INVESTIGATION OF THE $K^0\ell^+_3$ DECAYS PRESENT IN THE X 4 FILM

1. Introduction

The X 4 experiment is designed primarily for the investigation of the $K^0\rightarrow \pi^0 + \pi^0$ decay mode. The 300,000 pictures obtained will, however, contain many examples of the $K^0\ell^+_3$ leptonic decay modes and it is interesting to see what information we can hope to extract from these events concerning the nature of the interaction and the form factors involved.

In general the interaction leading to $K^0\ell^+_3$ decay can be of the vector, scalar or tensor type\(^1\). The data from previous experiments are consistent with the interaction being pure vector\(^2\). Assuming only vector coupling contributes the matrix element for the decay can be written:

\[ M = J_\lambda \cdot j_\lambda \]

where

\[ J_\lambda = \bar{u}_\ell \gamma_\lambda (1 + \gamma_5) U_\nu \]

is the V-A lepton current

and

\[ j_\lambda = f_+(q^2)(p_\ell + \gamma_\lambda) + f_-(q^2)(\gamma_\lambda - p_\pi) \]

is the weak $\pi - k$ current. The $f$'s are the $\pi - k$ weak current form factors, functions only of $q^2$ the momentum transfer from kaon to pion where

\[ q^2 = M_k^2 + M_\pi^2 - 2 M_k M_\pi \]

This form leads to a differential decay probability in the kaon rest frame for the leptonic decays of

**Notation**

$M_\ell$, $M_\pi$, $p_\ell$, $p_\pi$, $E_\ell$, $E_\pi$, $E_v$, $T_\ell$, $T_\pi$, $T_\nu$, $T_\nu'$ are the masses, momenta, total energies and kinetic energies of the lepton (used here as electron or muon), pion and neutrino respectively. The laboratory quantities are denoted by $P_\ell^L$, $E_\pi^L$, etc. $E'_{\pi} = E_{\pi} - E_\pi$ where $E_{\pi}^{Max}$ is the maximum pion energy possible in $K^0\ell^+_3$ decay. $p_{\pi \lambda}$ and $p_{k \lambda}$ are the particle four momenta.

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The form factors are, in general, complex quantities but if the interaction is invariant under time reversal they are constrained to be real.Muon-electron universality predicts that the form factors in $K_\mu^3$ and $K_e^3$ should be identical. However, the terms in $f_-$ are proportional to the lepton mass squared and hence in $K_e^3$ decay the influence of $f_-$ is negligible and the decay probability effectively only contains $f_+$. The $q^2$ dependence of the form factors is not predicted by weak interaction theory but depends on the model assumed for the strong interactions in the $\pi - k$ weak current. In general the $\pi - k$ weak current can be the sum of terms carrying isospin changes $|\Delta I|= 3/2$ or $1/2$. Assuming the $|\Delta I|= 3/2$ rule for the leptonic decays of the kaons the $|\Delta I|= 3/2$ part is absent and the form factors in $K_\ell^3$ decay are predicted to be the same as in $K_\mu^3$ decay. The $K^+$ data are compatible with the form factors being constants independent of $q^2$ with the ratio $\xi = f_-/f_+$ close to zero. By contrast if we assume the $\pi - k$ current is dominated by a sharp resonant state of mass $M$ and angular momentum $J = 1$ then dispersion theory predicts that

$$f_+ (q^2) = f_+ (0) / (\mu^2 - q^2)$$

$$f_- (q^2) = - f_+ (q^2) (\mu^2 - \pi^2) / \pi^2$$

Recently Carpenter et al from measurements on the particle energy spectra in $K^0\mu_3$ and $K^0\ell_3$ decay, have provided some evidence for such varying form factors. Their data are consistent with the assumptions of either form factors of the above type with $M = 540$ Mev. or constant form factors with $\xi = + 1.2 \pm 0.8$. Either case is considerably different from the predictions of the $\Delta I = 1/2$ rule and the $K^+$ data.

