Exotic Hadrons at LHCb

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On behalf of the LHCb Collaboration
• Since the inception of the quark model, hadrons beyond the usual quark, anti-quark mesons and three-quark baryons have been predicted.

• In recent years strong candidates for these exotic hadrons been observed with contributions from multiple experiments.

• In this talk I will cover results from LHCb for:


• Forward arm spectrometer designed for precision CP violation measurements and decays of bottom and charm hadrons.
• Rapidity coverage $2.0 < y < 4.5$
• Excellent particle identification:
  – Muons: $\varepsilon \sim 97\%$ for $1 - 3\% \pi \rightarrow \mu$ misidentification
  – Kaons: $\varepsilon \sim 95\%$ for $5\% \pi \rightarrow K$ misidentification
• Very good vertex resolution: $\sigma = 20\mu m$ impact parameter resolution
• Momentum resolution $\Delta p / p = 0.5\%$ at $20\text{GeV}$ to $0.8\%$ at $100\text{GeV}$
LHCb detector at LHC

Advantages over $e^+e^-$ B-factories:

- $\sim 1000x$ larger $b$ production rate
- **produce $b$-baryons at the same time as $B$-mesons**
- long visible lifetime of $b$-hadrons (no backgrounds from the other $b$-hadrons)

Advantages over GPDs:

- RICH detectors for $\pi/K/p$ discrimination (smaller backgrounds)
- Small event size allows large trigger bandwidth (up to 5 kHz in Run I); all devoted to flavor physics
The decay first observed by LHCb and used to measure $\Lambda_b^0$ lifetime:

- LHCb-PAPER-2013-032 (PRL 111, 102003)
$\Lambda_b^0 \rightarrow J/\psi p K^-$: unexpected structure in $m_{J/\psi p}$

- Unexpected, narrow peak in $m_{J/\psi p}$
- Many checks done to ensure it is not an “artifact” of selection:
  - Veto $B_s \rightarrow J/\psi K^- K^+$ & $B^0 \rightarrow J/\psi K^- \pi^+$ decays
  - Suppress fake tracks
  - Exclude $\Xi_b$ decays as a possible source
- Could it be a reflection of interfering $\Lambda^*$'s $\rightarrow p K^-$?
  - Full amplitude analysis absolutely necessary!
Amplitude Analysis of $\Lambda_b^0 \rightarrow J/\psi pK^-$, $J/\psi \rightarrow \mu^+\mu^-$

- Analyze all dimensions of the $\Lambda_b^0 \rightarrow J/\psi pK^-$, $J/\psi \rightarrow \mu^+\mu^-$ decay kinematics:
  - to maximize sensitivity to the decay dynamics
  - to avoid biases due to averaging over some dimensions in presence of the non-uniform detector efficiency

- Use 6D unbinned maximum likelihood fit of the matrix element parameters
- Two different background subtraction methods:
  - parametrized $m_{J/\psi pK}$ sidebands (cFit) or sWeighted log-likelihood (sFit)
Unbinned 6D maximum likelihood fit

- Fitted parameters (helicity couplings, $M_0, \Gamma_0$)

\[
P_{\text{sig}}(m_{Kp}, \Omega | \vec{\omega}) = \frac{1}{I(\vec{\omega})} |\mathcal{M}(m_{Kp}, \Omega | \vec{\omega})|^2 \Phi(m_{Kp}) \epsilon(m_{Kp}, \Omega), \tag{69}
\]

where $\Phi(m_{Kp})$ is the phase space function equal to $pq$, where $p$ is the momentum of the $Kp$ system (i.e. $\Lambda^*$) in the $\Lambda_0^0$ rest frame, and $q$ is the momentum of $K^\mp$ in the $\Lambda^*$ rest frame, and $I(\vec{\omega})$ is the normalization integral.

\[
I(\vec{\omega}) \equiv \int |\mathcal{M}(m_{Kp}, \Omega | \vec{\omega})|^2 \Phi(m_{Kp}) \epsilon(m_{Kp}, \Omega) \, dm_{Kp} \, d\Omega
\]

\[
\propto \sum_{j}^{N_{MC}} w_j^{MC} |\mathcal{M}(m_{Kp_j}, \Omega_j | \vec{\omega})|^2,
\]

Corrections improving MC simulations

In the fit we minimize:

\[
-2 \ln \mathcal{L}(\vec{\omega}) = -2s_W \sum_i^\Gamma W_i \ln \mathcal{P}(m_{Kp_i}, \Omega_i | \vec{\omega})
\]

\[
s_W \equiv \sum_i W_i / \sum_i W_i^2
\]

Possible data event weights (cFit or sFit).
$W_i$ are sWeights based on the fit to $m_{J/\psi pK}$ distribution. Negative weights correspond to background events, and are used to subtract the background in all 6D. The data in the extended $m_{J/\psi pK}$ range including the sidebands is passed to the amplitude fit.

Since the events are weighted in the log-likelihood this is “quasi” maximum-likelihood fit:

$$-2 \ln \mathcal{L}(\bar{\omega}) = -2s_W \sum_i W_i \ln \mathcal{P}_{\text{sig}}(m_{Kp\ i}, \Omega_i | \bar{\omega})$$

$$= -2s_W \sum_i W_i \ln |\mathcal{M}(m_{Kp\ i}, \Omega_i | \bar{\omega})|^2 + 2s_W \ln I(\bar{\omega}) \sum_i W_i - 2s_W \sum_i W_i \ln[\Phi(m_{Kp\ i})r(m_{Kp\ i}, \Omega_i)].$$

No need for parameterization of the signal efficiency.
\( W_i = 1 \) no event weights; true maximum likelihood fit

Data only in the \( \Lambda_b^0 \) peak region passed to the amplitude fit.

