THE LHC:
GENERAL PARAMETERS AND MAGNETIC ERRORS

J. Gareyte

Abstract

The main features of the LHC are described, with emphasis on the choices made to maximize energy and luminosity, and to reduce cost. The different sources of non linear errors in the field of the magnets are reviewed, and their consequences on particle dynamics and machine operation are outlined.

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THE LHC: GENERAL PARAMETERS AND MAGNETIC ERRORS

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I Introduction

The LHC\textsuperscript{1} is a high magnetic field twin aperture collider that CERN proposes to install on top of LEP in the LEP tunnel (fig. 1).

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure1.png}
\caption{L.H.C SUPERCONDUCTING MAGNETS INSTALLED ABOVE THE L.E.P MAGNETS}
\end{figure}

\textsuperscript{1} LHC
It will produce collisions between two counterrotating beams of protons at a maximum energy of about 8 TeV per beam, with a very high luminosity in the range of a few $10^{34} \text{cm}^{-2}\text{s}^{-1}$. It can also be used to collide heavier species, like for instance lead nuclei, in this case providing enormous amounts of energy in the center of mass system of the collisions, although with a lower luminosity around $10^{32} \text{cm}^{-2}\text{s}^{-1}$. And since the world largest electron storage ring, LEP, will be sitting about 1m below the LHC, it will be possible to organize collisions between electrons and protons at a modest additional cost.

The superconducting bending magnet is by far the most important single item in the project, and deserves special attention in this report. In fact, it is because it would be technologically difficult and economically prohibitive to produce superconducting magnets of the same field quality as classical ones that we, accelerator physicists, are present at this workshop seeking the help of other disciplines. The price to pay for getting compact, high field machines is a non linear, hostile environment for the particles.

II The LHC, a high field collider

The search for cost effectiveness irremediably pushes the design of a modern proton collider towards higher and higher magnetic fields. Fig. 2 shows how the cost of magnets per Tesla meter varies with field level. Considering that civil engineering, infrastructure and installation costs all decrease in proportion to the size of the machine when the magnetic field is increased for a fixed energy, the plot of global cost versus field would show an even larger advantage in going to high fields. In the case of CERN, it is imperative to make the best use of the existing infrastructure and tunnel and for reasons similar to those mentioned above this calls for the highest possible magnetic field.

However there are limitations. In order to reach high fields within a given magnet aperture using coils of a reasonable thickness, conductors capable of sustaining a high current density are needed. Unfortunately, the critical current density in type II superconductors strongly decreases at high fields, as is shown in Fig. 3, and this leads to a limit in the attainable field. Using NbTi cooled at 4.2 K, the limit lies around 6 to 7 Tesla, while with Nb$_3$S$_n$ cooled at 4.2 K or with NbTi cooled at 1.8 K one can reach 10 Tesla.
FIGURE 2
COST OF MAGNET + CRYOSTATS FOR 50 mm diam COIL APERTURE DIPOLES

FIGURE 3
CURRENT DENSITY IN COMMERCIAL SUPERCONDUCTORS (Jc) AND AN EXAMPLE OF THE NECESSARY COIL THICKNESS IN DIPOLE MAGNETS (W)
Another limitation arises in the case of the LHC from the fixed cross section of the LEP tunnel. At high field, magnet dimensions tend to increase, one of the reasons being that the iron shield which surrounds the magnet must be placed at a sufficient distance from the coils to minimize saturation effects. It was found that two independent magnetic channels operating at 10 T would be impossible to accommodate on top of LEP. The solution was the so called "two in one" concept, in which the two magnetic channels are embedded in the same iron yoke and cryostat (Fig. 4). This allows the design of 10T, compact magnets which in addition have the advantage of a reduced construction and installation cost. This design has unfortunately its drawbacks, namely a mutual influence of the two channels, a more complicated mechanical structure, and difficulties for the alignment.

