We show how the Higgs boson mass is protected from the potentially large corrections due to the introduction of minimal dark matter if the new physics sector is made supersymmetric. The fermionic dark matter candidate (a 5-plet of $SU(2)_L$) is accompanied by a scalar state. The weak gauge sector is made supersymmetric, and the Higgs boson is embedded in a supersymmetric multiplet. The remaining standard model states are nonsupersymmetric. Nonvanishing corrections to the Higgs boson mass only appear at three-loop level, and the model is natural for dark matter masses up to $15 \text{ TeV}$—a value larger than the one required by the cosmological relic density. The construction presented stands as an example of a general approach to naturalness that solves the little hierarchy problem which arises when new physics is added beyond the standard model at an energy scale around $10 \text{ TeV}$.

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I. INTRODUCTION

Minimal dark matter (MDM) [1] is an attractive model because the stability of the dark matter (DM) candidate is not enforced by an additional *ad hoc* symmetry, and the coupling to ordinary matter is fixed and equal to that of the weak gauge interaction. The model contains one or more new particles belonging to multiplets in a representation of the weak $SU(2)_L$ gauge group that makes their coupling to standard model (SM) particles by means of renormalizable operators impossible, except for the gauge interaction itself. A fermionic multiplet with $n = 5$ is singled out by the simultaneous requirements of containing a stable (neutral) state and preserving the perturbative running of the gauge coupling—that would be destroyed by too large a representation.

This model suffers a (mild) problem of naturalness [2] insofar as the mass of the DM candidate must be around $10 \text{ TeV}$ to satisfy the current relic abundance constraint [3]. Such a value turns into a correction to the Higgs boson mass roughly 1 order of magnitude larger than its value and therefore give rise to a little hierarchy problem.

In this work, we show how MDM can be made natural by making the DM sector and its interaction with the SM supersymmetric while leaving the SM itself nonsupersymmetric.

A. Naturalness

Naturalness [4] requires quantum corrections to the Higgs boson mass to be of the same order as the mass itself. It is not a physical principle as long as the Higgs boson mass is an input in the model and not a computable parameter. It is just a requirement we add on a model to satisfy a prejudice we entertain about the size of radiative corrections. It is best seen as a problem of decoupling in a theory with two separated energy scales for which we want the low-energy parameters not to depend on those at high-energy (i.e., no large thresholds in the effective theory).

The naturalness requirement cannot be stated in general because it depends on the specific kind of new physics one has introduced. It is best expressed in terms of finite corrections without reference to cutoff dependent quantities that render the issue moot.

In the absence of new physics the SM by itself is natural [2,5]. When new physics is added at an energy threshold significantly larger than the electroweak (EW) scale, corrections proportional to such a scale and the coupling of the new states to the Higgs boson mass appear, the size of which require an unnatural cancellation in the definition of the mass parameter.

For very large thresholds, a serious problem of hierarchy is present. As shown by the example of GUT [6], this problem can be solved by making the theory supersymmetric. For more modest energy scales, a little hierarchy problem might be identified and solved by a partial implementation of supersymmetry (SUSY) that only includes the new physics sector and the Higgs boson. This approach was recently emphasised in [7] and it is here applied to the MDM hierarchy problem (for a similar approach to the problem of naturalness and DM see also [8]).

The correction to the Higgs boson mass in the MDM model comes at the two-loop level because MDM only
interacts with the SM particles via the $SU(2)_L$ gauge bosons. It is given by
\begin{align*}
\delta m_{h|\psi}^2 &= 30 \frac{g^4 M^2}{(4\pi)^4} \left[ 6 \ln \frac{M^2}{\mu^2} - 1 \right], \\
\delta m_{h|\phi}^2 &= -30 \frac{g^4 M^2}{(4\pi)^4} \left[ \frac{3}{2} \ln \frac{M^2}{\mu^2} + 2 \ln \frac{M^2}{\mu^2} - \frac{7}{2} \right].
\end{align*}
respectively, for a Majorana $\psi$ and a scalar $\phi$ DM candidate, both with $n = 5$ and hypercharge $Y = 0$. Notice that the two contributions do not cancel against each other, not even in their nonpolynomial parts. Naturalness would require that this correction be of the same order as the Higgs boson mass $M_h$. By taking the matching scale at its natural value $\mu = m_h$, we find that the largest value of $M$ satisfying this requirement is 1.5 TeV in the fermionic case and 4.2 TeV in the scalar case.

