PERFORMANCE OF THE STRETCHED- AND VIBRATING-WIRE TECHNIQUES AND CORRECTION OF BACKGROUND FIELDS IN LOCATING QUADRUPOLE MAGNETIC AXES

Pasquale Arpaia 1,2, Domenico Caiazza 2,3, Carlo Petrone 2, Stephan Russenschuck 2

1 University of Napoli Federico II, Naples, Italy
2 European Organization for Nuclear Research (CERN), Geneva, Switzerland
3 University of Sannio, Benevento, Italy

Abstract - A systematic comparison of performance of stretched-wire and vibrating-wire methods for measuring the magnetic axis of a reference quadrupole magnet is presented. The compatibility of the two measurements is demonstrated and the precision of the two methods is compared as a function of the magnet gradient. Furthermore, a correction technique for the influence of non-uniform background fields is proposed for the vibrating-wire method. Experimental results are given for a normal-conducting quadrupole magnet.

Keywords: stretched wire, vibrating wire, magnetic axis, magnetic measurement

1. INTRODUCTION

The magnetic axis of a quadrupole magnet is defined as the locus of points within its aperture where the magnetic flux density is zero. One way to localize the axis is to measure by means of a single conducting wire, stretched through the aperture of the magnet 1,2. As the wire system is sensitive to the integral field along its entire length, a distinction has to be made between the average magnetic center and the tilt of the axis. The first one results in a vanishing first field integral, while the second one results in a vanishing second field integral 3. The magnetic axis is determined by both criteria together. In this paper only the measurement of the average center is discussed because the performance is the same for the tilt measurement.

The magnetic center can be found in two different ways. One is to displace the wire by moving its endpoints in the same direction (co-directional movement) and measuring the voltage induced in the wire loop caused by the magnetic flux variation 1,2. The measured voltage is then related to the offset between the initial position of the wire and the average center position. This method is referred to as the stretched-wire method and works well for large apertures (more than 10 mm radius) and high gradient fields (greater than 1 T integral gradient).

The second method makes use of a vibrating wire tuned at the resonance frequency and consists in measuring the vibration amplitudes and phases at different positions inside the magnet aperture, reached by displacing the wire ends co-directionally. The average center is then defined as the position where the vibration amplitude is zero 2,3. Due to the resonance condition, this method is characterized by a high sensitivity and is thus suitable for small apertures and low gradient fields.

The stretching-wire method was used at Fermilab for the alignment of quadrupole magnets for the Large Hadron Collider interaction regions, with an integral field strength greater than 5 T in DC mode, providing a precision of ±35 μm in measuring the center 1. The axis of the superconducting, 8-m long, quadrupoles for the Large Hadron Collider project at the European Organization for Nuclear Research (CERN) was found by the stretched wire within an uncertainty of ±20 μm 5. In this context, the additional uncertainty caused by the transfer of the wire position from the local frame of the wire stages to the frame of the magnet is not taken into account, as the main concern is the performance of the wire as a magnetic transducer.

The vibrating-wire method allowed the measurement of magnetic axis at the SLAC National Accelerator Laboratory of a resistive, 0.15-m long quadrupole with 0.14 T peak field within ±18 μm uncertainty and within ±4 μm for 1.9 T peak field 2. The axis of superconducting quadrupoles and solenoids for the Cornell Electron-positron Storage Ring experiment was measured within a precision of ±10 μm 6. A more recent setup, exploiting a phase-lock loop to follow the variations of the wire resonance frequency, allowed the axis of a quadrupole for the SwissFEL facility to be found within a reproducibility in the sub-micron range 7.

This short summary shows that the performance of the method depends on the particular setup and the magnet parameters. An effort to compare measurement methods of the magnetic axis was done in 6. In this work, the stretched-wire method and the vibrating-wire method are systematically compared when measuring the magnetic axis of the same quadrupole, and by using the same setup configuration. The aim is to check the compatibility of the measurements and to compare the precision as a function of the magnet gradient. The comparison is done only in terms of compatibility and precision and not of accuracy because the wire system is by itself the reference for the magnetic axis.

