Review of charmless $b$-hadron decays at LHCb

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on behalf of the LHCb Collaboration

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Why charmless B decays?

• They receive contributions from both tree and penguin diagrams → large direct CP violation

• Valid theoretical tool to test QCD calculations and to improve knowledge of CKM matrix

• Search for New Physics
  – As penguin topologies are generally sizeable, effects from New Physics in loops may be sizeable as well
  – CP-violating observables and branching fractions can differ from Standard Model predictions

• Theoretical interpretation is however not straightforward, because of unknown hadronic parameters entering the amplitudes
Direct CP asymmetries in \( B^0_{(s)} \to K\pi \) decays

Most precise measurement of this quantity to date, 10.5\( \sigma \) from zero

First observation of CP violation in \( B_s \) decays, with significance of 6.5\( \sigma \)

\[
A_{\text{raw}} = \frac{N(\bar{B} \to K\pi) - N(B \to K\pi)}{N(\bar{B} \to K\pi) + N(B \to K\pi)}
\]

\[
A_{CP} = A_{\text{raw}} - A_D - \kappa A_P
\]
From raw to physical asymmetries

\[ A_{\text{raw}} = \frac{N(B \rightarrow f) - N(B \rightarrow f)}{N(B \rightarrow f) + N(B \rightarrow f)} \]

\[ A_{CP} = A_{\text{raw}} - A_D - \kappa A_P \]

- \( \kappa \) is a selection-dependent dilution factor due to mixing when neutral \( B \) mesons are involved (otherwise \( \kappa = 1 \))

- Production asymmetry for neutral \( B \) mesons can be measured by means of untagged time-dependent studies of flavour-specific modes

Efficiency as function of the decay time

\[ \kappa = \frac{\int (e^{-\Gamma t} \cos \Delta m t \varepsilon(t)) \, dt}{\int (e^{-\Gamma t} \cosh \frac{\Delta m}{2} t \varepsilon(t)) \, dt} \]

Time-dependent CPV in $B^0 \to \pi^+\pi^-$ and $B_s \to K^+K^-$

$A(t) = \frac{-C_f \cos(\Delta m_d t) + S_f \sin(\Delta m_d t)}{\cosh\left(\frac{\Delta \Gamma_d}{2} t\right) - A_f \Delta \Gamma \sinh\left(\frac{\Delta \Gamma_d}{2} t\right)}$

$B^0 \to \pi^+\pi^-$
N = 9170 ± 144

$B_s \to K^+K^-$
N = 14650 ± 160

Results are compatible with previous measurements from BaBar and Belle

$C_{\pi\pi} = -0.38 \pm 0.15 \text{ (stat)} \pm 0.02 \text{ (syst)}$, $S_{\pi\pi} = -0.71 \pm 0.13 \text{ (stat)} \pm 0.02 \text{ (syst)}$, $C_{KK} = 0.14 \pm 0.11 \text{ (stat)} \pm 0.03 \text{ (syst)}$, $S_{KK} = 0.30 \pm 0.12 \text{ (stat)} \pm 0.04 \text{ (syst)}$, First measurement ever significance for $(C_{KK}, S_{KK})$ to differ from $(0, 0)$ is $2.7\sigma$

Work ongoing to update to full 3 fb$^{-1}$ statistics
\( \gamma \) and \(-2\beta_s\) from charmless two-body decays

- Determination of \( \gamma \) and \( \phi_s \) using \( B^0 \rightarrow \pi^+\pi^- \), \( B^0 \rightarrow \pi^0\pi^0 \), \( B^\pm \rightarrow \pi^\pm\pi^0 \) and \( B_s \rightarrow K^+K^- \)
  - based on use of isospin and U-spin symmetries
  - impact of non-factorisable U-spin breaking effects taken into account


Note: latest Belle result on \( \text{BR}(B \rightarrow \pi^0\pi^0) \) not included
Translating amplitudes to CPV coefficients and BRs

