Electroweak penguin $B$ decays

Thomas Nikodem
on behalf of the LHCb collaboration
including results from other experiments
Outline

1) Theory
2) Measurements
3) Interpretation
What is a penguin?

Standard Model Penguin

\[ b \rightarrow W t \rightarrow Z^0, \gamma \rightarrow \mu^+ \mu^- \]

New Physics

\[ b \rightarrow ? \rightarrow \mu^+ \mu^- \]

\[ b \rightarrow s(d) \rightarrow \mu^+ \mu^- \]

→ Indirect search for new heavy particles
Fermi’s theory of Beta Decay

- Beta decay described by 4-point interaction with coupling strength $G_F$:

\[ e^- \quad \bar{\nu}_e \]

Complete dynamics of heavy virtual particles described by 1 parameter!
Effective operators

- Effective operators correspond to different physical processes, i.e.:
  - Photon penguin $O_7$:
    \[
    \mathcal{H} = -4 G_F \bar{v}_{tb} \bar{v}_{ts} \left( C_i O_i + C_i' O_i' \right)
    \]
  - Electroweak penguin $O_9, O_{10}$:
Effective description

- Effective Hamiltonian for b-s transition (leading contribution):

\[ H_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \sum_i C_i O_i \]

Coupling Strength \( C_i = \) Wilson coefficient → Sensitive to New Physics

- Decays sensitive to different Wilson coefficients:

  \[ B \rightarrow X_s \gamma \quad C_7 \]
  \[ B \rightarrow (X_s, K^*) l^+ l^- \quad C_7, C_9, C_{10} \]
  \[ B \rightarrow \mu^+ \mu^- \quad C_{10}, C_s, C_P \]
Observables

- Most basic and possible with least statistics: Branching Ratio

But: Large theoretical uncertainties due to hadronical part (form factors)

In ratio measurements many uncertainties (theoretical and experimental) cancel:

- Ratio of different decays
- Ratio of different phase space regions
  - angular measurements (forward-backward asymmetry)
$B^0 \rightarrow K^{*0} \mu^+ \mu^-$

Results look like:

$$d\Gamma/dq^2$$

$C_7$, $C_7 C_9$ interference

$1-6$ GeV$^2$

$q^2 (= m_{\mu^+\mu^-}^2)$

$C_9$ and $C_{10}$

$J/\psi$, $\psi(2S)$
Charmonium Resonances

$B^0 \rightarrow K^{*0} \mu^+ \mu^-$

Rare decay:
$\approx 3000$ signal candidates

Tree level decay:
$\sim 100$ times more events than signal decay

$\psi(2S)$ preliminary

LHCB
Recent measurements

**Branching Ratio**

<table>
<thead>
<tr>
<th>Process</th>
<th>Date</th>
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<tbody>
<tr>
<td>$B^{0,+} \rightarrow K^{0,+,*+} \mu^+ \mu^-$</td>
<td>(LHCb, Mar 14)</td>
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<td>$B^0 \rightarrow K^{*0} \mu^+ \mu^-$</td>
<td>(CMS, Jul 15)</td>
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<td>$B_s^0 \rightarrow \phi \mu^+ \mu^-$</td>
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<td>$B^+ \rightarrow \pi^+ \mu^+ \mu^-$</td>
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<td>$\Lambda_b^0 \rightarrow \Lambda \mu^+ \mu^-$</td>
<td>(LHCb, Mar 15)</td>
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<tr>
<td>$B_{(s)}^0 \rightarrow \mu^+ \mu^-$</td>
<td>(CMS+LHCb, Jun 15)</td>
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**Lepton universality**

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**Angular**

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<td>$B_s^0 \rightarrow \phi \mu^+ \mu^-$</td>
<td>(BaBar, Aug 15)</td>
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**CP asymmetry**

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**Isospin asymmetry**

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$b \rightarrow (s/d)(\mu^+ \mu^-/e^+ e^-)$
Recent measurements

