EXPERIMENTAL EVIDENCE OF TRANSVERSE NONLINEAR PHENOMENA IN ACCELERATORS

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Abstract Selected experimental observations or measurements which illustrate the evolution of research in accelerator nonlinear dynamics are described.

INTRODUCTION
The first large accelerators based on the strong focusing principle were built in the years 1954 to 1959. One of the big concerns of that time was the tolerance of these new machines to the unavoidable nonlinear components of the magnetic fields. This problem was attacked by Moser, Schoch, Hagedorn and others (1957-58). It was shown that the betatron amplitudes of particles may grow if the horizontal tune $Q_x$ and the vertical tune $Q_z$ lie on resonance lines such that

$$m Q_x + n Q_z = p$$

with $m$, $n$, $p$ integers.

For a long time this picture of single isolated resonances dominated the scene as far as nonlinear effects in accelerators were concerned.

With the advent of the hadron colliders (the CERN ISR was proposed in 1964) the question of very long term stability of the particle orbits again raised general concern. In this case a powerful nonlinear perturbation is provided by the beam-beam interactions. Chirikov (1960 to 1964) showed that stochastic motion, which always exists close to the resonance separatrices can lead to large scale diffusion when many resonances overlap. It was also realised that a slow modulation of the tune may split the resonances into satellite sidebands, and that applying the Chirikov overlap criterion in this situation increases the domain of phase space where diffusion dominates.
Lately, the interest has been focused on very large hadron colliders using superconducting magnets. In these machines it is difficult and expensive to provide a linear magnetic field over all the space needed for beam manipulations. This has created a renewed interest in nonlinear dynamics in all three aspects of analytic calculations, computer simulations and experimental measurements. In this last domain new techniques are being developed to visualize the betatron phase space of real machines and to extract all relevant information to help understand the beam behaviour close to the limit of instability.

In this report the experimental observations or measurements which best illustrate the main advances in the understanding of nonlinear effects are described.

**ISOLATED RESONANCES**

In proton accelerators, and specially at low energy, the low order resonances\(^1\) cannot always be avoided because of the existence of a too large tune spread due to magnetic nonlinearities or space charge effects. In this case they have to be compensated by introducing in the machine special lenses (usually quadrupoles and sextupoles, more seldom octupoles) to cancel out dangerous azimuthal harmonics. A spectacular example is given by the CERN PS Booster\(^2\) where the space charge tune spread covers the region from the half integer down to the integer and where 3 sextupole resonances have to be simultaneously compensated (Fig. 1).

![Fig. 1 Improvement of transmission in the CERN PS Booster through resonance compensation](image)

- a) natural transmission
- b) \(QH + 2QV = 15\) compensated
- c) all 3 resonances compensated.
The dependence of the tune on the particle amplitude has important consequences on the dynamics. If this effect is strong enough it can stabilize the motion by shifting the tune out of resonance after a certain growth of amplitude. The phase space can then be closed, as for the 2nd order resonance displayed in Fig. 2.

Fig. 2 Phase plane trajectories in the vicinity of resonance of order 2 at various distances of the resonant tune.

However, even in this case particles can be lost if the tune is not held stable. For instance, in Fig. 2 sweeping the tune slowly from the right to the left initiates the growth of islands which can adiabatically trap particles and carry them to large amplitudes.

Particles trapped in such islands have been observed on many occasions. A spectacular example is the electron ring VEP 1 (Fig. 3). Here the emission of synchrotron light allows one to see directly the projection of the island structure in the real two dimensional transverse plane. The beam is excited externally and the resonance is controlled by the octupole term of the machine.

Fig. 3 Islands observed in VEP 1 (left) and destruction of islands as a second resonance is approached (right).
guide field. By varying the excitation frequency it is possible to capture the beam in stable islands at different amplitudes. In proton machines one can give a fast kick to a small beam to instantly displace it in the phase space so that it falls on top of one of the islands. Part of the beam is then trapped in the islands and continues to oscillate at the same amplitude for a long time whereas particles which have fallen outside the islands lose their coherence and become invisible to a beam observation monitor. Such an experiment has been done recently in the Fermilab Tevatron. By plotting the measured amplitude and the phase of the persistent signal one can reconstruct the motion of the trapped beam centroid in phase space (Fig. 4). This clearly reveals the centre of the five islands of the 5th order resonance \( 5q = 2(q = \text{non integer part of the tune}) \).