In Ref. [2], lower values for this limiting masses are found, namely 1.5 TeV in the fermionic case and 4.2 TeV in the scalar case.

We adopt the following parametrization (before EW symmetry breaking) for the four neutral and two charged degrees of freedom:
\begin{align*}
H_u &= \left( \frac{1}{\sqrt{2}} (H^0 c_\alpha - h_0 s_\alpha + iA^0 s_\beta - iG^0 c_\beta), \\
H^+ c_\beta - G^- s_\beta \\
H_d &= \left( \frac{1}{\sqrt{2}} (H^0 s_\alpha + h_0 c_\alpha + iA^0 c_\beta + iG^0 s_\beta),
\end{align*}
where $s_\alpha \equiv \sin \alpha$, $c_\alpha \equiv \cos \alpha$, $t_\alpha \equiv \tan \alpha$. In Eq. (4), $\alpha$, $\beta$ are two mixing angles and the neutral $h_0$ component is the physical Higgs of the SM. Finally, $F^A$ and $G_{u,d}^a$ are nondynamical complex auxiliary fields, and they feature the same $SU(2)_L$ transformation properties of the chiral superfield they belong to.

The supersymmetric Lagrangian is that of the gauged Wess-Zumino model [9]
\begin{align*}
L_{WZ} &= \int d^2 \theta \bar{\theta} \bar{\psi}_A \left( \delta \Phi_{DM} e^{2\Phi} + \Phi^\dagger H e^{2\Phi} \Phi_{DM} \right) + \frac{1}{2} \int d^2 \theta \bar{\psi}_A \left( W \cdot W + H \cdot H + \bar{W} \cdot \bar{W} + H \cdot \bar{H} + \bar{H} \cdot H \right),
\end{align*}
and it consists of three parts. The first term in $L_{WZ}$, in which the sum over $k = u, d$ is implicit, is the usual non-Abelian Kähler potential. The vector superfield is defined in the adjoint representation of $SU(2)_L$, $V_{AB} = V^\mu (T_3^A)_{AB}$. In the Cartesian basis, the generators of the adjoint representation of $SU(2)_L$ are explicitly given by $(T_3^A)_{AB} = -i \epsilon_{aAB}$. In the Wess-Zumino gauge, the vector superfield is
\begin{align*}
V^\mu = \theta \cdot (\sigma^\mu) W^\mu_a + \bar{\theta} \cdot \lambda^\mu + \theta \bar{\theta} \cdot \bar{\lambda}^\mu + \frac{1}{2} \theta \bar{\theta} \bar{\theta} D^a, \quad (6)
\end{align*}
where $W^a_{\mu}$ and $\lambda^a$ are, respectively, the $SU(2)_L$ gauge bosons and the corresponding gaugino triplet. Finally, $D^a$ is a nondynamical real auxiliary field. The supersymmetric field strength is $W_{\mu} = -(1/8\pi) D^{\mu}(e^{-2\Phi} D_{\nu} e^{-2\Phi})$, where $\nu$ is a spinorial index and the supersymmetric covariant derivative is $D_{\mu} = \partial_{\mu} - i(\sigma^\mu)_{\nu} \bar{\theta} \partial_{\nu}$.

The superpotential in Eq. (5) does not contain interactions and is given by
\begin{align*}
W(\Phi_{DM}) = \frac{M}{2} (\epsilon_5)_{AB} \Phi^A_{DM} \Phi^B_{DM}, \quad (7)
\end{align*}
where the parameter $M$ is the mass of the DM candidate. The isospin invariant coupling is guaranteed in Eq. (7) by the presence of the symmetric tensor $\epsilon_5$ realizing the equivalence between the 5-dimensional representation of $SU(2)_L$ and its conjugate. Considering the representation $n$ of $SU(2)_L$ with generators $t^a_n = 1, 2, 3$, the tensor $\epsilon_n$ is defined by the equivalence relation $\epsilon_n T^a_n (\epsilon_n)^{-1} = -(T^a_n)^\dagger$. If $n$ is even, $\epsilon_n$ is antisymmetric; in this case (pseudo-real representation of $SU(2)_L$) the mass term in Eq. (7) vanishes.

