal strength measurements at the LHC.
This also implies simplification of Monte Carlo simulation of collider signals, as relevant Higgs observables typically depend on a smaller number of parameters than in other bases. The advantages of the Higgs basis are especially pronounced when simplified approaches to LHC Higgs data are employed. The main point of the Higgs basis is to separate parameters affecting only Higgs observables at leading order from those that also affect electroweak precision observables. If the latter are neglected in an analysis, a small subset of Higgs basis parameters in Eq. (4.16) is adequate to describe all leading order effects of $D=6$ operators on Higgs observables.

Beyond tree level, advantages of using the Higgs basis are yet to be demonstrated. Indeed, one-loop corrections will introduce a dependence of the Higgs observables on a larger number of parameters, and the neat separation of parameters affecting precision observables is not maintained. As of this time, no one-loop EFT calculations using the Higgs basis formalism exists in the literature; the existing ones are typically performed in the SILH [45, 46, 47, 48, 49, 50] or Warsaw [51, 52, 53, 54, 55, 56] basis.

We will now comment on the relationship between the Higgs basis and other frameworks that also do not introduce new particles beyond the SM but are not equivalent to an EFT. The Higgs basis (and dimension-6 EFT in general) is an extension of the $\kappa$-formalism [57]. That formalism, widely used in LHC Run1 analyses, assumes that only the Higgs couplings already present in the SM receive corrections from new physics. This way, the kinematics of the Higgs production and decay in various channels is unchanged with respect to the SM, and only the signal strength is affected. Moreover, the standard approach allows for new effective Higgs coupling to gluons and photons, as they lead to subleading modifications of the Higgs kinematics when one restrict experimental analyses to inclusive signal strength observables. Recent applications of the $\kappa$-formalism include global fits to the Higgs data with 7 independent coupling modifiers [58].

This is still less general than the dimension-6 EFT, even in its restricted form with the free parameters Eq. (4.16). In particular, the $D=6$ operators may induce Higgs couplings with a different Lorentz structure than that present in the SM (see e.g. Eq. (3.18)) and thus they may violate the assumptions of the $\kappa$-formalism by modifying the Higgs kinematics. Therefore, the results obtained within the $\kappa$-formalism cannot be in general translated into the EFT language, whereas the translation is always possible in the opposite direction.\footnote{\footnote{Note however that, in the dimension-6 EFT, modifications of the relative Higgs coupling strength to $W_\mu W_\mu$ and $Z_\mu Z_\mu$ are always correlated with corrections to the W-boson mass, see Eq. (4.5), which is not taken into account in the $\kappa$-formalism. Strictly speaking, one can thus project general dim-6 EFT results onto a subset of 6 $\kappa$ parameters of Ref. [58]: $\kappa_{gZ}$, $\lambda_{Zg}$, $\lambda_{tg}$, $\lambda_{Z2}$, $\lambda_{W2}$, $\lambda_{WZ}$, with $\lambda_{WZ}$ set to zero. In the LO EFT, these 6 $\kappa$’s are in the 1-to-1 correspondence with a subset of 6 parameters in Eq. (4.16): $c_{gg}$, $\delta c_z$, $c_{\gamma\gamma}$, $\delta y_u$, $\delta y_d$, $\delta y_e$.}}

Pseudo-observables, introduced in Refs. [59, 60] and Section III.1 of [1], offer a more general approach than the SM EFT with $D=6$ operators discussed. Pseudo-observables are defined as form factors parametrizing amplitudes of physical processes subject to constraints from Lorentz invariance. These form factors are expanded in powers of kinematical invariants of the process around the known poles of SM particles, assuming poles from BSM particles are absent in the relevant energy regime. Such a framework involves a larger number of parameters, as it does not impose relations between different form factors or between amplitudes of different processes that are predicted by dimension-6 EFT. Constraints on pseudo-observables can always be projected into constraints on the Higgs basis parameters, provided the complete likelihood function (with correlations) is given; see Ref. [59] for a map between observables relevant for $h \rightarrow 4f$ decays and EFT parameters. The converse is in general not true: constraints on the Higgs basis parameters cannot always be translated into constraints on pseudo-observables.

In Section 3 we introduced the effective Lagrangian that arise when dimension-6 EFT is
rewritten in terms of mass eigenstates after electroweak symmetry breaking. The crucial feature of this Lagrangian is that various interaction terms are not independent but are instead related by the formulas summarized in Section 4.2. These relations are required by the SM gauge symmetry realized linearly at the level of operators with $D \leq 6$. However, one could consider the same Lagrangian without imposing the correlations listed in Section 4.2, and treating instead all parameters as independent. Such a construction is referred to as the Beyond-the-Standard Model Characterization (BSMC). The BSMC Lagrangian is more general than dimension-6 EFT, and involves more parameters. At leading order, it can be used to parametrize new physics effects on Higgs and other observables in a manner akin to pseudo-observables. Once the likelihood function for the parameters of the BSMC Lagrangian is provided by experiment, it can be projected into constraints on the Higgs basis parameters by imposing the relations of Section 4.2. At the same time, the BSMC likelihood can be used to constrain some more general theories that do not reduce to a SM EFT at low energies. The BSMC Lagrangian is a part of the Rosetta package [43].

Another well-known framework to describe Higgs observables is the so-called Higgs Characterization (HC) [24]. In the HC Lagrangian, one describes the effective Higgs couplings to the SM gauge bosons and fermions using 20 new parameters. The HC framework is distinct from the SM EFT. On the one hand, the relations between various 2-derivative Higgs couplings to gauge bosons required by dimension-6 EFT are not imposed. In this aspect HC is more general than the Higgs or other $D=6$ basis, where these relations follow automatically from the structure of the EFT Lagrangian. On the other hand, the HC Lagrangian does not include all possible deformations of the SM Lagrangian predicted in the presence of $D=6$ operators. For example, corrections to SM gauge boson couplings to fermions, dipole interactions, or contact Higgs interactions with one gauge boson and 2 fermions are not implemented. In this aspect, the HC framework is less general than the SM EFT.

Thus, it is in general not possible to translate the constraints from the HC framework to the Higgs basis or the other way around. However, it is possible to do so in certain situations when a simplified EFT description is employed. In particular, one can project constraints on the HC parameters onto the subset of the Higgs basis parameters in Eq. (4.16, assuming other parameters in the Higgs basis are not relevant for these constraints. For such a special case, the relation between the HC parameters and the Higgs basis parameters is given in Appendix B.3.

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A Map from the Warsaw basis

In this appendix we summarize the relations between the independent couplings defining the Higgs basis and the Wilson coefficients $w_i$ in the Warsaw basis. For the latter we use the original
notation of Ref. [16]. The procedure of relating the Wilson coefficients $w_i$ to the couplings in the mass eigenstate Lagrangian is exactly the same as the one described in Section 3 for the SILH basis. This way we can obtain the map between $w_i$ and the subset of independent couplings defining the Higgs basis. We find

$$\delta m = \frac{v^2}{\Lambda^2} \left[ \frac{1}{g^2 - g'^2} \left( -gg' w_{\phi W B} + \frac{g'^2}{4} \left( [w_{\phi \ell}]_{1221} - 2[w_{\phi \ell}^{(3)}]_{11} - 2[w_{\phi \ell}^{(3)}]_{22} \right) - \frac{g^2}{4} w_{\phi D} \right) \right],$$

(A.1)

$$\delta g_{W \ell}^L = \frac{v^2}{\Lambda^2} \left( w_{\phi \ell}^{(3)} + f(1/2, 0) - f(-1/2, -1) \right),$$

$$\delta g_{Z e}^L = \frac{v^2}{\Lambda^2} \left( -\frac{1}{2} w_{\phi \ell}^{(3)} - \frac{1}{2} w_{\phi \ell}^{(1)} + f(-1/2, -1) \right),$$

$$\delta g_{Z e}^R = \frac{v^2}{\Lambda^2} \left( -\frac{1}{2} w_{\phi e} + f(0, -1) \right),$$