FIGURE 4
LHC DIPOLE: STANDARD CROSS-SECTION
III A high luminosity collider

There is a wide consensus in the high energy physics community about the necessity of providing particle collisions in the near future at an energy scale of about 1 TeV at the parton level. This aim can be realized with protons provided the energy is larger than 10 TeV, which is well within reach of the LHC. However, as the energy E is increased so that parton collisions can probe matter properties at smaller and smaller distances, the cross section of the interesting reactions tend to decrease like $1/E^2$. For this reason very high luminosities are required to assure reasonable event rates.

The luminosity of a collider is given by:

$$L = \frac{N^2 k \gamma}{\pi \varepsilon^* \beta^*}$$

where $N$ is the number of particles per bunch, $k$ the number of bunches, $f$ the revolution frequency, $\gamma$ the energy divided by the rest energy, $\beta^*$ the betatron function at the interaction point and

$$\varepsilon^* = \frac{4 \sigma^2 \gamma}{\beta}$$

the normalized emittance, with $\sigma$ the rms beam radius (the beam cross section is supposed to be round).

The main limitation to the luminosity comes from the electromagnetic beam-beam interaction. This produces a change of the particle tune, which is for the central particle in a beam:

$$\xi = \frac{r_o N}{\pi \varepsilon^*}$$

where $r_o$ is the classical particle radius. It is know from experience with the CERN SPS collider that the total tune spread across the beam particle distribution must not increase beyond 0.02 to preserve reasonable beam lifetimes. In the LHC, as we will see later, some unavoidable tune spread will be produced by various non linearities in the guiding fields. If we share equally the allowed tune spread between these effects and the beam-beam effect, we arrive at the following condition:

$$n \xi \leq 0.01$$

where $n$ is the number of interaction regions.

-5-
For 3 interaction regions in the LHC one has \( \xi \leq 0.0033 \) and one obtains the maximum usable beam density:

\[
\frac{N}{e^*} \leq 6.16 \times 10^{13}
\]

When this beam-beam imposed limit is reached, one can only increase the luminosity by increasing at the same time the beam intensity and the emittance.

The beam intensity is limited by collective effects, by the amount of synchrotron radiation power emitted (which has to be absorbed at cryogenic temperatures and hence at high cost) and by the difficulty of handling beams with hundreds of megajoules of stored energy. The emittance is limited by the dynamic aperture of the machine. In a linear machine the limit to the oscillation amplitude of the particles is the radius of the vacuum chamber in which the beam circulates (in the LHC about 20 mm). In presence of non linear restoring forces particles may become unstable and subsequently hit the aperture for starting amplitudes considerably smaller than the chamber radius. The dynamic aperture corresponds to the maximum amplitude of the particles which are just stable for a certain duration (from half and hour at injection energy to about 10 hours at collision energy, that is for \( 2 \times 10^7 \) and \( 4 \times 10^8 \) revolutions respectively). We learned at this workshop that this is a physicist's definition of the "basin of infinity".

**IV Structure of the machine**

LHC being in the same tunnel as LEP will also have 8 arcs and 8 long straight sections. The two proton beams, horizontally separated by 180 mm in the arcs, alternate from the outside to the inside in the middle of each of the 8 long straight sections, where in principle they could interact (Fig. 5).

The LHC lattice of FODO type is constituted by:

- 8 arcs, each of them containing 50 half cells,

- 8 insertions, each of them containing one long straight section and two dispersion suppressors of a type that allows trajectories of identical length for the hadrons in LHC and for the leptons in LEP.
One half of the regular are cell (Fig. 6) consists of four dipoles D about 10 m long, a focussing (QF) and a defocussing (QD) quadrupole. On each beam stand individual correctors which are lumped near the main quadrupoles (a combined tuning quadrupole QT and octupole 0, a combined sextupole S, decapole D and closed orbit correction dipole COD, and a beam observation monitor BOM) and in the middle of the half cell (a combined sextupole, octupole and decapole).
V Sources of magnetic errors

The different magnetic multipoles can be generated by current sheets flowing outside a cylindrical vacuum chamber as shown in Fig. 7. A multipole of order m is generated by a current density of the form \( I(\Phi) = I_0 \cos(m\Phi) \). "Skew" multipoles are obtained in the same way by rotating the distribution \( I(\Phi) = I_0 \sin(m\Phi) \).