In components, the supersymmetric Lagrangian of the model takes the form
where the ordinary $SU(2)_L$ covariant derivatives are $D_\mu \phi = \partial_\mu \phi + igW^\mu_\alpha(T^\alpha_5 \phi)$, $D_\mu \Psi = \partial_\mu \Psi + igW^\mu_\alpha(T^\alpha_2 \Psi)$, $D_\mu H_k = \partial_\mu H_k + igW^\mu_\alpha(T^\alpha_1 H_k)$, and $D_\mu \rho_h = \partial_\mu \rho_h + igW^\mu_\alpha(T^\alpha_1 \lambda_{\alpha} H_k)$.

For the fundamental $SU(2)_L$ representation, we have $T^\psi_q = \sigma^\alpha/\sqrt{2}$, with $\sigma^\alpha = \{1, i\sigma^2\}$ the usual Pauli matrices. Thanks to the supersymmetric structure, the model—despite the introduction of new fields—preserves its simplicity: all the coupling are set by gauge interactions, and the only free parameter is the mass of the chiral supermultiplet $\Phi_{DM}$. The fermionic mass term in the first line of Eq. (8) is not diagonal. It can be easily diagonalized as follows,

$$\mathcal{L}_{\text{DM}} = \mathcal{L}_\text{SM} + \mathcal{L}_{WZ} - \frac{1}{2} \bar{m}_i (\lambda^i \cdot \lambda^j) + \text{H.c.},$$

where on the lhs we just rewrote the mass term in four-component notation while the Majorana mass eigenstates on the rhs are defined by $\Psi = U^T_\Psi L + (U^T_\Psi L)^c$. The unitary transformation matrix is implicitly defined via $U^T_\Psi e_3 U = 1$, and the charge conjugation is $\Psi^c = U_\Psi \bar{\Psi}^2$.

The complete Lagrangian for the model is given by

$$\mathcal{L}_{\mu_{\text{DM}}} = \mathcal{L}_\text{SM} + \mathcal{L}_{WZ} - \frac{1}{2} \bar{m}_i (\lambda^i \cdot \lambda^j) + \text{H.c.},$$

where $\mathcal{L}_{WZ}$ is the (nonsupersymmetric) SM Lagrangian, and the last term in the first line gives mass to the gauginos. The two Higgsinos are coupled via a $\mu$ term, with $\epsilon \equiv i\sigma^2$.

**A. Physical states and their masses**

The two mass parameters $\bar{m}_1$ and $\mu$ (taken to be real for simplicity) are of the order of the EW scale and do not therefore give rise to unnatural corrections to the Higgs boson mass. By defining $\bar{W}^0 \equiv \lambda^3$, $\bar{W}^\pm \equiv (\lambda^1 \mp i\lambda^2)/\sqrt{2}$, and introducing the analogous of the neutralino $\tilde{\chi}^0 \equiv (\bar{W}^0, \bar{\rho}_0, \bar{\nu}_0)^T$ and chargino $\tilde{\chi}^\pm \equiv (\bar{W}^+, \bar{\rho}_+)^T$, $\tilde{\chi}^- \equiv (\bar{W}^-, \bar{\rho}_-)^T$, states, we extract from Eq. (10) the following mass terms:

$$\mathcal{L}_{\chi^0} = -\frac{1}{2} (\tilde{\chi}^0)^T \left( \begin{array}{ccc} \bar{m}_1 & -\frac{\bar{m}_1}{2} & \frac{\bar{m}_1}{2} \\ -\frac{\bar{m}_1}{2} & 0 & -\mu \\ \frac{\bar{m}_1}{2} & -\mu & 0 \end{array} \right) \tilde{\chi}^0 + \text{H.c.},$$

$$\approx -\frac{1}{2} (\tilde{\chi}^0)^T \mathcal{M}_{\chi^0} \tilde{\chi}^0 + \text{H.c.},$$

