Angular analysis prospects in $b \rightarrow s\mu\mu$

Biplab Dey

Marseille Workshop 2015
**THE $b \to s \ell^+ \ell^-$ “INDUSTRY” AT THE LHC**

- Flavor-changing-neutral-current (FCNC).
- No tree-level diagram in the SM. Many ways where NP can enter.

Several ways to explore this:

- $B_s \to \mu^+ \mu^-$ BF @ LHCb/CMS
- $B \to K^{*J} \gamma_{\text{pol}}$ @ LHCb
- $B_d \to K^{(*)}\ell^- \ell^+$ @ LHCb/CMS/ATLAS
- $B_s \to \phi\mu^+ \mu^-$, $\Lambda_b \to \Lambda^{(*)}\mu^+ \mu^-$ ...

$b \quad W^- / ?\quad s$
$t / ?$

$\mu^+$
$\mu^-$

$Z^*/\gamma^* / ?$
Suite of anomalies...

$K^+$

$K_S$

$K^{*+}$

$B_s \rightarrow \phi \mu \mu$ (3.3 $\sigma$)

$\Lambda_b \rightarrow \Lambda \mu \mu$

$R_K$

2.6$\sigma$ tension

$B_{s} \rightarrow K^{+} \mu^{+} \mu^{-}$

$B_{s} \rightarrow K^{0} \mu^{+} \mu^{-}$

$B_{s} \rightarrow K^{*+} \mu^{+} \mu^{-}$

$B_{s} \rightarrow \phi \mu \mu$

$\Lambda_{b} \rightarrow \Lambda \mu \mu$

LHCb

PRL 112, 161801 (2014)

JHEP 06 (2014) 133

JHEP 09 (2015) 179

JHEP 06 (2015) 115

lepton asym.
Motivation for angular analyses

- Lesson from $P_5'$: anomalies can show up in hitherto unexpected places.

- Angular observables being interference terms have more sensitivity than rates. Good bet for NP hunting.
\( B_d \rightarrow K \pi \mu \mu \) AND FRIENDS

- \( b \rightarrow X(\rightarrow h_1 h_2)\ell_1 \ell_2 \) topology: \( \Phi \in \{m_X, q^2, \theta_\ell, \theta_\nu, \chi\} \)

Focus of this talk: wider window in the \( m_X \) spectrum.

Higher waves in the dihadron system \( \Rightarrow \) more observables to play around with.
The golden modes

- $\bar{B} \rightarrow K^- \pi^+ \mu\mu$
  - Flavor-tagged, but unpolarized.
  - After LHCb Run II $\sim 12000 \ K^*$. Everybody’s favorite EWP!

- $\bar{B} \rightarrow \{\pi\pi, D^*\} \ell^- \bar{\nu}_\ell$, hadronic tagged
  - Fully flavor-tagged and polarized.
  - Belle II $\rho \sim 50000$. $D^*$: $\times 20$.
  - RH currents, incl./excl. $|V_{ub}|/|V_{cb}|$ puzzle.

Prelim. LHCb MC:

Prelim. BABAR MC:

- Experimentally, easiest/cleanest. Percent-level resolutions in both.
The not-so optimal

- $B_s \rightarrow KK\mu\mu$: flavor-averaging is a dampener. Also, not polarized. After LHCb Run II, $\sim 2500 \ KK \ (\phi + f'(1525) + \ldots)$ events.

- For every two $B_d$'s in LHCb acceptance, we produce one $\Lambda_b$. Richer spinor angular structure than the pseudoscalars in $B_d/s$.
  
  - $\Lambda_b \rightarrow pK\mu\mu$: flavor-tagged, but unpolarized. Around 1500 signal, after Run II. Very poorly known $m(pK)$ spectrum.

  - $\Lambda_b \rightarrow \Lambda\mu\mu$: flavor-tagged, but unpolarized. Self-analyzing $\Lambda \rightarrow p\pi$ helps. Downstream tracks (no VeLo) have lower efficiency. Roughly 1500 $\Lambda$'s expected after Run II.
**Setup for** \( B \rightarrow X^J(\rightarrow h_1 h_2)\ell_1 \ell_2 \)**

- \( X^J \in \{\pi\pi, K\pi, KK, D\pi\} \) is in spin-\( J \).