A major issue when measuring magnetic fields by means of a conducting wire (stretched or vibrating) is the effect of background magnetic fields (e.g. Earth’s magnetic
field). A common way to compensate a background field for the vibrating-wire method is to place the magnet at \( \frac{L}{4} \) from the wire end (here \( L \) is the entire length of the wire [2]). However, if the background field is not homogeneous along the wire, this compensation scheme does not work and other solutions, such as rotating the magnet around the horizontal axis have to be adopted [3]. In this paper, a new correction technique of background fields for vibrating-wire axis measurements of strength adjustable magnets is proposed. This technique is appropriate for the cases in which the common compensation schemes fail because either the background field is not homogeneous or the rotation of the magnet is not practicable.

2. BACKGROUND

2.1. Stretched-wire method

The stretched-wire method exploits the Faraday’s induction law: when a single conducting wire is moved inside a magnetic field, the properties of this field are obtained by measuring the voltage induced across the wire. Suppose that the wire is moved in an ideal quadrupole field by displacing its ends co-directionally in the horizontal plane, i.e., first by a quantity \( +d \) and then by \( -d \) from an initial position \((x_0, y_0)\). The integral field gradient can then be evaluated as

\[
(gL_m)_z = \frac{\Phi(x_0, x_0 + d) - \Phi(x_0, x_0 - d)}{d^2},
\]

where \( \Phi \) is the measured magnetic flux, \( g \) is the magnetic field gradient, and \( L_m \) is the magnetic length. Measuring in one direction and then to the opposite allows the compensation of uniform external fields. The same applies for vertical displacements, unless a correction for the wire sagitta is required.

The magnetic center position \((x_c, y_c)\), defined as the position where the first longitudinal field integral is zero can then be calculated from

\[
\begin{align*}
x_c &= x_0 - \frac{d}{2} \left( \frac{\Phi(x_0, x_0 + d) - \Phi(x_0, x_0 - d)}{\Phi(x_0, x_0 + d) + \Phi(x_0, x_0 - d)} \right), \\
y_c &= y_0 - \frac{d}{2} \left( \frac{\Phi(y_0, y_0 + d) - \Phi(y_0, y_0 - d)}{\Phi(y_0, y_0 + d) + \Phi(y_0, y_0 - d)} \right).
\end{align*}
\]

The magnetic tilt can be measured in a similar way by moving the wire ends in opposite directions (counter-directional movement) and determining the position where the second field integral takes its minimum.

2.2. Vibrating-wire method

The vibrating-wire method consists of powering the wire with an alternating current such that the Lorentz force due to the current and the magnetic field determines the mechanical vibrations in the wire. The measurement of vibration amplitude and phase allows the magnetic field along the wire to be reconstructed. The excitation-current frequency can be set to match the mechanical resonance condition in order to increase the sensitivity.

When the wire is excited in its resonance, the vibration can be described approximately as a function of time \( t \) and of the longitudinal coordinate \( z \) by the following equation

\[
u(z, t) \approx \frac{I_0}{\rho \omega_m} \sin \left( \frac{m \pi}{L} z \right) \sin \left( \omega_m t - \frac{\pi}{2} \right) \int_0^L B_n \sin \left( \frac{m \pi}{L} z \right) dz
\]

where \( I_0 \) is the amplitude of the excitation current, \( \rho \) is the mass density per unit length, \( \alpha \) is the damping coefficient, \( \omega_m \) is the \( m \)th resonance frequency, \( T \) is the mechanical tension in the wire, \( L \) the length of the wire, and \( B_n \) is the component of the magnetic flux density, normal to the wire direction.

By positioning the magnet at the longitudinal center of the wire, the magnetic center is located when the vibration is zero for the first resonant mode. The offset from the wire initial position can be found by solving a zero-finding problem, i.e., find the wire position \( X = (x_A, y_A, x_B, y_B) \), the coordinates of its extremities, such that

\[
\begin{align*}
\langle \delta_x(X), \delta_y(X) \rangle &= 0, \\
\end{align*}
\]

where \( \delta = \text{max}_t \{u(z_0, t)\} \). The magnet tilt angle can be determined by exciting the second resonant mode and accomplishing counter-directional movements of the wire.

The effect of uniform background fields is commonly compensated for by displacing the magnet along the wire at \( L/4 \) rather than \( L/2 \), and exciting the second and fourth modes, where the forcing terms due to homogeneous fields cancel out.