Sideband data used to construct 6D model of the background:

\[
P^u_{\text{bkg}}(m_{K^p} j, \Omega_j)
\]

\[
P(m_{K^p}, \Omega | \vec{\omega}) = (1 - \beta) P_{\text{sig}}(m_{K^p}, \Omega | \vec{\omega}) + \beta P_{\text{bkg}}(m_{K^p}, \Omega)
\]

\[
\beta = 5.4\% \text{ background fraction}
\]

\[
-2 \ln L(\vec{\omega}) = 
-2 \sum_i \ln \left[ (1 - \beta) \frac{|M(m_{K^p i}, \Omega_i | \vec{\omega})|^2 \Phi(m_{K^p i}) \epsilon(m_{K^p i}, \Omega_i)}{I(\vec{\omega})} + \beta \frac{P^u_{\text{bkg}}(m_{K^p i}, \Omega_i)}{I_{\text{bkg}}} \right]
\]

\[
= -2 \sum_i \ln \left[ |M(m_{K^p i}, \Omega_i | \vec{\omega})|^2 + \frac{\beta I(\vec{\omega})}{(1 - \beta) I_{\text{bkg}}} \frac{P^u_{\text{bkg}}(m_{K^p i}, \Omega_i)}{\Phi(m_{K^p i}) \epsilon(m_{K^p i}, \Omega_i)} \right] + 2N \ln I(\vec{\omega}) + \text{constant},
\]

\[
I_{\text{bkg}} \equiv \int P^u_{\text{bkg}}(m_{K^p}, \Omega) \, dm_{K^p} \, d\Omega \propto \sum_j w^\text{MC}_j \frac{P^u_{\text{bkg}}(m_{K^p j}, \Omega_j)}{\Phi(m_{K^p j}) \epsilon(m_{K^p j}, \Omega_j)}.
\]

The background term is then efficiency-corrected so it can be added to the efficiency-independent signal probability expressed by \( |M|^2 \). This way the efficiency parametrization, \( \epsilon(m_{K^p}, \Omega) \), influences only the background component which affects only a tiny part of the total PDF.
Helicity Formalism

- We write down matrix element for these decays using the helicity formalism.
  - Allows for analyzing the data in all relevant kinematic variables, while taking into account the correlations between them.
  - This is formalism which allows us to measure quantum numbers of these states.
- In the helicity formalism, each sequential decay $A \to BC$ adds a term:

\[
\mathcal{H}_{\lambda_B, \lambda_C}^{A \to BC} \ D_{\lambda_A, \lambda_B - \lambda_C}^{J_A} (\phi_B, \theta_A, 0) \ * R_A(m_{BC})
\]

- $\lambda$ labels the helicity, and $R_A(m_{BC})$ is the resonance parametrization used if A has a non-negligible natural width.
- The three arguments of Wigner’s D-matrix are Euler angles describing the rotation of the initial coordinate system with the z-axis along the helicity axis of A to the coordinate system with the z-axis along the helicity axis of B.
\[ \Lambda^* \text{ Matrix Element} \]

\[
\mathcal{M}_{\lambda_0, \lambda_p, \Delta \lambda_\mu}^{\Lambda^*} \equiv \sum_n \sum_{\lambda_{\Lambda^*}} \sum_{\lambda_\psi} \mathcal{H}_{\lambda_0, \lambda_{\Lambda^*}, \lambda_\psi}^{\Lambda^* \rightarrow \Lambda_0, \psi} D \frac{1}{\Delta \lambda_{\Lambda^*}} D_{\lambda_\psi, \Delta \lambda_\mu} (0, \theta_{\Lambda^*}, 0)^* \]

4-6 independent **complex** helicity couplings per \( \Lambda_n^* \) resonance

6 independent data variables:
1 mass, 5 angles

\[
R_X (m) = B'_{L_X} \left( \frac{p}{M_{\Lambda^*_B}} \right)^{L_X} \]

Blatt-Weisskopf functions

Breit-Wigner
**Λ**\(^*\) resonance model

- Helicity couplings are rewritten in terms of LS couplings:

\[
\mathcal{H}_{A \rightarrow BC}^{L,B,\Lambda_C} = \sum_L \sum_S \sqrt{\frac{2L+1}{2J_A+1}} B_{L,S} \left( \begin{array}{cc} J_B & J_C \\ \lambda_B & -\lambda_C \end{array} \right) \times \left( \begin{array}{c} S \\ \lambda_B - \lambda_C \end{array} \right) \times \left( \begin{array}{c} L \\ 0 \end{array} \right) \left( \begin{array}{c} S \\ \lambda_B - \lambda_C \end{array} \right)
\]