- Helicity amplitudes \( H^J_\lambda;\eta \) tagged by \( J, \eta \equiv (\lambda_{\ell_1} - \lambda_{\ell_2}) = \pm 1 \), and \( \lambda \in \{0, \pm 1\} \).

- Amplitude squared reads:

\[
|\mathcal{M}|^2 = \sum_{\eta = \pm 1} \left| \sum_{\lambda \in \{0, \pm 1\}} \sum_{J} \sqrt{2J + 1} H^J_{\lambda;\eta} d^J_{\lambda,0}(\theta_V) d^1_{\lambda,\eta}(\theta_\ell) e^{i\lambda \chi} \right|^2
\]
**Difference between SL and EWP modes**

- **For SL decays**, the LH(RH) $\nu_L(\bar{\nu}_R)$ tags the polarization of both outgoing spinors.

- This is a “complete” measurement. Observables uniquely determine the underlying amplitudes.

- **For EWP case**, outgoing muon spins not known. Dilution of information due to incoherent summation over $\eta = \pm 1$.

- This is an “incomplete” measurement. Observables do not uniquely determine the amplitudes.
The two-fold ambiguity and bilinears

- Rate is invariant under the symmetry $\mathcal{H}_\lambda^{\eta,J} \rightarrow (\mathcal{H}_\lambda^{-\eta,J})^*$. 

- Generalization of the same “two-fold ambiguity” in determination of $\sin 2\beta_{(s)}$ from $B_d \rightarrow J/\psi K^*$ and $B_s \rightarrow J/\psi \phi$.

- Consider the complex matrices: $n_{\lambda}^J \equiv \begin{pmatrix} \mathcal{H}_\lambda^{\eta,J} \\ (\mathcal{H}_\lambda^{-\eta,J})^* \end{pmatrix}$

- All observables occur as bilinears $\Gamma \sim n_i^\dagger n_j$, respecting this symmetry.
Example: \textit{SPD waves in the $[K\pi]$ system}

- 7 two-component matrices that produce 56 real bilinears:

\[
\begin{align*}
    s &= \begin{pmatrix} S_L^* \\ S_R^* \end{pmatrix} \\
    h_{\parallel} &= \begin{pmatrix} H_{\parallel}^L \\ H_{\parallel}^R \end{pmatrix} \\
    h_{\perp} &= \begin{pmatrix} H_{\perp}^L \\ -H_{\perp}^R \end{pmatrix} \\
    h_0 &= \begin{pmatrix} H_0^L \\ H_0^R \end{pmatrix} \\
    d_{\parallel} &= \begin{pmatrix} D_{\parallel}^L \\ D_{\parallel}^R \end{pmatrix} \\
    d_{\perp} &= \begin{pmatrix} D_{\perp}^L \\ -D_{\perp}^R \end{pmatrix} \\
    d_0 &= \begin{pmatrix} D_0^L \\ D_0^R \end{pmatrix}
\end{align*}
\]

- Rate comprises 41 angular observables, like:

\[
\begin{align*}
    i &\quad f_i(\Omega) &\quad \Gamma_i(q^2) \\
    1 &\quad P_0^0 Y_0^0 &\quad |s|^2 + |h_0|^2 + |h_{\parallel}|^2 + |h_{\perp}|^2 + |d_0|^2 + |d_{\parallel}|^2 + |d_{\perp}|^2 \\
    27 &\quad P_3^0 \sqrt{2} \text{Im}(Y_2^2) &\quad -\frac{3}{5} \sqrt{\frac{3}{7}} \text{Im}(h_{\parallel}^\dagger d_{\perp} + d_{\parallel}^\dagger h_{\perp}) \\
    41 &\quad P_4^1 \sqrt{2} \text{Im}(Y_1^1) &\quad -\frac{3}{7} \sqrt{10} \text{Im}(d_0^\dagger d_{\parallel})
\end{align*}
\]
Relations amongst the observables

- I can sandwich a unitary matrix inside the product as: $n_i^\dagger U^\dagger Un_j$.
- 3 generators + 1 phase, so $n_{\text{gen}} = 4$ symmetry relations.
- 14 complex amplitudes mean $n_{\text{obs}} = 2 \times 14 - 4 = 24$ independent observables.