- \( C_{\pi^+\pi^-} = f_1(d, \vartheta, \gamma) \)
- \( S_{\pi^+\pi^-} = f_2(d, \vartheta, \gamma, \beta) \)
- \( C_{K^+K^-} = f_3(d', \vartheta', \gamma) \)
- \( S_{K^+K^-} = f_4(d', \vartheta', \gamma, \beta_s) \)
- \( C_{\pi^0\pi^0} = f_5(d, q, \vartheta, \vartheta_q, \gamma) \)
- \( B_{\pi^+\pi^-} = f_6(|D|, d, \vartheta, \gamma) \)
- \( B_{K^+K^-} = f_7(|D'|, d', \vartheta', \gamma) \)
- \( B_{\pi^0\pi^0} = f_8(|D|, d, q, \vartheta, \vartheta_q, \gamma) \)
- \( B_{\pi^+\pi^0} = f_9(|D|, q, \vartheta_q) \)

CKM angles \( \gamma, \beta, \beta_s \)

Hadronic parameters
\( d, \vartheta, d', \vartheta', q, \vartheta_q, |D|, |D'| \)

Too many unknowns → need flavour symmetry to reduce them
Effect of non-factorizable U-spin breaking

• Non-factorizable U-spin breaking effects are parameterized by

\[
|D'| = \left| \frac{D'}{D} \right| \text{fact} |D| \left| 1 + r_D e^{i \theta_D} \right|,
\]

\[
d' e^{i \varphi'} = d e^{i \varphi} \frac{1 + r_G e^{i \theta_G}}{1 + r_D e^{i \theta_D}}.
\]

• With unbroken U-spin one has \( r_D = r_G = 0 \)

• Flat priors on U-spin breaking parameters

  – \( r_D = [0, \kappa] \), \( r_G = [0, \kappa] \)
  – \( \theta_{r_D} = [-\pi, \pi] \), \( \theta_{r_G} = [-\pi, \pi] \)

• \( \kappa \) represents the maximum non-factorizable U-spin breaking allowed for in the analysis

  – Sloppily speaking: \( \kappa = 0.3 \) corresponds to up to 30% breaking

\( r_D \) and \( r_G \) are relative magnitudes and \( \theta_{r_D} \) and \( \theta_{r_G} \) are strong phase shifts caused by the breaking.
Effect of non-factorizable U-spin breaking
(U-spin method)

Determination of $\gamma$

- Sensitivity on $\gamma$ is good up to $\kappa = 0.6$
- For $\kappa > 0.6$ sensitivity on $\gamma$ deteriorates very quickly

Determination of $-2\beta_s$

- Sensitivity on $-2\beta_s$ almost unaffected up to $\kappa = 1$, but shifted toward decreasing values for $\kappa > 0.6$
Inclusion of $B^+ \rightarrow \pi^+ \pi^0$ and $B^0 \rightarrow \pi^0 \pi^0$

(following JHEP 10 (2012) 029)

Information from $B^+ \rightarrow \pi^+ \pi^0$ and $B^0 \rightarrow \pi^0 \pi^0$ helps to better control non-factorizable U-spin breaking effects.
Numerical results

- Within the current experimental precision, the two methods give identical results up to $\sim 50\%$ U-spin breaking.

- The inclusion of information from $B^+ \to \pi^+\pi^0$ and $B^0 \to \pi^0\pi^0$ gives less dependence against non-factorizable U-spin breaking values larger than 50%.

- Note that the current uncertainty on $2\beta_s$ is only 50% larger with respect to that obtained from the golden $B_s \to J/\psi KK$ mode, at equal luminosity wrt the $B_s \to K^+K^-$ measurement.
  - Update of $B_s \to K^+K^-$ CPV analysis to full 3 fb$^{-1}$ ongoing.

- Improvements in modes with $\pi^0$’s would be very important to better constrain U-spin breaking effects.
Observation of $B \to \rho^0 \rho^0$

- Amplitude analysis to determine the contribution from $B^0 \to \rho^0 \rho^0$ decays
- Decay observed with a significance of $7.1\sigma$
  - $N = 634 \pm 28 \pm 8$
- In the same $\pi^+\pi^-$ mass range, between 0.3 and 1.1 GeV/$c^2$, $B_s \to (\pi^+\pi^-)(\pi^+\pi^-)$ decays are also observed with a statistical significance of more than $10\sigma$

$$f_L = 0.745^{+0.048}_{-0.058} \text{ (stat)} \pm 0.034 \text{ (syst)}$$

$$\frac{\mathcal{B}(B^0 \to \rho^0 \rho^0)}{\mathcal{B}(B^0 \to \phi K^*(892)^0)} = 0.094 \pm 0.017 \text{ (stat)} \pm 0.009 \text{ (syst)}$$

$$\mathcal{B}(B^0 \to \rho^0 \rho^0) = (0.94 \pm 0.17 \text{ (stat)} \pm 0.09 \text{ (syst)} \pm 0.06 \text{ (BF)}) \times 10^{-6}$$