(A.2)

$$\delta g_{W q}^R = \frac{v^2}{\Lambda^2} \left( -\frac{1}{2} w_{\phi ud} \right),$$

$$\delta g_{Z u}^L = \frac{v^2}{\Lambda^2} \left( \frac{1}{2} w_{\phi q}^{(3)} - \frac{1}{2} w_{\phi q}^{(1)} + f(1/2, 2/3) \right),$$

$$\delta g_{Z u}^R = \frac{v^2}{\Lambda^2} \left( -\frac{1}{2} w_{\phi u} + f(0, 1/3) \right),$$

$$\delta g_{Z d}^R = \frac{v^2}{\Lambda^2} \left( -\frac{1}{2} w_{\phi d} + f(0, -1/3) \right),$$

(A.3)

where

$$f(T^3, Q) = -I_3 Q \frac{gg'}{g^2 - g'^2} w_{\phi W B} + I_3 \left( \frac{1}{4} [w_{\phi \ell}]_{1221} - \frac{1}{2} [w_{\phi \ell}^{(3)}]_{11} - \frac{1}{2} [w_{\phi \ell}^{(3)}]_{22} - \frac{1}{4} w_{\phi D} \right) \left( T^3 + Q \frac{g'^2}{g^2 - g'^2} \right),$$

(A.4)

and $I_3$ is the $3 \times 3$ identity matrix.

$$d_{G f} = -\frac{v^2}{\Lambda^2} \frac{2\sqrt{2}}{\sqrt{m_f m_f}} \frac{v}{\eta_f w_{f W} + w_{f B}},$$

$$d_{A f} = -\frac{v^2}{\Lambda^2} \frac{2\sqrt{2}}{\sqrt{m_f m_f}} \left( \eta_f w_{f W} + w_{f B} \right),$$

$$d_{Z f} = -\frac{v^2}{\Lambda^2} \frac{2\sqrt{2}}{g^2 + g'^2} \frac{v}{\sqrt{m_f m_f}} \left( g^2 n_f w_{f W} - g'^2 w_{f B} \right),$$

(A.5)

where $\eta_u = +1$, $\eta_{d,e} = -1$. 

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\[ \lambda_z = -\frac{v^2}{\Lambda^2} \frac{3}{2} g w W, \]
\[ \tilde{\lambda}_z = -\frac{v^2}{\Lambda^2} \frac{3}{2} g \tilde{w} W, \]
\[ c_{3G} = \frac{v^2}{\Lambda^2} \frac{w_G}{g_s^3}, \]
\[ \tilde{c}_{3G} = \frac{v^2}{\Lambda^2} \frac{w_G}{g_s^3}, \]
\[ [\delta y_f]_{ij} e^{\delta f} = \frac{v^2}{\Lambda^2} \left[ -\frac{v}{\sqrt{2} m_f m_f} [w_{f\phi}]_{ij} + \delta_{ij} \left( \frac{1}{4} [w_{\ell\ell}]_{1221} - \frac{1}{2} [w_{\phi\ell}]_{11} - \frac{1}{2} [w_{\phi\ell}]_{22} + w_{\phi\phi} - \frac{1}{4} w_{\phi\phi} \right) \right], \]
\[ \delta c_z = \frac{v^2}{\Lambda^2} \left( w_{\phi\phi} - \frac{1}{4} w_{\phi\phi} + 3 \frac{w_{\ell\ell}}{2} [w_{\phi\phi}]_{1221} - \frac{3}{2} [w_{\phi\phi}]_{11} - \frac{3}{2} [w_{\phi\phi}]_{22} \right), \]
\[ c_{z\phi} = \frac{v^2}{\Lambda^2} \frac{1}{g^2} \left( \frac{1}{2} [w_{\ell\ell}]_{1221} + [w_{\phi\phi}]_{11} + [w_{\phi\phi}]_{22} + \frac{1}{2} w_{\phi\phi} \right), \]
\[ c_{gg} = \frac{v^2}{\Lambda^2} \frac{4}{g_s^2} w_{\phi G}, \]
\[ c_{g\gamma} = \frac{v^2}{\Lambda^2} \frac{1}{4} \left( \frac{g^2 w_{\phi W}}{g^2} + \frac{g^2 w_{\phi B}}{g^2} - \frac{1}{2} g_{\phi W} \right), \]
\[ c_{zz} = \frac{v^2}{\Lambda^2} \frac{g^2 w_{\phi W}}{g^2 + g^2}, \]
\[ c_{z\gamma} = \frac{v^2}{\Lambda^2} \frac{4 w_{\phi W} - 4 w_{\phi B} - 2 g^2}{g^2 + g^2} - \frac{g_{\phi W}}{g^2}, \]
\[ \tilde{c}_{gg} = \frac{v^2}{\Lambda^2} \frac{4}{g_s^2} \tilde{w}_{\phi G}, \]
\[ \tilde{c}_{g\gamma} = \frac{v^2}{\Lambda^2} \frac{1}{4} \left( \frac{g^2 w_{\phi W}}{g^2} + \frac{g^2 w_{\phi B}}{g^2} - \frac{1}{2} g_{\phi W} \right), \]
\[ \tilde{c}_{zz} = \frac{v^2}{\Lambda^2} \frac{g^2 w_{\phi W}}{g^2 + g^2}, \]
\[ \tilde{c}_{z\gamma} = \frac{v^2}{\Lambda^2} \frac{4 w_{\phi W} - 4 w_{\phi B} - 2 g^2}{g^2 + g^2} - \frac{g_{\phi W}}{g^2}, \]
\[ \delta \lambda_3 = \frac{v^2}{\Lambda^2} \left[ \lambda \left( 3 w_{\phi\phi} - \frac{3}{4} w_{\phi\phi} + \frac{1}{4} [w_{\ell\ell}]_{1221} - \frac{1}{2} [w_{\phi\ell}]_{11} - \frac{1}{2} [w_{\phi\ell}]_{22} \right) - w_{\phi} \right], \]
\[ c'_{qq} = \frac{v^2}{\Lambda^2} u_{qq}^{(3)}, \]
\[ c_{qq} = \frac{v^2}{\Lambda^2} u_{qq}^{(1)}, \]
\[ c'_{\ell q} = \frac{v^2}{\Lambda^2} u_{\ell q}^{(3)}, \]
\[ c_{\ell q} = \frac{v^2}{\Lambda^2} u_{\ell q}^{(1)}, \]
\[ c'_{ud} = \frac{v^2}{\Lambda^2} u_{ud}^{(8)}, \]
\[ c_{ud} = \frac{v^2}{\Lambda^2} u_{ud}^{(1)}, \]
\[ c'_{qd} = \frac{v^2}{\Lambda^2} u_{qd}^{(8)}, \]
\[ c_{qd} = \frac{v^2}{\Lambda^2} u_{qd}^{(1)}, \]
\[ c'_{qu} = \frac{v^2}{\Lambda^2} u_{qu}^{(8)}, \]
\[ c_{qu} = \frac{v^2}{\Lambda^2} u_{qu}^{(1)}, \]
\[ c_{quqd} = \frac{v^2}{\Lambda^2} u_{quqd}^{(8)}, \]
\[ c_{quqd} = \frac{v^2}{\Lambda^2} u_{quqd}^{(1)}, \]
\[ c'_{\ell equ} = \frac{v^2}{\Lambda^2} u_{\ell equ}^{(3)}, \]
\[ c_{\ell equ} = \frac{v^2}{\Lambda^2} u_{\ell equ}^{(1)}, \]

and the relation is trivial,  \( c_i = w_i v^2 / \Lambda^2 \), for the remaining 4-fermion coefficients (except for \( [c_{\ell q}]_{1221} \) which does not enter into the definition of the Higgs basis).

This map can be used to translate to the Higgs basis formalism results of any tree level calculations using the Warsaw basis. Translating NLO EFT results between different bases requires specifying the renormalization scale for the SM couplings appearing in the dictionary. One simple choice is to use the running couplings; another natural choice is to use couplings defined at the scale \( \mu = m_h \).