![Normalized phase space obtained by computer simulation (left) and measured position from beam trapped in islands (right) in the Tevatron experiment E778.](image)

**STOCHASTICITY**

The excitation strength of resonances usually becomes smaller and smaller as the order of the resonance increases, so that the detuning dominates and the surface of the islands decreases. In the late 50's this lead people to conjecture that resonances of order higher than four would be stable and create no problems. This statement proved to be right for classical accelerators, but does not apply for storage rings. To understand why this is so let us look at Fig. 5 which represents a surface of section (the phase space) of a nonlinear...
transformation studied by Laslett. Successive zooms reveal finer and finer structures with island separatrices and chaotic regions close to the unstable fixed points. Such a "fractal" topology is characteristic of nonlinear phase space, and can also be seen in computer simulations of real accelerators.

In one dimension and for small enough values of the nonlinear terms the KAM theorem assures that there is always an external trajectory which is regular, and which prevents particles from diffusing to large amplitudes whatever the structure of the phase space inside. However, this breaks down when nonlinearities are increased to the point that the islands pertaining to different resonances begin to overlap, so that their stochastic layers communicate: this is Chirikov's criterion for the onset of stochastic motion. In the real two dimensional transverse plane the invariant KAM objects are 2-dimensional toroidal surfaces embedded in a 4-dimensional phase space, and they cannot prevent communication between stochastic regions. Therefore Arnold diffusion is always possible, but this is thought to be a very weak process compared to Chirikov's stochastic threshold. Interesting experiments have been made in VEP to test Chirikov's ideas. In conditions of Fig. 3 a second excitation is set up and its frequency gradually approached to that of the first. When the two resonances are close enough the islands disappear (Fig. 3, right) : the particles have equal access to a large area of phase space as if a strong diffusion mechanism was governing the dynamics.

An effect which can considerably promote stochastic motion is the modulation of parameters, for example of the tune. The influence of
modulation, which was also studied by Chirikov\textsuperscript{7}, falls into different categories depending on whether its frequency is larger or smaller than the oscillation frequency inside the resonance islands. For larger frequencies (fast crossings) it is possible to describe the dynamics by splitting the resonance into frequency-modulated satellite sidebands: if the width of these satellites exceeds their separation, there is overlap and the motion may become chaotic. For smaller frequencies the islands can adiabatically "follow" the changing parameter, and the motion should remain regular. However, we have seen that small stochastic regions always exist even in the absence of modulation, and therefore any change of parameter is bound to create some diffusion since the motion is infinitely slow near the unstable fixed points.

Again this was tested experimentally in VEP\textsuperscript{13} : a resonance controlled by an octupole field was excited externally and frequency modulated. By decreasing the modulation frequency one could observe successively the splitting of the resonance into sidebands, the onset of stochastic motion and finally the adiabatic régime.

The SPS was the first hadron collider to operate with head-on collisions and a bunched beam. In this machine a certain modulation of parameters due to synchrotron motion was unavoidable, and therefore the question of modulation enhanced diffusion was an important one. An experiment was set up using the Nonlinear Lens. This instrument was designed to excite many high order resonances at the same time with the aim of simulating the beam-beam effect in the ISR\textsuperscript{9}. In the SPS it was shown that with the Nonlinear Lens energized a 1 to 40 Hz tune modulation of amplitude $\Delta \Omega = 3 \times 10^{-3}$ produced an important reduction of beam lifetime, whereas almost no effect was noticed at the synchrotron frequency near 200 Hz\textsuperscript{10}. Satellite resonances produced by low frequency modulation are densely packed and therefore they can overlap even in the case of weak high order resonances. In this case one expects a stochastic region to be created over the tune modulation depth, whereas the motion stays regular for a larger modulation frequency. These results were an incentive to reduce the low frequency ripple of the SPS power supplies.