<table>
<thead>
<tr>
<th>State</th>
<th>(J^P)</th>
<th>(M_0) (MeV)</th>
<th>(\Gamma_0) (MeV)</th>
<th># Reduced</th>
<th>All states, (L)</th>
<th># Extended</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\Lambda(1405))</td>
<td>1/2(^-)</td>
<td>1405.1(^{+1.3}_{-1.0})</td>
<td>50.5 (\pm) 2.0</td>
<td>3</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>(\Lambda(1520))</td>
<td>3/2(^-)</td>
<td>1519.5 (\pm) 1.0</td>
<td>15.6 (\pm) 1.0</td>
<td>5</td>
<td>6</td>
<td></td>
</tr>
<tr>
<td>(\Lambda(1600))</td>
<td>1/2(^+)</td>
<td>1600</td>
<td>150</td>
<td>3</td>
<td>4</td>
<td></td>
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<tr>
<td>(\Lambda(1670))</td>
<td>1/2(^-)</td>
<td>1670</td>
<td>35</td>
<td>3</td>
<td>4</td>
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<tr>
<td>(\Lambda(1690))</td>
<td>3/2(^-)</td>
<td>1690</td>
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<td>(\Lambda(1800))</td>
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<td>1800</td>
<td>300</td>
<td>4</td>
<td>4</td>
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<tr>
<td>(\Lambda(1810))</td>
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<td>1810</td>
<td>150</td>
<td>3</td>
<td>4</td>
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<td>(\Lambda(1820))</td>
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<td>(\Lambda(1890))</td>
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<td>(\Lambda(2100))</td>
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<td>2100</td>
<td>200</td>
<td>1</td>
<td>6</td>
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<tr>
<td>(\Lambda(2110))</td>
<td>5/2(^+)</td>
<td>2110</td>
<td>200</td>
<td>1</td>
<td>6</td>
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<tr>
<td>(\Lambda(2350))</td>
<td>9/2(^+)</td>
<td>2350</td>
<td>150</td>
<td>0</td>
<td>6</td>
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<tr>
<td>(\Lambda(2585))</td>
<td>5/2(^-)</td>
<td>(\approx 2585)</td>
<td>200</td>
<td>0</td>
<td>6</td>
<td></td>
</tr>
</tbody>
</table>

No high-\(J^P\) high-mass states

All known \(\Lambda\(^*\)\) states

PRL 115, 07201 (2015)

# of fit parameters: 64, 146
Fit with $\Lambda^{*}\rightarrow pK^-$ contributions only

- Use extended model, so all possible known $\Lambda^*$ amplitudes: $m_{Kp}$ looks fine, but not $m_{J/\psi p}$
- Additions of non-resonant term, $\Sigma^*$’s or extra $\Lambda^*$’s doesn’t help
P\textsubscript{c}\textsuperscript{+} Matrix Element

1 mass (m\textsubscript{J/\psi}), 6 angles all derivable from the \Lambda\textsuperscript{*} decay variables

One more angle than in \Lambda\textsuperscript{*} decay: P\textsubscript{c}\textsuperscript{+} production angles must be defined relative to the \Lambda\textsubscript{b} reference frame established for \Lambda\textsubscript{b} \rightarrow J/\psi \Lambda\textsuperscript{*} decay

3-4 independent complex helicity couplings per P\textsubscript{c,j}\textsuperscript{+} resonance depending on its J\textsuperscript{P}

\[
\mathcal{M}_{\Lambda_0^b, \lambda_{P_c}, \Delta \lambda_{P_c}}^{P_c} = \sum_j \sum_{\lambda_{P_c}} \sum_{\lambda_{P_c}} \sum_{\lambda_{P_c}^0} \mathcal{H}_{\Lambda_0^b \rightarrow P_{c,j}} D^{1/2} \mathcal{H}_{\Lambda_0^b, \lambda_{P_c}, \lambda_{P_c}^0} (\phi_{P_c}, \theta_{P_c}, 0)^* R_{P_{c,j}} (m_{\psi}) D^{1/2} \mathcal{H}_{\lambda_{P_c}, \Delta \lambda_{P_c}^0} (\phi_{P_c}, \theta_{P_c}, 0)^*
\]

Breit-Wigner

Blatt-Weisskopf functions
\[ |\mathcal{M}|^2 = \sum_{\lambda_{\Lambda b}^0} \sum_{\lambda_p} \sum_{\Delta \lambda_\mu} |\mathcal{M}_{\Lambda b, \lambda_p, \Delta \lambda_\mu}^{\Lambda^*} \mathcal{M}_{\Lambda b, \lambda_p, \Delta \lambda_\mu}^{P_c^+}|^2 + e^{i \Delta \lambda_\mu \alpha_{\mu}} \sum_{\lambda_{Pc}} \frac{1}{2} d_{\lambda_p} \theta_p \rho_{\mu} |\mathcal{M}_{\Lambda b, \lambda_p, \Delta \lambda_\mu}^{P_c^+}|^2 \]

- Without this realignment can’t describe \( \Lambda^* \) plus \( P_c^+ \) interferences properly
- They integrate out to zero in full phase-space but present in the differential 6D fit-PDF

\[ \Lambda^* \text{ Plus } P_c^+ \text{ Matrix Element} \]
Fit with $\Lambda^*$'s and one $P_c^+ \rightarrow J/\psi p$ state

- Try all $J^P$ of $P_c^+$ up to $7/2^\pm$
- Best fit has $J^P = 5/2^\pm$. Still not a good fit

PRL 115, 07201 (2015)
Fit with $\Lambda^*$'s and two $P_c^+\rightarrow J/\psi p$ states

- Obtain good fits even with the reduced $\Lambda^*$ model
- Best fit has $J^P=(3/2^-, 5/2^+)$, also $(3/2^+, 5/2^-)$ & $(5/2^+, 3/2^-)$ are preferred

PRL 115, 07201 (2015)
Fit with $\Lambda^*$'s and two $P_c^+\rightarrow J/\psi p$ states

- Need for the 2$^{\text{nd}}$ broad $P_c^+$ state becomes visually apparent in the region where the $\Lambda^*\rightarrow pK^-$ background is the smallest.
Angular distributions

All data

- Good description of the data in all 6 dimensions!

