ATTAINING HIGH LUMINOSITY IN HADRON COLLIDERS

J. Gareye

Abstract

The beam-beam interaction limits the ultimate luminosity attainable in Hadron colliders to a few $10^{35}$ cm$^{-2}$s$^{-1}$ in one single interaction in the energy range covered by the LHC and the SSC. Although this limit increases with energy, it becomes more and more difficult and costly to reach at high energy, because of the increasingly large beam currents which are required. New technological progress might allow a further increase in performance beyond what is now possible.

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Attaining High Luminosity in Hadron Colliders

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ABSTRACT: The beam-beam interaction limits the ultimate luminosity attainable in Hadron colliders to a few $10^{35}$ cm$^{-2}$s$^{-1}$ in one single interaction in the energy range covered by the LHC and the SSC. Although this limit increases with energy, it becomes more and more difficult and costly to reach at high energy, because of the increasingly large beam currents which are required. New technological progress might allow a further increase in performance beyond what is now possible.

1. INTRODUCTION

Years ago the CERN ISR brightly demonstrated that high luminosities can be obtained in hadron colliders $^1$. Its world luminosity record of $1.4 \times 10^{32}$ cm$^{-2}$s$^{-1}$ established in 1982 still holds, only approached recently by the Cornell e$^+e^-$ collider. The generation of hadron colliders now under design, the LHC and the SSC, will operate at energies a few hundred times larger than the ISR, and to compensate for the reduction of the cross-sections at these energies, much higher luminosities are required from these machines.

Design studies for the LHC and SSC have shown that luminosities ranging from a few $10^{33}$ to a few $10^{34}$ cm$^{-2}$s$^{-1}$ can be obtained. This already puts a severe demand on physics detectors, since at this level each bunch collision generates several events, and also because the collisions themselves are a powerful source of damaging radiation. The possible detector limitations will be ignored in the following, to concentrate the discussion on accelerator issues related to very high luminosity in hadron colliders.

The luminosity is given by

$$L = \frac{N^2 \gamma}{4 \pi \epsilon \beta^* \Delta t} \quad (1)$$

where $N$ is the number of particles per bunch, $\gamma$ the relativistic factor, $\epsilon = \gamma \sigma^2 / \beta$ the normalized emittance, $\beta^*$ the betatron function at the interaction point and $\Delta t$ the bunch spacing in seconds.

The most fundamental limitation of a collider, the beam-beam effect, will be examined first. Problems related to the large beam currents which are necessary to reach high luminosity, essentially
synchrotron radiation and collective instabilities will then be discussed. Finally areas where technological progress could allow a further increase of the collider performance beyond the present limits will be mentioned.

2. BEAM-BEAM LIMITATIONS

2.1 Previous experience on the beam-beam effect

The effect of the beam-beam interaction in hadron colliders has been extensively studied in the CERN SPES$^{2}$ and in the Fermilab Tevatron$^{3}$, two machines which operate with a few dense bunches colliding head-on in 3 to 12 crossing points.

The most significant effect of this interaction is a slow diffusion of large amplitude particles into the tails of the transverse distributions. This eventually leads to particle losses which decrease the beam lifetime and create background in the experimental detectors. In the future colliders the occurrence of such losses in the superconducting part of the machine could lead to magnet quenches and jeopardize the operation. This diffusion is created by isolated high order betatron resonances excited by the strongly non linear beam-beam force. The resonance condition is given by

$$n_x Q_x + n_z Q_z = p$$

(2)

where $n_x$, $n_z$ and $p$ are integers and $Q_x$ and $Q_z$ are the horizontal and vertical tunes. The excitation strength of the resonances is proportional to the beam-beam parameter, which is for a round beam

$$\xi = N r_p / 4 \pi e_n$$

(3)

where $N$ is the number of particles per bunch, $r_p$ the classical proton radius and $e_n$ the normalized transverse emittance. In addition, it has a strong dependance on the particle amplitude expressed in r.m.s beam size of the opposing beam, as shown in Fig. 1. Resonances of high order ($n = |n_x| + |n_z|$) are only dangerous for large amplitude particles, and for a given amplitude the strength increases rapidly when resonances of lower and lower order are considered. This explains why the parameter which is of primordial importance in the beam-beam interaction is the resulting total tune spread, because it determines the lowest order of the resonances which cannot be avoided in the tune diagram.