We should like in the $X_4$ experiment to either confirm or refute this difference between the charged and neutral kaons.
2. Résumé of the Experimental Arrangement

In the X 4 experiment the $K^0_2$ beam collimated to a 2 cm. diameter will pass through the bubble chamber inside a 4 cm diameter aluminium vacuum pipe of wall thickness 3 mm. The $K^0_2$ in the beam will not be monoenergetic but will have a considerable momentum spread being about 800 MeV/c on the average. In addition to $K^0_2$ the beam will contain many neutrons and $\gamma$ rays. These should not prove troublesome since the beam is collimated to be well clear of the walls of the pipe. The background due to interactions in the wall is expected to be less than 0.1% of the real $K^0_2$ decays in the pipe. The chamber filling will be CF$_3$Br. The magnetic field will be 27 kGauss. 300'000 pictures will be taken and the $K^0_2$ beam will be adjusted to give an average of one decay per picture. The expected numbers of the kaons decaying into the various modes (excluding rare modes) are:

$$
\begin{align*}
K^+_{e-3} &= 108'000 \\
K^+_{\mu-3} &= 78'000 \\
K^+(\pi^+\pi^0) &= 36'000 \\
K^0(\pi^0\pi^0) &= 78'000
\end{align*}
$$

3. Information in the Events

We can obtain information on the form factors from the Dalitz plot density of the events, the $k_{\mu-3}/k_{e-3}$ branching ratio and the lepton polarization. The form of the Dalitz plot density has been given in equation 1. Integrating over this to obtain the total decay rate involves inserting some formula for the form factors. In the case of constant form factors performing the integrations leads to the following expression for the branching ratio $R$:

$$
R \left( \frac{k_{\mu-3}}{k_{e-3}} \right) = \left( .649 + .127 \text{ Re } \xi + .0193 | \xi |^2 \right) (f+\mu/f+e)
$$

If $\mu - e$ universality holds $f_{+\mu} = f_{+e}$.

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For the case of pole form factors a much more complicated formula holds. If, however, the characteristic mass $M$ in the form factor is greater than $M_k$ then expression 2 still holds with the addition of a correction term $\delta$, approximately given by

$$\delta = \left[ 0.007 + 0.066 \Re \xi + 0.012 \left| \xi \right|^2 \right] \frac{M_k}{M}$$

If $M > M_k$, the correction terms are evidently important and of the same order as the terms in $\Re \xi$ and $\left| \xi \right|^2$ in 2. Unless we have some independent estimate of the value of $M$ from the measurement of the Dalitz plot density then the interpretation of the branching ratio is uncertain.

In the case of constant form factors a measurement of $R$ enables us to restrict $\xi$ to a circular region in the complex $\xi$ plane. If $\xi$ is assumed real as required by time reversal invariance we obtain two values from the branching ratio. Unfortunately $R$ is not very sensitive to changes in $\xi$ as diagram 1 shows. The present experimental limit of $0.73 \pm 0.15$ on $R$ enables only a very poor estimate of $\xi$ to be made. Evidently to discriminate between the result obtained by Abashian et al. and the result expected from $K^+$ decay and the $\Delta I = \frac{1}{2}$ rule the branching ratio must be measured more precisely.

4. Experimental Difficulties in Measuring the Branching Ratio

The beam layout for the experiment ensures that neutron and $\gamma$ ray interactions in the walls of the pipe should be less frequent than $K^0$ decays by a factor $10^{-4}$. Under these circumstances we may attribute any particles coming out of the pipe as being due to $K^0$ decays. A $K^0$ decay will be identified by two oppositely charged tracks originating in the pipe. Just counting the decays in which a pion and electron or a pion and muon are emitted will give a good estimate of the relative branching ratios. A number of difficulties are evident.

1) Events of each type must be identified unambiguously. For the electronic decay this should be easy due to the characteristic nature of electron tracks in heavy freon. For the muonic mode the possibility of background arises from $\tau$ events ($K^0 \rightarrow \pi^+ \pi^- \pi^0$) in which both $\gamma$-rays do not convert.
With an expected detection efficiency of 90 o/o per γ only 1 o/o of the γ decays will be ambiguous. Since γ decay is only half as probable as K_2^0μ_3 decay the contamination from this source will be only about .5 o/o.

ii) Some small loss of events is expected due to loss of particles down the pipe. A crude estimate of this effect neglecting the magnetic field and simply rejecting events in which the particle goes down the pipe and does not enter the liquid, shows that for K_2^0μ_3 decay approximately 4.5 o/o of the events will lose one particle like this. The estimate depends on the Dalitz plot density assumed for the decay varying from 3 o/o for γ = 0 to 6 o/o for γ = + 1.2 and constant form factors. For K_2^0e_3 the loss is about 3 o/o. The effect is small and can probably be reliably estimated including the magnetic field effect to within ± 2 o/o.