3. PROPOSAL

3.1. Comparison procedure

The proposed comparison procedure of the stretched-wire and vibrating-wire methods is illustrated in Fig. 1 (a) and consists of the following steps:

- powering the magnet by a given DC current;
- positioning the wire at an initial position \((x_0, y_0)\);
- performing a stretched-wire measurement of the magnetic center by measuring the quantities defined in eq. (2) and evaluating \((x_0^{sw}, y_0^{sw})\);
- repositioning the wire back to \((x_0, y_0)\);
- performing a vibrating-wire measurement of the magnetic center, by measuring the wire vibration amplitude at different positions for each axis and interpolating the measured vibration amplitudes in order to find the solution of problem (4);
- comparing the measured center in terms of compatibility and repeatability.
3.2. Background field correction for the vibrating-wire method

In this section, a correction of non-uniform background fields for the center measurement by the vibrating-wire method is proposed. The method is based on a variation of the gradient with respect to the quadrupole current, and is given by

\[
\delta x(x, z) = \kappa(I_0, \omega, z) \int_0^L \left[(g(z)(x - x_c) + B_y^E(z))\sin\left(\frac{m\pi L}{L}z\right)\right] dz, \tag{5}
\]

where \(\kappa(I_0, \omega, z)\) is the transfer function from the magnetic field to the local wire vibration, depending on the driving current amplitude and frequency. The local vibration amplitude in eq. (5) is zeroed at an apparent center, whose horizontal coordinate is given by

\[
x'_c = x_c - \frac{\int_0^L B_y^E(z) \sin\left(\frac{m\pi L}{L}z\right) dz}{\int_0^L g(z) \sin\left(\frac{m\pi L}{L}z\right) dz}. \tag{6}
\]

Eq. (6) shows that there is a hyperbolic dependence of the apparent center coordinate \(x'_c\) with respect to the quadrupole gradient. The offset of this hyperbolic function coincides with the real center \(x_c\). A similar relationship applies to the vertical coordinate \(y_c\). In order to extrapolate the real center \((x_c, y_c)\), two independent measurements must be taken and then fitted to the equation

\[
x'_c = x_c - \frac{C_{yE}^E}{I_M k_y}, \tag{7}
\]

where \(x_c\) and \(C_{yE}^E/\tilde{k}_y\) are parameters to be identified. \(I_M\) is the current supplied to the magnet. In particular,

\[
C_{yE}^E = \int_0^L B_y^E(z) \sin\left(\frac{m\pi}{L}z\right) dz \tag{8}
\]

and

\[
\tilde{k}_y = \int_0^L k_y(z) \sin\left(\frac{m\pi}{L}z\right) dz, \tag{9}
\]

where the assumption has been made that the gradient is a linear function in \(I_M\): \(g(z, I_M) = k_y(z) I_M\). This assumption can be checked independently with a probe or rotating-coil measurement.

Note that this way of correcting background fields excludes permanently magnetized regions inside the magnet, which are not a function of the excitation current.

4. EXPERIMENTAL RESULTS

4.1. Setup

The system used for the experiment is the same as described in [9], which can be used both for stretched-wire and vibrating-wire measurements. The measured magnet is a normal-conducting quadrupole magnet, which was constructed for the LEP machine at CERN (reference name "LEP-LI-QS"). The magnet has an aperture of 10 cm and a length of 10 cm. The length of the wire was 1870 mm, the mechanical tension 973 g. The magnet was positioned in the longitudinal center of the wire.

For the stretched-wire measurement, a distance \(d\) of 20 mm was chosen for both horizontal and vertical displacements. For the vibrating wire, 9 equidistant measurement points inside a range of \(\pm 1\) mm around the initial position \((x_0, y_0)\) were fixed. The driving current amplitude was set to 50 mA, the frequency to 73.1 Hz, that is slightly below the resonance frequency (73.7 Hz) to minimize instabilities of the resonance condition [7]. The measurement was repeated also for different values of the magnet current.

For the investigation of background field effect on the vibrating-wire method the magnet was moved to a longitudinal position \(L/4\) from one of the wire ends and the second resonance was excited to find the center. The mechanical tension was in this case 870 g, the fundamental frequency 74.46 Hz. The driving current frequency was set to 148.1 Hz, the amplitude to 50 mA.