FIGURE 6
LAYOUT OF THE STANDARD LHC HALF-CELL

FIGURE 7
GENERATION OF MULTIPOLE FIELDS
In order to build an accelerator the minimum one needs is dipoles to bend the particle trajectories and quadrupoles to focus them. In addition, in order to achieve high performance it is necessary to add sextupoles to correct the chromatic aberrations which arise from the variation of the focal length of quadrupoles with the particle momentum, and skew quadrupoles to cancel the horizontal-vertical coupling effects generated by unavoidable tilts of the main quadrupoles. In proton machines octupoles are also necessary to control the tune spread in the beam and in this way damp instabilities due to collective effects.

Therefore the particle motion in accelerators or storage rings is by necessity fundamentally non linear. In addition unavoidable errors in the magnetic fields introduce further important non linearities, and in superconducting machines these are generally dominant.

It is customary to expand the magnetic field in the main bending dipoles in the following way:

\[ B_x + iB_y = B_0 \sum_{i=1}^{\infty} (b_n + ia_n) \left( \frac{Z}{R_r} \right)^{n-1} \]

where \( B_0 \) is the wanted vertical bending field and \( B_x \) and \( B_y \) are the actual components of the field in the horizontal and vertical planes respectively. The coefficients \( b_n \) correspond to the normal and \( a_n \) to the skew multipole components. The reference radius \( R_r \) is usually chosen close to the half aperture of the magnet coils. For LHC as well as SSC studies \( R_r = 1 \ \text{cm} \) is assumed and \( a_n \)'s and \( b_n \)'s are expressed in units of \( 10^{-4} \). The above formula corresponds to the European notation where \( n = 1 \) describes the dipole, whereas in the US the dipole is \( n = 0 \).

The first, systematic source of errors comes from the fact that the ideal \( \cos (\phi) \) current distribution is approximated by current shells of constant thickness as shown in Fig. 8. This generates unwanted normal multipoles of odd order. The lowest order multipoles are minimized by a suitable choice of the angle subtented by the coils. For instance with two layers as in Fig. 8 one can cancel out both the sextupole \( (n = 3) \) and the decapole \( (n = 5) \) terms. By inserting non-conducting wedges one can minimize higher order multipoles. In the LHC magnet, residual \( n = 7 \) and \( n = 9 \) terms are large enough to pose problems, as we will see later on, and ways are sought to reduce their values.
Mechanical tolerances on the dimensions and positions of the coils are responsible for additional errors, although precisions as high as 0.02 mm can be reached in practice. At high field the enormous magnetic forces which tend to compress the coils (about 400 Tons/meter for the LHC) are potential sources of deformation. Adequate prestress has to be applied in order to diminish their effect.

At low field, by far the most important source of errors is the persistent currents. These are eddy currents which establish themselves as soon as a magnetic field is applied to the superconducting wires. They flow in opposite directions on each side of the wires so as to oppose the applied magnetic field inside the superconductor. (Fig. 9).
Their net effect at the center of the coil is the generation of odd order (for a dipole magnet) and even order (for a quadrupole) high multipoles. In the dipoles the strongest component is sextupole, while in the quadrupoles the dodecapole is dominant. Their strength depend on the magnetization history and vary slowly (time constants of about half an hour) when the main field is held constant, like for instance during the injection process. For these reasons it is very difficult to compensate this effect accurately.

The only way to diminish the effect of the persistent currents is to use conductors of extremely small diameter. The Hera magnet coils are wound with wires made of 15µm diameter superconducting filaments, and it seems technologically possible now to go down to diameters of 5µm or even 2.5µm for the LHC and the SSC. The limit is set by the tunnelling of the Cooper pairs in between filaments, which could possibly be prevented by a suitable doping of the copper matrix in which the filaments are embedded.