In the SPES operating at values of $\xi$ between 0.003 and 0.006 and with 3 crossing points resonances of order 10 or less produce a strong diffusion which reduces the beam lifetime to unacceptably low values, while the influence of resonances of order 13 and 16, which cannot be avoided, is clearly noticed. In the Tevatron, resonances of order 12 seem tolerable, but at a smaller value of $\xi$ than in the SPES. Considering this experience, it seems prudent to base the design of the future colliders on a total tune spread in the beam smaller than 0.02; this is about the largest value
which allows operation outside resonances of order 12 or less, without going extremely close to integer tune values. The future colliders will operate with a large number of closely spaced bunches and in order to prevent them from colliding in many places all along that portion of the trajectories which is common to both beams around each interaction region, the beams cross at a small angle $\phi$. This slightly reduces the luminosity by a factor $(1 + (\phi \sigma_s/2\sigma)^2)^{-1/2}$ where $\sigma_s$ is the r.m.s. bunch length and $\sigma$ the r.m.s. beam radius at the collision point. It provides a separation of the order of 6 to 7 $\sigma$ of the beam trajectories, sufficient to prevent particle collisions but not to suppress the electromagnetic interactions between the bunches, the so-called long range interactions.

The beam-beam resonances are excited mainly by the quasi head-on collisions and in this respect the situation in the future colliders will be very similar to that of the SPPS, since both the beam-beam parameter and the number of interaction points are similar in the two cases. However there is a difference as far as the total tune spread is concerned. In the SPPS the tune spread is roughly $\xi$ times the number of collision points, whereas in the future colliders an additional contribution comes from the long range interactions. This is illustrated in Fig. 2 for the case of the LHC. The combination of the long range and the head-on interactions produces an average tune shift which can be compensated by adjusting the machine tune, and a tune spread which has to be accommodated in between the dangerous resonances. The situation is complicated by the existence of gaps in the beam azimuthal distribution which are required by the finite risetimes of the injection and abort kickers. A few bunches at the beginning and at the end of the bunch trains suffer only half the long range interactions; these edge bunches are therefore displaced in the tune diagram, and as a consequence the total area occupied by the particles is increased. This effect will be neglected in the following to simplify the analysis.

2.2 Beam-beam limitations in future colliders

The total allowed tune spread has to be shared by the various interaction points. As a consequence the luminosity attainable in a hadron collider depends on the number of experiments simultaneously active, and is maximum in the case of a single experiment. It is proposed to concentrate the following discussion on the potential of hadron colliders in this simple case.

There is yet no experience available concerning the permissible $\xi$ and total tune spread for a single interaction. Stronger resonances have to be expected in this case for the same total head-on tune shift since there can be no cancellation of the driving beam-beam force from one point to another. It seems prudent in this case to limit the maximum tune shift parameter to 0.01 and the total tune spread to 0.015, a choice comparable to that already made in similar studies. This leaves 0.005 for the tune spread arising from the long range interactions, which is given by

$$\Delta Q_{LR} = \frac{3r_n^2\varepsilon_s N D}{2\pi c \gamma^2 \beta^2 \phi^4 \Delta t}$$ (4)
where $n$ is the particle amplitude at which $\Delta Q$ is evaluated divided by the r.m.s. beam radius, $D$ the effective distance over which the two beams interact and $c$ the velocity of light.

Using relations (3) to express $N/\epsilon_n$ and (4) to express $N/\Delta t$ the luminosity can be written

$$L = \frac{2}{3} \frac{\pi c}{r_p^2} \xi \Delta Q_{RL} \frac{\gamma^3 \beta^* \phi^4}{n^2 \epsilon_n D}$$  (5)

The values of $D$ and $\beta^*$ depend both on the maximum gradient achievable in the quadrupoles of the final focusing triplet, and on the length of the experimental set up. With the maximum quadrupole gradient of 250 T m$^{-1}$ allowed by present technology and a free length for experiments of 20 m the following approximate scaling laws apply in the range of the LHC and SSC energies:

$$D = 100 \left[ \frac{\gamma}{10^4} \right]^{1/2} \text{ m}$$  (6)

$$\beta^* = 0.3 \left[ \frac{\gamma}{10^4} \right]^{1/2} \text{ m}$$  (7)

