We discuss the conditions for an effective field theory (EFT) to give an adequate low-energy description of the underlying physics beyond the Standard Model (SM). Starting from the EFT where the SM is extended by dimension-6 operators, experimental data can be used without further assumptions to measure (or set limits on) the EFT parameters. The interpretation of these results requires instead a set of broad assumptions on the UV dynamics. This allows one to establish, in a bottom-up approach, the validity range of the EFT description, and to assess the error associated with the truncation of the EFT series. We give a practical prescription on how experimental results should be reported, so that they admit a maximally broad range of theoretical interpretations. Namely, the experimental constraints on dimension-6 operators should be reported as functions of the kinematic variables that set the relevant energy scale of the studied process. This is especially important for hadron collider experiments where collisions probe a wide range of energy scales.

1 Introduction

We consider an EFT where the SM is extended by a set of higher-dimensional operators, and assume that it reproduces the low-energy limit of a more fundamental UV description. The theory has the same field content and the same linearly-realized $SU(3) \times SU(2) \times U(1)$ local symmetry as the SM. The difference is the presence of operators with canonical dimension $D$ larger than 4. These are organized in a systematic expansion in $D$, where each consecutive term is suppressed by a larger power of a high mass scale. Assuming baryon and lepton number conservation, the Lagrangian takes the form

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{SM}} + \sum_i c_i^{(6)} O_i^{(6)} + \sum_j c_j^{(8)} O_j^{(8)} + \cdots,$$

(1.1)
where each $\mathcal{O}_i^{(D)}$ is a gauge-invariant operator of dimension $D$ and $c_i^{(D)}$ is the corresponding coefficient. Each coefficient has dimension $4 - D$ and scales like a given power of the couplings of the UV theory; in particular, for an operator made of $n_i$ fields one has

$$c_i^{(D)} \sim \frac{\text{(coupling)}^{n_i-2}}{(\text{high mass scale})^{D-4}}.$$  

(1.2)

This scaling holds in any UV completion which admits some perturbative expansion in its couplings [1]. An additional suppressing factor $(\text{coupling}/4\pi)^{2L}$ may arise with respect to the naive scaling if the operator is first generated at $L$ loops in the perturbative expansion. If no perturbative expansion is possible in the UV theory because this is maximally strongly coupled, then Eq. (1.2) gives a correct estimate of the size of the effective coefficients by replacing the numerator with $(4\pi)^{n_i-2}$ (i.e. setting coupling $\sim 4\pi$) [2].

The EFT defined by Eq. (1.1) is able to parameterize observable effects of a large class of beyond the SM (BSM) theories. In fact all decoupling $B - L$ conserving BSM physics where new particles are much heavier than the SM ones and much heavier than the energy scale at which the experiment is performed can be mapped to such a Lagrangian. The main motivation to use this framework is that the constraints on the EFT parameters can be later re-interpreted as constraints on masses and couplings of new particles in many BSM theories. In other words, translation of experimental data into a theoretical framework has to be done only once in the EFT context, rather than for each BSM model separately. Moreover, the EFT can be used to establish a consistent picture of deviations from the SM by itself and thus can provide guidance for constructing a UV completion of the SM.

The EFT framework contains higher-dimensional operators (non-renormalizable in the traditional sense). As a consequence, physical amplitudes in general grow with the energy scale of the process, and therefore the EFT inevitably has a limited energy range of validity. In this note we address the question of the validity range at the quantitative level. We will discuss the following points:

- Under what conditions does the EFT give a faithful description of the low-energy phenomenology of some BSM theory?
- When is it justified to truncate the EFT expansion at the level of dimension-6 operators?
- To what extent can experimental limits on dimension-6 operators be affected by the presence of dimension-8 or higher operators?

