SIXTRACK
A Single Particle Tracking Code

F. Schmidt

Abstract

The new single particle tracking code SIXTRACK, an offspring of the well known RACETRACK code written by A. Wrulich, has been installed at CERN as a general tool to study non-linear effects in large hadron colliders like the LHC. The main aim of this program is to carry one particle through those complicated structures over large number of turns taking into account the full six-dimensional phase space including synchrotron oscillations in a symplectic manner. It allows to the prediction of the long-term dynamic aperture by evaluating the Lyapunov exponent. Parameters of interest like non-linear detuning and smear are determined via a post-processing of the tracking data. An analysis of the first order resonances can be done and correction schemes for several of those resonances can be calculated. Moreover there is the feature to calculate a one-turn map of such complicated structures as the LHC to very high order, using the differential algebra techniques of M. Berz. This map allows a subsequent theoretical analysis like normal form procedures which are provided by E. Forest.

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SIXTRACK - A Single Particle Tracking Code

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Abstract

The new single particle tracking code SIXTRACK [1], an offspring of the well known RACETRACK code written by A. Wurllich [2], has been installed at CERN as a general tool to study non-linear effects in large hadron colliders like the LHC [3]. The main aim of this program is to carry one particle through those complicated structures over large number of turns taking into account the full six-dimensional phase space including synchrotron oscillations in a symplectic manner. It allows to the prediction of the long-term dynamic aperture by evaluating the Lyapunov exponent. Parameters of interest like non-linear detuning and smear are determined via a post-processing of the tracking data. An analysis of the first order resonances [4] can be done and correction schemes for several of those resonances can be calculated. Moreover there is the feature to calculate a one-turn map of such complicated structures as the LHC to very high order, using the differential algebra techniques of M. Berz [5]. This map allows a subsequent theoretical analysis like normal form procedures which are provided by E. Forest [6].

1 Introduction

For the new generation of large proton accelerators like the LHC superconducting magnets have to be used to achieve the high fields needed to bend protons in the energy regime of 10 TeV. These magnets have unavoidable high order multipole errors which limit the dynamic aperture and thereby the usable regime of transverse amplitudes where the protons are stable. SIXTRACK is one of the programs to tackle these single particle stability problems. The aim is to estimate the dynamic aperture sufficiently in a safe and fast way, by making use of today's most powerful computers with their vectorization facilities. After the description of the features of SIXTRACK, the next section shows how the tracking studies can be done most efficiently and finally a small tracking example is given.

2 Description of SIXTRACK

The Single Particle Tracking Code SIXTRACK is optimized to carry two particle \(^1\) through an accelerator structure over a large number of turns.

The main features of SIXTRACK are:

1. Treatment of the full six-dimensional motion including synchrotron motion in a symplectic manner [7]. The energy can be ramped at the same time considering the relativistic change of the velocity [8].

2. Detection of the onset of chaotic motion and thereby the long-term dynamic aperture by evaluating the Lyapunov exponent.

3. Postprocessing procedure allowing
   
   • calculation of the Lyapunov exponent

\(^1\)Two particles are needed for the detection of chaotic behavior.
• calculation of the average phase advance per turn
• FFT analysis
• resonance analysis
• calculation of the average, maximum and minimum values of the Courant Synder emittance and the invariants of linearly coupled motion
• calculation of smear
• plotting using the CERN packages HBOOK, HLOT and HIGZ [9, 10, 11]

4. Calculation of first-order resonances and of correction schemes for the resonances [4].

5. Calculation of the one turn map using the differential algebra techniques of M.Berz [5].

6. A vectorized version is available, where the two particles, with a number of different amplitudes, the different momentum deviations and several seeds for the random distribution of multipole errors can be treated in parallel [12].

7. Operational improvements:
   • free format input
   • optimization of the calculation of multipole kicks
   • improved treatment of random errors

To save storage space and to facilitate program development the program is kept in Patchy [13] format. There are different versions of SIXTRACK for Apollo,Cray,IBM and the VAX.

Very large accelerator structures like the LHC with its 2500 multipoles demand large memory space (8 Mbytes). On top of that the differential algebra package needs another 12 Mb to allow the calculation of a one-turn map of the accelerator to 15th order. This was made manageable by a special treatment of large common blocks [14].