- The measured $f_L$ value is consistent with that obtained from BaBar, and differs at the $2.3\sigma$ level from that from Belle
- Dedicated analysis for $B_s$ decays to be done
Angular analysis of $B^0 \rightarrow \phi K^*$

- Sensitive to NP contributions in the loop
  - $1655 \pm 42$ signal candidates
- Measurement of the longitudinal polarisation fraction
  $$f_L = 0.497 \pm 0.019 \pm 0.015$$
- Direct rate CP asymmetry, measured w.r.t $J/\psi K^*$
  $$\Delta A_{CP} = (+1.5 \pm 3.2 \pm 0.5)\%$$

Results compatible with BaBar and Belle with a factor 2 higher precision
Angular analysis of $B_s \rightarrow K^{*0} \overline{K}^{*0}$

- Measurement of the longitudinal polarisation fraction
  \[ f_L = 0.201 \pm 0.057 \text{ (stat.)} \pm 0.040 \text{ (syst.)} \]

- Triple product and direct CP asymmetries are determined to be compatible with zero

- Measurement of the branching fraction
  \[ \frac{\mathcal{B}(B_s^0 \rightarrow K^{*0} \overline{K}^{*0})}{\mathcal{B}(B^0 \rightarrow \phi K^{*0})} = 1.080 \pm 0.182 \text{ (stat.)} \pm 0.081 \text{ (syst.)} \pm 0.063 (f_d/f_s) \]

  \[ \mathcal{B}(B_s^0 \rightarrow K^{*0} \overline{K}^{*0}) = (10.5 \pm 1.8 \text{ (stat.)} \pm 1.1 \text{ (syst.)} \pm 0.6 (f_d/f_s)) \times 10^{-6} \]
**CP violation in B_s → φφ**

- Gluonic b → s̅s̅s̅ss penguin
  - Measurements of triple product asymmetries and of φ_s
- Latest LHCb result with full Run 1 data set

\[ A_U = -0.003 \pm 0.017 \text{ (stat)} \pm 0.006 \text{ (syst)} \]

\[ A_V = -0.017 \pm 0.017 \text{ (stat)} \pm 0.006 \text{ (syst)} \]

\[ φ_s = -0.17 \pm 0.15 \text{ (stat)} \pm 0.03 \text{ (syst) rad} \]

- Precision comparable to golden b → c̅c̅s̅s̅ modes

![Graph showing pull distributions for different masses and a peak at φ_s = -0.17 with statistical and systematic uncertainties.]
Observation of $B_s \rightarrow \eta' \eta'$

- First observation of the $B_s \rightarrow \eta' \eta'$ decay
  - $N = 36.4 \pm 7.8 \pm 1.6$
  - Significance of $6.4\sigma$

$$\frac{B(B_s^0 \rightarrow \eta'\eta')}{B(B^\pm \rightarrow \eta'K^\pm)} = 0.47 \pm 0.09 \text{ (stat)} \pm 0.04 \text{ (syst)}$$

$$B(B_s^0 \rightarrow \eta'\eta') = [3.31 \pm 0.64 \text{ (stat)} \pm 0.28 \text{ (syst)} \pm 0.12 \text{ (norm)}] \times 10^{-5}$$

- In addition, the charge asymmetries of the $B^\pm \rightarrow \eta'K^\pm$ and $B^\pm \rightarrow \phi K^\pm$ control modes are measured

$$A^{CP}(B^\pm \rightarrow \eta'K^\pm) = [-0.2 \pm 1.2 \text{ (stat)} \pm 0.1 \text{ (syst)} \pm 0.6 \text{ (norm)}] \times 10^{-2}$$

$$A^{CP}(B^\pm \rightarrow \phi K^\pm) = [+1.7 \pm 1.1 \text{ (stat)} \pm 0.2 \text{ (syst)} \pm 0.6 \text{ (norm)}] \times 10^{-2}$$
B$^\pm \rightarrow K^\pm h^+ h^-$, $\pi^\pm h^+ h^-$ signals from Run I