**Branching Ratio**

- $B^0, + \rightarrow K^{0,+,*+} \mu^+ \mu^-$ (LHCb, Mar 14)
- $B^0 \rightarrow K^{*0} \mu^+ \mu^-$ (CMS, Jul 15)
- $B_s^0 \rightarrow \phi \mu^+ \mu^-$ (LHCb, Jun 15)
- $B^+ \rightarrow \pi^+ \mu^+ \mu^-$ (LHCb, Sep 15)
- $\Lambda_b^0 \rightarrow \Lambda \mu^+ \mu^-$ (LHCb, Mar 15)
- $B_{(s)}^0 \rightarrow \mu^+ \mu^-$ (CMS+LHCb, Jun 15)

**CP asymmetry**

- $B^+ \rightarrow \pi^+ \mu^+ \mu^-$ (LHCb, Sep 15)

**Isospin asymmetry**

- $B^{0,+} \rightarrow K^{0,+,*+} \mu^+ \mu^-$ (LHCb, Mar 14)

**Lepton universality**

- $B^+ \rightarrow K^+ l^+ l^-$ (LHCb, Jun 14)

**Angular**

- $B^{0} \rightarrow K^{*0} l^+ l^-$ (LHCb, Jan 15)
- $B^{+} \rightarrow K^{*+} l^+ l^-$ (LHCb, Mar 15)
- $B_s^{0} \rightarrow \phi \mu^+ \mu^-$ (LHCb, Jun 15)
- $\Lambda_b^{0} \rightarrow \Lambda \mu^+ \mu^-$ (LHCb, Mar 15)

$>2\sigma$ deviation from SM prediction
Branching Ratio
BR of $B^0 \to K^{*0} \mu^+\mu^-$

- Compatible to SM predictions
- Compatible to previous results
- 8TeV update from LHCb soon
BR of $B^{0,+} \rightarrow K^{0,+,*+} \mu^+ \mu^-$

- In low $q^2$ region below SM prediction
BR of $B_s^0 \rightarrow \phi \mu^+ \mu^-$

- Narrow $\phi$ resonance:
  - $\rightarrow$ clean signal
- In low $q^2$ region below SM

\[
\text{LHCb, arXiv:1506.08777}
\]

- transition: b-s
  spectator: s

\[
\text{SM (wide): Altmannshofer, Straub, arXiv:1411.3161}
\]
BR of $B^+ \rightarrow \pi^+ \mu^+ \mu^-$

- $\rho^0/\omega$ resonance clearly visible
- Only in HKR15 calculation low $q^2$ resonances taken into account
- Agreement with SM, however again everywhere slightly below
In combination with $B^+ \rightarrow K^+ \mu^+ \mu^-$, measure:

$$\left| \frac{V_{td}}{V_{ts}} \right| = 0.24^{+0.05}_{-0.04}$$

Most precise measurement in process with penguin AND box diagrams.

More about CKM angles:
Today 11:40, James Libby
Two spectator quarks: complex theory, different form factors

In low $q^2$ region below SM predictions, even compatible with zero

Transition: $b$-$s$ spectator: $u+d$
BR of $B_{s,d}^0 \rightarrow \mu^+ \mu^-$

CMS+LHCb, Nature 522, 68-72 (June 2015)

$BR(B_{s,d}^0 \rightarrow \mu^+ \mu^-)$ transition: b-s,d spectator: -

- b-s(d) transition without spectator quark
- sensitive to other $C_i$ then decays presented until now

SM: PRL 112(2014) 101801

$BR(B_s^0 \rightarrow \mu^+ \mu^-) = (3.66 \pm 0.23) \cdot 10^{-9}$

$BR(B_d^0 \rightarrow \mu^+ \mu^-) = (1.06 \pm 0.09) \cdot 10^{-10}$

Measured (CMS+LHCb, 7+8TeV)

$BR(B_s^0 \rightarrow \mu^+ \mu^-) = (2.8^{+0.7}_{-0.6}) \cdot 10^{-9}$

$BR(B_d^0 \rightarrow \mu^+ \mu^-) = (3.9^{+1.6}_{-1.4}) \cdot 10^{-10}$

Ratio:

$R = \frac{BR(B_s^0 \rightarrow \mu \mu)}{BR(B_d^0 \rightarrow \mu \mu)} = 0.14^{+0.08}_{-0.06}$

17.09.2015

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BR ratio and asymmetry
Lepton Universality

• Ratio of Branching ratios:

\[ R_K = \frac{\text{BR}(B_u^+ \rightarrow K^+ \mu^+ \mu^-)}{\text{BR}(B_u^+ \rightarrow K^+ e^+ e^-)} \]

• Very precise Standard Model prediction:

\[ R_K = 1.00030^{+0.00010}_{-0.00007} \]

• Measurement from LHCb in \( q^2 \) 1-6 GeV²:

\[ R_K = 0.745^{+0.090}_{-0.074} \text{(stat.)} \pm 0.036 \text{(syst.)} \]

→ 2.6σ from SM prediction
Isospin asymmetry

- Test effect of charge of spectator quark
- Consistent with SM

\[
A_I(B \rightarrow K^{(*)}\mu^+\mu^-) = \frac{\Gamma(B^0 \rightarrow K^{(*)0}\mu^+\mu^-) - \Gamma(B^+ \rightarrow K^{(*)+}\mu^+\mu^-)}{\Gamma(B^0 \rightarrow K^{(*)0}\mu^+\mu^-) + \Gamma(B^+ \rightarrow K^{(*)+}\mu^+\mu^-)}
\]

LHCb, JHEP 06 (2014) 133

SM=0(1%)
CP asymmetry

\[ A_{CP}(B \rightarrow \pi \mu^+ \mu^-) = \frac{\Gamma(B^- \rightarrow \pi^- \mu^+ \mu^-) - \Gamma(B^+ \rightarrow \pi^+ \mu^+ \mu^-)}{\Gamma(B^- \rightarrow \pi^- \mu^+ \mu^-) + \Gamma(B^+ \rightarrow \pi^+ \mu^+ \mu^-)} \]

- Sensitive to interference effects

- Measured in \( q^2 \) 1-6 GeV²:

\[ A_{CP}(B \rightarrow \pi \mu^+ \mu^-) = -0.11 \pm 0.12 \text{(stat.)} \pm 0.01 \text{(syst.)} \]

- SM prediction (Hambrock et al., arXiv:1506.07760):

\[ A_{CP}(B \rightarrow \pi \mu^+ \mu^-) = -0.14^{+0.04}_{-0.03} \]

\( \rightarrow \) Consistent with SM
Angular measurements
Past: $e^+ e^- \rightarrow \mu^+ \mu^- \ @ \text{LEP}$

- Forward-Backward asymmetry:

\[ A_{FB} = \frac{N_{\text{forward}} - N_{\text{backward}}}{N_{\text{forward}} + N_{\text{backward}}} \]

- by counting:

\[ \frac{1}{\sigma} \frac{d\sigma}{d \cos \theta} = \frac{3}{8} (1 + \cos^2 \theta) + A_{FB} \cos \theta \]

DELPHI, Nuclear Physics B 367 (1991) 511-574
Present: $B \rightarrow K^{*0} \mu^+ \mu^-$

- Decay fully described by three angles $\theta_K$, $\theta_l$, $\phi$ and $q^2$

- Normalized differential decay rate:

$$\frac{1}{d(\Gamma + \bar{\Gamma})/dq^2} \left. \frac{d^3(\Gamma + \bar{\Gamma})}{d\Omega} \right|_P = \frac{9}{32\pi} \left[ \frac{3}{4} (1 - F_L) \sin^2 \theta_K + F_L \cos^2 \theta_K ight. \\
+ \frac{1}{4} (1 - F_L) \sin^2 \theta_K \cos 2\theta_l \\
- F_L \cos^2 \theta_K \cos 2\theta_l - S_3 \sin^2 \theta_K \sin^2 \theta_l \cos 2\phi \\
+ S_4 \sin 2\theta_K \sin 2\theta_l \cos \phi + S_5 \sin 2\theta_K \sin \theta_l \cos \phi \\
+ \frac{4}{3} A_{FP} \sin^2 \theta_K \cos \theta_l + S_7 \sin 2\theta_K \sin \theta_l \sin \phi \\
+ S_8 \sin 2\theta_K \sin 2\theta_l \sin \phi + S_9 \sin^2 \theta_K \sin^2 \theta_l \sin 2\phi \]