P_{c} enriched region

- LHCb all \( m_{K\pi} \)

- LHCb \( m_{K\pi} > 2 \) GeV

\( \Lambda^{*} \) interferences

\( \Lambda_{b}^{0} \)

\( \phi_{K} \)

\( \phi_{\mu} \)

\( \cos\theta_{\Lambda^{*}} \)

\( \cos\theta_{J/\psi} \)

\( \cos\theta_{\Lambda_{b}^{0}} \)

\( \cos\theta_{J/\psi} \)

PRL 115, 07201 (2015)
No need for exotic J/ψK⁻ contributions

- J/ψK⁻ system is well described by the Λ⁺ and Pₖ⁺ reflections.

\[ m_{Kp} < 1.55 \text{ GeV} \]
\[ 1.55 < m_{Kp} < 1.70 \text{ GeV} \]
\[ 1.70 < m_{Kp} < 2.00 \text{ GeV} \]
\[ m_{Kp} > 2.00 \text{ GeV} \]
Data preference for opposite parity $P_c^+$ states

- $m_{Kp} < 1.55$ GeV
- $1.55 < m_{Kp} < 1.70$ GeV
- $1.70 < m_{Kp} < 2.00$ GeV

Positive interference between the $P_c$ states

Negative interference between the $P_c$ states

This interference pattern only for states with opposite parity
## Systematic uncertainties

<table>
<thead>
<tr>
<th>Source</th>
<th>$M_0$ (MeV) low</th>
<th>$M_0$ high</th>
<th>$\Gamma_0$ (MeV) low</th>
<th>$\Gamma_0$ high</th>
<th>Fit fractions ($%$) high</th>
<th>$\Lambda(1405)$</th>
<th>$\Lambda(1520)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Extended vs. reduced</td>
<td>21</td>
<td>0.2</td>
<td>54</td>
<td>10</td>
<td>3.14</td>
<td>0.32</td>
<td>1.37</td>
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<tr>
<td>$\Lambda^*$ masses &amp; widths</td>
<td>7</td>
<td>0.7</td>
<td>20</td>
<td>4</td>
<td>0.58</td>
<td>0.37</td>
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<td>1</td>
<td>2</td>
<td>0.27</td>
<td>0.14</td>
<td>0.20</td>
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<tr>
<td>$10 &lt; p_p &lt; 100$ GeV</td>
<td>0</td>
<td>1.2</td>
<td>1</td>
<td>1</td>
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<td>Nonresonant</td>
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<td>2</td>
<td>2.35</td>
<td>0.13</td>
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<td>Separate sidebands</td>
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<td>0</td>
<td>0.24</td>
<td>0.14</td>
<td>0.02</td>
</tr>
<tr>
<td>$J^P$ (3/2$^+$, 5/2$^-$) or (5/2$^+$, 3/2$^-$)</td>
<td>10</td>
<td>1.2</td>
<td>34</td>
<td>10</td>
<td>0.76</td>
<td>0.44</td>
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</tr>
<tr>
<td>$d = 1.5 - 4.5$ GeV$^{-1}$</td>
<td>9</td>
<td>0.6</td>
<td>19</td>
<td>3</td>
<td>0.29</td>
<td>0.42</td>
<td>0.36</td>
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<tr>
<td>$L_{P_c}^{P_c} \Lambda_b^0 \rightarrow P_c^+(\text{low/high}) K^-$</td>
<td>6</td>
<td>0.7</td>
<td>4</td>
<td>8</td>
<td>0.37</td>
<td>0.16</td>
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<tr>
<td>$L_{P_c}^{P_c} P_c^+(\text{low/high}) \rightarrow J/\psi \Lambda$</td>
<td>4</td>
<td>0.4</td>
<td>31</td>
<td>7</td>
<td>0.63</td>
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<tr>
<td>$L_{P_c}^{Lambda^<em>} \Lambda_b^0 \rightarrow J/\psi \Lambda^</em>$</td>
<td>11</td>
<td>0.3</td>
<td>20</td>
<td>2</td>
<td>0.81</td>
<td>0.53</td>
<td>3.34</td>
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<td>Efficiencies</td>
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<td>4</td>
<td>0</td>
<td>0.13</td>
<td>0.02</td>
<td>0.26</td>
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<tr>
<td>Change $\Lambda(1405)$ coupling</td>
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<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1.90</td>
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<td>Overall</td>
<td>29</td>
<td>2.5</td>
<td>86</td>
<td>19</td>
<td>4.21</td>
<td>1.05</td>
<td>5.82</td>
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<td>sFit/cFit cross check</td>
<td>5</td>
<td>1.0</td>
<td>11</td>
<td>3</td>
<td>0.46</td>
<td>0.01</td>
<td>0.45</td>
</tr>
</tbody>
</table>

- Uncertainties in the $\Lambda^*$ model dominate
Additional cross-checks

- Many additional cross-checks have been done. Some are listed here:
  - The same $P_c^+$ structure found using very different selections by different LHCb teams
  - Two independently coded fitters using different background subtractions (cFit & sFit)
  - Split data shows consistency: 2011/2012, magnet up/down, $\Lambda_b/\Lambda_b$, $\Lambda_b(p_T \text{ low})/\Lambda_b(p_T \text{ high})$
  - Extended model fits tried without $P_c$ states, but with two additional high mass $\Lambda^*$ resonances allowing masses & widths to vary, or 4 non-resonant terms of $J$ up to $3/2$
Significances and the results
• Fit improves greatly, for 1 $P_c \Delta(-2\ln\mathcal{L})=14.7^2$, adding the 2nd $P_c$ improves by 11.6$^2$, for adding both together $\Delta(-2\ln\mathcal{L})=18.7^2$
• Simulations of pseudoexperiments are used to turn the $\Delta(-2\ln\mathcal{L})$ values to significances:
  – significance of $P_c(4450)^+$ state is 12$\sigma$
  – significance of $P_c(4380)^+$ state is 9$\sigma$
  – combined significance of the two $P_c^+$ states is 15$\sigma$
• This includes the dominant systematic uncertainties, coming from difference between extended and reduced $\Lambda^*$ model results.
• Parameters of the $P_c^+$ states (and F.F. of well isolated $\Lambda^*$’s )