It is important to realize that addressing the above questions cannot be done in a completely model-independent way, but requires a number of (broad) assumptions about the new physics. An illustrative example is that of the Fermi theory, which is an EFT for the SM degrees of freedom below the weak scale after the $W$ and $Z$ bosons have been integrated out. In this language, the weak interactions of the SM fermions are described at leading order by 4-fermion operators of $D=6$, such as:

$$\mathcal{L}_{\text{eff}} \supset c^{(6)} (\bar{e}_{\alpha} P_L \nu_{\nu}) (\bar{\nu}_\mu \gamma_\rho P_L \mu) + \text{h.c.} , \quad c^{(6)} = -\frac{g^2/2}{m_W^2} = -\frac{2}{v^2} .$$

(1.3)
This operator captures several aspects of the low-energy phenomenology of the SM, including for example the decay of the muon, \( \mu \rightarrow e\nu\nu \), and the inelastic scattering of neutrinos on electrons \( \nu e \rightarrow \nu\mu \). It can be used to adequately describe these processes as long as the energy scale involved (i.e. the momentum transfer between the electron current and the muon current) is well below \( m_W \). However, the information concerning \( m_W \) is not available to a low-energy observer. Instead, only the scale \( |c^{(6)}|^{-1/2} \sim v = 2m_W/g \) is measurable at low energies, which is not sufficient to determine \( m_W \) without knowledge of the coupling \( g \). For example, from a bottom-up viewpoint, a precise measurement of the muon lifetime gives indications on the energy at which some new particle (i.e. the W boson) is expected to be produced in a higher-energy process, like the scattering \( \nu e \rightarrow \nu\mu \), only after making an assumption on the strength of its coupling to electrons and muons. Weaker couplings imply lower scales: for example, the Fermi theory could have ceased to be valid right above the muon mass scale had the SM been very weakly coupled, \( g \approx 10^{-3} \). On the other hand, a precise measurement of the muon lifetime sets an upper bound on the mass of the W boson, \( m_W \lesssim 1.5 \text{ TeV} \), corresponding to the limit in which the UV completion is maximally strongly coupled, \( g \sim 4\pi \).

This example illustrates the necessity of making assumptions (in this case on the value of the coupling \( g \), see also Section 3 for another BSM example) when assessing the validity range of the EFT, that is, when estimating the mass scale at which new particles appear. On the other hand, the very interest in the EFT stems from its model-independence, and from the possibility of deriving the results from experimental analyses using Eq. (1.1) without any reference to specific UV completions. In this note we identify under which physical conditions Eq. (1.1), and in particular its truncation at the level of dimension-6 operators, can be used to set limits on, or determine, the value of the effective coefficients. Doing so, we also discuss the importance that results be reported by the experimental collaborations in a way which makes it possible to later give a quantitative assessment of the validity range of the EFT approach used in the analysis. As we will discuss below, this entails estimating the energy scale characterizing the physical process under study. Practical suggestions on how experimental results should be reported will be given in this note. A more theoretical discussion of the EFT validity issues and scaling of higher-dimensional operators can be found in Ref. [3].

2 General discussion

2.1 Model-independent experimental results

Let us first discuss how an experimental analysis can be performed in the context of the EFT. We start considering Eq. (1.1) truncated at the level of \( D = 6 \) operators, and assume that it gives an approximate low-energy description of the UV theory. Below we discuss the theoretical error associated with this truncation and identify the situations where the truncation is not even possible. Physical observables are computed from the truncated EFT Lagrangian.
in a perturbative expansion according to the usual rules of effective field theories [4]. The
perturbative order to be reached depends on the experimental precision and on the aimed
theoretical accuracy, as we discuss in the following. Theoretical predictions obtained in this
way depend on the coefficients \( c_i^{(6)} \) and can be used to perform a fit to the experimental data.
The fit to the coefficients \( c_i^{(6)} \) should be performed by correctly including the effect of all the
theoretical uncertainties (such as those from the PDFs and missing SM loop contributions)\(^1\)
not originating from the EFT perturbative expansion. The errors due to the truncation at the
\( D = 6 \) level and higher-loop diagrams involving insertions of different effective operators, on
the other hand, are not quantifiable in a model-independent way and should thus be reported
separately. Below we discuss how the neglected contributions from \( D \geq 8 \) operators can be
estimated; the effects of EFT loops are discussed elsewhere [NLO note].