with $\mathcal{M}_{\chi^0} = (\bar{m}_1, \mu)^T$. The off-diagonal entry in $\mathcal{M}$ is due, after EW symmetry breaking with $H_d = (0, \bar{v}_S^2/\sqrt{2})^T$, to the symmetric Yukawa interactions in Eq. (8). The neutralino mass matrix can be diagonalized via the unitary transformation $\mathcal{N}^T \mathcal{M}_{\chi^0} \mathcal{N} = \text{diag}(m_1, m_2, m_3)$, and we denote the corresponding mass eigenstates as $\tilde{\chi}^0 = N^T \tilde{\chi}^0$. Since $\mathcal{M}^T \neq \mathcal{M}$ (unless $\bar{t}_h = 1$) two distinct unitary transformations, $V$ and $W$, are needed for its diagonalization. We denote the corresponding mass eigenstates as $\tilde{\chi}^0 = V^T \tilde{\chi}^0$, $\tilde{\chi}^- = W^T \tilde{\chi}^0$, with $\tilde{\chi}^\pm = (\tilde{\chi}^+, \tilde{\chi}^0)^T$.

At the tree level, and before the EW symmetry breaking, the scalar and fermion multiplets have the same mass $M$. At one loop, SUSY preserves the degeneracy between the multiplets, as a consequence of the non-renormalization theorem [10]. More explicitly, it is possible to show that the mass renormalization induced by the one-loop diagram in the first two rows of Fig. 1 is exactly the same for the scalar and fermion multiplet. The (nonsupersymmetric) SM Lagrangian introduces an explicit breaking of SUSY. All the cancellations and properties inherent to the supersymmetric structure of the model fail when higher-order interactions are important.
corrections involved nonsupersymmetric SM particles are considered. At two loops, the mass degeneracy between $\phi^A$ and $\psi^A$ is broken, and we show in the lower row of Fig. 1 the typical diagrams responsible for this effect.

The size of this two-loop correction is

$$\Delta M_{\text{SUSY}}^{(\phi,\psi)} = \frac{g^2 M}{16\pi^2} = 2.5 \times 10^{-3} M.$$  \hspace{1cm} (13)

The actual value and sign of this correction depend on the details of the two-loop computation, and in principle it can be different for the scalar and fermion multiplet. The mass shift $\Delta M_{\text{SUSY}}^{(\phi,\psi)}$ is of the entire multiplet—weather scalar or fermion—and it does not introduce any split between the single components.

A mass splitting within the components of the multiplet is introduced (i) at one-loop by SM EW interactions [1], (ii) at one-loop by supersymmetric interactions, and (iii) at the tree level, after EW symmetry breaking, by the presence of the operators $(H^\dagger u_d T^a_{H u,d})(\phi^\dagger T^a \phi)$ in Eq. (8) [1,11]. The first correction—generated by the diagrams in the first row in Fig. 1—does not depend on the spin, and it splits the components of $\phi^A$ and $\psi^A$ in the same way; the third correction, on the other hand, only affects the scalar multiplet. We have

$$M_{\phi}^{\text{(Q)}} = M_{\phi} + Q^2 \Delta M_{\phi} + \Delta M_{\text{SUSY}}^{(\phi)} - Q(s_\beta^2 - c_\beta^2) \frac{g^2 v^2}{8M_\phi},$$

$$M_{\psi}^{\text{(Q)}} = M_{\psi} + Q^2 \Delta M_{\psi} + \Delta M_{\text{SUSY}}^{(\psi)},$$  \hspace{1cm} (14)

where the electric charge $Q = \pm 2, \pm 1, 0$ distinguishes the components of the multiplets, and

$$\Delta M_{\phi} = 166 \text{ MeV}$$  \hspace{1cm} (15)

is the splitting induced by the EW interactions; finally, as discussed before,

$$M_{\phi} = M + \Delta M_{\text{SUSY}}^{(\phi)} \text{ and } M_{\psi} = M + \Delta M_{\text{SUSY}}^{(\psi)}.$$  \hspace{1cm} (16)

The mass splitting in Eq. (14) induced by supersymmetric interactions—the diagrams in the second row in Fig. 1—is given by

$$\Delta M_{\text{SUSY}}^{(\phi)} = \frac{g^2 M}{16\pi^2} \left[ \frac{1}{2} (Q^2 - Q) V_{1k} V_{k1}^\dagger R_{x_k}^2 (1 + \ln R_{x_k}^2) + \frac{1}{2} (Q^2 + Q) W_{1k} W_{k1}^\dagger R_{x_k}^2 (1 + \ln R_{x_k}^2) - Q^2 (N_{1k} N_{k1}^\dagger) R_{x_k}^2 (1 + \ln R_{x_k}^2) \right],$$  \hspace{1cm} (17)