<table>
<thead>
<tr>
<th>State</th>
<th>Mass (MeV)</th>
<th>Width (MeV)</th>
<th>Fit fraction (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_c(4450)^+$</td>
<td>4380 ±8±29</td>
<td>205±18±86</td>
<td>8.4±0.7±4.2</td>
</tr>
<tr>
<td>$P_c(4380)^+$</td>
<td>4449.8±1.7±2.5</td>
<td>39± 5±19</td>
<td>4.1±0.5±1.1</td>
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<tr>
<td>$\Lambda(1405)$</td>
<td></td>
<td></td>
<td>15±1±6</td>
</tr>
<tr>
<td>$\Lambda(1520)$</td>
<td></td>
<td></td>
<td>19±1±4</td>
</tr>
</tbody>
</table>
Resonance Phase Motion

- The Breit-Wigner shape for individual $P_c$'s is replaced with 6 independent amplitudes in $M_0 \pm \Gamma_0$

- $P_c(4450)$: shows resonance behavior: a rapid counter-clockwise change of phase across the pole mass

- $P_c(4380)$: does show large phase change, but is not conclusive

![Breit-Wigner Prediction Fitted Values](image)

Plot fitted values for amplitudes in an Argand diagram
$Z(4430)^+$
\( \mathbf{Z}^+ \mathbf{K}^* \)  previous measurements

\[ \mathbf{B} \to \psi' \pi^+ \mathbf{K} \]

Belle 2008

1D M(\( \psi' \pi^- \)) mass fit

("\( \mathbf{K}^* \) veto region")

1D M(\( \psi' \pi^- \)) mass fit

PRD 79, 112001 (2009)

\[ \mathbf{Z}(4430)^+ \]

\( \mathbf{K}^* \to \mathbf{K} \pi^+ \) bkg.

non-B bkg

M(\( \psi' \pi^- \)) GeV

Bkg. subtracted efficiency corrected

Almost \textbf{model independent} approach to \( \mathbf{K}^* \to \mathbf{K} \pi^+ \) backgrounds.

Ad hoc assumption about the \( \mathbf{K}^* \to \mathbf{K} \pi^+ \) background shape.

\[ \text{M}(\mathbf{Z}) = 4433 \pm 4 \pm 2 \text{ MeV} \]

\[ \text{\( \Gamma(\mathbf{Z}) = 45^{+18}_{-13}^{+30}_{-13} \text{ MeV} \)} \]

significance 6.5\( \sigma \)

BaBar 2009

Harmonic moments of \( \mathbf{K}^* \)s (2D) reflected to M(\( \psi' \pi^+ \))

Belle 1D 4D

PRD 88, 074026 (2013)

(2D amplitude fit in 2009)

\[ 0.996 \text{ GeV/c}^2 < \text{M}(\mathbf{K}, \pi) < 1.332 \text{ GeV/c}^2 \]

("\( \mathbf{K}^* \) veto region")

\[ \psi(\mathbf{2S}) \pi^+ \mathbf{K} \text{ moments} \]

\[ \mathbf{J}/\pi \mathbf{K} \text{ moments} \]

BaBar did not confirm \( \mathbf{Z}(4430)^+ \) in B sample comparable to Belle.

Did not numerically contradict the Belle results.

\[ \text{M}(\mathbf{Z}) = 4485^{+22}_{-22}^{+28}_{-11} \text{ MeV} \]

\[ \text{\( \Gamma(\mathbf{Z}) = 200^{+41}_{-46}^{+26}_{-11} \text{ MeV} \)} \]

6.4\( \sigma \) (5.6\( \sigma \) with sys.)

J\( ^P = 1^+ \) preferred by >3.4\( \sigma \)

Model dependent approach to \( \mathbf{K}^* \to \mathbf{K} \pi^+ \) backgrounds.

Higher statistical sensitivity.

Belle 2013

(4D amplitude fit (subsample with \( \psi \to l^+ l^- \))

PRD 100, 142001 (2008)

PRD 79, 112001 (2009)

("\( \mathbf{K}^* \) veto region")

With \( \mathbf{Z}(4430)^+ \)

No Z

M(\( \psi' \pi^- \)) GeV

Bkg. subtracted efficiency corrected

Almost \textbf{model independent} approach to \( \mathbf{K}^* \to \mathbf{K} \pi^+ \) backgrounds.
Z(4430)\(^+\) At LHCb

- An order of magnitude larger signal statistics than in Belle or BaBar thanks to larger $\bar{B}B$ cross section in pp collisions

- Even smaller non-B background than at the e+e- experiments thanks to excellent performance of the LHCb detector (vertexing, PID).