**B$^\pm$ → K$^\pm$ h$^+$ h$^-$, π$^\pm$ h$^+$ h$^-$ signals from Run I**

**b → sūu**

penguin dominated

**b → dūu**

tree dominated

181069 ± 404  $B^\pm \rightarrow K^\pm \pi^+ \pi^-$

24907 ± 222  $B^\pm \rightarrow \pi^\pm \pi^+ \pi^-$

109240 ± 354  $B^\pm \rightarrow K^\pm K^+ K^-$

6161 ± 172  $B^\pm \rightarrow \pi^\pm K^+ K^-$
Integrated CP asymmetries


- Global (phase space integrated) asymmetry is computed from observed signal yields
  \[ A_{\text{raw}} \equiv \frac{N_{B^-} - N_{B^+}}{N_{B^-} + N_{B^+}} \]

- CP asymmetry obtained correcting for the \( B^\pm \) production asymmetry and asymmetry in the detection of unpaired hadron \( (B^\pm \to K^\pm h^+h^-, B^\pm \to \pi^\pm h^+h^-) \)
  \[ A_{\text{raw}} \approx A_{CP} + A_{P} + A_{D}^{h'} \]

- \( A_{\text{prod}} \) and \( A_{\text{det}} \) determined from control samples:
  \( B^\pm \to J/\psi[\mu^+\mu^-] K^\pm \), \( D^{*+} \to D^0[K^-\pi^-\pi^+\pi^+] \pi^+ \)

- Global asymmetries are typically small, not the most sensitive observable
  \[ A_{CP}(B^\pm \to K^\pm \pi^+\pi^-) = +0.025 \pm 0.004 \pm 0.004 \pm 0.007 \]
  \[ A_{CP}(B^\pm \to K^\pm K^+K^-) = -0.036 \pm 0.004 \pm 0.002 \pm 0.007 \]
  \[ A_{CP}(B^\pm \to \pi^\pm \pi^+\pi^-) = +0.058 \pm 0.008 \pm 0.009 \pm 0.007 \]
  \[ A_{CP}(B^\pm \to \pi^\pm K^+K^-) = -0.123 \pm 0.017 \pm 0.012 \pm 0.007 \]
Asymmetries over the Dalitz plane

\[ B^{\pm} \rightarrow K^{\pm} \pi^+ \pi^- \]

\[ B^{\pm} \rightarrow K^{\pm} K^+ K^- \]

\[ B^{\pm} \rightarrow \pi^{\pm} \pi^+ \pi^- \]

\[ B^{\pm} \rightarrow \pi^{\pm} K^+ K^- \]

Rich resonance structure, plus a large non-resonant component (plots are not background subtracted)
Asymmetries over the Dalitz plane


- Large asymmetries in local regions of phase space
- Different mechanisms in action, possibly related to different sources of strong phase

\[ B^\pm \rightarrow K^\pm \pi^+ \pi^- \]
Large local asymmetries

Large asymmetries observed in $B^\pm \to K\pi\pi$ and $B^\pm \to \pi\pi\pi$...
...mirrored when compared with $B^\pm \to KK\pi$ and $B^\pm \to KK\pi$

Evidence for large $\pi^+\pi^- \leftrightarrow K^+K^-$ rescattering at low mass?
Asymmetry from S- and P-wave interference in $B^\pm \to \pi^\pm \pi^+ \pi^-$

- Projections on $m_{\pi\pi}$ split according to the sign of the cosine of the angle between the momenta of the unpaired $\pi$ and the resonance daughter $\pi$ with the same charge.
- For a decay involving one vector resonance, the interference term is multiplied by $\cos \theta$, which is a linear function of the other Dalitz variable.

$$A_{CP} \propto (c_+^2-c_-^2)|A_\rho|^2+(c_-^{NR^2}-c_+^{NR^2})+\cos \theta (s_{low}-m_\rho^2)2\text{Re}(c_+^{NR^2}-c_-^{NR^2})f_\rho+...$$

- The charge asymmetry changes sign near the $\rho \to$ dominance of long-distance interference effects.
- The change occurs for both $\cos \theta > 0$ and $\cos \theta < 0 \to$ asymmetry related to the real part of the long-distance interference.