where $R_a = m_a/M$. In the square brackets, the first two terms (last term) are (is) generated by loop exchange of charginos (neutralinos). Notice that in the absence of EW symmetry breaking there is no mixing in the mass matrices $M_{\phi}$ and $M_{\psi}$; as a consequence, the neutral and charged contributions in Eqs. (17) and (18) cancel out each other. An analogous cancellation is valid also in the pure EW sector, and $\Delta M_{\phi} = 0$ in the unbroken EW phase. On the other hand, after EW symmetry breaking, $\Delta M_{\text{SUSY}}^{(\phi,\psi)} = 0$ vanishes in the limit $t_\beta = 1$ since neutral and charged degrees of freedom are diagonalized in the same way.

The splitting $Q^2 \Delta M_{\phi}$ makes all the charged components heavier than the neutral one; the mass splitting induced by the Higgs vacuum expectation value $v$, on the contrary, depends on the sign of the difference $s_\beta^2 - c_\beta^2$, and for $s_\beta^2 - c_\beta^2 > 0$ makes the positively (negatively) charged components lighter (heavier) than the neutral one. The coupling in front of the operators $(H^\dagger u_d T^a_{H u,d})(\phi^\dagger T^a \phi)$ is fixed by SUSY to be equal to $g^2$, and cannot be neglected as usually assumed in MDM-inspired scalar models. Furthermore, the correction induced by $(H^\dagger u_d T^a_{H u,d})(\phi^\dagger T^a \phi)$ dominates if compared to $\Delta M_{\text{SUSY}}^{(\phi,\psi)}$ since the latter has an extra suppression of order $gv/M$.

As far as the scalar multiplet is concerned, therefore, in order to avoid the presence of a charged particle as the lightest component of the multiplet, it is necessary that $M_\phi > (s_\beta^2 - c_\beta^2)g^2 v^2/8\Delta M_{\phi}$. In turn, this condition can be recast into a constraint on $\beta$ for a given $M_\phi$.

The model with only one chiral multiplet $\Phi^\dagger_{\phi,\psi}$ (with the Higgs boson as its scalar component), while more attractive because simpler, receives a tree-level EW correction that is not suppressed by the mixing we have in the presence of a second scalar doublet and therefore may produce a scalar DM candidate with a charged component lighter than the neutral one.

Finally, we remark that all the splittings generated by the Higgs vacuum expectation value—hence either $\Delta M_{\phi}$, $\Delta M_{\text{SUSY}}^{(\phi,\psi)}$ or the splitting induced by $(H^\dagger u_d T^a_{H u,d})(\phi^\dagger T^a \phi)$—depend on the temperature, since they vanish for $T > T_c$—with $T_c$ the critical temperature of the EW phase transition—when the $SU(2)_L$ symmetry is restored. We shall return to this point when we compute the relic density.
B. Stability of the DM candidates

Nonrenormalizable operators could in principle open new decay channels and make MDM unstable. In the framework we are following, this is not possible without introducing a new scale and therefore negate the entire approach.

By assuming a unique threshold scale, we know the UV completion of the SM and therefore know what nonrenormalizable operators can or cannot be present. In our case, there is no way to construct operators leading to DM decay like

\[
\frac{1}{M^2} \phi HHH^* H^* \text{ or } \frac{1}{M^2} \psi LHH^* .
\]

by means of the Lagrangian in Eq. (10), and both \(\phi\) and \(\psi\) are stable. Therefore, as opposed to the original MDM model, both multiplets may contain a DM candidate (the neutral component of \(\psi^a\) and \(\phi^3\)), and it is their combined abundance that has to be compared to the cosmological bound.

As discussed in the previous section, interactions mediated by nonsupersymmetric SM particles break the degeneracy between the multiplets \(\psi^a\) and \(\phi^3\). As a consequence, the trilinear Yukawa interaction \(\sqrt{2}g((\phi^3)^T T^a \psi) \cdot \lambda^a + \text{H.c.}\) in Eq. (8) could lead to decays \(\psi \to \phi \lambda\) (or \(\phi \to \psi \lambda\), depending on which one between the two multiplets turns out to be the lightest once the corrections in Eq. (13) are properly computed) with final state gauginos. However, the size of the typical mass splitting in Eq. (13) is far too small to kinematically open these decay channels since the mass of the gaugino in the final state is of the order of the EW scale.

