Flux of atmospheric neutrinos

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We have repeated a one-dimensional calculation of neutrino fluxes, including the effect of muon polarization on neutrinos from the decay of muons. Fluxes have been calculated as a function of energy and angle with cutoffs appropriate for locations of five deep nucleon-decay experiments. Overall, the $\nu_\tau/\nu_\mu$ ratio increases by 5% (20%) for neutrinos with energies around 200 MeV (2–3 GeV), in agreement with a previous analytic estimate of the effect.

I. INTRODUCTION

Motivated by the recent measurement of the ratio of atmospheric $\nu_\tau$ to $\nu_\mu$ reported by the Kamiokande Collaboration,\textsuperscript{1} we have repeated our earlier Monte Carlo calculation\textsuperscript{2} of the flux of atmospheric neutrinos, now including muon polarization. In view of the reported\textsuperscript{1} relative deficit of muonlike events, the muon polarization becomes an important detail because it causes the $\nu_\tau$'s from muon decay to be thrown forward relative to the $\nu_\mu$'s from muon decay. This in turn increases the ratio

$$R_\nu \equiv (\nu_\tau + \bar{\nu}_\tau)/\nu_\mu \bar{\nu}_\mu$$

at a fixed neutrino energy.\textsuperscript{3} The direction of the effect is the same for neutrinos and antineutrinos.\textsuperscript{4}

An analytic calculation of neutrino ratios,\textsuperscript{4} including muon polarization, showed that for a differential pion spectrum

$$\phi(E_\pi) \propto E^{-\alpha},$$

$R_\nu$ increases by 20% at high energy for $\alpha = 2.65$ when muon polarization is included. This value of $\alpha$ corresponds to the primary spectrum of highly relativistic cosmic rays at the top of the atmosphere. At a fixed neutrino energy, the increase in $R_\nu$ due to polarization is smaller for smaller values of $\alpha$. This is because the forward kinematic region is emphasized less in the convolution of the parent spectrum with the pion and muon decay distributions. The production spectrum of pions has the same spectral index as the primary nucleon spectrum for pion energies high enough to be in the forward c.m. hemisphere of the hadron-nucleon collisions in which they are produced. For neutrinos with low energies the parent pion spectrum is flatter, reflecting both the rollover of the primary nucleon spectrum below ~10 GeV and the importance of pions produced backward in the c.m. frame of an interaction. Thus for low-energy neutrinos the effect of polarization is smaller, and at very low energies ($< m_\mu/2$) the effect on $R_\nu$ is reversed. Because of the importance of cascading and energy loss in the atmosphere the Monte Carlo simulation is required to evaluate the fluxes quantitatively.

In this Rapid Communication we first describe how the muon polarization is added to the Monte Carlo program, and we then summarize the results. The simulation has been described in detail in Ref. 2, and many of the results remain essentially unchanged. We have calculated the neutrino fluxes in such a way that the same simulated cascades are used for each detector location. The only source of difference between the fluxes at different sites is that the neutrino yields are folded with different geomagnetic cutoffs on the primary spectrum as appropriate for each geographical site. The calculated fluxes can therefore be used self-consistently as the basis for comparing neutrino interaction rates and neutrino ratios in different experiments.

II. MUON POLARIZATION

For decay in flight of a pion, the resulting $\nu_\mu$ and $\mu$ are produced isotropically in the pion rest frame, which transforms into a flat distribution in the laboratory frame. The limits are

$$\frac{1+r}{2} - \beta \frac{1-r}{2} \leq E_\mu/E_\pi \leq \frac{1+r}{2} + \beta \frac{1-r}{2},$$

$$\frac{1-r}{2} (1-\beta) \leq E_\pi/E_\pi \leq \frac{1-r}{2} (1+\beta),$$

where $r = m_\mu^2/m_\pi^2$. These distributions are of course unaffected by the fact that the muon is produced fully polarized along its direction of motion in the pion rest frame for decay of $\pi^-$ and against it for decay of $\pi^+$. There is, however, a correlation between the laboratory energy of the muon and its polarization, which affects the distribution of the decay products of the muon.