Full amplitude analysis is needed, not an easy task.
B$^\pm \to p\bar{p}K^\pm$ and B$^\pm \to p\bar{p}\pi^\pm$

- Similar studies with baryonic decays
- Hadron rescattering not expected to play a large role compared to $\pi\pi \leftrightarrow KK$: expect smaller CPV

Signals include charmonium states

Enhancement near $p\bar{p}$ threshold at low $pK^\pm$ and high $p\pi^\pm$ mass
CP violation in $B^\pm \rightarrow p\bar{p}K^\pm$

First evidence of CPV in baryonic B decays

$A_{CP} = A_{obs} - A_{prod} - A_{det}$

$A_{prod}, A_{det}$ from $B^\pm \rightarrow J/\psi K^\pm$

<table>
<thead>
<tr>
<th>mode</th>
<th>$A_{CP}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\eta_c[p\bar{p}]K^+$</td>
<td>+0.040 ± 0.034</td>
</tr>
<tr>
<td>$\psi(2S)[p\bar{p}]K^+$</td>
<td>+0.092 ± 0.058</td>
</tr>
<tr>
<td>$p\bar{p}K^+, m_{p\bar{p}} &lt; 2.85$ GeV/c$^2$</td>
<td>+0.021 ± 0.020</td>
</tr>
<tr>
<td>$p\bar{p}K^+, m_{p\bar{p}} &lt; 2.85$ GeV/c$^2, m_{pK}^2 &lt; 10$ GeV$^2$/c$^4$</td>
<td>$-0.036 ± 0.023$</td>
</tr>
<tr>
<td>$p\bar{p}K^+, m_{p\bar{p}} &lt; 2.85$ GeV/c$^2, m_{pK}^2 &gt; 10$ GeV$^2$/c$^4$</td>
<td>+0.096 ± 0.024</td>
</tr>
<tr>
<td>$p\bar{p}\pi^+, m_{p\bar{p}} &lt; 2.85$ GeV/c$^2$</td>
<td>$-0.041 ± 0.039$</td>
</tr>
</tbody>
</table>
**B^0_0(s) → K_S h^+h^- decays**

- CP violation relevant for the measurement of γ angle from B→Kππ decays
- Can also measure B^0 and Bs mixing phases
- Event sample split according to where the K_s is reconstructed
- First observation of Bs → K_s K^+π^- and Bs → K_s π^+π^- decays
- Branching ratios normalised to B^0 → K_S π^+π^-

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\[
\text{BR}(B_s \to K_s K^+\pi^-) = (73.6 \pm 5.7 \pm 6.9 \pm 3.0) \times 10^{-6}
\]

\[
\text{BR}(B_s \to K_s \pi^+\pi^-) = (14.3 \pm 2.8 \pm 1.8 \pm 0.6) \times 10^{-6}
\]

Amplitude analysis ongoing
$\Lambda_b \rightarrow \bar{K}_S \rho \pi^-$ decays

- First observation of $\Lambda_b \rightarrow \bar{K}_S \rho \pi^-$
- No significant evidence of $\Lambda_b \rightarrow \bar{K}_S \rho K^-$
- Branching fraction measured relative to $B^0 \rightarrow \bar{K}_S \pi^+ \pi^-$

$$B(\Lambda_b^0 \rightarrow \bar{K}_S^0 \rho \pi^-) = (1.26 \pm 0.19 \pm 0.09 \pm 0.34 \pm 0.05) \times 10^{-5}$$

$$A_{CP}(\Lambda_b^0 \rightarrow \bar{K}_S^0 \rho \pi^-) = 0.22 \pm 0.13 \text{ (stat)} \pm 0.03 \text{ (syst)}$$
Observation of $B_s \to K^{*+}K^-$ and evidence of $B_s \to K^{*-}\pi^+$

- First measurements of these decays
  - Significances of 7.8$\sigma$ and 3.4$\sigma$
- Branching fractions determined relative to $B^0 \to K^{*+}\pi^-$

\[
\frac{\mathcal{B}(B^0_s \to K^{*\pm}K^{\mp})}{\mathcal{B}(B^0 \to K^{*+}\pi^-)} = 1.49 \pm 0.22\text{ (stat)} \pm 0.18\text{ (syst)}
\]
\[
\frac{\mathcal{B}(B^0_s \to K^{*-}\pi^+)}{\mathcal{B}(B^0 \to K^{*+}\pi^-)} = 0.39 \pm 0.13\text{ (stat)} \pm 0.05\text{ (syst)}
\]
\[
\mathcal{B}(B^0_s \to K^{*\pm}K^{\mp}) = (12.7 \pm 1.9 \pm 1.9) \times 10^{-6}
\]
\[
\mathcal{B}(B^0_s \to K^{*-}\pi^+) = (3.3 \pm 1.1 \pm 0.5) \times 10^{-6}
\]