The IBM at CERN needs for one turn of two particles and for the full LHC lattice with all multipole errors in the dipoles and correction elements approximately 125 ms. Therefore $10^4$ turns take 24 minutes of cpu time which is the upper limit that can be reached without going to dedicated emulators that can run 24 hours a day. There is, however, a vectorized version of the program, which allows a speedup by approximately a factor of 10 when 60 particles are treated in parallel [12]. The time needed to determine with the differential algebra option the one-turn map with four parameters (four-dimensional transverse motion) of the same LHC lattice is given in the table below as a function of the order to which the map is calculated.

<table>
<thead>
<tr>
<th>order</th>
<th>cpu time (seconds)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>5</td>
<td>26</td>
</tr>
<tr>
<td>7</td>
<td>133</td>
</tr>
<tr>
<td>9</td>
<td>565</td>
</tr>
</tbody>
</table>

Increasing the order by two takes five times as much cpu time, which agrees with the increase of the number of differential algebra multiplications and additions [15].
3 Tracking Recipe

The following rather crude recommendations for getting answers about specific non-linear problems in the design phase of an accelerator reflect (partly) cumbersome experiences during such a design phase (HERA).

1. There is a tendency to produce large numbers of tracking studies for the sake of statistics. This is probably good for the final version of a lattice. In case of unexpected new effects which appear (!) it is hard to judge if the old data is still valid. It is therefore preferable to limit the studies to a small (<< 1000) but sufficient set of parameters, so that there is a chance to redo the analysis! It goes without saying that a careful judgment of what is sufficient is mandatory.

   • The variation of the amplitude in phase space is good enough. Don’t vary the phase space angle! Tracking roughly 1000 turns will scan the phase space sufficiently, so that the angle information comes for free.
   • A worst but still realistic case should be studied. For instance tracking at one maximum relative momentum deviation.
   • An optimization of the tunes should be done first. This fixes the tunes to one pair of values. These optimized tunes might also diminish the effect of different random seeds.
   • If the distribution of machines with different seeds is narrow, study only one machine.
   • A thorough study for a low number of turns (10^4) should be done first, followed by a few well chosen cases at a large number of turns (10^5 - 10^7).
   • After a coarse variation of the amplitudes, a fine scan is necessary to find the dynamic aperture to the desired precision. The long-term runs should be done in this step of the analysis including a thorough study of the phase space. Complicated resonance structures and the dynamic aperture as a function of phase space angles can be found that way.

2. The tracking data are obtained over hours or even days of CPU-time and they hold a surprising amount of information, so that they all should be saved on tape. These data should be analyzed in any possible way and should be at hand at any time. Moreover during a longer period of tracking studies it often happens that new ways of analyzing the data emerge.

3. Criteria for early detection of instability should be searched, checked and used to reduce the tracking effort or even reach beyond the normal tracking scope. SIXTRACK offers the possibility to detect the onset of chaotic motion. It uses the evolution of the distance in phase space of two particles (example see next section). New direct methods are being investigated [16] to evaluate the Lyapunov exponent. Whether the predictive power can thereby be increased remains to be seen. The pragmatic approach followed here has undergone exhaustive testing:

   • The onset of chaotic motion can normally be detected after only 10^4 turns. In addition there are early indications that suggest chaotic behavior, so that a prolonged run can clarify the situation.
   • Even when the maximum distance in phase space is reached (opposite positions in phase space), the 2 types of motions can be easily distinguished. For regular particles the distance in phase space decreases to its original value, while in the chaotic case the distance in phase space remains at a statistical mean value.
   • The method needs the information of only 2 particles tracked in parallel, which can be done fast especially when the vectorization is used.

3
• Only one of the two particles has to be chaotic to find the chaotic behavior.

• The initial distance is dictated by rounding errors to stay above roughly $10^{-6}$. Setting this distance to its minimum value allows to check for chaotic behavior just above the granularity of phase space imposed by these rounding errors.

• The effect of rounding errors is self-compensated, as long as the initial distance stays above its minimum value. The influence of the unavoidable presence of rounding errors has to be considered in any kind of method used to predict the long-term behavior.

4. Finally one should ask the theorists for complementary analytic tools and rules. These should be simple and take advantage of the strength of today's computer power. I would like to mention two examples that fulfill these requirements:

• The D.A. package of M. Berz [5]
• The Bengtsson rule for FFT's [17]
4 Tracking example

A simple tracking example is shown with its input file (4.1), its output file (4.2) and some corresponding plots in (4.3).