The crossing angle $\phi$ is limited by geometrical considerations. Assuming an inner coil diameter of about 50 mm in the final focusing quadrupoles the separation between the two beam centres at a distance $D/2$ from the interaction point should not exceed 10 mm, which gives a maximum possible value

$$\phi = \frac{0.02}{D}$$  (8)

Using relations (5) to (8), $\xi = 0.01$ and $\Delta Q_{LR} = 0.005$ for $n = 4$ gives

$$L = 4.026 \times 10^{25} \frac{\gamma}{\epsilon_n} \text{ cm}^{-2} \text{ s}^{-1}$$  (9)

Assuming $\epsilon_n = 1 \times 10^{-6}$ m, which is the value considered in the SSC design, this formula gives a maximum beam-beam limited luminosity of $3.3 \times 10^{35}$ cm$^{-2}$ s$^{-1}$ for a machine with the energy of the LHC and $8.5 \times 10^{35}$ for a machine with the energy of the SSC. The main parameters of these two machines are given in Table 1. Formula (9) deserves the following comments. Considering only the head-on beam-beam interaction, which sets a higher limit on $N/\epsilon_n$, one would increase the luminosity by increasing both $N$ and $\epsilon_n$ because of the $N^2$ dependance of the luminosity. However, when the long range effect is taken into account, a small emittance is favored because it reduces the long range tune spread for the same crossing angle, so that the total beam intensity proportional to $N/\Delta t$ can be increased. In reality, the edge bunch effect shown in Fig. 2 complicates the situation
since this component does not depend on emittance. Formula (9) is therefore an approximation useful for scaling purposes, but should not be used for a detailed design.

The lower limit of the normalized emittance is determined by the space charge tune spread in the low energy injectors, given by

$$\Delta Q_{sc} = r_p N / 4\pi \beta y^2 B \epsilon_n$$

(10)

where the bunching factor \( B \) is for a Gaussian bunch \( \sqrt{2\pi} \sigma_s / (2\pi R) \) with \( \sigma_s \) the r.m.s. bunch length and \( 2\pi R \) the machine circumference.

TABLE 1

Beam-beam limited performance of two machines optimized at different energies for \( \epsilon_n = 1 \times 10^{-6} \) m

<table>
<thead>
<tr>
<th>Energy ([E], \text{TeV})</th>
<th>7.7</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>Luminosity ([L], \text{cm}^2\text{s}^{-1})</td>
<td>3.3 \times 10^{35}</td>
<td>8.5 \times 10^{35}</td>
</tr>
<tr>
<td>Particles per bunch ([N])</td>
<td>8.2 \times 10^{10}</td>
<td>8.2 \times 10^{10}</td>
</tr>
<tr>
<td>Bunch spacing ([\Delta t], \text{s})</td>
<td>4.9 \times 10^{-9}</td>
<td>3.1 \times 10^{-9}</td>
</tr>
<tr>
<td>Beam current ([I], \text{A})</td>
<td>2.7</td>
<td>4.2</td>
</tr>
<tr>
<td>Total number of particles per beam ([N_t])</td>
<td>(1.5 \times 10^{15})</td>
<td>(7.7 \times 10^{15})</td>
</tr>
<tr>
<td>Beam stored energy ([U], \text{GJ})</td>
<td>1.7</td>
<td>23</td>
</tr>
<tr>
<td>Synchrotron radiation power (two beams) ([P_s], \text{kW})</td>
<td>60</td>
<td>1000</td>
</tr>
<tr>
<td>Synchrotron radiation power per unit length (1 beam), (W)</td>
<td>1.8</td>
<td>8.3</td>
</tr>
</tbody>
</table>

Both \(\Delta Q_{sc}\) in the injectors and \(\xi\) in the collider are proportional to \(N/\epsilon_n\). The parameters shown in Table 1 imply a value of \(N/\epsilon_n\) three times larger than that of the LHC design and twelve times larger than that of the SSC design. Such a value can probably be achieved with an adequate design of the injector chain, but at an increased cost.