At this point we have a 1-to-1 map between the Higgs basis and the Warsaw basis, as well as one between the Higgs basis and the SILH basis derived in Section 3. Using these two, we can eliminate the Higgs basis coefficients, and derive the map between the Warsaw and SILH basis. We find the following relations between the Wilson coefficients \( w_i \) in the Warsaw basis and the
Wilson coefficients $\bar{c}_i$ in the SILH basis:

\[
\begin{align*}
\frac{v^2}{\Lambda^2} w_\phi &= \lambda (-c_6 + 8\bar{c}_2W + 8\bar{c}_W + 8\bar{c}_{HW}), \\
\frac{v^2}{\Lambda^2} w_\phi^{\Box} &= 3\bar{c}_2W + 3\bar{c}_W + 3\bar{c}_{HW} + \frac{g'^2}{g^2} (\bar{c}_{2B} + \bar{c}_B + \bar{c}_{HB}) - \frac{1}{2} \bar{c}_H - \frac{1}{2} \bar{c}_T, \\
\frac{v^2}{\Lambda^2} w_\phi D &= \frac{4g'^2}{g^2} (\bar{c}_{2B} + \bar{c}_B + \bar{c}_{HB}) - 2\bar{c}_T, \\
\frac{v^2}{\Lambda^2} w_\phi G &= \frac{4g'^2}{g^2} \bar{c}_g, \\
\frac{v^2}{\Lambda^2} w_\phi W &= -\bar{c}_{HW}, \\
\frac{v^2}{\Lambda^2} w_\phi B &= \frac{4g'^2}{g^2} \bar{c}_g - \frac{g'^2}{g^2} \bar{c}_{HB}, \\
\frac{v^2}{\Lambda^2} w_\phi WB &= -\frac{g'}{g} (\bar{c}_{HW} + \bar{c}_{HB}), \\
\frac{v^2}{\Lambda^2} w_\phi G &= \frac{4g'^2}{g^2} \bar{c}_{3G}, \\
\frac{v^2}{\Lambda^2} w_\phi W &= 4g \bar{c}_{3W}, \\
\frac{v^2}{\Lambda^2} [w_{\phi}]_{ij} &= \sqrt{\frac{2m_f}{m_f}} \left( [\bar{c}_f]_{ij} + 2\delta_{ij} (\bar{c}_{2W} + \bar{c}_W + \bar{c}_{HW}) \right), \\
\frac{v^2}{\Lambda^2} [w_{\phi}]^{(3)}_{ij} &= [\bar{c}_{H'f}]_{ij} + \delta_{ij} (2\bar{c}_{2W} + \bar{c}_W + \bar{c}_{HW}), \\
\frac{v^2}{\Lambda^2} [w_{\phi}]^{(1)}_{ij} &= [\bar{c}_{Hf}]_{ij} + 2Y_f \frac{g'^2}{g^2} \delta_{ij} (2\bar{c}_{2B} + \bar{c}_B + \bar{c}_{HB}), \\
\frac{v^2}{\Lambda^2} [w_{\phi}]_{ij} &= [\bar{c}_{Hf}]_{ij} + 2Y_f \frac{g'^2}{g^2} \delta_{ij} (2\bar{c}_{2B} + \bar{c}_B + \bar{c}_{HB}), \\
\frac{v^2}{\Lambda^2} [w_{\ell\ell}]_{iii} &= [\bar{c}_{\ell\ell}]_{iii} + g'^2 \frac{g^2}{g^2} \bar{c}_2B + \bar{c}_{2W}, \\
\frac{v^2}{\Lambda^2} [w_{\ell\ell}]_{ijj} &= [\bar{c}_{\ell\ell}]_{ijj} + \frac{2g'^2}{g^2} \bar{c}_{2B} - 2\bar{c}_{2W}, \quad i < j, \\
\frac{v^2}{\Lambda^2} [w_{\ell\ell}]_{ijji} &= [\bar{c}_{\ell\ell}]_{ijji} + 4\bar{c}_{2W}, \quad i < j,
\end{align*}
\]

(A.15)

where it is implicit that $[\bar{c}_{H'f}]_{11} = [\bar{c}_{Hf}]_{11} = 0$, and $[\bar{c}_{\ell\ell}]_{122} = [\bar{c}_{\ell\ell}]_{1122} = 0$. The same relations can be obtained by directly transforming the SILH operators to the Warsaw basis using equations of motion and integration by parts.

## B More dictionaries

In this section we quote the linear transformation between the parameters defining the Higgs basis and the Wilson coefficients in two other bases of dimension-6 operators utilized in the literature.\footnote{On request, translation to other bases may be added in the future.}
For simplicity, we assume here (unlike in the rest of this note) that the parameters are flavor blind. Moreover, we give the dictionary only for the subset of the Higgs basis parameters that can give observable contributions to single Higgs and electroweak diboson processes, given the constraints from electroweak precision tests. That set consists of 10 CP-even and 8 CP-odd parameters:

\begin{align}
&c_{gg}, \delta c_z, c_{\gamma\gamma}, c_{zz}, c_{z\Box}, \delta y_u, \delta y_d, \delta y_e, \lambda_z, \\
&\tilde{c}_{gg}, \tilde{c}_{\gamma\gamma}, \tilde{c}_{zz}, \tilde{c}_{z\Box}, \tilde{\phi}_u, \tilde{\phi}_d, \tilde{\phi}_e, \tilde{\lambda}_z.
\end{align}

The dictionaries below allow one to translate results of any complete EFT Higgs analyses into constraints on the Higgs basis parameters (and, by consequence, between any pair of bases), as long as the full likelihood function in the space of Wilson coefficients is given.

### B.1 SILH’ basis

The original SILH basis discussed in the main text includes the operators \(O_{2W}, O_{2B}\) and \(O_{2G}\), which lead to 4-derivative corrections to the kinetic terms of the gauge fields. This may be inconvenient for some applications. A simple fix is to remove these operators in favor of the 4-fermion operators \([O_{\ell\ell}]_{1221}, [O_{\ell\ell}]_{1122}, \text{ and } [O_{u}]_{3333}\). This construction was used in Ref. [26] and we refer to it as the SILH’ basis. One advantage of this choice is that electroweak precision constraints take a particularly simple form. Namely, the vanishing of the vertex correction \(\delta g\) and the W mass correction \(\delta m\) corresponds to setting \(\tilde{c}_T = [\tilde{c}_{\ell\ell}]_{1221} = \tilde{c}_{Hf} = \tilde{c}'_{Hf} = 0\), and \(\tilde{c}_B = -\tilde{c}_W\).

The CP even Higgs basis parameters in Eq. [B.1] are related to the Wilson coefficients in the SILH’ basis by

\begin{align}
&c_{gg} = \frac{16}{g^2} \tilde{c}_g, \\
&\delta c_z = -\frac{1}{2} \tilde{c}_H + \frac{3}{4}[\tilde{c}_{\ell\ell}]_{1221} - \frac{3}{2}[\tilde{c}'_{H\ell}]_{22}, \\
&c_{\gamma\gamma} = \frac{16}{g^2} \tilde{c}_\gamma, \\
&c_{zz} = \frac{2}{g^2} \left( \tilde{c}_{HB} - \tilde{c}_{HW} - 8s_\theta^2 \tilde{c}_\gamma \right), \\
&c_{z\Box} = \frac{2}{g^2} \left[ \tilde{c}_W + \tilde{c}_{HW} + \frac{g'^2}{g^2} (\tilde{c}_B + \tilde{c}_{HB}) - \frac{1}{2} \tilde{c}_T + \frac{1}{2}[\tilde{c}'_{H\ell}]_{22} - \frac{1}{4}[\tilde{c}_{\ell\ell}]_{1221} \right], \\
&\lambda_z = -6g^2 \tilde{c}_{3W}.
\end{align}

The CP odd Higgs basis parameters in Eq. [B.2] are related to the Wilson coefficients in the
SILH' basis by
\[ \tilde{c}_{gg} = 16 g^2 \tilde{c}_g, \]
\[ \tilde{c}_{\gamma \gamma} = 16 g^2 \tilde{c}_\gamma, \]
\[ \tilde{c}_{z \gamma} = 2 g^2 (\tilde{c}_{HB} - \tilde{c}_{HW} - 8 s_\theta^2 \tilde{c}_\gamma), \]
\[ \tilde{c}_{zz} = -4 g^2 + g'^2 \left[ \tilde{c}_{HW} + \frac{g'^2}{g^2} \tilde{c}_{HB} - 4 \frac{g'^2}{g^2} s_\theta^2 \tilde{c}_\gamma \right], \]
\[ \tilde{\lambda}_z = -6 g^2 \tilde{c}_{3W}. \]  
(B.4)