iii) If we use all the pictures then in many cases we will have two or more decays per picture and it will be necessary to tie each pion and lepton to the same origin in the pipe to avoid confusion of one event with another. This will be difficult to do in many cases since one of the particles may be of low energy and might be badly scattered on passing through the pipe and may be twisted in the liquid. Difficulties of this nature can be avoided if we use only those pictures containing one event. We can obtain a wrong interpretation in this case only if we have a picture on which two decays occur and a particle from each event goes down the pipe. Given the small (approx. 5 o/o) chance of one particle going down the pipe the probability of two doing so is negligible. Approximately 1/e of the pictures will contain one decay and so we expect

40'000 K_2^0e_3
29'000 K_2^0μ_3

Restricting ourselves to events in the 50 cm fiducial region chosen for the K_2^0 → 2 π^0 investigations, to give a high detection probability for the γ-rays from decays, the number of events is reduced by half.
The statistical errors on the number of events are then .7 o/o and 1 o/o. The signature for an acceptable event is quite clear so scanning efficiency should be high and the same for both decay modes.

Considering the smallness of all the errors we should be able to determine the branching ratio to better than 5 o/o. This will allow a determination of (assuming constant form factors and $I^\mu - 0$) to ± 0.35 if $\xi$ is between 0 and 1.2.

5. Measurement of the Dalitz Plot Density

To establish the Dalitz plot density the momentum of each of the particles must be measured in the laboratory frame and then transformed to the rest frame of the decaying kaon. This leads to a number of difficulties:

i) In addition to identifying the $K^0_{e3}$ decays the identity of each of the two particles in the decay must be established. Again this is simple for $K_{e3}$ decays. For the $K_{\mu3}$ decays, however, it means that only those events in which the pion interacts in the chamber fluid revealing its nature will be useful. In addition for $K^0_2 \rightarrow \pi^+ \mu^- \nu$ there will be some events in which the pion stops showing a clearly visible $\pi - \mu - e$ decay chain and others in which the muon stops and decays.

ii) The two tracks from a decay are seen only after they emerge from the pipe. In order to transform the measured momenta to the rest system of the decaying kaon the angles and momenta of the tracks at the decay vertex must be known. This means finding the decay point by projecting back the tracks into the pipe. This should be relatively easily done. Although the decay vertex is hidden in the pipe the points of emergence of the tracks from the pipe will be visible and measurable as corresponding points in all views. Treating each of these points as a separate vertex the two tracks can be reconstructed in DRAT as separate events. The fitted helices taken from DRAT can then be projected back to find their point of closest approach in the pipe and this taken as the decay vertex. In the case of low momentum tracks which may undergo considerable multiple scattering in the pipe and liquid this can lead to
a bad approximation to the vertex being obtained and large errors on
the angles.

iii) The momentum of the decaying kaon is unknown and must be established
from the measured momenta of the two charged particles. Unfortunately
two solutions for the K momentum are possible.

The invariant mass of the pion and lepton \( M^2 \) is given by

\[
M^2 = \left( E_\pi + E_\mu \right)^2 - \left( P_\pi^L + P_\mu^L \right)^2
\]

Hence, in the rest frame of the kaon, the neutrino momentum is given by

\[
P_\nu = (M^2 - M_k^2)/2 M_k
\]

The kaon beam direction is known and we may resolve \( P_\nu \) into components
along and parallel to it. For the transverse component, \( P_{\nu\perp} \) we have

\[
P_{\nu\perp} = P_{\perp} = (P_\pi^L + P_\mu^L)
\]

The longitudinal component is then

\[
P_{\nu\parallel} = \pm \sqrt{P_\nu^2 - P_{\perp}^2}
\]

The \( \pm \) sign means the pion and muon laboratory energies are compatible
with \( K^0 \rightarrow \pi^\pm \mu^\mp \) decay at rest in which the neutrino can go forward or
backwards. The ambiguity in the neutrino direction leads directly
to an ambiguity in the kaon laboratory energy which is given by

\[
\frac{E_K^L}{M_k} = \gamma_k \frac{(M_k - P_\nu)(E_\nu^L + E_\mu^L) \pm P_{\nu\|}(P_\pi^L + P_\mu^L)}{M^2 \mp (P_\pi^L + P_\mu^L)^2}
\]