Let us consider a situation in which no new physics effect is observed in future data (the
discussion follows likewise in the case of observed deviations from the SM). In this case, the
experimental results can be expressed into the limits\(^2\)

\[
\delta_i^{\exp}(M_{\text{cut}}) < c_i^{(6)} < \delta_i^{\text{u,exp}}(M_{\text{cut}}).
\]  

(2.1)

The functions \( \delta_i^{\exp} \) depend on the values of the kinematic variables (such as transverse mo-
menta or invariant masses), here collectively denoted by \( M_{\text{cut}} \), which set the typical maximum
energy scale characterizing the process and which may be subject to cuts in a collider anal-
ysis. For example, when the EFT is applied to describe inclusive on-shell Higgs decays one
has \( M_{\text{cut}} \approx m_h \). Another example is \( e^+e^- \) collisions at a fixed center-of-mass energy \( \sqrt{s} \), in
which case \( M_{\text{cut}} \approx \sqrt{s} \). For certain physically important processes these considerations are
less trivial, especially in the context of hadron collider experiments. The relevant scale for
the production of two on-shell particles in proton-proton collisions, for example, is the center-
of-mass energy of the partonic collision \( \sqrt{\hat{s}} \); this varies in each event and may not be fully
reconstructed in practice. Important examples of this kind are the vector boson scattering
(e.g. with final states \( WW \rightarrow 2l2\nu \) and \( ZZ \rightarrow 4l \)), and Higgs production in association with
a vector boson (\( Vh \)) or a jet (\( hj \)). In all these processes the relevant energy is given by the
invariant mass of the final pair; when this cannot be fully reconstructed, other correlated
variables such as the transverse momentum of the Higgs or a lepton, or the transverse invari-
ant mass can be considered.\(^3\) Since the energy scale of the process determines the range of
validity of the EFT description, it is extremely important that the experimental limits \( \delta_i^{\exp} \) are
reported by the collaborations for various values of \( M_{\text{cut}} \). For processes occurring over a wide
energy range (unlike inclusive Higgs decays or \( e^+e^- \) collisions), knowledge of \( \delta_i^{\exp} \) for just one

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1 These latter can be estimated as usual by varying the factorization and renormalization scales.
2 In general, the experimental constraints on different \( c_i^{(6)} \) may have non-trivial correlations. Depending on
a chosen basis, the left-hand-side of Eq. (2.1) may contain linear combinations of several Wilson coefficients.
If a deviation from the SM is observed, Eq. (2.1) turns into a confidence interval,
\( \delta_i^{\text{d,exp}}(M_{\text{cut}}) < c_i^{(6)} < \delta_i^{\text{u,exp}}(M_{\text{cut}}) \).
3 However, one needs to be aware that it is the former which determines if one is within the validity range
of the EFT.
kinematic point severely limits the interpretation of the EFT results in terms of constraints on specific BSM models. If the relevant energy of the process cannot be determined (e.g. because the kinematics cannot be closed), setting consistent bounds requires a more careful procedure, similar to the one proposed in Ref. [5] in the context of DM searches (see also Refs. [6,7] for related discussions).

2.2 EFT validity and interpretation of the results

Extracting bounds on the EFT coefficients can be done by experimental collaborations in a completely model-independent way. However, the interpretation of these bounds is always model-dependent. In particular, whether or not the EFT is valid in the parameter space probed by experiment depends on further assumptions about the (unknown) UV theory. These assumptions correspond, in the EFT language, to a choice of power counting, i.e. a set of rules to estimate the coefficients of the effective operators in terms of the couplings and mass scales of the UV dynamics.

The simplest situation, which we discuss in some detail here, is when the microscopic dynamics is characterized by a single mass scale $\Lambda$ and a single new coupling $g_*$ [1]. This particular power counting prescription smoothly interpolates between the naive dimensional analysis ($g_* \sim 4\pi$) [2,8], the case $g_* \sim 1$ as e.g. in the Fermi theory, and the very weak coupling limit $g_* \ll 1$. While this is not a unique prescription, it covers a large selection of popular scenarios beyond the SM. In this class falls the Fermi theory described previously, as well as other weakly coupled models where a narrow resonance is integrated out. Moreover, despite the large number of resonances, also some theories with a strongly-interacting BSM sector belong to this category (e.g. the holographic composite Higgs models [9] or, more generally, theories where the strong sector has a large-N description). The scaling of the effective coefficients with $g_*$ is then determined by Eq. (1.2) and by symmetries and selection rules.