Finally, the corrections to the Yukawa couplings are given by
\[ \delta y_f e^{i \phi_f} = -[\tilde{c}_f]_{ij} - \delta_{ij} \frac{1}{2} \left[ \tilde{c}_H + [\tilde{c}_H^H]_{22} - \frac{1}{2} [\tilde{c}_{H^H}]_{1221} \right], \quad f \in \{u, d, e\}. \]  
(B.5)

### B.2 HISZ basis

We consider a subset of bosonic operators introduced by Hagiwara et al. (HISZ) in Ref. [31]:

\[ \hat{O}_{H,2} = \frac{1}{2} \left( \partial^{\mu} (H^{\dagger} H) \right)^2, \]
\[ \hat{O}_{GG} = -\frac{g^2}{32 \pi^2} H^{\dagger} H G^a_{\mu \nu} G^a_{\mu \nu}, \]
\[ \hat{O}_{WW} = H^{\dagger} W_{\mu \nu} W_{\mu \nu} H, \]
\[ \hat{O}_{BB} = H^{\dagger} B_{\mu \nu} B_{\mu \nu} H, \]
\[ \hat{O}_W = D_{\mu} H^{\dagger} W_{\mu \nu} D_{\nu} H, \]
\[ \hat{O}_B = D_{\mu} H^{\dagger} B_{\mu \nu} D_{\nu} H, \]
\[ \hat{O}_{WWW} = \text{Tr} \left[ W_{\mu \nu} W_{\nu \rho} W_{\rho \mu} \right], \]  
(B.6)

\[ \hat{O}_{\tilde{G} G} = -\frac{g^2}{32 \pi^2} H^{\dagger} H G^a_{\mu \nu} \tilde{G}^a_{\mu \nu}, \]
\[ \hat{O}_{\tilde{W} W} = H^{\dagger} W_{\mu \nu} \tilde{W}_{\mu \nu} H, \]
\[ \hat{O}_{\tilde{B} B} = H^{\dagger} B_{\mu \nu} \tilde{B}_{\mu \nu} H, \]
\[ \hat{O}_{\tilde{W}} = D_{\mu} H^{\dagger} \tilde{W}_{\mu \nu} D_{\nu} H, \]
\[ \hat{O}_{\tilde{W} \tilde{W} \tilde{W}} = \text{Tr} \left[ \tilde{W}_{\mu \nu} \tilde{W}_{\nu \rho} \tilde{W}_{\rho \mu} \right], \]  
(B.7)

where the electroweak field strength tensors are related to the one used in this note via
\[ B_{\mu \nu} = -\frac{i}{2} g' B_{\mu \nu}, \quad \tilde{W}_{\mu \nu} = -\frac{i}{2} g' W'_{\mu \nu}. \]  
(B.8)

---

\[ ^{13}\text{The additional minus sign in Eq. (B.8) is due to the fact that the covariant derivatives in Refs. [31] are defined with the opposite sign to that used here. This amounts to rescaling the gauge fields as } W_{\mu} \rightarrow -W_{\mu}, B_{\mu} \rightarrow -B_{\mu} \text{ in the translation.} \]
We also consider the Yukawa operators

\[ \hat{O}_u = H^\dagger H \bar{q}_L \mathcal{M}_u \gamma^0 u_R, \quad \hat{O}_d = H^\dagger H \bar{q}_L \mathcal{M}_d \gamma^0 d_R, \quad \hat{O}_e = H^\dagger H \bar{\ell}_L \mathcal{M}_e \gamma^0 e_R, \]

where \( m_f \) are 3 \times 3 diagonal fermion mass matrices. The dimension-6 Lagrangian is given by

\[ \mathcal{L}^{D=6}_{\text{HISZ}} = \frac{1}{\Lambda^2} \left[ \sum_i f_i \hat{O}_i + \sum_j \left( f_j \hat{O}_j + \text{h.c.} \right) + \ldots \right], \]

where the first sum goes over the bosonic operators in Eqs. (B.6) and (B.7), the second sum goes over the fermionic operators in Eq. (B.9), and the dots stands for remaining operators that complete the dimension-6 basis. The CP-even operators from this set (except \( \hat{O}_{WWW} \)) are used by SFitter \cite{SFitter} to describe constraints on dimension-6 operators from LHC Higgs data. Ref. \cite{HISZ} proposes to use the HISZ operators \( \hat{O}_{W}, \hat{O}_{B}, \hat{O}_{WWW}, \hat{O}_{\bar{W}}, \) and \( \hat{O}_{\bar{W}WWW} \) to describe constraints on dimension-6 operators from the pair production of electroweak gauge bosons.

The CP even Higgs basis parameters in Eq. (B.1) are related to the Wilson coefficients in the HISZ basis by

\[
\begin{align*}
    c_{gg} &= -\frac{1}{8\pi^2} \hat{f}_{GG} \frac{v^2}{\Lambda^2}, \\
    \delta c_z &= -\frac{1}{2} \hat{f}_{H,2} \frac{v^2}{\Lambda^2}, \\
    c_{\gamma\gamma} &= (\hat{f}_{WW} - \hat{f}_{BB}) \frac{v^2}{\Lambda^2}, \\
    c_{z\gamma} &= \left( \frac{1}{4} \hat{f}_W - \frac{1}{4} \hat{f}_B - \frac{1}{4} f_{WW} + \frac{1}{4} f_{BB} \right) \frac{v^2}{\Lambda^2}, \\
    c_{zz} &= \left( \frac{1}{2} \hat{f}_W - \frac{1}{4} f_{WW} - \frac{1}{4} f_{BB} \right) \frac{v^2}{\Lambda^2}, \\
    c_{z\Box} &= \left( \frac{1}{4} \hat{f}_W - \frac{1}{4} f_{WW} - \frac{1}{4} f_{BB} \right) \frac{v^2}{\Lambda^2}, \\
    \lambda_z &= \frac{3g^4}{8} \frac{v^2}{\Lambda^2} \hat{f}_{WWW}.
\end{align*}
\]

The CP odd Higgs basis parameters in Eq. (B.2) are related to the Wilson coefficients in the HISZ basis by

\[
\begin{align*}
    \tilde{c}_{gg} &= -\frac{1}{8\pi^2} \tilde{f}_{GG} \frac{v^2}{\Lambda^2}, \\
    \tilde{c}_{\gamma\gamma} &= (\tilde{f}_{WW} - \tilde{f}_{BB}) \frac{v^2}{\Lambda^2}, \\
    \tilde{c}_{z\gamma} &= \left( \frac{1}{4} \tilde{f}_W - \frac{1}{4} \tilde{f}_B - \frac{1}{4} \tilde{f}_{WW} + \frac{1}{4} \tilde{f}_{BB} \right) \frac{v^2}{\Lambda^2}, \\
    \tilde{c}_{zz} &= \left( \frac{1}{2} \tilde{f}_W - \frac{1}{4} \tilde{f}_{WW} - \frac{1}{4} \tilde{f}_{BB} \right) \frac{v^2}{\Lambda^2}.
\end{align*}
\]

Finally, the corrections to the Yukawa couplings are given by

\[
\delta y_j e^{i\phi_j} = \left( -\frac{1}{2} f_{H,2} - \frac{f_j}{\sqrt{2}} \right) \frac{v^2}{\Lambda^2}, \quad j \in \{u, d, e\}. \]
For completeness, we also give the relation between the anomalous TGCs and the HISZ basis Wilson coefficients:

\[
\delta g_1 z = \frac{g^2 + g'^2}{8} f_W \frac{v^2}{\Lambda^2}, \\
\delta \kappa \gamma = \frac{g^2}{8} (f_W + f_B) \frac{v^2}{\Lambda^2}, \\
\delta \kappa^\gamma = \frac{g^2}{8} f_W \frac{v^2}{\Lambda^2}, \\
\lambda_z = \frac{3g^4}{8} f_{WW} \frac{v^2}{\Lambda^2}, \\
\tilde{\lambda}_z = \frac{3g^4}{8} \tilde{f}_{WW} \frac{v^2}{\Lambda^2}.
\]

(B.14)