There are therefore two values of \( \gamma_k \) with which to transform the pion
and muon energies to the K rest system. Three ways of dealing with this
situation present themselves.
a) We can arbitrarily choose one of these solutions as the correct one and correct the Dalitz plot for this statistically by a Monte Carlo method. Hopefully the smearing introduced into the Dalitz plot will not entirely obscure any form factor dependent shape.

b) To make sure we obtain a solution which is almost correct we can choose events in which the two solutions lie close together. Physically this means we choose events in which the neutrino is emitted almost perpendicular to the kaon direction so that the K momentum is determined by the sum of the pion and lepton longitudinal momenta. Inevitably only a small fraction of the events will fulfil this requirement.

c) We can arbitrarily choose the lower solution (or higher) as the K momentum and by making some cut on the data try to ensure that we obtain the correct solution more often than the wrong one. If we choose events in which the pion and lepton have large energy in the laboratory system we preferentially pick out events in which both particles are projected forward in the kaon rest system and the neutrino goes backwards. In this case the lower solution will be the correct one.

The calculation of the kaon momentum is also complicated by the large errors expected on the pion and lepton momenta due to multiple scattering. In practice this means that in many cases no physically meaningful value will be obtained for $P_k$.

To investigate the effects of the momentum errors and the three methods for choosing $P_k$ on the experimentally observed Dalitz plot density, the decay of the $K^0_2$ beam into the chamber was simulated by a Monte Carlo method. Figure 3 is a block diagram of the programme simulating the $K^0_2 \rightarrow \mu^- \pi^+ \nu$ mode. It is in three parts. Part I generates the decays, the particle momenta and the orientation of the tracks in the chamber. Part II distorts the momenta to take account of the multiple scattering error and selects events in which the pion or muon identifies itself by interaction or decay. Part III calculates the two possible kaon solutions and rejects events with unphysical solutions. Some choice of the "correct" $P_k$ is then made and the pion and muon momenta transformed to the K rest system.
Part I

The kaons were assumed to decay at points uniformly distributed along the axis of the pipe. Decays near the beam exit end of the pipe are less useful than decays earlier in the pipe since the pion and muon have less potential length in which to interact or stop and therefore only decays more than 30 cm from the end of the pipe were considered. Events with a given pion and muon energy in the K rest frame were generated with a frequency proportional to the Dalitz plot density given by (1). The muon direction was chosen at random in the K rest frame and the pion direction calculated from kinematics. The kaon momentum was then chosen according to the beam momentum distribution of Figure 2. This is the decaying kaon momentum distribution found by Carpenter et al. The beam line in their experiment was similar to that proposed for X 4 and so the momentum spread of the beam should be quite similar. Using this kaon momentum the pion and muon momenta and directions were transformed to the laboratory system.

Part II

Events with one particle going down the pipe (neglecting magnetic curvature) were rejected. The potential lengths available to the pion and muon were calculated taking account of magnetic field curvature. If the potential length of the particle was greater than its range it was assumed to have stopped. In this case the error on the particle momentum would be very small and hence the momentum was not distorted to take account of measurement errors. The moments of non-stopping tracks was perturbed by a fraction (in 1/p) chosen from a gaussian with mean 0 and variance 0.15. The latter was assumed to be an average variance on pion and muon track momenta in CF-Br due to multiple scattering.

Events were acceptable as $K_\mu_3$ decays if the pion interacted or stopped showing a $\pi - \mu - e$ decay chain or the muon stopped and decayed. Using the momentum dependent pion interaction length indicated in Fig. 7 the probability of the pion interaction within its potential length (or range) was evaluated and compared with a random number from 0 to 1 generated by the computer. If the interaction probability was greater than the random number the pion was assumed to interact. Stopping tracks which interacted before attaining that range were assigned the same momentum error as non-stopping tracks. In the same way, assuming a $\pi - \mu - e$.
visibility of .5 and a muon decay probability in CF Jr of .46, events in which the pion or muon stopped were given the additional chance of being accepted.