$F_L$: fraction of longitudinal polarization of $K^*$

$S_i$: angular observables

Attention: Different definitions!
Angular $B^0 \rightarrow K^{*0} \mu^+ \mu^-$

4D+1D Maximum Likelihood Fit to $m(K\pi\mu\mu), m(K\pi)$, 3 angles

Besides decay over $K^*$ (P-wave) also decay over spin 0 resonances (S-wave)

Fit of 8 P-wave observables and 6 (S-wave) nuisance parameters

Fit projections in $q^2$ bin 1.1–6 GeV$^2$

Blue: signal
Red: combinatorial background
Angular $B^0 \rightarrow K^{*0} \mu^+ \mu^-$

- In general good agreement with SM prediction
- In $A_{FB}$ and $S_5$ in lower $q^2$ region below/above predictions

Angular $B^0 \rightarrow K^{*0} \mu^+ \mu^-$

- Also fit different observable:
  
  $P_5' = \frac{S_5}{\sqrt{F_L(1 - F_L)}}$

- Form factor uncertainties cancel out to a certain extent
  -> more precise SM prediction

- In lower $q^2$ region 3.7σ deviation to SM observed

SM: Descotes-Genon, Hofer, Matias, Virto JHEP 1412 (2014) 125
Angular $B^0 \rightarrow K^{*0} \mu^+ \mu^-$

- fit 2D angular distribution $\theta_K, \theta_l$
- fit S-wave contribution
- in good agreement with SM and measurement by LHCb
- Full angular analysis on its way!
Angular $B^{0,+} \to K^{*0,*+} l^+ l^-$

- Studying in total 5 channels
- Subsequent fits of 1D projections of $\theta_K, \theta_l$ (→ study $F_L, A_{FB}$)

First angular analysis of $B^+ \to K^{*+} l^+ l^-$

- At low $q^2$: $F_L$ of $B^+ \to K^{*+} l^+ l^-$ below SM.
  → Otherwise consistent with SM

$q^2$ 1-6 GeV$^2$

fig modified from 1508.07960

SM:
- Altmannshofer et. al.
  JHEP 0901, 019 (2009)
- Belle
  PRL 103, 171801 (2009)
- CDF
  PRL 108, 081807 (2012)
- ATLAS
  ATLAS-CONF-2013-038
Angular $B_s^0 \to \phi \mu^+ \mu^-$

- Described very similar to $B_d^0 \to K^* \mu^+ \mu^-$
- However, decay not self-tagging
- Measure instead $T$-odd asymmetries $A_{5,6,8,9}$
- Measurement of $A_5$ and $A_8$ for $b \to sll$ for first time

→ Consistent with SM


transition: $b$-$s$
spectator: $s$
Angular: $\Lambda_b^0 \rightarrow \Lambda\mu^+\mu^-$

- As proton has spin, measure two forward-backward asymmetries:
  - $A_{FB}^l$ same as in $B_d^0 \rightarrow K^*\mu^+\mu^-$
  - $A_{FB}^h$ in hadronic part

- At high $q^2$ $A_{FB}^l$ above SM prediction
Summary

- Several 2-3σ deviations
- 2.6σ deviation for lepton universality
- Most striking:
  at low q², sensitive to $C_7 - C_9$ interference
On the path to New Physics?