For a given power counting, it is relatively simple to derive limits on the theoretical parameter space that are automatically consistent with the EFT expansion, provided the relevant energy of the process is known. Considering the case of a single scale $\Lambda$ and a single coupling strength $g_*$, the bounds (2.1) can be recast as limits on these two parameters by using the power counting to estimate $c_i^{(6)} = \tilde{c}_i^{(6)}(g_*)/\Lambda^2$, and setting the maximum relevant energy scale to $M_{\text{cut}} = \kappa \Lambda$. Here $\tilde{c}_i^{(6)}(g_*)$ is a (dimensionless) polynomial of $g_*$ and of the SM couplings, while $0 < \kappa < 1$ controls the size of the tolerated error due to neglecting higher-derivative operators (the value of $\kappa$ can be chosen according to the sensitivity required in the analysis). One finds

$$\frac{\tilde{c}_i^{(6)}(g_*)}{\Lambda^2} < \delta_{\text{exp}}(\kappa \Lambda). \quad (2.2)$$

These inequalities determine the region of the plane $(\Lambda, g_*)$ which is excluded consistently with the EFT expansion, with a relative error of order $\kappa^2$. They give a useful indication of how effective are the experimental data in constraining the class of theories under consideration (i.e.
those respecting the assumed power counting). A detailed re-analysis of experimental results based on the $M_{\text{cut}}$ technique that we propose here, was performed in Ref. [10] for processes with $Vh$ associated production. The same reasoning can be applied to more complicated theories following a different power counting than the simple $g_*$-scaling discussed above.

The usefulness of power counting stems from a number of reasons. First of all it provides a physically motivated range in which the coefficients $c_i^{(D)}$ are expected to vary. Secondly, and very importantly, it allows one to estimate the relative importance of higher-order terms in the EFT series. As an example, consider a $2 \to 2$ scattering process, where the SM contribution to the amplitude is at most of order $g_{\text{SM}}^2$ at high energy ($g_{\text{SM}}$ denotes a SM coupling). The correction from $D = 6$ operators involving derivatives will in general grow quadratically with the energy and can be as large as $g_{\text{SM}}^2 (E^2/\Lambda^2)$.\footnote{Effects growing with energy can also be induced by operators without additional derivatives, if they yield new contact interactions relevant for the process, or if they disrupt cancellations between $O(E^2)$ contribution of different SM diagrams, see e.g. [11,12]. The following discussion is unchanged in these cases.} If the coupling strength $g_*$ is much larger than $g_{\text{SM}}$, then the BSM contribution dominates over the SM one at sufficiently high energy (i.e. for $\Lambda > E > \Lambda (g_{\text{SM}}/g_*)$), while the EFT expansion is still valid. The largest contribution to the cross section in this case comes from the square of the $D = 6$ term, rather than from its interference with the SM. The best sensitivity to $c_i^{(6)}$ is thus expected to come from the highest value of the relevant energy scale accessible in the experiment. In this example the contribution of $D = 6$ derivative operators is enhanced by a factor $(g_*/g_{\text{SM}})^2$ compared to the naive expansion parameter $(E/\Lambda)^2$; such enhancement is a consequence of the fact that the underlying strong coupling $g_*$ only appears at the level of $D = 6$ operators, while SM operators mediate weaker interactions. In this example, no further enhancement exists between $D = 6$ and $D = 8$ operators, i.e. $D = 8$ operators are subdominant and the EFT series is converging. In other words, although the contributions to the cross section proportional to $(c_i^{(6)})^2$ and $c_i^{(8)}$ are both of order $1/\Lambda^4$, the latter (generated by the interference of $D = 8$ operators with the SM) is smaller by a factor $(g_{\text{SM}}/g_*)^2$ independently of the energy, and can thus be safely neglected. A well known process where the above situation occurs is the scattering of longitudinally-polarized vector bosons. Depending on the UV dynamics, the same can happen in other $2 \to 2$ scatterings, such as Higgs associated production with a $W$ or $Z$ boson (VH) [10, 13] or dijet searches at the LHC [14]. A simple illustrative example is discussed in the next section. Finally, the domination of $(c_i^{(6)})^2$ terms can also happen when $g_*$ is moderate or small but at the same time $g_{\text{SM}}$ is even more suppressed. One possible example concerns flavor-changing neutral current processes which in the SM are strongly suppressed by a loop and CKM factors, see e.g. [15]. An even sharper example is lepton-flavor violating processes (e.g. $h \to \mu \tau$) for which $g_{\text{SM}} = 0$ exactly.