Events in which the pion or muon momenta were less than 100 Mev/c were rejected. This cut off was applied because in many of such events one track will be highly curved and undergo large multiple scattering in the pipe and liquid, and consequently only a poor approximation to the decay vertex will be obtained on projecting the tracks back. The 100 Mev/c cut off is arbitrary since it is difficult to say how bad this effect will be before the experiment is done. For higher momentum tracks the error on the angles at the decay vertex due to the projection back of the tracks should be small compared with the momentum error and is neglected.

Part III

The two kaon momentum solutions are calculated. If an unphysical solution is found the event is rejected. Three ways of choosing the "correct" kaon momentum were then applied.

i) The lower solution was taken

ii) The lower solution was taken and only events with a pion and muon having laboratory energies greater than 250 Mev were selected.

iii) Only events with the two solutions closer than 100 Mev/c were selected and the average of the two solutions taken.

For each selection of Fk the laboratory energies of the pion and muon were transformed back to the K rest frame and the event stored at an appropriate position in the experimental Dalitz plot array.

The programme for the electronic mode was similar but with the requirement of the pion interacting or stopping showing a π-μ- e chain removed and a larger error assigned to the electron momentum to take account of Bremsstrahlung losses.

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6. Results

6.1 The electronic mode

The analysis of the electronic mode is somewhat easier due to the absence of the f-form factor. 15'000 decays were generated starting with, as hypotheses for the form factor.

a) \( f + = \text{constant} \)

b) \( f + = \frac{1}{1 - q^2/M^2} \), \( M = 540 \text{ MeV} \)

Two liquids were considered CF\(_5\)Br and C\(_2\)F\(_5\)Cl. The errors on the electron and pion momenta were, assumed to be 45 o/o and 15 o/o and 30 o/o and 10 o/o respectively for the two liquids.

i) Choosing the lower momentum solution

51 o/o of the events generated in C\(_2\)F\(_5\)Cl and 46 o/o of those in CF Br lead to identified \( K^0 e^\pm \) decays with physical solutions for \( P_k \). The experimental Dalitz plots for cases a) and b) show considerable differences both in CF\(_5\)Br and C\(_2\)F\(_5\)Cl. The easiest way to see this is by examining the pion spectrum which, since \( q^2 = P_k^2 + P_\pi^2 - 2 M P_k E_\pi \), is more affected by the pole form factor than the electron spectrum. Figures 4 a) and b) show the pion spectra for form factor hypotheses a) and b) generated in the two liquids. The effect of a pole form factor is to shift events to lower pion energies. This effect is clearly visible in the spectra for both liquids. The numbers of events in the experiment will be approximately five times the numbers in the spectra. (The \( K^e\) events can be taken together for the Dalitz plot density measurement). The differences between a) and b) in each bin are several times the statistical errors on the numbers of events expected so the effect of a pole form factor with a characteristic mass considerably higher than 540 MeV should be

* The spectra are generated using the same set of random numbers for the events in each case so the differences are not subject to the statistical fluctuations associated with the numbers in each bin.

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detectable if statistics are the limiting factor. The effect of a pole with mass 800 Mev is indicated in Figure 4 b). The spectra in C2F5Cl are, as expected, slightly less smeared and the differences between a) and b) are statistically more significant.

The method of determining the form factor from the experimental data will be to generate events with a Monte Carlo programme similar to that described and to adjust the form factor in the starting Dalitz plot density until the generated Dalitz plot density matches as closely as possible the shape of the experimentally determined one. The determining factor in the precision with which the form factor can be fitted may well be not the statistical error on the number of real events but the systematic errors in the programme. An obvious source of such error is the variance and distribution assumed for the errors on the electron and pion momenta. Obviously a more correct procedure than in the present programme is to take into account the orientation, measurable length and momentum of the track before assigning the error to it. The gaussian distribution assumed for the errors is also only an approximation.

To estimate the importance of a systematic error in the assumed precision of the electron and pion momentum measurements the "f+ = constant" spectrum was generated again in C2F5Cl with the assumed width of the error distributions increased by 20 o/o. The increased errors result in 4 o/o less events having physical solutions for Pk and smears the Dalitz plot slightly. The change in shape induced in the spectrum is indicated in Figure 4 c). The two spectra have been normalized to the same number of events to display the change in shape. The increased error results in events being shifted to lower pion momenta simulating the effect of the pole form factor. With an uncertainty of 20 o/o on the standard deviation of the error distribution the systematic shifts in the spectrum are at about the same level as the statistical errors. An accurate knowledge of the precision of momentum measurements is therefore required if the systematic error is not to dominate.
ii) Choosing the lower \( P_k \) solution and events with \( T_{\pi}^{L}, T_{\mu}^{L} > 250 \text{ MeV} \).