Amplitude Analysis of $B^0 \rightarrow \psi' \pi^+ K^-, \psi' \rightarrow \mu^+ \mu^-$

- Analyze all dimensions of the decay kinematics:
  - 1 mass $m_{K\pi}$ and three angles $\theta_{K^*}, \theta_\psi, \phi$.
  - Analogous (not independent) Z decay variables are $m_{\psi\pi}, \theta_Z, \theta_{\psi Z}, \phi^Z$.

\[
|M|^2 = \sum_{\Delta \lambda_\mu = -1,1} \left| \sum_{\lambda_\psi = -1,0,1} \sum_{n} \mathcal{H}^{B \rightarrow \psi K^*}_{\lambda_\psi} R(m_{K\pi}) d^{J_k}_{\lambda_\psi,0} (\theta_{K^*}) e^{i \lambda_\psi \phi} d^{1}_{\lambda_\psi,\Delta \lambda_\mu} (\theta_\psi) \right|^2
\]

\[
+ \sum_{\lambda_{Z_\psi} = -1,0,1} \mathcal{H}^{B \rightarrow Z K}_{\lambda_{Z_\psi}} R(m_{\psi\pi}) d^{J_{Z}}_{0,\lambda_{Z_\psi}} (\theta_Z) e^{i \lambda_{Z_\psi} \phi^Z} d^{1}_{\lambda_{Z_\psi},\Delta \lambda_\mu} (\theta_{Z_\psi}) e^{i \Delta \lambda_\mu \alpha}
\]

\[
R_X(m) = B_{L_B}^\prime (p, p_0, d) \left( \frac{p}{M_B} \right)^{L_X^B} \text{BW}(m|M_{0X}, \Gamma_{0X}) B_{L_X}^\prime (q, q_0, d) \left( \frac{q}{m} \right)^{L_X}
\]

cFit technique used to account for the background.

Perform a 4D unbinned maximum likelihood fit of the matrix element parameters.
\( K\pi \) resonant model

<table>
<thead>
<tr>
<th>Resonance</th>
<th>( J^P )</th>
<th>Likely ( n^{2S+1}L_J )</th>
<th>Mass (MeV)</th>
<th>Width (MeV)</th>
<th>( \mathcal{B}(K^{*0} \rightarrow K^+\pi^-) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( K_0^*(800)^0 ) (( \kappa ))</td>
<td>0(^+)</td>
<td>—</td>
<td>682 ( \pm ) 29</td>
<td>547 ( \pm ) 24</td>
<td>( \sim 100% )</td>
</tr>
<tr>
<td>( K^*(892)^0 )</td>
<td>1(^-)</td>
<td>( 1^3S_1 )</td>
<td>895.94 ( \pm ) 0.262</td>
<td>48.7 ( \pm ) 0.7</td>
<td>( \sim 100% )</td>
</tr>
<tr>
<td>( K_0^*(1430)^0 )</td>
<td>0(^+)</td>
<td>( 1^3P_0 )</td>
<td>1425 ( \pm ) 50</td>
<td>270 ( \pm ) 80</td>
<td>( (93 \pm 10)% )</td>
</tr>
<tr>
<td>( K_1^*(1410)^0 )</td>
<td>1(^-)</td>
<td>( 2^3S_1 )</td>
<td>1414 ( \pm ) 15</td>
<td>232 ( \pm ) 21</td>
<td>( (6.6 \pm 1.3)% )</td>
</tr>
<tr>
<td>( K_2^*(1430)^0 )</td>
<td>2(^+)</td>
<td>( 1^3P_2 )</td>
<td>1432.4 ( \pm ) 1.3</td>
<td>109 ( \pm ) 5</td>
<td>( (49.9 \pm 1.2)% )</td>
</tr>
</tbody>
</table>

\( B^0 \rightarrow \psi(2S)K^{+}\pi^- \) phase space limit: 1593

\( K_1^*(1680)^0 \) 1\(^-\) \( 1^3D_1 \) | 1717 \( \pm \) 27 | 322 \( \pm \) 110 | \( (38.7 \pm 2.5)\% \) |
\( K_3^*(1780)^0 \) 3\(^-\) \( 1^3D_3 \) | 1776 \( \pm \) 7  | 159 \( \pm \) 21  | \( (18.8 \pm 1.0)\% \) |

• The above \( K^* \) resonances plus an additional \( K\pi \) non-resonant S-wave contribution cannot fully reproduce the data.

• As \( K_3^*(1780) \) is spin 3 and well above threshold, it is unlikely to be present in the data and is excluded from the default model.

• Alternative S-wave model uses LASS parametrization.

Z(4430)$^+$ 4D Amplitude Analysis in LHCb

- Data are well described in all dimensions when $1^+ Z(4430)$ is included!

- Improvement in fit quality is $\Delta(-2 \ln \mathcal{L}) = 18.7^2$. The significance with systematic uncertainties included is $13.9\sigma$.

Z(4430)$^+$ Comparison with Belle

- The fit results and $Z(4430)^+$ properties agree well with Belle.

- A spin-parity analysis also overwhelmingly confirms $J^P = 1^+$

<table>
<thead>
<tr>
<th></th>
<th>LHCb</th>
<th>Belle</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M(Z)$ [MeV]</td>
<td>$4475 \pm 7^{+15}_{-25}$</td>
<td>$4485 \pm 22^{+28}_{-11}$</td>
</tr>
<tr>
<td>$\Gamma(Z)$ [MeV]</td>
<td>$172 \pm 13^{+37}_{-34}$</td>
<td>$200^{+41+26}_{-46-35}$</td>
</tr>
<tr>
<td>$f_Z$ [%]</td>
<td>$5.9 \pm 0.9^{+1.5}_{-3.3}$</td>
<td>$10.3^{+3.0+4.3}_{-3.5-2.3}$</td>
</tr>
<tr>
<td>$f_Z^I$ [%]</td>
<td>$16.7 \pm 1.6^{+2.6}_{-5.2}$</td>
<td>$16.1\sigma$</td>
</tr>
<tr>
<td>Significance</td>
<td>$&gt; 13.9\sigma$</td>
<td>$&gt; 5.2\sigma$</td>
</tr>
</tbody>
</table>

Including systematic variations:

<table>
<thead>
<tr>
<th>Disfavored $J^P$</th>
<th>LHCb Rejection level relative to $1^+$</th>
<th>Belle Rejection level relative to $1^+$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0-</td>
<td>$9.7\sigma$</td>
<td>$3.4\sigma$</td>
</tr>
<tr>
<td>1-</td>
<td>$15.8\sigma$</td>
<td>$3.7\sigma$</td>
</tr>
<tr>
<td>2+</td>
<td>$16.1\sigma$</td>
<td>$5.1\sigma$</td>
</tr>
<tr>
<td>2-</td>
<td>$14.6\sigma$</td>
<td>$4.7\sigma$</td>
</tr>
</tbody>
</table>
Resonance Phase Motion

- Replace the Breit-Wigner shape with 6 independent amplitudes.