Crivellin, D’Ambrosio, Heeck

Descotes-Genon, Hofer, Matias, Virto
arXiv:1503.03328

Altmannshofer, Straub
EPJC (2015) 75: 382

Beaujean, Bobeth, van Dyk
EPJC 74 (2014) 2897

Hurth, Mahmoudi
JHEP 1404 (2014) 097

...
• Wilson coefficients determined in $\chi^2$ fit of observables:
  
  91 measurements,
  18 different observables

Decay Channels:

- $B^{(+) \rightarrow K^{(*)} l^+ l^-}$
- $B_s \rightarrow \phi \mu^+ \mu^-$
- $B(s) \rightarrow \mu^+ \mu^-$

Not presented today:

- Measured by BaBar, Belle and CLEO
- In agreement with SM, important to constrain physics!
Combined Fit: Results

Altmannshofer, Straub, EPJC (2015) 75: 382

New Physics preferred over Standard Model: $3.7\sigma$

\[ C_i^{\text{obs}} = C_i^{\text{NP}} + C_i^{\text{SM}} \]
One of the New Physics Models : $Z'$

b-s(d) transition at tree level:

$\begin{array}{c}
 b \\
\downarrow Z' \\
\mu^+ \\
\mu^-
\end{array}$

Constrained by:

- Dilepton/Dijet production measurements at ATLAS and CMS
- $B_s$ mixing
- ....

$\rightarrow Z'$ mass at least $O(\text{TeV})$

Exclusion limits depend on coupling

Especially, if $Z'$ couples differently to muons and electrons **still** allowed!
Underestimated Theory Uncertainties?

- Charm Loop could mimic effect of $C_{9}^{NP}$, both: vector like coupling
- Cannot explain deviation from lepton universality
- Hadronic effects expected to be $q^2$ dependent!

Straub, Moriond EW, 2015
New Physics expected to be $q^2$ independent

→ No final answer possible yet ...
The Future

More precise theory
• hadronic uncertainties
  ...

More observables/decays
• asymmetries
• flavour violating decays: $B \rightarrow K\mu e$
  ...

More statistics
• LHCb: $8fb^{-1}$ by 2019, $50fb^{-1}$ by 2030
• CMS/ATLAS: $100fb^{-1}$ by 2019, $300fb^{-1}$ by 2023
• Belle II: $50ab^{-1}$ by 2024
  ...
## Conclusion

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<tr>
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<th>Compatible with SM</th>
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<tr>
<td>Angular</td>
<td>?</td>
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<td>Branching Ratio</td>
<td>?</td>
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BACKUP
CMS Detector

CMS Detector

- Weight: 14,000 tonnes
- Diameter: 15.0 m
- Length: 28.7 m
- Magnetic field: 3.8 T

Silicon Trackers

Superconducting Solenoid

Muon Chambers

Preshower

Forward Calorimeter

Steel Return Yoke

Electromagnetic Calorimeter

Hadron Calorimeter
LHCb detector

LHCb Detector
- Weight: 5,600 tonnes
- Height: 10 m
- Length: 20 m

Electromagnetic Calorimeter

RICH1

Vertex Locator

Tracker Turicensis

Dipole Magnet

Tracking Stations

RICH2

Hadron Calorimeter

Muon Chambers
BaBar detector

**BABAR Detector**

- Muon/Hadron Detector
- Magnet Coil
- Electron/Photon Detector
- Cherenkov Detector
- Tracking Chamber
- Support Tube
- Vertex Detector

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2.2. Preliminary considerations

Before turning to the numerical analysis, it is instructive to make some analytical considerations as to which Wilson coefficients have to be modified to explain the tensions in the data. An immediate observation is that all three tensions occur in CP-averaged observables, so there is no need to invoke non-standard CP violation, i.e. the Wilson coefficients can be kept real.