In this case 21 o/o of the events in \( C_{\text{F}, \text{Cl}} \) and 17.5 o/o in \( C_{\text{F}, \text{Br}} \) are useful. Figures 3 a) and b) show the pion and spectra for both these events for both form factor hypotheses. The effect of the form factor is again evident but the differences on the spectra have about the same statistical significance as when all events are taken. The same considerations concerning the systematic effect of the imprecise knowledge of the errors still applies but if anything the systematic effects are worse. (Figure 5 c)).

iii) Choosing events with the two \( P_k \) solutions within 100 Mev/c

Only approximately .9 o/o of the events in \( C_{\text{F}, \text{Cl}} \) and .8 o/o in \( C_{\text{F}, \text{Br}} \) survive this criterion. The differences in shape between the pion spectra for the two form factor hypotheses described above are again apparent. The lack of events, however, makes the differences in this case less significant statistically than for the other choices of \( P_k \).

Conclusion

To analyse the events the crude method of choosing the lower solution for \( P_k \) seems to be the best of the three tried. Because of the smaller error on the momentum determinations \( C_{\text{F}, \text{Cl}} \) is to be preferred as the chamber filling. However, on account of the high \( \gamma \) conversion efficiency required for the \( K^0 \rightarrow 2 \pi^0 \) experiment the major part of the exposure must be done in \( C_{\text{F}, \text{Br}} \). Since the improvement offered by \( C_{\text{F}, \text{Cl}} \) is only small it is not worth doing a short run, with the resultant small statistics, in this fluid but it is better to do the whole run in \( C_{\text{F}, \text{Br}} \). In view of the marked effect on the spectrum shape of a systematic error in the estimated precision of the momentum determinations it is important to establish the variance and shape of the error distributions as accurately as possible. If we can be certain of the width of the error distributions to better than 20 o/o then it seems possible to detect in the Dalitz plot density the effects of a pole form factor with a mass in excess of 800 Mev.

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6.2 The muonic mode

30'000 events were generated for the $K_2^0 \mu^- \pi^+ \nu$ mode starting with as hypotheses for the form factors:

a) $f_+ = \text{constant}$\hspace{1cm} $\tau = 0$

b) $f_+ = \text{constant}$\hspace{1cm} $\tau = 1.2$

c) $f_+ = \frac{1}{1 - q^2/M^2}$\hspace{1cm} $M = 540 \text{ MeV}$\hspace{1cm} $\gamma = -0.8$

1) Choosing the lower momentum solution for $P_k$

45 o/o of the events reveal their nature by interaction of the pion in the chamber etc, and give physical solutions for $P_k$. The effect of form factor b) is to shift events to lower pion momenta and this can be clearly seen on comparing the pion spectra of the generated events for hypotheses a) and b). (Figure 7 a)). The differences between the two spectra are in general much bigger than the statistical errors on the numbers of events in each bin (the number of events generated is approximately the number expected in the experiment) showing that the effect of a much smaller $\gamma$ can be detected if statistics are the limiting factor. Again systematic errors in the generating programme may be a bigger source of uncertainty than the statistical error. Obvious sources of systematic error are the precision assumed for the momentum measurements, the pion interaction length, the $\pi - \mu - e$ visibility and the muon decay probability. The $\pi - \mu - e$ visibility should be accurately known in $X \pi$ from observations on the $\pi$ decays. The effects of the other three factors were investigated by generating the "$f_+ = \text{constant}, \tau = 0$" spectrum again with

1) The assumed error on momentum measurements increased by 20 o/o

2) The muon decay probability decreased by 15 o/o

The variation in pion interaction length with energy distorted as indicated in Fig. 6. The uncertainty in interaction length was estimated by considering the accuracy with which it can be determined at various momenta from measurements on the $\pi$ decays ($K \rightarrow \pi^+ \pi^- \pi^0$) present in the film.
The effects of the three changes are indicated in Fig. 7 b). The spectra have been normalised to the same numbers of events to display the changes in shape. Clearly the latter two sources of uncertainty produce very small effects and, compared with the level of the statistical errors, are negligible. The biggest source of systematic error is likely to be the precision assumed for the momentum measurements. The effect of a 20 o/o increase in the standard deviation on the momentum determinations is quite clearly to displace events to lower pion momenta simulating the effect of a positive $\gamma^2$. It is obviously imperative to know the variance and distribution of the momentum error distribution to better than this if the systematic error is not to seriously interfere with the determination of $\gamma^2$.