3 Example

In this section we apply our arguments, and compare it with a specific BSM model, in the analysis of the $q\bar{q} \to Vh$ process at the LHC, along the lines of Ref. [10]. The purpose of
the example presented here is to show that, as in the Fermi theory, the knowledge of the
$D = 6$ coefficients of an effective Lagrangian is not enough to determine the validity range of
the EFT approximation. Therefore the theoretical errors incurred from the truncation of the
EFT Lagrangian cannot be quantified in a model-independent way.

We consider the SM extended by a triplet of vector bosons $V^i_\mu$ with mass $M_V$ transforming
in the adjoint representation of the SM $SU(2)_L$ symmetry. Its coupling to the SM fields is
described by [16,17]

$$\mathcal{L} \supset ig_H V^i_\mu H^\dagger \sigma^i \bar{D}_\mu H + g_q V^i_\mu \bar{q}_L \gamma_\mu \sigma^i q_L, \quad (3.1)$$

where $q_L = (u_L, d_L)$ is a doublet of the 1st generation left-handed quarks. In this model $V^i_\mu$
couples to light quarks, the Higgs boson, and electroweak gauge bosons, and it contributes
to the $q\bar{q} \to V h$ process at the LHC. Below the scale $M_V$, the vector resonances can be inte-
grated out, giving rise to an EFT where the SM is extended by $D=6$ and higher-dimensional
operators. Thus, $M_V$ plays the role of the EFT cut-off scale $\Lambda$. Using the language of the
Higgs basis [18], the EFT at the $D=6$ level is described by the parameter $\delta c_z$ (relative cor-
rection to the SM Higgs couplings to $WW$ and $ZZ$) and $\delta g^Z_{q}$ (relative corrections to the $Z$
and $W$ boson couplings to left-handed quarks), plus other parameters that do not affect the
$q\bar{q} \to V h$ process at tree level. The relevant EFT parameters are matched to those in the UV
model as

$$\delta c_z = -\frac{3v^2}{2M_V^2} g_H^2, \quad [\delta g^Z_{q}]_{11} = -\frac{v^2}{2M_V^2} g_H g_q. \quad (3.2)$$

When these parameters are non-zero, certain EFT amplitudes grow as the square of the center-
of-mass energy $s \equiv M^2_{W_h}$ of the analyzed process, $\mathcal{M} \sim M^2_{W_h}/M_V^2$. Then, for a given value
of the parameters, the observable effects of the parameters become larger at higher energies.

However, above a certain energy scale, the EFT may no longer approximate correctly the UV
theory defined by Eq. (3.1), and then experimental constraints on the EFT parameters do
not provide any information about the UV theory.