- This means, 17 relations amongst the 41 $\Gamma_i$.

- Some are simple: $\Gamma_{25} = -\sqrt{\frac{7}{3}}\Gamma_{27}$. Some are messy:

\[
0 = \left[ \left( \sqrt{\frac{5}{3}} f_{23} + \frac{\sqrt{5} f_{10} + f_5}{3} \right) \left( f_{14}^2 + \frac{f_{41}^2}{5} \right) - \left( \frac{f_5/2 - \sqrt{5} f_{10}}{54} \right) \left( (f_{29} + \sqrt{5} f_{31})^2 + 5(f_{24} + \sqrt{5} f_{24})^2 \right) \right] \\
+ \left( \frac{2}{3\sqrt{15}} \right) \left[ (f_{37} f_{14} + f_{18} f_{41}) \left( f_{24} + \sqrt{5} f_{31} \right) + (f_{37} f_{41} - 5 f_{18} f_{41}) \left( f_{24} + \sqrt{5} f_{24} \right) \right] \\
- \left( \sqrt{\frac{5}{3}} f_{23} - \frac{\sqrt{5} f_{10} + f_5}{3} \right) \left[ \left( \frac{f_5/2 - \sqrt{5} f_{10}}{2} \right) \left( \sqrt{\frac{5}{3}} f_{23} + \frac{\sqrt{5} f_{10} + f_5}{3} \right) + \left( \frac{f_{37}^2}{5} + f_{18}^2 \right) \right]
\]
The pure $P$-wave case is special. 6 real bilinears, directly solvable from the observables.

Aside from the two-fold ambiguity, things are determined.

“Almost” true for $SP$-wave case (see Matias). $Im(s^\dagger h_0)$ absent in the observables, but all relations known.

For $SPD$ waves (and higher), several problems:

- Unlike, $SP$ case, the pure $S$, $P$, $D$-waves do not decouple. Eg. $(|S|^2 + |H_0|^2)$ occurs together, so we might not have $F_S$.
- 56 real bilinears, but only 41 observables. Many missing.
- Deriving the relations between observables yet unsolved.
The Moments Method: introduction

- Recap: higher waves and full 4-D fit difficult because we don’t know the minimal set of independent observables.
- Constrained fits could be one way: take FF predictions and float a few Wilson coefficients.
- Other way is to bypass doing a fit at all: the moments method.
- Rewrite $|M|^2 = \sum_i \Gamma_i(q^2) f_i(\theta_\ell, \theta_V, \chi)$ in an orthonormal $f_i$ basis.
- Orthonormality guarantees $\langle f_i \rangle \equiv \Gamma_i$.
- Convenient basis: products of $Y_l^m \equiv Y_l^m(\theta_\ell, \chi)$ and $P_l^m \equiv \sqrt{2\pi} Y_l^m(\theta_V, 0)$. 
Moments: what results we will provide

- For $B_d \rightarrow K\pi\mu\mu$ SPD analysis, we will provide the 40 normalized $\Gamma_i$’s and $40 \times 40$ cov. matrix in “some” $\{q^2, m(K\pi)\}$ binning.

- Straightforward to compare to these to theory $\Rightarrow$ core results.

- Some specific components extractable:

\[
|d_0|^2 = \frac{7}{9} \left( \frac{f_5}{2} - \sqrt{5}f_{10} \right)
\]

\[
|d_\parallel|^2 = \frac{7}{4} \left( \sqrt{\frac{5}{3}} f_{23} - \frac{1}{3} \left( \sqrt{5}f_{10} + f_5 \right) \right)
\]

\[
|d_\perp|^2 = \frac{7}{4} \left( - \sqrt{\frac{5}{3}} f_{23} - \frac{1}{3} \left( \sqrt{5}f_{10} + f_5 \right) \right)
\]
Moments: toy studies

- Many toy studies done, both LHCb and *BABAR*. Method works beautifully.
- Covariance matrix element between $\Gamma_i$ and $\Gamma_j$ checked by looking at pulls in ($\Gamma_i + \Gamma_j$).
More advertisement

- Not having to do a fit is a big deal! Just count.