The differences between hypotheses b) and c) are not so marked in the pion spectrum as between a) and b) since the effect of form factor c) on the Dalitz plot is not so simple as a shifting of events to lower pion momenta. The differences become apparent if the complete Dalitz plot is inspected. Figure 8 shows the differences between b) and c) in various regions of the plot compared with the expected statistical errors. The two cases are quite distinct and if statistics are the limiting factor should be readily differentiated. The systematic effects discussed above will again complicate the interpretation of the Dalitz plot.

ii) Choosing the lower Pk solution and events with $T_L^\pi$ and $T_L^\mu > 250$ Mev. 18 o/o of the events fall into this category. The pion spectra for hypotheses a) and b) are shown in Figure 9 a). The effect of $\mathbf{f^+}$ is again evident but the differences between c) and b) are not so significant statistically as when all the events are taken. The systematic effects discussed above have no less effect when this choice of events is made (Fig. 9 b)).

iii) Choosing events with the two momentum solutions within 100 Mev/c

Only 1.4 o/o of the events survive this requirement. The pion spectra show the same differences apparent in the spectra for the other event selections but the differences are statistically much inferior.
Conclusion

The method of choosing the lower $P_k$ seems to be the best way of circumventing the double solution. The differences between the Dalitz plot densities generated for the three form factor hypotheses are statistically highly significant, and should enable form factor effects of the magnitude reported by Abashian et al to be easily detected, if the various systematic effects can be reduced to a low enough level. In particular the level of the errors attached to momentum measurements must be well known. Unless this can be done with confidence the interpretation placed on the experimentally observed Dalitz densities will be open to serious doubt.

6.3 Measurement of the muon polarization

A measurement of the direction of the muon polarization in $K_2^{\mu 3}$ is particularly interesting since the presence of a component of the polarization perpendicular to the decay plane in the kaon rest frame indicates a violation of time reversal invariance in the decay. Violation of time reversal invariance, which means $\gamma$ has an imaginary part, leads to a component of the muon polarization transverse to the decay plane, $\delta^\mu$, given by

$$\delta^\mu = \frac{m_\mu}{m_\pi} (p_\pi \times p_\mu) \text{Im} \gamma / \sqrt{W (E_\mu, E_\pi, \gamma)}$$

To estimate the direction of muon polarization we measure the direction of emission of the decay electrons from muons stopping in the chamber which are preferentially emitted parallel to the muon spin. Due to the presence of the magnetic field in the bubble chamber, which causes the muon spin vector to precess about the field direction, only the component of the muon polarization along the magnetic field direction is measurable. To estimate $\text{Im} \gamma$, we need the component of muon polarization perpendicular to the decay plane in the kaon rest system. We have decays in flight, the muon polarization vector measured in the laboratory system must therefore be transformed to the K rest system. Since we can only measure one component of the polarization in the laboratory system the transformation cannot be carried out and a measurement of $\text{Im} \gamma$ by this method is not possible. Even if the polarization vector in the laboratory frame we know completely there would still remain the problem of the double solution for the K momentum making the transformation to the K rest system ambiguous.

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Dependence of Branching Ratio, $R$, on $\xi$

$\text{Im } \xi = 0$, form factors constant

$$R = 0.649 + 0.127 \xi + 0.0193 \xi^2$$

Fig. 1
Energy Spectrum of Decaying Kaons

\[ \chi_k = \frac{E_k}{M_k} \]
Choose $E_k$ and $E_\mu$ according to Dalitz Plot density.

Generate orientation of tracks in $k$ rest frame.

Pick $K$ momentum and transform pion and muon to lab system.

Does pion or muon go downpipe?

Choose $K$ decay point and calculate potential lengths.