To illustrate this point, we compare the UV and EFT descriptions of $q\bar{q} \to V h$ for three
benchmark points:

- **Strongly coupled**: $M_V = 7$ TeV, $g_H = -g_q = 1.75$;
- **Moderately coupled**: $M_V = 2$ TeV, $g_H = -g_q = 1/2$;
- **Weakly coupled**: $M_V = 1$ TeV, $g_H = -g_q = 1/4$;

Clearly, all three benchmarks lead to the same EFT parameters at the $D=6$ level. How-
ever, because $M_V = \Lambda$ varies, these cases imply different validity ranges in the EFT. This is
illustrated in Fig. 1. While, as expected, in all cases the EFT description is valid near the
production threshold, above a certain point $M_{W_h}^{\max}$ the EFT is no longer a good approxima-
tion of the UV theory. Clearly, the value of $M_{W_h}^{\max}$ is different in each case. For the moderately
coupled case, it coincides with the energy at which the linear and quadratic EFT approxima-
tions diverge. From the EFT perspective, this happens because $D=8$ operators can no longer
Figure 1: **Left:** The partonic $u\bar{d} \rightarrow W^+ h$ cross section as a function of the center-of-mass energy of the parton collision. The black lines correspond to the $SU(2)_L$ triplet model with $M_V = 1$ TeV, $g_H = -g_q = 1/4$ (dashed), $M_V = 2$ TeV, $g_H = -g_q = 1/2$ (dotted), and $M_V = 7$ TeV, and $g_H = -g_q = 1.75$ (solid). The corresponding EFT predictions are shown in the linear approximation (red), and when quadratic terms in $D=6$ parameters are included in the calculation of the cross section (purple). **Right:** The theory errors defined as the relative difference between the constraints on $\sigma \equiv \sigma_H^2 = \sigma_q^2$ obtained by recasting the limits obtained in the framework of a $D=6$ EFT and the limits obtained directly by comparing the predictions of the resonance model with experimental observations. The limits come from re-interpreting experimental constraints on the partonic $\sigma(u\bar{d} \rightarrow W^+ h)$ cross section using the hypothetical data described in the text up to $M_{cut} = 1$ TeV (blue) and up to $M_{cut} = 2$ TeV (red). The dotted lines correspond to the naive estimate $\sim \kappa^2$ where $\kappa \equiv M_{cut}/\Lambda = M_{Wh}/M_V$. The theory errors defined in this way depend mostly on $\kappa^2$ and very weakly on the experimental precision: the curves obtained by recasting limits for another value of $\sigma/\sigma_{SM}$ practically overlap with the ones plotted here.

be neglected. However, for the strongly coupled case, the validity range extends beyond that point. In this case, it is the quadratic approximation that provides a good approximation of the UV theory. As discussed in the previous section, that is because, for strongly-coupled UV completions, the quadratic contribution from $D=6$ operators dominate over that of $D=8$ operators in a larger energy range.

As an illustration of our discussion of setting limits on EFT parameters and estimating associated theoretical errors, consider the following example of an idealized measurement. Suppose an experiment makes the following measurement of the $\sigma(u\bar{d} \rightarrow W^+ h)$ cross section at different values of $M_{Wh}$:

<table>
<thead>
<tr>
<th>$M_{Wh}$[GeV]</th>
<th>250</th>
<th>500</th>
<th>1000</th>
<th>1500</th>
<th>2000</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma/\sigma_{SM}$</td>
<td>1 ± 0.1</td>
<td>1 ± 0.05</td>
<td>1 ± 0.1</td>
<td>1 ± 0.15</td>
<td>1 ± 0.2</td>
</tr>
</tbody>
</table>

This is meant as a simple proxy for more realistic measurements at the LHC, for example measurements of a fiducial $\sigma(pp \rightarrow W^+ h)$ cross section in several bins of $M_{Wh}$. For simplicity, we assume that the errors are Gaussian and uncorrelated. These measurements can be
recast as constraints on $D=6$ EFT parameters for different $M_{\text{cut}}$ identified in this case with the maximum $M_{Wh}$ bin included in the analysis. For simplicity, in this discussion we only include $[\delta g_L^{Zq}]_{11}$ and ignore other EFT parameters (in general, a likelihood function in the multi-dimensional space of EFT parameters should be quoted by experiments). Then the “measured” observable is related to the EFT parameters as

$$\frac{\sigma}{\sigma_{SM}} \approx \left( 1 + 160 \delta g_L^{Wq} \frac{M_{Wh}^2}{\text{TeV}^2} \right)^2,$$

where $\delta g_L^{Wq} = [\delta g_L^{Zq}]_{11} - [\delta g_L^{Zq}]_{11}$. Using this formula, one can recast the measured cross sections as 95% CL confidence intervals on $\delta g_L^{Wq}$:

<table>
<thead>
<tr>
<th>$M_{\text{cut}}$ [GeV]</th>
<th>250</th>
<th>500</th>
<th>1000</th>
<th>1500</th>
<th>2000</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta g_L^{Wq} \times 10^{-3}$</td>
<td>[-10.4, 9.6]</td>
<td>[-1.3, 1.2]</td>
<td>[-0.57, 0.53]</td>
<td>[-0.34, 0.31]</td>
<td>[-0.24, 0.21]</td>
</tr>
</tbody>
</table>

Suppose these constraints are quoted by experiment. A theorist may try to interpret them as constraints on the vector resonance model with $g_q = -g_H \equiv g_*$ using the map in Eq. (3.2). This way one would obtain the constraints on $g_*$ as a function of $M_V$: for example, for $M_V = 3$ TeV one would find $g_* \leq 0.28(0.18)$ for $M_{\text{cut},1} = 1$ TeV ($M_{\text{cut},2} = 2$ TeV). Note that, for our (arbitrary) choice of data points, the limits on $g_*$ obtained from the measurement with $M_{\text{cut},2}$ are stronger from the one with $M_{\text{cut},1}$. However, the result for $M_{\text{cut},1}$ is also useful for theorists. First, it can be used also for $M_V \approx 2$ TeV, whereas the one with $M_{\text{cut},2} = 2$ TeV does not have a meaningful interpretation in this mass range. Furthermore, the theory error is smaller for $M_{\text{cut},1}$ ($\sim 10\%$ for $M_V = 3$ TeV) than for $M_{\text{cut},2}$ ($\sim 60\%$ for $M_V = 3$ TeV). Here, we define the theory error as the fractional difference between the bound on $g_*^2$ interpreted from the EFT constraints, and the true bound obtained by fitting the full resonance model to the experimentally “measured” cross sections using the bins up to a given $M_{\text{cut}}$. The theory errors are plotted as solid lines in the right panel of Fig. 1. By general arguments discussed in the previous section, one expects the theory error to scale as $\kappa^2$ for $M_V \gg M_{\text{cut}}$, where $\kappa = M_{\text{cut}}/\Lambda = M_{Wh}/M_V$, and this expectation, which is shown as dotted lines in the same plot, is confirmed. While the limits on $g_*$ obviously depend on the experimental central values and errors we assumed, the theory errors as defined here are very weakly dependent on it.

4 Summary

In this note we have discussed the validity of an EFT where the SM is extended by $D=6$ operators. We have argued that the validity range cannot be determined using only low-energy information. The reason is that, while the EFT is valid up to energies of order of the mass $\Lambda$ of the new particles, low-energy observables depend on the combinations $c^{(6)}/\Lambda^2$, where the Wilson coefficients $c^{(6)}$ of $D=6$ operators are function of the couplings of the UV theory.

The question of a theoretical uncertainty due to the truncation of the EFT at the level of $D=6$ operators depends on the impact of $D > 6$ operators on the studied processes. The
relative size of the contribution of $D=8$ operators is controlled by $\tilde{c}^{(8)}/\tilde{c}^{(6)}(E_{\text{exp}}^{2}/\Lambda)$, where $E_{\text{exp}}$ is the typical energy scale of the process. We have discussed the physical assumptions that lead to a situation with $\tilde{c}^{(8)} \approx \tilde{c}^{(6)}$. In this situation the energy at which the EFT breaks down coincides with the scale at which the contribution of $D=8$ and higher-dimensional operators is of the same order as that of $D=6$ operators. Conversely, when the EFT expansion is well convergent at the LHC energies, the effects of $D=8$ operators can be neglected. We have also shown that the power counting is necessary to estimate the range of variation of the effective coefficients $\tilde{c}_{i}^{(6)}$, and to identify situations in which departures from the SM can be sizable (even bigger than the SM itself), compatibly with the EFT expansion. Exceptions from this rule, in the form $\tilde{c}^{(8)} \gg \tilde{c}^{(6)}$, may arise in a controlled way as a consequence of symmetries and selection rules or for certain well-defined classes of processes. The concrete examples where this occurs are discussed in Ref. [3]. The inclusion of $D=8$ operators in experimental analyses is justified only when dealing with these special cases, and would represent an inefficient strategy in a generic situation.