- Data points clearly show a resonant behavior on the Argand Plot.

This was the first demonstration of this type for an exotic hadron candidate.
Z(4430)$^+$ in LHCb: 2D model independent analysis (a la BaBar)

Decompose into Legendre moments

Pass only moments with $l$ not more than $l_{\text{max}} = J_{\text{max}}/2$

Excess of events over the $K^*$ $J_{\text{max}}=2$ filtered distribution in the Z(4430)$^-$ region is apparent!

Quantitative results from the model independent approach

- The procedure on the previous slide is repeated for different \( l_{\text{max}} \) contributions.
Quantitative results from the model independent approach

Test significance of implausible \( l_{\text{max}} < l < 30 \cos(\theta_{K^*}) \) moments using the log-likelihood ratio:

\[
\Delta(-2\text{NLL}) = -2 \log \frac{\mathcal{L}_{l_{\text{max}}}}{\mathcal{L}_{30}} = -2 \log \frac{\prod_i \mathcal{F}_{l_{\text{max}}}(m_i^{\psi'\pi})}{\prod_i \mathcal{F}_{30}(m_i^{\psi'\pi})}
\]

Statistical simulations of pseudo-experiments were generated from the \( l < l_{\text{max}} \) hypotheses, and \( \Delta(-2\text{NLL}) \) calculated:

**Explanation of the data with plausible \( K^* \) contributions is ruled out at high significance without assuming anything about \( K^* \) resonance shapes or their interference patterns!**
Possibility of an additional $Z^+$


- An additional $0^-$ state has $6\sigma$ significance. $0^-$ is preferred over $1^-, 2^+, 2^-$ by $8\sigma$. $1^+$ cannot be ruled out, although it is unreasonably wide at $660 \pm 150$ MeV.

- Argand diagram for this state is inconclusive, and there is no evidence for it in the model independent approach.
- More data is needed to clarify.
X(3872)
X(3872) quantum numbers

- Quantum numbers were narrowed down by CDF to be $1^{++}$ or $2^{-+}$.
- LHCb used $B^+ \to X(3872)K^+$, with $X(3872) \to J/\psi \pi^+\pi^-$, and $J/\psi \to \mu^+\mu^-$ to pin down the $J^{PC}$ to be $1^{++}$.
- All analyses had assumed the presence of only the lowest possible orbital angular momentum in the $X(3872) \to J/\psi \pi^+\pi^-$ decays.

- However a significant $L > L_{\text{min}}$ could invalidate $J^P=1^+$ assignment.
  - It is important to check this!

- Using improved statistics from the full 3 fb$^{-1}$ of Run 1 data, the analysis was done without any assumptions on orbital angular momentum present.
Determination of $X(3872)$ $J^{PC}$: helicity formalism

Matrix element

$$|M|^2 = \sum_{\Delta\lambda_\mu=-1,1} \sum_{\lambda_{J/\psi}, \lambda_\rho=-1,0,1} \mathcal{H}_{\lambda_{J/\psi}, \lambda_\rho}^{X(3872) \rightarrow J/\psi \rho} \times D_{0, \lambda_{J/\psi} - \lambda_\rho}^{J_X}(0, \theta_X, 0)^* \times D_{\lambda_\rho, 0}^{1}(\phi_\rho, \theta_\rho, 0)^* \times D_{\lambda_{J/\psi}, \Delta\lambda_\mu}^{1}(\phi_{J/\psi}, \theta_{J/\psi}, 0)^*$$

$$\mathcal{H}_{\lambda_{J/\psi}, \lambda_\rho}^{X(3872) \rightarrow J/\psi \rho} = \sum_L \sum_S B_{LS} \times \left( \begin{array}{c|cc} J_{J/\psi} & J_\rho & \lambda_\rho \\ \hline \lambda_{J/\psi} & -\lambda_\rho & \lambda_{J/\psi} - \lambda_\rho \\ \end{array} \right) \times \left( \begin{array}{c|cc} S & \lambda_{J/\psi} - \lambda_\rho \\ \hline L & \lambda_{J/\psi} - \lambda_\rho \\ \end{array} \right)$$

$$P_X = P_{J/\psi} P_{\rho} (-1)^L = (-1)^L$$

<table>
<thead>
<tr>
<th>$J^{PC}$</th>
<th>all $L$</th>
<th>$B_{LS}$</th>
<th>minimal $L$</th>
<th>$B_{LS}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0$^+$</td>
<td>$B_{11}$</td>
<td>$B_{11}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0$^+$</td>
<td>$B_{00}, B_{22}$</td>
<td>$B_{00}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1$^+$</td>
<td>$B_{10}, B_{11}, B_{12}, B_{32}$</td>
<td>$B_{10}, B_{11}, B_{12}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1$^+$</td>
<td>$B_{01}, B_{21}, B_{22}$</td>
<td>$B_{01}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2$^-$</td>
<td>$B_{11}, B_{12}, B_{31}, B_{32}$</td>
<td>$B_{11}, B_{12}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2$^+$</td>
<td>$B_{02}, B_{20}, B_{21}, B_{22}, B_{42}$</td>
<td>$B_{02}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3$^-$</td>
<td>$B_{12}, B_{30}, B_{31}, B_{32}, B_{52}$</td>
<td>$B_{12}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3$^+$</td>
<td>$B_{21}, B_{22}, B_{41}, B_{42}$</td>
<td>$B_{21}, B_{22}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4$^+$</td>
<td>$B_{31}, B_{32}, B_{51}, B_{52}$</td>
<td>$B_{31}, B_{32}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4$^+$</td>
<td>$B_{22}, B_{40}, B_{41}, B_{42}, B_{62}$</td>
<td>$B_{22}$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