$B \to K_s K^{*0}$ analysis coming soon
Conclusions

• LHCb has obtained several results (not all of them shown today) regarding charmless $b$-hadron decays in Run I, but in many sectors is not yet exploiting the full power of the data
  – Important analyses to be updated to full Run I statistics
  – Full three-body amplitude analyses being worked out
  – $B_s$ and $\Lambda_b$ decay modes still largely unexplored

• Many new results to come with increased statistics in Run II

• Watch this space!
Additional slides
Bringing fractions and CP asymmetries for $B^{\pm} \rightarrow K_{S} \pi^{\pm}$ and $B^{\pm} \rightarrow K_{S} K^{\pm}$ decays

$A_{raw} = \frac{N(B^{-} \rightarrow K_{S}^{0} h^{-}) - N(B^{+} \rightarrow K_{S}^{0} h^{+})}{N(B^{-} \rightarrow K_{S}^{0} h^{-}) + N(B^{+} \rightarrow K_{S}^{0} h^{+})}$

$A_{CP} = A_{raw} - A_{Det.+Prod.} - A_{K_{S}^{0}}$

Results are compatible and competitive with B-Factories

No evidence for CP violation
Evidence for the baryonic decay $B_{(s)}^0 \rightarrow p\bar{p}$

Still unobserved decays

- Any measurement of $\text{BR} < 10^{-7}$ will rule out all the current theoretical predictions
  

\[ L = 0.9 \text{ fb}^{-1} \ @ \sqrt{s} = 7 \text{ TeV} \]

Profile likelihood

- Stat. only
- Syst. included

\[
\mathcal{B}(B^0 \rightarrow p\bar{p}) = (1.47^{+0.62}_{-0.51} +0.35_{-0.14}) \times 10^{-8} \quad \text{at} \quad 68.27\% \ \text{CL}
\]

\[
\mathcal{B}(B^0 \rightarrow p\bar{p}) = (1.47^{+1.09}_{-0.81} +0.69_{-0.18}) \times 10^{-8} \quad \text{at} \quad 90\% \ \text{CL}
\]

\[
\mathcal{B}(B_s^0 \rightarrow p\bar{p}) = (2.84^{+2.03}_{-1.68} +0.85_{-0.18}) \times 10^{-8} \quad \text{at} \quad 68.27\% \ \text{CL}
\]

\[
\mathcal{B}(B_s^0 \rightarrow p\bar{p}) = (2.84^{+3.57}_{-2.12} +2.00_{-0.21}) \times 10^{-8} \quad \text{at} \quad 90\% \ \text{CL}
\]

Evidence at more than 3$\sigma$

Values of $O(10^{-7})$ are excluded.
U-spin method

- $B^0 \rightarrow \pi^+\pi^-$ and $B_s \rightarrow K^+K^-$ are U-spin symmetry partners
- Assuming perfect U-spin symmetry one gets
  - $d = d', \theta = \theta', |D| = |D'|$
- The equalities $d = d'$ and $\theta = \theta'$ do not receive U-spin breaking corrections within factorization
  - But non-factorizable U-spin contributions may still be large
- By contrast, the U-spin equality $|D| = |D'|$ is already broken in factorization
- Using QCD sum rules one has $\left|\frac{D'}{D}\right|_{\text{fact}} = 1.41^{+0.20}_{-0.11}$. 

[PRD 78 (2008) 054015]
U-spin method

[PLB 459 (1999) 306 and others]

- Using CP asymmetries and BRs of $B^0 \rightarrow \pi^+ \pi^-$ and $B_s \rightarrow K^+ K^-$
  - 9 unknowns: $d, d', \theta, \theta', |D|, |D'|, \gamma, \beta, -2\beta_s$
  - 6 constraints: $C_{\pi\pi}, S_{\pi\pi}, B_{\pi\pi}, C_{KK}, S_{KK}, B_{KK}$
  - Constrain the sign of $A_{KK}^{\Delta \Gamma}$

$$A_{KK}^{\Delta \Gamma} = -\frac{\cos(-2\beta_s + 2\gamma) + 2d' \cos(\vartheta') \cos(-2\beta_s + \gamma) + \tilde{d}'^2 \cos(-2\beta_s)}{1 + 2\tilde{d}' \cos(\vartheta') \cos(\gamma) + \tilde{d}'^2} < 0$$