4.2. Results

The center coordinates measured by the two methods at a magnet current of 10 A are shown in Tab. [1]. The standard deviation was calculated for the stretched-wire over a set of ten measurements. For the vibrating-wire, according
to aforementioned measurement procedure, the magnetic center was obtained by linear fitting of the measured vibration amplitudes as a function of the wire displacement. Then the measurement standard deviation was calculated as the standard deviation of the fitting residuals. The results are compatible but the repeatability of the vibrating-wire measurement is an order of magnitude better, owing to the high sensitivity arising from the resonance.

Table 1. Offset between magnetic center and wire initial position measured by the stretched-wire and vibrating-wire methods.

<table>
<thead>
<tr>
<th>Method</th>
<th>$x_c$ [µm]</th>
<th>$\sigma_{x_c}$ [µm]</th>
<th>$y_c$ [µm]</th>
<th>$\sigma_{y_c}$ [µm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stretched</td>
<td>-203.0</td>
<td>62.2</td>
<td>47.7</td>
<td>64.7</td>
</tr>
<tr>
<td>Vibrating</td>
<td>-209.0</td>
<td>2.2</td>
<td>57.8</td>
<td>3.0</td>
</tr>
</tbody>
</table>

In a successive experiment with the same setup, the measurements were repeated at different magnet currents by assessing the repeatability of the methods as function of the gradient (Fig. 2 in logarithmic scale). The integral gradient values used for this estimation were measured by the stretched-wire method. Based on the obtained results, a raw estimation of the repeatability as a function of the gradient was carried out. Assuming an exponential dependence of the repeatability, the stretched-wire method would be repeatable within the micron range for integral gradient values above 0.8 T.

4.3. Background field correction for the vibrating wire

In Fig. 3, the apparent average centers measured at different magnet currents, also with inverted polarity, show a satisfying agreement with eq. (5). By fitting the measurements to eq. (7), the actual average center is obtained. The resulting coordinates are reported in Tab. 2 with the rms of the fit residuals. The difference between the actual and the apparent center is 30 µm in $x$ and 10 µm in $y$ at 10 A magnet current.

The configuration where the magnet is positioned at $L/4$ was also studied. As the background field was not homogeneous along the wire, its effect was noticeable also and therefore the correction procedure was applied to the results; see Fig. 3 (c). The difference between the actual and the apparent centers is in this case 46 µm in $x$ and 15 µm in $y$ at 10 A magnet current, which is even slightly higher than for the first configuration. This can be attributed to the fringe-field generated by the stepper motor used for tensioning the wire. This fringe field is more effective on the second resonant mode, because it is located closely to one of the wire ends. This non-uniformity of the background field is compatible with previous observations carried out on the same system [10].

5. CONCLUSIONS

A comparison procedure of the stretched-wire and vibrating-wire methods for measuring the magnetic center of a quadrupole was proposed and employed on a test magnet by means of a wire system developed at CERN. The compatibility of the two methods was demonstrated, and the precision was compared as a function of the magnetic field gradient. It was shown that for the given setup, with 0.3 T integral field gradient and 10 cm aperture, the repeatability of the vibrating-wire measurement is at least 20 times better than for the stretched wire.

Moreover, a correction technique of non-uniform background fields was proposed for the vibrating-wire method, avoiding the rotation of the magnet. The procedure was employed to correct the measurements done at different longitudinal positions of the magnet. At 0.3 T integral field gradient the maximum difference between the corrected and non-corrected center is 46 µm.

This technique has a relevance within the Particle Accelerator Components Alignment and Metrology to the Nanometre scale project (PACMAN) [11], where a wire system will be used to measure the axis of the main-beam quadrupoles for the Compact Linear Collider (CLIC) [12].

REFERENCES


[11] The PACMAN project is funded by the European Union’s Seventh Framework Programme for research, technological development and demonstration.
Fig. 3. Measured magnetic center by the vibrating wire as a function of the magnet current. (a) $x$ coordinate measured with the magnet at $z_M = L/2$; (b) $y$ coordinate measured with the magnet at $z_M = L/2$; (c) $x$ coordinate measured with the magnet at $z_M = L/4$ and second wire resonance.


[12] The Compact Linear Accelerator’s webpage. URL: http://clic-study.org/