Does muon stop?  
  Yes  
  Does muon decay?  
    Yes  
    Is $P_\mu > 100\text{MeV/c}$?
      No
    No  

Is $P_\pi > 100\text{MeV/c}$?

Perturb pion momentum.

Does pion interact or stop showing $K^\pi_\mu$ decay?

Calculate $K$ momentum.

Is $P_k$ Physical?

Event selection hypothesis.

Transform $P_\pi$ and $P_\mu$ to $K$ rest system.

Store event in experimental Dalitz plot array.

Monte Carlo Programme
Generating $K^0\mu_3$ Decays

Fig. 3
Pion Spectra \( K^0_{\pi} \) decay

The lower solution for \( Pk \) in chosen

- \( f^+ = \text{constant} \)
- \( f^+ = \frac{1}{1-a^2/M^2} \) \( M = 540 \text{ Mev} \)
- \( f^+ = \) \( M = 800 \text{ Mev} \)

b) \( \text{CF}_3 \) Br

These numbers are the differences between ① and ② in each bin divided by the statistical errors on numbers of events expected in the experiment.
Effect of Systematic error in Estimated Accuracy of Pion Electron Momenta

1. $f + \text{constant}$

2. $f^+ = \frac{1}{1 - q^2/M^2}$ $M = 540$ Mev

3. $\rightarrow$ Indicates systematic shift in shape of spectrum

on increasing error on Electron and Pion momenta by 20%

Fig. 4c
Pion Spectra from $K^0\pi e^+\nu$ lower solution for $p_\pi$ chosen events with pion and muon lab kinetic energies > 250 Mev selected

1. $f_\pi$ const.
2. $f_\pi = \frac{1}{1 - q^2/M^2}$, $M=540$ Mev

Fig. 5
Pion Spectrum in $C_2 F_5 Cl$, Effect of a Systematic error in Estimated Precision of Pion and Electron Momenta

- lower $P_l$ solution chosen, Events with Pion and Electron lab energies > 250 Mev selected
- $f +$ constant
- $f +$ with errors on electron $q$ Pion momenta increased by 20%
- $f + = \frac{1}{1-q^2/M^2}$, $M = 540$ Mev

Fig. 5c
Normalised to asymptotic limit given by geometrical cross-section of 68 cms.

Variation of Pion Interaction Length with Energy.

Estimated accuracy on interaction length determinations using $I$ decays.

Fig. 6
Pion Spectrum from $K^0 \mu_3$ Decay

lower solution for $P_k$ chosen

1. $f^+ = \text{const } \xi = 0$
2. $f^+ = \text{const } \xi = 1.2$
3. $f^+ = \frac{1}{1-q^2/M^2}, M = 540 \text{ Mev}, \xi = -8$

Events / bin

$X10^3$

These numbers are the differences between 1 and 2 in each bin divided by the statistical errors

Fig. 7a
Effect of Systematic Errors

1. \( f + \text{const} \xi = 0 \) spectrum
2. \(--\) form of Pion Interaction length variation with energy distorted
3. \( \cdots \cdot \) momentum errors increased by 20%
4. \( \cdots \cdot \) \( f + \text{const} \xi = 1.2 \) spectrum

Events/bin
\( \times 10^2 \)

Fig. 7 b
\( \xi = 1.2 \) constant fs

\( \xi = -0.8 \) \( f_+ = \frac{1}{1 - q^2/M^2} \), \( M = 540 \) Mev

( ) are the differences between 1 and 2 in each region

\( \circ \) the statistical errors

The numbers are the differences divided by the statistical errors

Dalitz Plot Density for \( K^0 \mu_3 \) Decay

Fig 8.
Pion Spectrum from $K_2^+ \mu_3$ Decay

Events with $T_\pi^b \sigma T_\mu^b > 250$ Mev chosen

lower solution for $P_k$ taken

--- $f + = \text{const } \xi = 0$

----- $f + = \text{const } \xi = 1.2$

Fig 9a
Effect of Systematic Errors

Events with $T_R^+, T_\alpha^+ > 250$ Mev
lower solution for $P_k$ taken

$\sim f + \text{const } \xi = 0.$

form of pion interaction
length variation with energy
distorted

$\rightarrow \leftarrow$ momentum errors increased by 20%

Fig. 9b