- Need to assume U-spin symmetry to reduce the number of unknown parameters
Experimental status

<table>
<thead>
<tr>
<th>Quantity</th>
<th>BaBar</th>
<th>Belle</th>
<th>CDF</th>
<th>LHCb</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_{\pi^+-\pi^-}$</td>
<td>$-0.25 \pm 0.08 \pm 0.02$</td>
<td>$-0.33 \pm 0.06 \pm 0.03$</td>
<td></td>
<td>$-0.38 \pm 0.15 \pm 0.02$</td>
</tr>
<tr>
<td>$S_{\pi^+-\pi^-}$</td>
<td>$-0.68 \pm 0.10 \pm 0.03$</td>
<td>$-0.64 \pm 0.08 \pm 0.03$</td>
<td></td>
<td>$-0.71 \pm 0.13 \pm 0.02$</td>
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<tr>
<td>$\rho(C_{\pi^+-\pi^-}, S_{\pi^+-\pi^-})$</td>
<td>$-0.06$</td>
<td>$-0.10$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\mathcal{B}_{\pi^+-\pi^-} \times 10^6$</td>
<td>$5.5 \pm 0.4 \pm 0.3$</td>
<td>$5.04 \pm 0.21 \pm 0.18$</td>
<td></td>
<td>$5.02 \pm 0.33 \pm 0.35$</td>
</tr>
<tr>
<td>$C_{K^+K^-}$</td>
<td>$-$</td>
<td>$-$</td>
<td></td>
<td>$0.14 \pm 0.11 \pm 0.03$</td>
</tr>
<tr>
<td>$S_{K^+K^-}$</td>
<td>$-$</td>
<td>$-$</td>
<td></td>
<td>$0.30 \pm 0.12 \pm 0.04$</td>
</tr>
<tr>
<td>$\rho(C_{K^+K^-}, S_{K^+K^-})$</td>
<td>$-$</td>
<td>$-$</td>
<td></td>
<td>$0.02$</td>
</tr>
<tr>
<td>$\mathcal{B}_{K^+K^-} \times 10^6$</td>
<td>$-$</td>
<td>$38^{+10}_{-9} \pm 7$</td>
<td></td>
<td>$25.8 \pm 2.2 \pm 1.7$</td>
</tr>
<tr>
<td>$A_{\pi^+\pi^0}$</td>
<td>$-0.03 \pm 0.08 \pm 0.01$</td>
<td>$-0.025 \pm 0.043 \pm 0.007$</td>
<td></td>
<td>$-$</td>
</tr>
<tr>
<td>$\mathcal{B}_{\pi^+\pi^0} \times 10^6$</td>
<td>$5.02 \pm 0.46 \pm 0.29$</td>
<td>$5.86 \pm 0.26 \pm 0.38$</td>
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</tr>
<tr>
<td>$C_{\pi^0\pi^0}$</td>
<td>$-0.43 \pm 0.26 \pm 0.05$</td>
<td>$0.44^{+0.53}_{-0.52} \pm 0.17$</td>
<td></td>
<td>$-$</td>
</tr>
<tr>
<td>$\mathcal{B}_{\pi^0\pi^0} \times 10^6$</td>
<td>$1.83 \pm 0.21 \pm 0.13$</td>
<td>$2.3^{+0.4+0.2}_{-0.5-0.3}$</td>
<td></td>
<td>$-$</td>
</tr>
</tbody>
</table>

- LHCb results are published in
  - JHEP 10 (2012) 037 (Branching ratios)
  - JHEP 10 (2013) 183 (CP asymmetries)

Note: latest Belle result on \( BR(B \to \pi^0\pi^0) \) not included
Inclusion of $B^+ \rightarrow \pi^+\pi^0$ and $B^0 \rightarrow \pi^0\pi^0$

(following JHEP 10 (2012) 029)

- Better control of non-factorizable U-spin breaking effects for $\kappa > 0.6$ but still multiple nonzero probability regions appear

- The shift towards decreasing values of $-2\beta_s$ for $\kappa > 0.6$ disappears completely $\rightarrow$ robust determination up to 100% non-factorizable U-spin breaking