- $B^- \rightarrow \pi^+ \pi^- \ell^- \bar{\nu}_\ell$ in BABAR with $S + P$ waves under the $\rho$.

- Highly statistics limited, yet toys seem to perform very well. Full moments paper in the pipeline.

- Without the moments method, pulling out $F_S$ is semi-impossible.
The $\Lambda_b \rightarrow \{pK, \Lambda\} \mu\mu$ case

- In addition to the muons, we now have a proton whose polarization is being averaged over.

- Exacerbates “incompleteness” of the measurement.

- $\Lambda_b \rightarrow \Lambda\mu\mu$: BF, $A_{FB}^{\ell,h}$ published.

- $\Lambda_b \rightarrow pK\mu\mu$: BF and $A_{FB}^{\ell,h}$ in the pipeline.
\( \Lambda_b \rightarrow \Lambda \mu \mu \) MOMENTS ANALYSIS

- Go beyond \( A_{FB}^{\ell,h} \). Assuming \( \Lambda_b \) almost unpolarized, full 4-D rate in the Korner paper.

- 12 complex amplitudes: \( H_{\lambda_A,\lambda}^{V,A} \), where \( \lambda \) is the “usual” helicity of the spin-1 dimuon.

- Very preliminary calculation gives 10 moments.

<table>
<thead>
<tr>
<th>( i )</th>
<th>( f_i(\Omega) )</th>
<th>( \Gamma_i(q^2) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( P_0^0 Y_0^0 )</td>
<td>( 2\sqrt{2\pi}(U + L) + 2\sqrt{2\pi}(U - L)/3 )</td>
</tr>
<tr>
<td>5</td>
<td>( P_0^0 Y_2^0 )</td>
<td>( 4\sqrt{2\pi}(U - L)/5 )</td>
</tr>
<tr>
<td>10</td>
<td>( \sqrt{2}P_1^1 \text{Im}(Y_1^1) )</td>
<td>( -4\sqrt{\pi}/3l_3 )</td>
</tr>
</tbody>
</table>

- As before, the moments are not independent, with possible very complicated inter-relations.
Summary: $B_d \to K\pi\mu\mu$

- Run I analysis till $m(K\pi) = 1530$ MeV, including $SPD$-waves making good progress.

- $\{q^2, m(K\pi)\}$ binning under discussion.

- Run II will include $F$ and $G$-waves as well. Moments calculations in progress.

- Same tools for $B_s \to KK\mu\mu$ in Run II.

- We already have reasonable statistics. Need theory predictions.
Summary: $\Lambda_b \rightarrow \Lambda \mu\mu$ and $\Lambda_b \rightarrow pK\mu\mu$

- Extend the Run I paper with the full set of moments.
- From MC studies, expect $\sim 400 \ [pK]$ events in Run I.
- Large suite of poorly known $\Lambda^*$'s makes the moments derivation complicated.
- Thrust is to retain the full set of available moments and come up with a global $\chi^2$ against theory.
**More ideas...**

- **Charm-loop effects:**
  - can we go closer to the $c\bar{c}$ resonances?
  - any specific observable sensitive to the non-factorizable part?
  - can we measure the relative phase between pen. and $c\bar{c}$?

- **$ee$ analyses (lots of ongoing effort):**
  - $R(K^*)$, $R(\phi)$ and $R(K\pi)$. $R(K)$ in $q^2 > 1 \text{ GeV}^2$.
  - joint $ee$ and $\mu\mu$ angular analyses? Bin-migration.

- I assumed $4 \times$ more statistics after Run II. Of course, this can increase as well.
The Zwicky paper considers non-$P$-wave in the dilepton system as well.

For the $\tau$ this is well known (see Ligeti), or even at $q^2 \to m_{\mu}^2$.

For the massless case $\eta \equiv (\lambda_{\ell_1} - \lambda_{\ell_2}) = \pm 1$ for spin-1, and $\eta = 0$ for spin-0.

In addition to helicity suppression, the $\eta = 0$ component can add only incoherently to $\eta = \pm 1$. No interference means NLO effect.