Inverting the transformations, the relation between the Wilson coefficients in the HISZ basis and the Higgs basis parameters reads

\[
f_{GG} \frac{v^2}{\Lambda^2} = -8\pi^2 c_{gg}, \\
f_{H} \frac{v^2}{\Lambda^2} = -2\delta c_z, \\
f_W \frac{v^2}{\Lambda^2} = -\frac{4}{g^2 - g'^2} \left[ g^2 c_{zz} + g'^2 c_{zz} - s_\theta^2 c_{\gamma\gamma} - s_\theta^2 (g^2 - g'^2) c_{z\gamma} \right], \\
f_B \frac{v^2}{\Lambda^2} = \frac{4}{g^2 - g'^2} \left[ g^2 c_{zz} + g'^2 c_{zz} - c_\theta^2 c_{\gamma\gamma} - c_\theta^2 (g^2 - g'^2) c_{z\gamma} \right], \\
f_{WW} \frac{v^2}{\Lambda^2} = -\frac{1}{g^2 - g'^2} \left[ 2g^2 c_{zz} + (g^2 + g'^2) c_{zz} - s_\theta^2 g'^2 c_{\gamma\gamma} \right], \\
f_{BB} \frac{v^2}{\Lambda^2} = \frac{1}{g^2 - g'^2} \left[ 2g^2 c_{zz} + (g^2 + g'^2) c_{zz} - c_\theta^2 g^2 c_{\gamma\gamma} \right], \\
f_{WWW} \frac{v^2}{\Lambda^2} = \frac{8}{3g^4} \lambda_z, \\
f_{j} \frac{v^2}{\Lambda^2} = \sqrt{2} \delta c_z - \sqrt{2} \delta y_j e^{-i\phi_j}, \quad j \in \{u, d, e\}, \\
\tilde{f}_{GG} \frac{v^2}{\Lambda^2} = -8\pi^2 \tilde{c}_{gg}, \\
\tilde{f}_W \frac{v^2}{\Lambda^2} = 4\tilde{c}_{zz} - 4\frac{g^2 - g'^2}{g^2 + g'^2} \tilde{c}_{z\gamma} - 4\frac{e^2}{g^2 + g'^2} \tilde{c}_{\gamma\gamma}, \\
\tilde{f}_{WW} \frac{v^2}{\Lambda^2} = \tilde{c}_{zz} - 2c_\theta \tilde{c}_{z\gamma} - \frac{g'^2 (2g^2 + g'^2)}{(g^2 + g'^2)^2} \tilde{c}_{\gamma\gamma}, \\
\tilde{f}_{BB} \frac{v^2}{\Lambda^2} = -\tilde{c}_{zz} + 2c_\theta \tilde{c}_{z\gamma} - c_\theta^2 \tilde{c}_{\gamma\gamma}, \\
\tilde{f}_{WWW} \frac{v^2}{\Lambda^2} = \frac{8}{3g^4} \tilde{\lambda}_z.
\]

(B.15)

B.3 Higgs Characterization framework

The Higgs Characterization (HC) framework [24] in general cannot be mapped to the Higgs basis or any other dimension-6 EFT basis. However, it is possible to to related the HC and EFT parameters in certain situations when a simplified EFT description is employed. In particular,
the HC parameters can be related to the subset of the Higgs basis parameters in Eq. (4.16), assuming other parameters in the Higgs basis are set to zero. In such a case, the relation between the HC parameters (as defined in Section II.3.1 of [1]) and the Higgs basis parameters reads:\[14\]

\[
\begin{align*}
  c_{\alpha} \kappa_{Hff} - 1 &= \delta y_f \cos \phi_f, \\
  -s_{\alpha} \kappa_{Aff} &= \delta y_f \sin \phi_f, \\
  c_{\alpha} \kappa_{SM} - 1 &= \delta c_z, \\
  -\frac{1}{12\pi^2} c_{\alpha} \kappa_{H\gamma\gamma} &= c_{\gamma\gamma}, \\
  -\frac{1}{3\pi^2} s_{\alpha} \kappa_{A\gamma\gamma} &= \tilde{c}_{\gamma\gamma}, \\
  \frac{1}{12\pi^2} c_{\alpha} \kappa_{Hgg} &= c_{gg}, \\
  -\frac{1}{8\pi^2} s_{\alpha} \kappa_{Agg} &= \tilde{c}_{gg}, \\
  -\frac{v}{(g^2 + g'^2) \Lambda} c_{\alpha} \kappa_{HZZ} &= c_{zz}, \\
  -\frac{v}{(g^2 + g'^2) \Lambda} s_{\alpha} \kappa_{AZZ} &= \tilde{c}_{zz}, \\
  -\frac{v}{g^2 \Lambda} c_{\alpha} \kappa_{HWZ} &= c_{zz} + 2s_\theta^2 c_{z\gamma} + s_\theta^4 c_{\gamma\gamma}, \\
  -\frac{v}{g'^2 \Lambda} s_{\alpha} \kappa_{AWZ} &= \tilde{c}_{zz} + 2s_\theta^2 c_{z\gamma} + s_\theta^4 \tilde{c}_{\gamma\gamma}, \\
  \frac{v}{g \Lambda} c_{\alpha} \kappa_{H\partial Z} &= c_{z\Box}, \\
  \frac{v}{g'} \Lambda c_{\alpha} \kappa_{H\partial \gamma} &= \frac{gg'}{g^2 - g'^2} \left[ 2g'^2 c_{zz} + g'^2 (g^2 + g'^2)c_{zz} - e^2 c_{\gamma\gamma} - (g^2 - g'^2)c_{z\gamma} \right], \\
  \frac{v}{\Lambda} c_{\alpha} \kappa_{H\partial \gamma W} &= \frac{g^2}{g^2 - g'^2} \left[ g^2 c_{zz} + g'^2 c_{zz} - e^2 s_\theta^2 c_{\gamma\gamma} - (g^2 - g'^2)s_\theta^2 c_{z\gamma} \right], \\
  -\frac{(94 - 13)}{144\pi^2} c_{\alpha} \kappa_{HZ\gamma} &= c_{z\gamma}, \\
  -\frac{(8c_\theta^2 - 5)}{24\pi^2} s_{\alpha} \kappa_{AZ\gamma} &= \tilde{c}_{z\gamma}. 
\end{align*}
\]

\[B.18\]

C Goldstone bosons and gauge fixing

In the main body of this note we worked in the unitary gauge where the Goldstone boson degrees of freedom in the Higgs doublet are set to zero. This is enough for the sake of tree-level EFT calculations. However, in order to extend the calculations to a loop level, retrieving the Goldstone degrees of freedom may be convenient, as it allows one to perform the standard gauge fixing procedure. The procedure is sketched in this appendix.

We parametrize the Higgs doublet as

\[
H = \begin{pmatrix} \frac{1}{\sqrt{2}} (v + h - iG_3) \end{pmatrix}
\]

\[C.1\]

\[14\] Thanks to Rostislav Konoplich for pointing out several mistakes in an earlier version of this equation.
where $G_\pm$ and $G_3$ are three Goldstone fields, that will be eaten by the W and Z bosons. In the presence of dimension-6 operators, in general one may need to rescale

$$G_\pm \rightarrow G_\pm (1 + \delta G_\pm), \quad G_3 \rightarrow G_3 (1 + \delta G_3), \quad (C.2)$$

in order to bring the Goldstone kinetic terms into a canonically normalized form. Other fields and couplings are also rescaled to match the conventions specified earlier in this note, see Eq. (3.1).

Once this has been done, the quadratic terms containing Goldstone fields are given by

$$\mathcal{L}_{G}^{\text{kinetic}} = \partial_\mu G_+ \partial_\mu G_+ + \frac{1}{2} (\partial_\mu G_3)^2 - \frac{g v}{2} (1 + \delta m) (\partial_\mu G_+ W^-_\mu + \text{h.c.}) - \frac{\sqrt{g^2 + g'^2}}{2} \partial_\mu G_3 Z_\mu. \quad (C.3)$$

As usual, the Goldstones kinetically mix with the electroweak gauge bosons with the mixing strength proportional to the gauge boson mass.