Applying formula (9) with the emittance of the LHC, namely \(\epsilon_n = 3.75 \times 10^{-6} \) m, gives a luminosity of \(8.8 \times 10^{34} \text{ cm}^2\text{s}^{-1}\) which is very close to \(7.8 \times 10^{34} \text{ cm}^2\text{s}^{-1}\), the luminosity quoted in the LHC design report for a single, high luminosity experiment.
3. BEAM INTENSITY LIMITATIONS

3.1 Synchrotron Radiation

The total power radiated by the two beams is

\[ P_s = \frac{2}{3} \frac{Z_o e^3 c N \gamma^4}{\rho \Delta t} \]  \hspace{1cm} (11)

where \( Z_o \) is the impedance of free space, \( e \) the elementary charge, \( c \) the velocity of light and \( \rho \) the bending radius. Since the magnetic field \( B \) is fixed by the technology available the following formulation is also of interest

\[ P_s = \frac{2}{3} Z_o \frac{e^3}{m} \frac{N}{\Delta t} B \gamma^3 \]  \hspace{1cm} (12)

where \( m \) is the proton mass.

This power has to be absorbed at cryogenic temperature, and the total power required for that is \( P_s / \eta \) if \( \eta \) is the efficiency of the refrigeration system. Typical values for \( 1/ \eta \) are 250 if the synchrotron radiation is absorbed at 5 K and 60 if it is absorbed at 20 K. For several reasons, in particular the necessity to provide a beam pipe with a very low resistivity, it seems difficult to use a radiation shield at a temperature higher than 20 K. Since there are financial and social limitations to the total power used by the collider, the maximum allowed value of \( P_s \) may limit the luminosity. Another limit may arise from the inability to evacuate the power produced by each beam per unit length

\[ P / 2 \pi \rho = \frac{Z_o e^4}{6 \pi m^2 c} \frac{N}{\Delta t} B^2 \gamma^2 \]  \hspace{1cm} (13)

The LHC radiation shield is designed to absorb 0.6 W/m at 5K. If it were possible to increase this value to 1.8 W/m at 5 K for the LHC and 8.3 W/m at 20 K for the SSC, synchrotron radiation would not limit the performance indicated in Table 1 for these two machines.

However the increase in cost of the facility due to the more complicated design and the larger power consumption will in practice impose a limit on the value of \( P_s \). When this limit is reached the energy can only be increased by sacrificing luminosity.

This is illustrated in Fig. 3 with the case of a hypothetical 40 TeV collider assuming a magnetic field of 10 T. Expression (10) with \( \epsilon_n = 1 \times 10^{-6} \) gives a maximum luminosity of \( 1.7 \times 10^{36} \) cm\(^{-2}\)s\(^{-1}\) at maximum energy. However above 23.7 TeV the power radiated per unit length would
exceed 8 W and the total power would exceed 1.3 MW. If larger values are not allowed the luminosity decreases like $\gamma^{-3}$ above this energy to keep the radiated power constant.

Operating the machine at a lower energy requires the value of $\beta^*$ to be increased proportionally to $\gamma^{-1}$ to keep the divergence of the beam in the interaction region constant. This divergence is given by $n \left[ \epsilon_n / \gamma \beta^* \right]^{1/2}$ for a particle at $n\sigma$ and is limited by the maximum possible crossing angle. As a consequence the luminosity decreases like $\gamma^2$ at lower energies.

3.2 Collective instabilities

Attaining the beam-beam limited luminosity indicated by equation (9) requires very high beam intensities, especially at the higher energies. Assuming that such beam intensities can be furnished by the injectors at reasonable cost, a number of potential instabilities have to be mastered in the collider itself.

These instabilities arise from the electromagnetic interaction of the beam with the surrounding structure. The main contributions to the beam-environment coupling impedance comes from the finite resistivity of the beam pipe, from the bellows, monitors, kickers, and from the RF cavities. The surface resistivity of the beam pipe can be lowered by copper-coating and maintaining the pipe walls at low temperature, while the bellows and kickers can be partially shielded from the beam electromagnetic fields. The coupling impedance of the RF structures can be considerably lowered by using superconducting cavities with a large bore. All these measures are being applied to the LHC, with the aim of reducing the coupling impedance to about 0.7 $\Omega$ for the resistive wall and 0.1 $\Omega$ for the broad band component. In this case the thresholds of the single bunch instabilities, which are determined by the broad band component, are of the order of a few $10^{11}$ protons per bunch and these effects do not limit the performance. The growth rates of the coupled bunch modes which are excited by the beam pipe resistivity and the high order modes of the RF cavities depend on the total beam current. These modes are unstable for large beam currents and must be damped by feedback systems, but this is relatively easy in the LHC up to the current mentioned in Table 1, for which the resistive-wall growth time is about 300 turns.