In the muon rest frame, the distribution of the decay products is

$$g(x) = f_0(x) + f_1(x) \cos \theta,$$

for $\mu \to e \pm + \nu_\tau (\bar{\nu}_\tau) + \bar{\nu}_\mu (\nu_\mu)$, where $\theta$ is the angle between the muon polarization and the direction of the lepton in the muon rest frame and $x = 2E_\pi^e/m_\mu$ is the fractional energy of the decay product. The functions $f_i(x)$ are given\textsuperscript{4} in Table I. In Ref. 4 the distribution (2) is transformed first to the pion rest frame and then to the
TABLE I. Functions for muon decay.

<table>
<thead>
<tr>
<th></th>
<th>$f_0(x)$</th>
<th>$f_1(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\nu_e$</td>
<td>$2x^2(3-2x)$</td>
<td>$2x^2(1-2x)$</td>
</tr>
<tr>
<td>$\nu_\tau$</td>
<td>$12x^2(1-x)$</td>
<td>$12x^2(1-x)$</td>
</tr>
</tbody>
</table>

laboratory frame, integrating over the muon energies and angles. For the complete Monte Carlo calculation it is necessary to compute the decay in the muon rest frame so that muon energy-loss processes can be accounted for.

To do this, we project the muon polarization along a fixed $z$ axis which is the direction of the muon in the laboratory (see Fig. 1). If the projection is $P_\mu$ then

$$g(x) = f_0(x) \mp f_1(x) P_\mu \cos \theta_\ell,$$

where $\theta_\ell$ is the angle between the lepton and the fixed $z$ axis. We need to evaluate $P_\mu$ in terms of $E_\mu$ and $E_\pi$, the laboratory energies of the pion and muon. This is done by noting that the muon polarization is parallel (antiparallel) to the direction of a $\pi^+$ ($\pi^-$) in the muon rest frame. Then one can find the relation between quantities in the muon rest frame and quantities in the laboratory by evaluating the four-vector scalar product $p_\pi \cdot u_{lab}$ in both frames. Here

$$u_{lab} = (E_\mu, -p_\mu)/m_\mu$$

is the four-velocity of the laboratory. The result is

$$\pm P_\mu = \frac{1}{\beta_\mu} \left[ \frac{E_\pi}{E_\mu} \frac{2r}{1-r} - \frac{1+r}{1-r} \right] \cos \theta_\pi,$$

where $\theta_\pi$ is the angle in the muon rest frame between the pion and the fixed $z$ axis. The final expression for the distributions of the decay products of the muon in the muon rest frame is

$$\frac{dn}{dx \, d\cos \theta_\ell} = \frac{1}{2} \left[ f_0(x) - f_1(x) \cos \theta_\ell \cos \theta_\pi \right].$$

Note that, because of the opposite relation between spin and kinematics for the two charges, the final distribution is the same for particles and antiparticles.

The simulation of the neutrino yields proceeds by calculating $\cos \theta_\pi$ for each muon in a cascade from Eq. (5) with $E_\mu$ selected randomly from the step-function distribution of Eq. (1). The value of $E_\mu$ at the point of decay of a parent pion of energy $E_\pi$ is used to evaluate $\cos \theta_\pi$. Next the lifetime of the muon in its rest frame is chosen from the appropriate exponential distribution. The muon propagates, losing energy, until it decays. At decay the energies $(x)$ and angles $(\theta_\ell)$ of the neutrinos from the muon are chosen randomly from Eq. (6). The reduced energy of the muon at decay is then used to transform the energies of its decay products to the laboratory system. It is assumed that the energies of interest for current measurements of $R_\nu$ are high enough ($p_\mu > 100 \text{ MeV/c}$) so that physical depolarization can be neglected.

As in our earlier paper a linear approximation is made in which all particles are propagated along the direction of the incident cosmic rays. Thus the only change in the calculation is the addition of polarization. The linear approximation is satisfactory for lepton momenta above 100 MeV/c.