All Goldstone boson couplings are dependent ones, that is they can be expressed by the independent couplings defining the Higgs basis. As an illustration, below we display Goldstone a subset of CP-even interaction terms with up to 4-fields and with one or two electroweak gauge fields. The relevant part of the Lagrangian can be written as

$$\mathcal{L} = \mathcal{L}_{G}^{S^2 V} + \mathcal{L}_{G}^{S^2 V^2} + \mathcal{L}_{G}^{SdV^2} + \mathcal{L}_{G}^{S^2 dV^2} + \mathcal{L}_{G}^{S^2 dV^2}.$$ \quad (C.4)

where

$$\mathcal{L}_{G}^{S^2 V} = \beta_{hew} \frac{g}{2} \partial_\mu h (G_+ W^-_\mu + \text{h.c.}) + \beta_{h3z} \frac{g^2 + g'^2}{2} \partial_\mu h G_3 Z_\mu + \frac{g v}{2} (1 + \delta m) \partial_\mu G_+ W^-_\mu + \partial_\mu G_3 Z_\mu.$$ \quad (C.5)

$$\mathcal{L}_{G}^{S^2 V^2} = i \beta_{cw} \frac{eg v}{2} (G_+ W^-_\mu - \text{h.c.}) A_\mu - i \beta_{cw} \frac{eg' v}{2} (G_+ W^-_\mu - \text{h.c.}) Z_\mu.$$ \quad (C.6)

$$\mathcal{L}_{G}^{SdV^2} = i \frac{\eta_{cwz}}{2 v} (G_+ W^-_\mu - \text{h.c.}) A_\mu - i \frac{\eta_{cwz}}{2 v} (G_+ W^-_\mu - \text{h.c.}) Z_\mu.$$ \quad (C.7)

$$\mathcal{L}_{G}^{S^2 dV^2} = \frac{g^2}{2} (G_+ W^-_\mu + \text{h.c.}) A_\mu + \frac{g^2}{2} (G_+ W^-_\mu + \text{h.c.}) Z_\mu.$$ \quad (C.8)
\[ v^2 L_{G}^{S^2} = G_+ G_- \left( \eta_{cc\gamma} e^2 A_{\mu\nu} A_{\mu\nu} + \eta_{ccz} g' A_{\mu\nu} Z_{\mu\nu} + \eta_{cc\zeta z} (g^2 + g'^2) Z_{\mu\nu} Z_{\mu\nu} + \eta_{ccwz} g^2 W^+_{\mu\nu} W^-_{\mu\nu} \right) + G_3 G_3 \left( \eta_{33\gamma} e^2 A_{\mu\nu} A_{\mu\nu} + \eta_{33z} g' A_{\mu\nu} Z_{\mu\nu} + \eta_{33z\zeta z} (g^2 + g'^2) Z_{\mu\nu} Z_{\mu\nu} + \eta_{33ww} g^2 W^+_{\mu\nu} W^-_{\mu\nu} \right) + \eta_{c3w} e g \left( G_+ W^-_{\mu\nu} + h.c. \right) G_3 A_{\mu\nu} + \eta_{c3w} e g' \left( G_+ W^-_{\mu\nu} + h.c. \right) G_3 Z_{\mu\nu}. \]

(C.9)

The coefficients of the Goldstone interaction terms are related as follows to those of the Higgs and gauge boson interaction terms in Eq. 3.3:

\[ \beta_{hcw} = 1 + g^2 c_{w\Box} + \delta c_z + 3 \delta m, \]
\[ \beta_{h3z} = 1 + g^2 c_{w\Box} + \delta c_z + 2 \delta m, \]
\[ \beta_{3cw} = 1 - 2 g^2 c_{w\Box} + \frac{3}{2} g^2 c_{z\Box} - 3 \delta m, \]
\[ \beta_{3hz} = 1 - g^2 c_{w\Box} + g^2 c_{z\Box} + \delta c_z - 2 \delta m, \]
\[ \beta_{ccz} = 1 + \frac{g^2 + g'^2}{2 (g^2 - g'^2)} (-g^2 c_{z\Box} + 4 \delta m), \]
\[ \beta_{cchw} = 1 + \delta c_z + 3 \delta m, \]
\[ \beta_{3cw} = 1 - \frac{g^2}{2} c_{z\Box} + \delta m, \]

(C.10)

\[ \beta_{cwy} = 1 + \delta m, \]
\[ \beta_{cwx} = 1 + \frac{g^2 (g^2 + g'^2)}{2 g'^2} (c_{z\Box} - c_{w\Box}) - \frac{2 g^2 + g'^2}{g^2} \delta m, \]

(C.11)

\[ \eta_{cwy} = \frac{g^2}{g' \sqrt{g^2 + g'^2}} ((g^2 - g'^2) c_{w\Box} - g^2 c_{z\Box}), \]
\[ \eta_{cwx} = \frac{g}{\sqrt{g^2 + g'^2}} ((g^2 - g'^2) c_{w\Box} - g^2 c_{z\Box}), \]

(C.12)

\[ \beta_{cc\gamma} = 1 + \frac{g^2 + g'^2}{2 (g^2 - g'^2)} (-g^2 c_{z\Box} + 4 \delta m), \]
\[ \beta_{ccz} = 1 + \frac{g^2 + g'^2}{g^2 - g'^2} (-g^2 c_{z\Box} + 4 \delta m), \]
\[ \beta_{ccw} = 1 + 2 g^2 c_{z\Box} - 3 g^2 c_{w\Box} - 4 \delta m, \]
\[ \beta_{33z} = 1 - g^2 c_{w\Box} - 2 \delta m, \]
\[ \beta_{33w} = 1 - g^2 c_{z\Box} + 2 \delta m, \]
\[ \beta_{chw} = 1 + \delta c_z + 3 \delta m, \]
\[ \beta_{chw} = 1 + \frac{3 g^2 (g^2 + g'^2)}{2 g'^2} (c_{z\Box} - c_{w\Box}) + \delta c_z - \frac{3 g^2 + g'^2}{g'^2} \delta m, \]
\[ \beta_{c3w} = 1 - \frac{g^2}{2} c_{z\Box} + \delta m, \]
\[ \beta_{33w} = 1 + \frac{g^4}{2 g'^2} c_{z\Box} - \frac{g^2 (g^2 + g'^2)}{2 g'^2} c_{w\Box} - \frac{2 g^2 + g'^2}{g'^2} \delta m, \]
\[ \eta_{ccw} = g^2 (c_{w\Box} - c_{z\Box}) + 2 \delta m, \]

(C.13)
\[ \eta_{c\gamma\gamma} = c_{zz} - \frac{g^2 - g'^2}{g^2 + g'^2} c_{z\gamma} + \frac{(g^2 - g'^2)^2}{4(g^2 + g'^2)} c_{\gamma\gamma}, \]

\[ \eta_{c\gamma\gamma} = \frac{g^2 - g'^2}{g^2 + g'^2} c_{zz} - \frac{g^4 - 6g^2 g'^2 + g'^4}{2(g^2 + g'^2)^2} c_{z\gamma} - \frac{e^2 (g^2 - g'^2)}{(g^2 + g'^2)^2} c_{\gamma\gamma}, \]

\[ \eta_{czzz} = \frac{(g^2 - g'^2)^2}{4(g^2 + g'^2)^2} c_{zz} - \frac{e^2 (g^2 - g'^2)}{(g^2 + g'^2)^2} c_{z\gamma} + \frac{e^4}{(g^2 + g'^2)^2} c_{\gamma\gamma}, \]

\[ \eta_{ccww} = \frac{1}{2} c_{zz} + s_\theta^2 c_{z\gamma} + \frac{s_\theta^4}{2} c_{\gamma\gamma}, \]

\[ \eta_{cc\gamma\gamma} = \frac{c_{\gamma\gamma}}{8}, \]

\[ \eta_{cczz} = \frac{c_{zz}}{4}, \]

\[ \eta_{\delta \gamma} = \frac{c_{zz}}{8}, \]

\[ \eta_{\delta \gamma} = \frac{1}{2} c_{zz} + \frac{s_\theta^2}{2} c_{z\gamma} + \frac{s_\theta^4}{4} c_{\gamma\gamma}, \]

\[ \eta_{\delta \gamma} = \frac{1}{2} c_{zz} - \frac{g^2 - g'^2}{2(g^2 + g'^2)^2} c_{z\gamma} + \frac{e^2}{2(g^2 + g'^2)} c_{\gamma\gamma}, \]

\[ \eta_{\delta \gamma} = \frac{1}{2} c_{zz} - \frac{g^2 - g'^2}{2(g^2 + g'^2)^2} c_{z\gamma} - \frac{e^2}{2(g^2 + g'^2)} c_{\gamma\gamma}. \] (C.14)