To get an analytical understanding of the dependence of the relevant observables on the Wilson coefficients, we can derive approximate expressions, valid for small NP contributions, neglecting interference terms between NP effects in different coefficients. We find\(^4\)

\[
\langle F_L \rangle_{[1,6]} \simeq +0.77 + 0.25 C_7^{\text{NP}} + 0.05 C_9^{\text{NP}} - 0.04 C_9' + 0.04 C_{10}' ,
\]

\[
\langle S_4 \rangle_{[14,18,16]} \simeq +0.29 - 0.02 C_9' + 0.03 C_{10}' ,
\]

\[
\langle S_5 \rangle_{[1,6]} \simeq -0.14 - 0.59 C_7^{\text{NP}} - 0.49 C_7' - 0.09 C_9^{\text{NP}} - 0.03 C_9' + 0.10 C_{10}' .
\]
Relations

Altmannshofer, Straub, EPJ C73 (2013) 2646

\[ A_{FB} \] vs. \( q^2 [\text{GeV}^2] \)

\[ S_5 \] vs. \( q^2 [\text{GeV}^2] \)

**Parameters:**
- \( C_{NP} = -0.1 \), \( C_1 = -0.2 \)
- \( C_{NP} = -1.5 \), \( C_1 = 1.5 \)
- \( C_{NP} = -2 \), \( C_1 = -1 \)
Branching Ratio measurement: Usual Strategy

- In each $q^2$ bin extract number of signal candidates with Maximum Likelihood Fit to $K\mu^+\mu^-$ invariant mass.

- Differential Branching Ratio:

$$\frac{dBR}{dq^2} = \frac{N(B \rightarrow K\mu^+\mu^-)}{N(B \rightarrow J/\psi(\rightarrow \mu^+\mu^-)K)} \cdot \frac{\varepsilon(B \rightarrow J/\psi(\rightarrow \mu^+\mu^-)K)}{\varepsilon(B \rightarrow K\mu^+\mu^-)} \cdot \frac{BR(B \rightarrow J/\psi K)BR(J/\psi \rightarrow \mu^+\mu^-)}{q_{\text{max}}^2 - q_{\text{min}}^2}$$

Normalize to control channel

Relative reconstruction and selection efficiency from simulation

Branching Ratio of normalization channel from PDG
Angular Observables

Differential Decay Rate

\[
\frac{d^4 \Gamma[B^0 \rightarrow K^{*0} \mu^+ \mu^-]}{dq^2 \ d\Omega} = \frac{9}{32\pi} \sum_{j(s,c)} I_{j(s,c)}(q^2) f_j(\Omega)
\]

\[
\frac{d^4 \bar{\Gamma}[B^0 \rightarrow K^{*0} \mu^+ \mu^-]}{dq^2 \ d\Omega} = \frac{9}{32\pi} \sum_{j(s,c)} \bar{I}_{j(s,c)}(q^2) f_j(\Omega)
\]

CP-averaged

\[
S_i = (I_j + \bar{I}_j)/(\frac{d\Gamma}{dq^2} + \frac{d\bar{\Gamma}}{dq^2})
\]

Asymmetry

\[
A_i = (I_j - \bar{I}_j)/(\frac{d\Gamma}{dq^2} + \frac{d\bar{\Gamma}}{dq^2})
\]

\[
\frac{1}{d(\Gamma + \bar{\Gamma})/dq^2 \ d\Omega} \bigg|_p = \frac{9}{32\pi} \left[\frac{3}{4}(1 - F_L) \sin^2 \theta_K + F_L \cos^2 \theta_K \right.
\]

\[
+ \frac{1}{4}(1 - F_L) \sin^2 \theta_K \cos 2\theta_t
\]

\[
- F_L \cos^2 \theta_K \cos 2\theta_t + S_3 \sin^2 \theta_K \sin^2 \theta_t \cos 2\phi
\]

\[
+ S_4 \sin 2\theta_K \sin 2\theta_t \cos \phi + S_5 \sin 2\theta_K \sin \theta_t \cos \phi
\]

\[
+ \frac{4}{3} A_{FB} \sin^2 \theta_K \cos \theta_t + S_7 \sin 2\theta_K \sin \theta_t \sin \phi
\]

\[
+ S_8 \sin 2\theta_K \sin 2\theta_t \sin \phi + S_9 \sin^2 \theta_K \sin^2 \theta_t \sin 2\phi \bigg]\]