VI Effect of magnetic errors and their corrections

Table I gives the values of the error coefficients which are expected for the LHC bending magnets. The systematic errors (those which are the same in all magnets of a production line) originate mainly from the persistent currents (b₃ and b₅) and the coil design (b₇ and b₉). The random errors are due to geometrical variations from magnet to magnet.

Systematic errors drive structure resonances which are excited by harmonics of the machine periodicity (8 for the LHC) and in addition produce a tune variation with the betatron amplitude and the momentum of the particles.

Structure resonances can be avoided by adequately choosing the integer part of the tune, but the tune variation remains and constitutes the dominant effect of systematic errors. In order to assure a sufficient beam lifetime all resonances up to a rather high order have to be avoided for all beam particles. This is only possible if the tune dispersion in the beam is kept below a certain limit, around 0.005.

Sextupole, octupole and decapole correctors are inserted in the lattice (see Fig. 6) to reduce the tune dispersion caused by the systematic errors. However, at large amplitudes and large momentum deviations the high order multipoles b₇ and b₉ dominate and their effect cannot be corrected.
Random errors mainly drive non-linear resonances, and experience shows that even high order ones (up to 7th or 8th order) can produce a slow increase of particle amplitudes, leading to the subsequent loss of the particles. Since the largest error is the sextupole, it is possible after measuring it in each single magnet to install the magnets in a determined sequence so as to minimize the excitation of the more dangerous resonances. Such "sorting" schemes have been found interesting, although their efficiency is limited by the presence of the higher order terms.

The detrimental effect of random errors reveals itself by an increase of the "Smear", a value easy to calculate from computer simulation of the particle trajectories. The Smear is defined as the variance of the square of the amplitude of a particle measured turn after turn; in a linear machine the amplitude is constant and the value of the Smear is zero. It is assumed that the motion will be "sufficiently linear" if the Smear stays below 0.1. The "linear aperture" of a machine is defined as the region of the vacuum chamber where the tune shift is smaller than 0.005 and the Smear smaller than 0.1.

The optimization of a machine is made using a computer "tracking" programme. Tracking over a few hundred turns is sufficient to obtain the short term dynamic aperture (where the particles stay in the machine) as well as the linear aperture defined above. One can also make a few selected long term trackings (over say $10^6$ turns) or evaluate the Lyapunov exponents to determine a "chaotic boundary" inside which the motion appears regular. The result of these studies is a cascade of successive "apertures" (as in Fig. 10) which are useful to judge whether the magnet quality is sufficient or not.

![Diagram of aperture concepts](image)

**FIGURE 10**

**DIFFERENT CONCEPTS OF "MACHINE APERTURES"**

-12-
TABLE I

ESTIMATED ERRORS FOR THE LHC \( \left( \text{coil} \phi = 5\, \text{cm} \right) \) \( \left( \text{fil.} \phi = 5\, \mu\text{m} \right) \) Injection at 450 GeV

<table>
<thead>
<tr>
<th>Multipole</th>
<th>Systematic</th>
<th>Random (rms)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a_2 )</td>
<td>( \pm 0.6 ) ( \pm 1.6 )</td>
<td>1.7 \ 1.2</td>
</tr>
<tr>
<td>( b_2 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( a_3 )</td>
<td>( \pm 0.1 ) ( -3.7 )</td>
<td>0.5 \ 1.7</td>
</tr>
<tr>
<td>( b_3 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( a_4 )</td>
<td>( \pm 0.03 ) ( \pm 0.05 )</td>
<td>0.2 \ 0.15</td>
</tr>
<tr>
<td>( b_4 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( a_5 )</td>
<td>( \pm 0.03 ) ( 0.45 )</td>
<td>0.07 \ 0.22</td>
</tr>
<tr>
<td>( b_5 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( a_7 )</td>
<td>( \pm 0.01 ) ( 0.13 )</td>
<td>0.04 \ 0.02</td>
</tr>
<tr>
<td>( b_7 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( a_9 )</td>
<td>( \pm 0.001 ) ( 0.024 )</td>
<td>0.002 \ 0.005</td>
</tr>
<tr>
<td>( b_9 )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

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