With the Goldstone bosons degrees of freedom present in the Lagrangian, gauge fixing can be implemented as in any gauge theory. Below we show how to implement the linear \( R_\xi \) gauge. For the electroweak sector, we introduce the following gauge fixing Lagrangian

\[ L_{gf} = -\frac{1}{2\xi} [F_A^2 + F_Z^2 + 2F_+ F_-], \] (C.15)

where

\[ F_A = (1 + \alpha_{AA}) \partial_\mu A_\mu + \alpha_{AZ} \partial_\mu Z_\mu, \]

\[ F_Z = (1 + \alpha_{ZZ}) \partial_\mu Z_\mu + \xi \sqrt{g^2 + g'^2} \theta (1 + \alpha_{Z3}) G_3 \]

\[ F_\pm = (1 + \alpha_{WW}) \partial_\mu W^\pm_\mu + \xi \frac{g_\gamma}{2} (1 + \alpha_{Wc}) G_\pm. \] (C.16)

Above the fields and couplings are the ones before the rescaling in Eqs. (3.1) and (C.2) that bring the Lagrangian to the canonical form. To derive the ghost action later, we will need the \( SU(2) \times U(1) \) gauge transformations acting on these fields

\[ \delta A_\mu = \partial_\mu \alpha_\gamma + ie (W_{\mu}^- \alpha_+ - W_{\mu}^+ \alpha_-), \]

\[ \delta Z_\mu = \partial_\mu \alpha_Z + ig s_\theta (W_{\mu}^- \alpha_+ - W_{\mu}^+ \alpha_-), \]

\[ \delta W^+_\mu = \partial_\mu \alpha_+ - ig s_\theta (c_\theta Z_\mu + s_\theta A_\mu) + ig (c_\theta \alpha_Z + s_\theta \alpha_\gamma) W^+_\mu, \] (C.17)

\[ \delta h = -\frac{\sqrt{g^2 + g'^2}}{2} G_3 \alpha_Z - \frac{g}{2} (G_+ \alpha_- + G_- \alpha_+), \]

\[ \delta G_3 = \frac{\sqrt{g^2 + g'^2}}{2} (v + h) \alpha_Z - \frac{ig}{2} (G_+ \alpha_- - G_- \alpha_+), \]

\[ \delta G_+ = \frac{g}{2} (v + h - iG_3) \alpha_+ + ie G_+ \alpha_\gamma + i\frac{g^2 - g'^2}{2\sqrt{g^2 + g'^2}} G_+ \alpha_Z. \] (C.18)
The coefficients \( \alpha_{XY} \) in Eq. (C.16) are \( O(1/A^2) \) in the EFT expansion. One can choose them such that, after the rescaling in Eqs. (3.1) and (C.2), the gauge fixing Lagrangian becomes

\[
L_{gf} = -\frac{1}{2\xi} \left[ (\partial_\mu A_\mu)^2 + \left( \partial_\mu Z_\mu + \xi \frac{\sqrt{g^2 + g'^2} v}{2} G_3 \right)^2 \right. + \left. 2 \left| \partial_\mu W_\mu^+ + \frac{\xi g v}{2} (1 + \delta m) G_- \right|^2 \right].
\] (C.19)

This choice of the gauge fixing terms ensures that the kinetic mixing between the Goldstone bosons and massive vector bosons in Eq. (C.3) is canceled after gauge fixing. At the same time, the Goldstone bosons acquire the gauge dependent masses:

\[
m_{G_\pm} = \sqrt{\xi \frac{g v}{2}} (1 + \delta m) \equiv \sqrt{\xi} m_W, \quad m_{G_3} = \sqrt{\xi \frac{\sqrt{g^2 + g'^2} v}{2}} \equiv \sqrt{\xi} m_Z.
\] (C.20)

Finally, the ghost Lagrangian can be obtained by the usual Fadeev-Popov procedure. In the \( R_\xi \) gauge introduced above

\[
L_{\text{ghost}} = -\sum_{n \in (+, -, Z, \gamma)} \left[ \bar{c}_- \frac{\partial \delta F_+}{\partial \alpha_n} + \bar{c}_+ \frac{\partial \delta F_-}{\partial \alpha_n} + \bar{c}_Z \frac{\partial \delta F_{Z}}{\partial \alpha_n} + \bar{c}_\gamma \frac{\partial \delta F_A}{\partial \alpha_n} \right] c_n,
\] (C.21)

where \( \delta F \) is the variation of the gauge fixing terms \( F \) in Eq. (C.16) under the infinitesimal \( SU(2) \times U(1) \) gauge symmetry transformations in Eq. (C.17). At this point the ghost kinetic terms are not canonically normalized and diagonal. To this end one needs to perform the transformation

\[
c_\gamma \rightarrow (1 - \alpha_{AA}) c_\gamma - \alpha_{AZ} c_Z, \\
c_Z \rightarrow (1 - \alpha_{ZZ}) c_Z, \\
c_\pm \rightarrow (1 - \alpha_{WW}) c_\pm.
\] (C.22)

After this transformation (and the rescaling Eq. (3.1)) the ghost kinetic and mass terms are diagonal and the ghost kinetic terms are canonically normalized. The gauge dependent masses of the ghosts are given by

\[
m_{c_\pm} = \sqrt{\xi \frac{g v}{2}} (1 + \delta m) \equiv \sqrt{\xi} m_W, \quad m_{c_Z} = \sqrt{\xi \frac{\sqrt{g^2 + g'^2} v}{2}} \equiv \sqrt{\xi} m_Z, \quad m_{c_\gamma} = 0.
\] (C.23)

The ghost interactions with electroweak vector bosons take the form

\[
\mathcal{L}_{ccV} = i \left( e A_\mu + \omega_{ccw} g c_\theta Z_\mu \right) \left( \partial_\mu \bar{c}_+ c_- - \partial_\mu \bar{c}_- c_+ \right) + i \left( \omega_{ccw} e \partial_\mu c_+ + \omega_{ccw} g c_\theta \partial_\mu c_Z \right) \left( W_\mu^+ c_+ - W_\mu^- c_- \right) + i \left( W_\mu^+ \partial_\mu \bar{c}_- - W_\mu^- \partial_\mu \bar{c}_+ \right) \left( e c_\gamma + \omega_{ccw} g c_\theta c_Z \right),
\] (C.24)

where

\[
\omega_{ccw} = 1 - \frac{g'^2}{2 c_w c_\theta}, \\
\omega_{c\gamma w} = 1 + \frac{g^2 (g^2 - g'^2)}{2 g'^2 c_w c_\theta} - g^4 c_w c_\theta, \\
\omega_{c泽} = 1 - g^2 c_w + \frac{g^2}{2} c_\theta, \\
\omega_{泽w} = 1 + \frac{g^2 + g'^2}{2 c_w}.
\] (C.25)
D Examples

D.1 On eliminating $h\partial h^2$ interactions

We consider an effective Lagrangian for a Dirac fermion $f$ and a scalar $h$ with the following terms:

$$\mathcal{L} = i\bar{f}\gamma_\mu \partial_\mu f + \frac{1}{2}(\partial_\mu h)^2 - \frac{m_h^2}{2} h^2 - \frac{y}{\sqrt{2}} h\partial_\mu h \partial_\mu h.$$  \hspace{1cm} (D.1)

The interactions terms are the Yukawa coupling between the scalar and fermions, and a 2-derivative self-interaction term of the scalar. In this effective theory, we consider the scattering process $ff \rightarrow hh$ at tree level. The amplitude for this process can be written as

$$\mathcal{M}(f\bar{f} \rightarrow hh) = -\frac{y}{\sqrt{2}} (\bar{v}(p_2) u(p_1)) \frac{2\alpha (k_1 + k_2)^2 - k_1 k_2}{\Lambda (k_1 + k_2)^2 - m_h^2} \left[ 1 + \frac{3m_h^2}{s - m_h^2} \right],$$  \hspace{1cm} (D.2)

where $p_1, p_2, k_1, k_2$ are the momenta of the incoming fermions and the outgoing scalars, $s = (k_1 + k_2)^2$, and $u, v$ are spinor wave functions for the fermions. Since $\bar{v}(p_2) u(p_1) \sim \sqrt{s}$, the $h(\partial_\mu h)^2$ interaction leads to amplitudes growing with energy as $\alpha\sqrt{s}/\Lambda$ which eventually violates perturbative unitarity for large enough $\sqrt{s}$.