III. CORRECTIONS TO THE HIGGS BOSON MASS

Because of the supersymmetric structure, the first correction to the Higgs boson mass comes at three-loop level (see Fig. 3) as opposed to the two-loop result in Eq. (2). At two loops, the cancellation of large quadratic corrections proportional to \(M^2\) is guaranteed by the supersymmetric structure of the theory.

For simplicity, we illustrate the cancellation mechanism in the limit \(\alpha = 0, \beta = \pi/2\). In this limit, there is no mixing between the components of the two Higgs doublets, and the SM Higgs can be identified with the real part of the neutral component of \(H_d\). The two-loop diagrams involved in the cancellation are shown in Fig. 2. Since we are only interested in the renormalization of the Higgs boson mass induced by the MDM scale \(M\), we compute the two-point function of the Higgs boson setting the external momentum to zero, and we work in the massless EW theory. Moreover, we work in the Landau gauge, where diagrams with gauge bosons attached to external scalar lines with zero momentum vanish. We find the following contributions (see Fig. 2 for the corresponding notation):

\[
i\Pi_s^{(1)} = -\frac{ig^4 C_s}{4} \int \frac{d^4k_1}{(2\pi)^4} \frac{d^4k_2}{(2\pi)^4} \frac{k_1^2 + 4k_1 \cdot k_2}{D_{k_1}^{[1,M]} D_{k_2}^{[1,M]}},
\]

\[
i\Pi_s^{(2)} = \frac{ig^4 C_s}{4} \int \frac{d^4k_1}{(2\pi)^4} \frac{d^4k_2}{(2\pi)^4} \frac{k_1 + k_2}{D_{k_1}^{[2,0]} D_{k_2}^{[1,M]}},
\]

\[
i\Pi_s^{(3)} = \frac{ig^4 C_s}{4} \int \frac{d^4k_1}{(2\pi)^4} \frac{d^4k_2}{(2\pi)^4} \frac{1}{D_{k_1}^{[1,M]} D_{k_2}^{[1,M]}},
\]

\[
i\Pi_s^{(4)} = \frac{ig^4 C_s}{4} \int \frac{d^4k_1}{(2\pi)^4} \frac{d^4k_2}{(2\pi)^4} \frac{8k_2 \cdot k_2}{D_{k_1}^{[2,0]} D_{k_2}^{[1,M]} D_{k_2-k_1}^{[1,M]}},
\]

\[
i\Pi_s^{(5)} = \frac{ig^4 C_s}{4} \int \frac{d^4k_1}{(2\pi)^4} \frac{d^4k_2}{(2\pi)^4} \frac{1}{D_{k_1}^{[1,M]} D_{k_2}^{[1,M]} D_{k_2-k_1}^{[1,M]}},
\]

\[
x \frac{-4(k_1 \cdot k_2)k_1^2 + 6k_1 \cdot k_2 - 2k_2^2 + 6M^2}{D_{k_1}^{[2,0]} D_{k_2}^{[1,M]} D_{k_2-k_1}^{[1,M]}},
\]

with \(C_s \equiv \delta_{M} \text{Tr}(T^A T^B)\), where \((T^A T^B)_{ij}\) is the quadratic Casimir operator for the generic irreducible representation \(R\). For the quintuplet, we find \(\text{Tr}(T^A T^B) = 10\delta_{AB}\). We used the shorthand notation \(D_{ij}^{[n,M]} \equiv (k_2^2 - M^2)^n\). Notice that the class of double-bubble corrections like the last diagram in the second row of Fig. 2 gives a vanishing contribution since it involves the trace in the isospin space.
Planck data [12] to be repeated by the thermal relic abundance Eq. (22) is saturated by a value \( M = 3 \) TeV of their common mass (lighter lines, see caption). This result does not take into account the Sommerfeld enhancement of the cross section. The Sommerfeld enhancement is a nonrelativistic effect generated by the exchange of light force carriers between the two annihilating DM particles [13]. In the MDM scenario, the EW gauge bosons are the mediators responsible for this effect [3,14]. After including this correction—which we compute following closely [14]—we conclude that the relic abundance Eq. (22) is saturated by a value \( M = 7 \) TeV (see Fig. 4). Such a value is inside the naturalness limits found in the previous section.