4.1 Input Example

For the description of the different input blocks see ref [1].

```
FREE FORMAT TITLE: EXAMPLE
PRINTOUT OF INPUT PARAMETERS------------------------
NEXT-----------------------------------------------
SINGLE ELEMENTS-------------------------------------
B 0 0.000000 0.00000 50.00000
CD2 2 0.000000 0.009536 0.77000
QF2 2 0.000000 -0.009536 0.77000
MU 11 1. 1.00000 0.00000
SEX 3 0.050000 0.00000
NEXT
BLOCK DEFINITIONS
1 1
B1 QD2 B QF2
B2 QF2 B QD2
NEXT
STRUCTURE INPUT
MU B1 SEX B2
NEXT
MULTIPOLY COEFFICIENTS-----------------------------
MU 10.0 3.5765
0. 0.00000 0. 0.00000
0. 0.00000 0. 0.00000
-5E-3 0.00000 0. 0.00000
-5E-4 0.00000 0. 0.00000
0.0 0.00000 0. 0.00000
0.3E-5 0.00000 0. 0.00000
0. 0.00000 0. 0.00000
NEXT
TRACKING PARAMETERS-----------------------------
10000 0 2 11.0 11.5 0 1
1 0 0 1 1 1 50000
NEXT
INITIAL COORDINATES-----------------------------
2 0. 0. 1. 0.
0. 0.
0.
0.
0.
0.
0.
0.
0.
0.
0.
0.
0.
0.
0.
450000.
450000.
450000.
NEXT
ITERATION-ACCURACY-------------------------------
50 1D-14 1D-15
10 1D-10 1D-10
10 1D-5 1D-6
1D-8 1D-12 1D-10
NEXT
POSTPROCESSING------------------------------------
EXAMPLE
1000 0 0 1 .08 .08
0. 0. 11 20 .005 1 10
7878 -1 0 1 1 1 1
NEXT
ENDE---------------------------------------------
```
### 4.2 Output Example

The pre-processing part is shown first.

---

<table>
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<tr>
<th>TUNE</th>
<th>CLO</th>
<th>CLOP</th>
<th>BETO</th>
<th>ALF0</th>
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<td>0.000000</td>
</tr>
</tbody>
</table>

REL. MOMENTUM DEVIATION: 0.00000

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Followed by the initial coordinates and the final coordinates for a regular (right side) and chaotic (left side) case.

### INITIAL COORD. OF TWIN-TRAJECTORIES

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<tbody>
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### INITIAL COORD. OF TWIN-TRAJECTORIES

<table>
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</tr>
</tbody>
</table>
Finally part of the postprocessing for the two particles are shown (chaotic on the left and regular on the right respectively) and a summary of the postprocessing is given.

**Summary of the Postprocessing**

| TURN  | LINEAR | SETA+ | AMPLITUDES | MOMENTUM | PHASESPACE | OF THE | NONLINEAR | NEAREST | SHEAR OF | OF | ORG. | $|$ | |
|-------|--------|-------|------------|----------|------------|--------|-----------|---------|---------|----|------|-----|---|
| 10000X | 0.12221X | 22.9575X | 11.00000000.X | 0.00000000.X | 0.307380-03 | 9.0100X | 1.002410-02X | 0.7200X | 52.5400X | 10.16970-02 | 0.05 | 0.00 | 22.9575X |
| 2X  | 0.12221X | 203.58121X | 203.58121Z | 10.278681 | 1.002410-02X | 0.7200X | 0.307380-03 | 52.5400X | 0.00000000.X | 0.00000000.X | 10.16970-02 | 0.05 | 0.00 | 203.58121Z |
| 10000X | 0.12221X | 22.9575X | 11.00000000.X | 0.00000000.X | 0.307380-03 | 9.0100X | 1.002410-02X | 0.7200X | 52.5400X | 10.16970-02 | 0.05 | 0.00 | 22.9575X |
| 2X  | 0.12221X | 203.58121X | 203.58121Z | 10.278681 | 1.002410-02X | 0.7200X | 0.307380-03 | 52.5400X | 0.00000000.X | 0.00000000.X | 10.16970-02 | 0.05 | 0.00 | 203.58121Z |
4.3 Plot Example

In figure 1 a typical example of the evolution of the distance in phase space is shown of a regular and chaotic particle. Figure 2 and figure 3 show the corresponding horizontal phase space and the physical phase space projections respectively.

Figure 1: Evolution of the distance of phase space for regular (upper part) and chaotic (lower part) motion
Figure 2: Horizontal phase space projections for the regular (upper part) and the chaotic (lower part) cases
Figure 3: Physical phase space projections for the regular (upper part) and the chaotic (lower part) cases
References


[12] F. Schmidt, M. Vaenttinen, Vectorization of SIXTRACK, to be published


[14] H.Renshall, private communication


[16] F. Zimmermann, private communication