We have stressed that the ratio $\tilde{c}^{(8)}/\tilde{c}^{(6)}$, which controls the theoretical uncertainty of the EFT predictions, depends on the assumptions about the UV theory that generates the $D=6$ and $D=8$ operators. Only when a particular power counting is adopted, for example the $g_{\star}$-scaling discussed in this note, can the contributions from $D=6$ and $D=8$ relative to the SM be estimated in a bottom-up approach, and the error associated with the series truncation be established. For this reason we suggest to report the estimated uncertainty due to the truncation separately from the other errors, and to clearly state on which assumptions the estimate is based.

If no large deviations from the SM are observed at the LHC Run-2, stronger constraints on $D=6$ operators can be set, i.e., $\delta_{\exp}$ decreases. This will extend the EFT validity range to a larger class of UV theories, i.e. those with weaker coupling $g_{\star}$ and, for a fixed $g_{\star}$ and given experimental energy scale, will leave less room for contributions of $D=8$ operators. As a consequence, the internal consistency and the validity range of the LO $D=6$ EFT will only increase. On the other hand, if a deviation from the SM is observed, efforts to estimate the effects of $D > 6$ operators may be crucial to better characterize the underlying UV theory.

Most of the discussion in this note is relevant at the level of the interpretation of the EFT results, rather than at the level of experimental measurements. However, there are also practical conclusions for experiments. In particular, the experimental constraints on the effective coefficients of $D=6$ operators should be reported as functions of $M_{\text{cut}}$, where $M_{\text{cut}}$ collectively denotes the value of the relevant kinematic variables such as transverse momenta or invariant masses. This is especially important for hadron collider experiments, such as those performed at the LHC, where collisions probe a wide range of energy scales. It is also useful to present the experimental results both with and without the contributions to the measured cross sections and decay widths that are quadratic in the effective coefficients.

5The validity range can also be improved by means of a global analysis combining different measurements, which often lifts flat directions in the parameter space [19, 20] and leads to stronger constraints on $D=6$ effective coefficients, see e.g. [21].
With this way of presentation, the experimental results can be applied to constrain a larger
class of theories beyond the SM in a larger range of their parameter space. Other frameworks
to present results, for example template cross-sections discussed elsewhere, should also be
pursued in parallel, as they may address some of the special situations discussed in this note.

Note that even in the case of BSM discoveries in the next LHC runs, the EFT approach and
the results presented here remain still useful. For measurements with a characteristic scale
\( M_{\text{cut}} \) considerably below the new physics threshold, the new particle(s) can be integrated
out (in analogy to the Fermi Theory) and deviations from the predicted values of the \( D=6 \)
coefficients can be probed. Such an EFT approach may give a more economical description
of the relevant precess, with fewer parameters (the effective coefficients) that can be directly
measured from low-energy data. For processes involving higher scales, an EFT including the
BSM degrees of freedom can be set up and all results generalize straightforwardly.

A final comment is in order when it comes to constrain explicit models from the bounds
derived in an EFT analysis of the data. Although EFT analyses aim at a global fit with
all the operators included, it is important to ensure that the reported results are complete
enough to later consider more specific scenarios where one can focus on a smaller set of
operators. Reporting the full likelihood function, or at the very least the correlation matrix,
would be a way to address this issue. Furthermore, for the purpose of estimating the validity
of the EFT approach, it might be useful to compare the EFT constraints obtained with and
without including the quadratic contributions of \( D=6 \) operators in the theoretical calculations
of observables: significant differences between these two procedures will indicate that the
results apply only in the case of strongly-coupled UV theories, where quadratic terms can
give the dominant effect at large energies.

References


[5] D. Racco, A. Wulzer, and F. Zwirner, Robust collider limits on heavy-mediator Dark