\[\Omega_{\text{DM}} h^2 = \frac{Y_{\phi} M_{S0}}{\rho_\chi h^2} = 0.1188 \pm 0.0022, \]

where \( s_0 = 2.71 \times 10^3 \text{ cm}^{-3} \) and \( \rho_\chi h^2 = 1.05 \times 10^{-5} \text{ GeV cm}^{-3} \) are the present entropy and critical energy density of the Universe while \( Y_{\phi} \) is the asymptotical value of the DM comoving density. In the usual freeze-out scenario, the value of \( Y_{\phi} \) is controlled by a system of coupled Boltzmann equations describing, as the Universe gradually expands and cools, the evolution of the DM density driven by its interactions with SM particles.

In our case, we have two DM candidates, the scalar \( \phi \) and the fermion \( \psi \), with the same mass \( M \), both \( SU(2)_L \) multiplets with \( n = 5 \); the thermally averaged cross sections for coannihilation into SM states are given by, respectively, [1]

\[\langle \sigma v \rangle_\phi = \frac{g^4}{64\pi M^2} \left[ 3 - 4n^2 + n^4 + \frac{n^2 - 1}{2} \right], \]

\[\langle \sigma v \rangle_\psi = \frac{g^4}{64\pi M^2} \left[ 2n^4 + 17n - 19 \right]. \]

The problem reduces to the solution of two separated Boltzmann equations for the two multiplets since the only interactions between \( \phi^A \) and \( \psi^A \) are those mediated by gaugino exchange, and the phase space of these interactions is kinematically closed if the mass splitting is neglected. For the values of masses we will find, the freeze-out temperature \( T_f = M/25 \) is above the EW phase transition and we can neglect the mass correction in Eq. (14) induced by the Higgs vacuum expectation value.

By solving the Boltzmann equations (Fig. 4), we find that the relic density is saturated by the two DM states for a value \( M = 3 \) TeV of their common mass (lighter lines, see caption). This result does not take into account the Sommerfeld enhancement of the cross section. The Sommerfeld enhancement is a nonrelativistic effect generated by the exchange of light force carriers between the two annihilating DM particles [13]. In the MDM scenario, the EW gauge bosons are the mediators responsible for this effect [3,14]. After including this correction—which we compute following closely [14]—we conclude that the relic abundance Eq. (22) is saturated by a value \( M = 7 \) TeV (see Fig. 4). Such a value is inside the naturalness limits found in the previous section.

\[\text{FIG. 4. Thermal relic density of DM as a function of its mass } M \text{ for the two DM components } \phi \text{ and } \psi \text{ taken one at the time and together. Darker (lighter) curves with (without) the Sommerfeld enhancement. In the inset, the DM comoving density as a function of the variable } x = M_{\text{DM}}/T \text{ shows the freeze-out temperature } T_f.\]
The scalar components of the chiral multiplets $\Phi^a_{\mu\nu}$ give rise to a two Higgs doublet model (2HDM), the phenomenology of which is actively under study at the LHC—primarily in the decay modes of the Higgs boson. Limits on the parameter space can be found in the literature (see, for instance, [17]).

The model, contrary to most SUSY models, does not require the existence of any colored super-partner, in particular there is no partner for the top quark.

VI. OUTLOOK

We have shown that a model with a distinctive phenomenology can be defined by requiring MDM to be natural. A partial implementation of SUSY solves the little hierarchy problem which arises when new physics is added beyond the SM at an energy scale around 10 TeV.

The same approach can be extended to other hierarchy problems. First, the SM Higgs boson is protected by some custodial symmetry (SUSY in our case) under which the new physics sector is invariant. After the introduction of new physics determines the energy threshold, this scale will decide the degree of SUSY (or of whatever custodial symmetry one is using) required within the SM particles to make the new physics sector natural.

We thus obtain a class of telescoping models for which a little hierarchy, like the one discussed here, requires the introduction of only Higgsinos and weak gauginos (or sleptons in the model of [7]). Little hierarchies originating in weakly interacting new physics, only require weakly interacting SUSY partners to be introduced. An intermediate hierarchy, as one generated by a threshold between 100 and $10^3$ TeV, also requires—in addition to the states introduced for the case of a the little hierarchy—the presence of squarks. A large hierarchy, like that of GUT models, requires the full supersymmetrization of the SM.

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