III. RESULTS

Many of the results of Ref. 2 remain essentially unchanged. The flux of atmospheric muons and of those $\nu_\mu$ which come directly from decay of pions and kaons is of course unaffected by polarization. The ratios of neutrino fluxes at different parts of the solar cycle, as well as the relative angular distributions of neutrinos at the various sites, are virtually unchanged. We show in Fig. 2 the ratio $R_\nu$ with and without polarization. The average fluxes per bin are shown for $E_\nu > 60 \text{ MeV}$ in Table II. In comparing to experiment, the ratio should be emphasized because many systematic effects largely cancel in the ratio. These

![FIG. 1. The decay $\pi^+ \rightarrow \mu^+ \nu_\mu$ as viewed from the muon rest frame. Also shown is one of the leptons ($\ell$) from the subsequent muon decay. The dashed arrow is the direction of the muon in the laboratory system. This direction and the $\pi^+ \rightarrow \nu_\mu$ direction lie in the plane of the page. The lepton $\ell$ in general is out of this plane.](image)

![FIG. 2. $R_\nu$ as a function of the neutrino energy. Boxes show the result of this calculation, crosses are from Ref. 2. The error bars and heights of the boxes include not only statistical errors from the simulation, but also variations of $R_\nu$ with the solar epoch and variations among the different locations. For each location, variations of $R_\nu$ are significantly smaller.](image)
include related effects to the primary spectrum, to the
linear nature of the present calculation, and to the interaction model.

The ratio of electronlike to muonlike events of Ref. 1 is
1.09 ± 0.16, which is 2.9 r higher than the expected value of
0.62 that the Kamioka group obtained from their detector Monte Carlo simulation with the fluxes of Ref. 2 as input.
Because of the complexity of the analysis of the data,
and because the change in R, due to polarization is energy dependent, the effect on the interpretation of the data of including muon polarization must await the full analysis with the detector Monte Carlo simulation. If, however, the ratio of electron to muon events is proportional to Rμ,
we can use the new calculation of Rμ around the max-
imum of the flux times cross section to estimate an upper limit to the size of the effect. In the 1–2-GeV region of neutrino energy Rμ, is increased by about 18%. This leads to an estimate of ∼0.73 for the ratio of electron to muon

<table>
<thead>
<tr>
<th>Site</th>
<th>Northern U.S.</th>
<th>Kamiokande</th>
<th>Mt. Blanc/Frejus</th>
<th>Kolar Gold Fields (India)</th>
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<tbody>
<tr>
<td>E (GeV)</td>
<td>νμ, ντ, ντ, ντ</td>
<td>νμ, ντ, ντ</td>
<td>νμ, ντ, ντ</td>
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<tr>
<td>0.06–0.08</td>
<td>165 150 340 338</td>
<td>92 87 189 190</td>
<td>141 130 290 290</td>
<td>75 70 153 153</td>
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<tr>
<td>0.08–0.10</td>
<td>133 120 272 271</td>
<td>76 72 156 156</td>
<td>115 107 236 235</td>
<td>62 58 126 126</td>
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<tr>
<td>0.10–0.14</td>
<td>194 175 390 389</td>
<td>116 109 236 235</td>
<td>170 158 346 345</td>
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<td>0.14–0.20</td>
<td>185 167 360 360</td>
<td>118 111 235 236</td>
<td>167 154 329 329</td>
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<tr>
<td>0.20–0.30</td>
<td>172 153 337 336</td>
<td>117 110 238 237</td>
<td>160 145 317 316</td>
<td>96 90 194 194</td>
</tr>
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<td>96 85 184 184</td>
<td>70 64 138 139</td>
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<td>0.40–0.60</td>
<td>98 85 188 187</td>
<td>75 68 150 149</td>
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<td>0.60–0.80</td>
<td>47 40 89 88</td>
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<td>0.80–1.00</td>
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<tr>
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<td>6.9 5.6 13 13</td>
<td>6.2 5.2 13 13</td>
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<td>4.5 3.8 9.0 8.8</td>
<td>4.9 4.1 9.5 9.3</td>
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<td>6.3 5.0 15 14</td>
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