However since the coupling impedance scales like the machine radius and the inverse cubic power of the beam pipe radius serious problems have to be expected at higher beam energies; these can only be obtained with larger machines for which there is a strong incentive to reduce the magnet coil diameter for economic reasons. As an example in the "SSC like" machine of Table 1 the growth time of the resistive wall instability is only three turns. This is clearly too fast to be damped by a conventional feedback. Although one can imagine several feedback systems transmitting signals across a diameter of the ring, this instability is a serious limitation to the beam current. In the colliders now under design the high order bunch modes, which cannot be damped by feedback, are
naturally stabilized by Landau damping. This may no longer be the case at extreme values of the beam current, and it may constitute another limitation.

3.3 Other limitations to the beam current

In addition to the synchrotron radiation and collective instabilities, the problems linked to the enormous amount of energy stored in the beam may limit the collider performance at high energy. Already at the level of the LHC and SSC the design of the beam dumping system is not trivial. An elaborate set of collimators is needed to protect the superconducting magnets from unavoidable continuous beam losses. The environmental and machine irradiation problems should not be forgotten either.

4. FURTHER TECHNOLOGICAL PROGRESS

4.1 Low beta insertion

At present the performance of the low beta insertion is limited at the higher energies by the available pole tip magnetic field $\hat{B}$ in the quadrupole of the final focusing triplet. In the LHC a maximum gradient of about 250 T/m is obtained with a 56 mm inner coil diameter. An increase of the pole tip field can be used either to increase the gradient at constant aperture, thus getting a more compact triplet which reduces the long range beam-beam interaction, or increase the aperture at constant gradient and allow a larger crossing angle. The minimum crossing angle is proportional to the angular divergence of the beam at the interaction point, $[\epsilon_n / \gamma \beta^*]^{1/2}$. Using this, formula (5) gives

$$L \propto \xi \Delta Q_{LR} \frac{\epsilon_n \gamma}{\beta^* D}$$

Using the scaling rule for an optimized triplet design$^5\text{, } \epsilon_n \gamma / \beta^* \propto \hat{B} D^2$ we get

$$L \propto \xi \Delta Q_{LR} \hat{B}^2 D$$

Technological progress in this field should therefore be encouraged.

4.2 Crab-crossing

Instead of passing the two beams through the same, large aperture quadrupole, one could by increasing sufficiently the crossing angle use two separate, small aperture channels as in the "two in one" LHC magnets. This would eliminate the long range interaction but would also considerably decrease the luminosity unless "crab-crossing" is used to align the bunches at the interaction point$^6$. 
This technique has been proposed for electron colliders, and has not yet been demonstrated in these machines. Its possible use in hadron machines should be investigated.

5. ACKNOWLEDGEMENTS

In July 1990 a miniworkshop on problems linked to high luminosity in the SSC and the LHC took place in the framework of the DPF Summer Study on Physics at High Energy held at Snowmass, Co, USA. The participants were: A. Chao, R. Palmer, R. Sieman and the author, with the part-time participation of K. Brown and L. Evans. Most of the results presented in this report have been discussed at length during the meeting, and very similar views will be found in the proceedings of the Summer Study.
References


Fig. 1 Strength of high order beam-beam induced resonances as a function of the particle amplitude normalized to the r.m.s radius of the opposing beam.
Fig. 2 Beam-beam induced tune shifts per interaction normalized to $\xi$ for particles with horizontal, vertical amplitudes $(x, y)$ and $N = 10^{11}$, $\Delta t = 15$ ns, $\phi = 200 \, \mu\text{rad}$, $\varepsilon_a = 3.75 \times 10^{-6}$ m in the LHC.
Fig. 3 Maximum attainable luminosity as a function of energy in a 40 TeV collider with $B = 10\,\text{T}$ and $\varepsilon_n = 10^{-6}\,\text{m}$. Below 23.7 TeV the luminosity is limited by beam beam effects and increases like $\gamma^2$. Above 23.7 TeV the luminosity decreases like $\gamma^3$ if the synchrotron radiation power emitted by the two beams is restricted to 1.3 MW.