The effect of the beam-beam interaction which was observed
somewhat later in this machine\textsuperscript{11} is essentially a tune dependent diffusion mechanism affecting primarily the tails of the transverse distributions. For values of the linear tune shift per crossing around 0.005 the influence of 16th order resonances can be clearly seen\textsuperscript{12} while it is impossible to operate the machine close to resonances of order 10 or less. This puts a severe limit on the total tune spread which can be tolerated in the beam.

**LARGE HADRON COLLIDERS**

Whereas the research done over the past 20 years on nonlinear effects in accelerators has provided at least a qualitative understanding of the behaviour of colliding beams, a new challenge is posed by the design of Large Hadron Colliders. In these machines nonlinearities of the guide fields play an important role, since for reasons of cost one cannot overdesign the aperture. In contrast with the situation for lepton colliders and light sources, particle tracking by computer is not yet fast enough to simulate the operation of these machines, and therefore it is of the utmost importance to find practical and reliable criteria to set tolerance limits. Two parameters which are easy to measure both in computer experiments (tracking) and in experiments on real machines are the tune shift $\Delta Q$ and the smear $S$, this latter representing the dispersion turn after turn of the normalized amplitudes\textsuperscript{13}. Both are believed to represent adequately the "degree of nonlinearity" of the accelerator, the tune shift being mainly produced by systematic errors, and the smear by random variations.

In recent years experiments have been carried out both at the Fermilab Tevatron and at the CERN SPS to evaluate the aperture which remains available for operating the machine when strong nonlinearities are introduced. It is convenient to define a short term dynamic aperture inside which particles survive for at least a few seconds (this is necessary for beam injection, measurements and steering) and a long term dynamic aperture inside which particles can circulate for many hours without emittance dilution (necessary during slow energy ramping and collisions). The nonlinear phase space is explored by kicking a small beam to increasing amplitudes and acquiring the signals of two different position monitors per plane.
Provided the monitors in each plane are separated by about 90° of betatron phase this allows not only to calculate the tune but also to visualize the phase space structure and evaluate the smear. A high accuracy can be obtained by using modern acquisition and storage techniques. Figure 6 displays some pioneering work done years ago in Stanford. 

Fig. 6  SPEAR phase space measured with two orthogonal monitors

Both at Fermilab and CERN strong sextupoles were energized to simulate conditions which may prevail in superconducting machines. Figure 7 shows measurements of the smear at Fermilab and Fig. 8 measurements of the tune shift at CERN. Both measurements agree very well with tracking results, showing the reliability of the

Fig. 7  Smear in the Tevatron perturbed by sextupoles
computer models and the measurement techniques. In Fig. 8 one can compare the short term dynamic aperture found experimentally (the amplitude at which particle losses occur) with the onset of chaotic motion found by tracking: both agree remarkably well. Long term measurements in coast mode have shown that inside the short term aperture particles diffuse slowly outwards at a speed which is strongly amplitude dependent. This observation could not be reproduced by extended computer tracking until a small tune

Fig. 8 Tune shift and dynamic aperture in the CERN SPS perturbed with sextupoles

Fig. 9 Effect of tune modulation ($\Delta Q = 2 \times 10^{-3}$) on beam lifetime in the Tevatron
modulation was introduced in the computer model. In this case and for low modulation frequency (10 to 50 Hz) chaotic motion could be detected after tracking for about $10^6$ turns. The corresponding Liapunov exponents were very small, in qualitative agreement with the experimental observations of a slow diffusion.\(^{15}\)

A recent experiment at Fermilab confirms the importance of modulation in the presence of strong sextupoles\(^{16}\). Figure 9 shows the reduction in beam lifetime due to the introduction of a small tune modulation. Switching off the sextupoles suppresses the particle losses, even with modulation present.

**CONCLUSION**

Due to collective effects accelerators would essentially be unstable without some nonlinearity. However, single particle dynamics is very sensitive to nonlinear disturbances, and this demands a severe control of the nonlinear terms of the magnetic fields. Despite considerable progress in the understanding of these phenomena, empirical rules based on experience are still necessary for this purpose.

**REFERENCES**

1. A. Schoch, CERN 57–23.