We can equivalently work with an effective Lagrangian where the 2-derivative $h(\partial_\mu h)^2$ interaction is eliminated via field redefinitions. To this end we redefine the scalar field as

$$h \rightarrow h + \frac{\alpha}{2\Lambda} h^2.$$  \hspace{1cm} (D.3)

After this redefinition the effective Lagrangian of Eq. (D.1) takes the form

$$\mathcal{L} = i\bar{f}\gamma_\mu \partial_\mu f + \frac{1}{2}(\partial_\mu h)^2 - \frac{m_h^2}{2} h^2 - \frac{y}{\sqrt{2}} \left( h + \frac{\beta}{\Lambda} h^2 \right) \bar{f} f - \lambda_3 \frac{m_h^2}{\Lambda} h^3 + \ldots,$$  \hspace{1cm} (D.4)

where

$$\beta = \lambda_3 = \frac{\alpha}{2},$$  \hspace{1cm} (D.5)

and the dots stand for $h^4$ interactions that are not important for the present discussion. Seemingly, the effective Lagrangians in Eqs. (D.1) and (D.4) are different, as they contain different interaction terms. However, we know that field redefinitions cannot change the physical content of the theory. Thus, the two Lagrangians must give exactly the same predictions for physical observables. We will verify this explicitly for the $f\bar{f} \rightarrow hh$ process. Indeed, calculating the amplitude using the Lagrangian in Eq. (D.4) we find

$$\mathcal{M}(f\bar{f} \rightarrow hh) = -\frac{y}{\sqrt{2}} (\bar{v}(p_2) u(p_1)) \frac{1}{\Lambda} \left[ 2\beta + 6\lambda_3 \frac{m_h^2}{s - m_h^2} - \frac{3m_h^2}{s - m_h^2} \right].$$  \hspace{1cm} (D.6)

This is exactly the same as the amplitude in Eq. (D.2) upon using Eq. (D.5). In particular, the amplitude is growing as $\sqrt{s}$ even though the 2-derivative interaction $h(\partial_\mu h)^2$ have been redefined away. In Eq. (D.6), this behavior is due the contact term $h^2 f\bar{f}$ in Eq. (D.4) which appeared as a consequence of the redefinition.
An analogous method to eliminate the $h(\partial_{\mu}h)^2$ interaction of the Higgs boson was applied in the mass eigenstate Lagrangian for dimension-6 EFT in Section 3, see Eq. (3.1). In that case, the motivation for doing so is stronger than in the current example. The point is that dimension-6 operators in popular bases generate corrections to all the 3 types of interaction terms relevant for double Higgs production: $h^3$, $h^2\bar{f}f$ and $h(\partial_{\mu}h)^2$. By eliminating the $h(\partial_{\mu}h)^2$ interaction we reduce the number of interaction vertices in the theory, without changing at all the EFT predictions for double Higgs production amplitudes. Similarly, the purpose of other redefinitions in Eq. (3.1) is to fix the redundancies in the mass eigenstate Lagrangian, so as to reduce the number of interaction terms. This way one arrives at a simpler and more convenient form of the mass eigenstate Lagrangian without changing the physical predictions of the dimension-6 EFT.

D.2 On eliminating $hV\bar{f}f$ interactions

We consider an effective Lagrangian for a massless Dirac fermion $f$, a massive scalar $h$, and a massive $U(1)$ vector $V_\mu$ with the following terms:

\begin{equation}
\mathcal{L} = -\frac{1}{4}V_{\mu\nu}V^{\mu\nu} + \frac{1}{2}m_V^2 V_\mu V^\mu + if\bar{\gamma}_\mu \partial_\mu f + \frac{1}{2}(\partial_\mu h)^2 - \frac{m^2}{2}h^2 + gV_\mu \bar{f}\gamma_\mu f + c_{hVf} \frac{h}{\Lambda} gV_\mu \gamma_\mu f.
\end{equation}

The interaction terms are the renormalizable interaction between the vector and fermion fields with the coupling strength $g$, and the non-renormalizable contact interaction between the scalar, vector and fermions with the dimensionful coupling strength $c_{hVf}/\Lambda$. In this effective theory, we consider the decay process $h \rightarrow f\bar{f}V$ at tree level. The amplitude for this process can be written as

\begin{equation}
\mathcal{M}(h \rightarrow f\bar{f}V) = \frac{g_{c_{hVf}}}{\Lambda} (\bar{u}(k_1)\gamma_\mu v(k_1)) \epsilon^*(k_3),
\end{equation}

where $k_1, k_2, k_3$ are the momenta of the outgoing fermion, anti-fermion, and vector, respectively. Furthermore, $u, v$ are spinor wave functions for the fermions, and $\epsilon$ is the polarization vector for $V$.

We can equivalently work with an effective Lagrangian where the Higgs contact interaction is eliminated in favor of other Higgs couplings. To this end, we can use the equation of motion for the vector field:

\begin{equation}
\partial_\nu V_{\mu\nu} = m_V^2 V_\mu + g\bar{f}\gamma_\mu f + O(1/\Lambda).
\end{equation}

Solving this equation for $\bar{f}\gamma_\mu f$ and plugging back the solution into the last term of Eq. (D.7) one obtains the effective Lagrangian

\begin{equation}
\mathcal{L} = -\frac{1}{4}V_{\mu\nu}V^{\mu\nu} + \frac{1}{2}m_V^2 V_\mu V^\mu + if\bar{\gamma}_\mu \partial_\mu f + \frac{1}{2}(\partial_\mu h)^2 - \frac{m^2}{2}h^2 + gV_\mu \bar{f}\gamma_\mu f + c_{hVf} \frac{h}{\Lambda} gV_\mu \gamma_\mu f + O(1/\Lambda^2).
\end{equation}

where

\begin{equation}
c_V = -c_{hVf}, \quad c_{V\Box} = c_{hVf}.
\end{equation}

Seemingly, the effective Lagrangians in Eqs. (D.7) and (D.10) are different, as they contain different interaction terms. However, much like field redefinitions, replacing interactions terms by
others related via equations of motion does not change physics. Therefore, observables calculated with the effective Lagrangian (D.10) must be the same. We can verify it explicitly for the decay process $h \rightarrow f \bar{f} V$ at tree level. Unlike with Eq. (D.7), there is no direct vertex connecting $h$ to the final state $f \bar{f} V$. Instead, with the vertices in Eq. (D.10), the process occurs via a decay of $h$ to one on-shell and one off-shell $V$, with the latter decaying to a fermion pair. One finds

$$\mathcal{M}(h \rightarrow f \bar{f} V) = \frac{g}{\Lambda} (\bar{u}(k_1) \gamma_\mu v(k_1)) \epsilon^*(k_3) \left( \frac{2c_V m_V^2 + c_V \Box (m_V^2 + q^2)}{q^2 - m_V^2} \right),$$  \hspace{1cm} (D.12)$$

where $q = k_1 + k_2$, and the denominator comes from the propagator of the off-shell $V$. This exactly matches the amplitude in Eq. (D.8) upon using Eq. (D.11).

This example demonstrates that contact $hVf\bar{f}$ interactions can be equivalently represented by $hVV$ interactions with zero and two derivatives. This is taken advantage of in this note to simplify the mass eigenstate Lagrangian for the SM EFT with $D=6$ operators in Section 3. Namely, it turns out that $D=6$ operators generate two kinds of $hVf\bar{f}$ terms, with $V = W, Z$ bosons, and $f$ the SM fermions. One is universal, in the sense that it depends only on the quantum numbers of $f$, and the corresponding direction in the EFT parameter space is relatively unconstrained by experiment so far. For the other kind, the coefficients of $hVf\bar{f}$ terms are in general non-universal and equal to vertex corrections to SM $Vf\bar{f}$ interactions. The latter are strongly constrained by electroweak precision measurements. The condition #6 below Eq. (3.1) amounts trading the universal $hVf\bar{f}$ terms for zero- and two-derivative $hWW$, $hZZ$ and two-derivative $hZ\Box A$ interactions. The motivation is that, in this way, a small number of bosonic $hVV$ terms in the Lagrangian represents universal $hVf\bar{f}$ terms for all flavors of the SM fermions. One could of course, equivalently, eliminate $hV\Box V$ interactions in favor of the fermionic contact terms, without changing the observable predictions of the EFT.
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