LHCb 2013

CDF 2007

LHCb 2015

Many more amplitudes to fit

LS amplitudes to be determined from the data
Projection of all angles with $1^{++}$ hypothesis

- Best fit to the data is obtained with the $1^{++}$ hypothesis.
- Good description in all dimensions.
Likelihood ratio tests were performed to quantify level of rejection of other hypotheses.

- Data unambiguously prefers $1^{++}$ hypothesis (new: no assumptions about $L$)
D-wave fraction in $X(3872) \rightarrow \rho^0 J/\psi$ for $J^{PC}=1^{++}$

- Fit to the data:

$$\frac{|B_{21}|^2}{|B_{10}|^2} = 0.0018 \pm 0.0042 \quad \frac{|B_{22}|^2}{|B_{10}|^2} = 0.0066 \pm 0.0081$$

D-wave amplitudes are consistent with zero
D-wave significance using Wilk’s theorem applied to the likelihood ratio with/without D wave amplitudes: 0.8σ

The likelihood was profiled as a function of the D-wave fraction:

$$f_D = \frac{\int |\mathcal{M}(\Omega)_D|^2 d\Omega}{\int |\mathcal{M}(\Omega)_{S+D}|^2 d\Omega}$$

$f_D<4\%$ at 95\% CL
Different types of tetra- and penta-quarks

- "plain"
- Diquark model
- Triquark model
- Hydro-charmonium model
- Molecular model

[Diagram showing different models of quarks and mesons]
Conclusions

• Two pentaquark candidates decaying to J/ψp have been observed by LHCb with overwhelming significance in an amplitude analysis.

• The existence of Z(4430) has been decisively confirmed in both a model independent and model dependent way, the latter of which was used to measure its properties.

• The quantum numbers of X(3872) have been measured without any assumptions on the angular momentum present in its decays. The D wave fraction has also been measured.

• With the Run 1 data set, very exciting measurements have been made that will serve to teach us much about exotic hadrons. Many new questions have also been raised.

• As we continue to fully utilize the Run 1 data, increased statistics on the way will allow for more sensitive tests.
  • LHCb expects 8 fb$^{-1}$ in Run 2 (-2018) followed by the detector/luminosity upgrade which will bring ~50 fb$^{-1}$ by 2028.

• We of course also look forward to hints from theory and other experiments.
BACKUP SLIDES
Complete set of fit fractions

Table 3: Fit fractions of the different components from cFit and sFit for the default \((3/2^-, 5/2^+)\) model. Uncertainties are statistical only.

<table>
<thead>
<tr>
<th>Particle</th>
<th>Fit fraction (%) cFit</th>
<th>Fit fraction (%) sFit</th>
</tr>
</thead>
<tbody>
<tr>
<td>(P_c(4380)^+)</td>
<td>8.42 ± 0.68</td>
<td>7.96 ± 0.67</td>
</tr>
<tr>
<td>(P_c(4450)^+)</td>
<td>4.09 ± 0.48</td>
<td>4.10 ± 0.45</td>
</tr>
<tr>
<td>(\Lambda(1405))</td>
<td>14.64 ± 0.72</td>
<td>14.19 ± 0.67</td>
</tr>
<tr>
<td>(\Lambda(1520))</td>
<td>18.93 ± 0.52</td>
<td>19.06 ± 0.47</td>
</tr>
<tr>
<td>(\Lambda(1600))</td>
<td>23.50 ± 1.48</td>
<td>24.42 ± 1.36</td>
</tr>
<tr>
<td>(\Lambda(1670))</td>
<td>1.47 ± 0.49</td>
<td>1.53 ± 0.50</td>
</tr>
<tr>
<td>(\Lambda(1690))</td>
<td>8.66 ± 0.90</td>
<td>8.60 ± 0.85</td>
</tr>
<tr>
<td>(\Lambda(1800))</td>
<td>18.21 ± 2.27</td>
<td>16.97 ± 2.20</td>
</tr>
<tr>
<td>(\Lambda(1810))</td>
<td>17.88 ± 2.11</td>
<td>17.29 ± 1.85</td>
</tr>
<tr>
<td>(\Lambda(1820))</td>
<td>2.32 ± 0.69</td>
<td>2.32 ± 0.65</td>
</tr>
<tr>
<td>(\Lambda(1830))</td>
<td>1.76 ± 0.58</td>
<td>2.00 ± 0.53</td>
</tr>
<tr>
<td>(\Lambda(1890))</td>
<td>3.96 ± 0.43</td>
<td>3.97 ± 0.38</td>
</tr>
<tr>
<td>(\Lambda(2100))</td>
<td>1.65 ± 0.29</td>
<td>1.94 ± 0.28</td>
</tr>
<tr>
<td>(\Lambda(2110))</td>
<td>1.62 ± 0.32</td>
<td>1.44 ± 0.28</td>
</tr>
</tbody>
</table>
Extended Model with Two $P_c$ Resonances