CALCULATING THE STANDING WAVE RATIO OF A PASSIVE, LOSS-FREE MICROWAVE JUNCTION USING SUPERFISH

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Abstract

In designing the RF circuitry around an accelerating cavity, one often encounters loss-free two-port junctions between sections of homogeneous lines or waveguides. The design of such a junction requires the evaluation of the junction Standing Wave Ratio (SWR). For simply constructed junctions the SWR can be determined from the known geometry, but for most practical constructions one must build a model and make the measurement. Typically the model is then modified to minimize the SWR, and the process is repeated. Here is presented an outline of the process using the cavity code SUPERFISH. The method is equally applicable to other cavity codes like URMEL and MAFIA.

1. GENERAL EXPERIMENTAL BASIS

Consider the general arrangement of a pair of transmission lines joined by a passive loss-free junction as in fig. 1. One end is matched to an RF generator while the other has a movable short. This is the typical arrangement one sets up to measure the SWR of the junction in the lab. When the short is moved to a distance $x_1$, a corresponding shift in the location of the node in the slotted line is observed, $x_2$. The relationship between the two is

$$\tan (k_2 x_2 - a) = s \tan (k_1 x_1 - b),$$

where $k_1 = 2\pi/\lambda_1$ and $k_2 = 2\pi/\lambda_2$ are the propagation constants of the two sections of the line, and $s$ is the SWR of the junction. The constants $a$ and $b$ relate to the origins of $x_1$ and $x_2$ and are established when the right side terminates into a matched load [1] (derivation of this relationship is given in the Appendix).

A measurement of $x_2$ and $x_1$, normalized by their propagation constants, gives a curve like that shown in fig. 2. The data points wiggle about the line $k_2 x_2 = k_1 x_1 + (a - b)$, representing the matched junction ($s = 1$). The slopes, as the data points cross this reference line, are $s$ and $1/s$.

![Fig. 1 Set-up for locating the nodes in a slotted line.](image1)

![Fig. 2 Typical data set for the measurement discussed in the text.](image2)

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Alternatively one may examine \( \eta = k_2 x_2 - k_1 x_1 \). The data in fig. 2 are redisplayed in fig. 3 in just such a manner. Now the SWR \( s \) is given by [2]

\[
s = \left( \frac{1 + \sin(\Delta_e)}{\cos(\Delta_e)} \right)^2,
\]

where \( \Delta_e \) is the maximum excursion of the data from the \( \eta = (a-b) \) line.

This method is particularly useful in that one can place a short at both ends of the transmission line, and drive it somewhere in the middle. Now the measurement consists of moving one short by \( x_1 \), then locate the corresponding distance of the other short, \( x_2 \), required to re-establish the resonance. The methods are equivalent, differing only in the apparatus used.

2. APPLICATION OF THE CAVITY CODE

With this model for an experimental arrangement, we can now define the problem for a cavity code like SUPERFISH. A simple coaxial line with some disturbance in the middle is set up with shorts at the two ends, and its resonant frequency \( f_0 \) found. Now, while moving the disturbing section back and forth in the resonator, the overall length is adjusted to re-establish \( f_0 \). This is an interactive process, but is easily automated with some computer code.

We have tested this method on two problems with axial symmetry. The first is a 4 cm thick \( \text{Al}_2\text{O}_3 \) window (\( \varepsilon = 9 \) assumed) in an 8 cm \( \phi \), 50 \( \Omega \) coaxial line. This problem can be solved using circuit theory for waveguiding systems. One finds \( s = 6.925 \) at 400 MHz. Calculating the various resonators in SUPERFISH, we obtained the data of fig. 4. With \( k_1 = k_2 = 8.38 \text{ m}^{-1} \), the data yields \( \Delta_e = k 0.017 \text{ m} = 0.844 \). Then

\[
s = \left( \frac{1 + \sin(0.844)}{\cos(0.844)} \right)^2 = 6.91.
\]

Fig. 4 Relative node shifts for the solid ceramic block discussed in the text.

A more complicated and perhaps more practical, ceramic window is shown in fig. 5.

Fig. 5 A matched coaxial line with a conical ceramic window. SWR is calculated to be 1.002.
Here we have "tuned" the radius of the inner conductor, its taper, and its overall length to minimize \( s \). The data in Fig. 6 show several curves of \( \eta \) for different trials in the design. The open circles (○) and squares (□) have a constant inner radius \( r_1 = 1.35 \) and 1.20 cm, respectively. Both the open triangles (△) and the solid squares (■) have tapered inner conductors, with \( r_c = 1.0 \) cm for both. The triangles represent \( r_s = 1.50 \) cm and the solid squares \( r_s = 1.45 \) cm. That configuration has the best SWR, 1.002.

![Data for variations of the coax window of fig. 5.](image)

**CONCLUSION**

We have shown a method to calculate the SWR of a passive, loss-free microwave junction using cavity codes like URMEL and SUPERFISH. In the examples used, the two-dimensional code SUPERFISH was used, but the method may be generalized to fully three-dimensional problems, using the appropriate computer code, e.g. MAFIA.

**REFERENCES**


A loss-free two-port junction like that in fig. A.1, transforms a load $R_L$ at its output into an input impedance $R_i$ according to

$$R_i = \frac{a_{11}R_L + ja_{12}}{a_{21} + ja_{22}R_L} \tag{A1}$$

where $a_{11}$, $a_{12}$, $a_{21}$ and $a_{22}$ are real and $a_{11}a_{22} + a_{12}a_{21} = 1$. If we imbed the junction in transmission lines with the characteristic impedance $Z_1$ and $Z_2$, as in fig.A.2, match the output side into $Z_1$ and connect the input line to a generator, a standing wave pattern on the input line will result. Let its VSWR be $s$.

![Fig. A.1](image)

![Fig. A.2](image)

We now locate a voltage maximum (at a distance $a$ from some reference position on the input line) and choose it as the new input of the junction. The input impedance there is $sZ_2$ (a real number) and

$$sZ_2 = \frac{a_{11}Z_1 + ja_{12}}{a_{22} + ja_{21}Z_1} \tag{A2}$$

Now suppose the line is terminated by a short as in the lower part of fig. A.2. The short is located at a position (distance $b$ from the reference) which produces a voltage zero at $a$. In this case, a zero load impedance at $b$ is transformed to a zero input impedance at $a$,

$$R_i \equiv 0 = j \frac{a_{12}}{a_{22}} \Rightarrow a_{12} = 0 \text{ and } a_{11}a_{22} = 1 \tag{A3}$$

Now eq. (A2) becomes

$$sZ_2 = \frac{a_{11}Z_1}{1/\frac{a_{11}}{a_{21}} + j/\frac{a_{21}}{a_{22}}Z_1} \tag{A4}$$

but since both $sZ_2$ and $Z_1$ are real, $a_{21} = 0$. Hence between the new ports at $a$ and $b$ generally $R_i = \frac{s^2}{a_{11}}R_L$ and since, in particular, $sZ_2 = a_{11}Z_1$, then

$$R_i = \frac{sZ_2}{Z_1} R_L \tag{A5}$$

If the short is moved to a position $x_1 \neq b$, there is a corresponding shift of the voltage node on the input line from $a$ to $x_2$. The short at $x_1$ corresponds to a reactive load $X(b)$ at $b$. 

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APPENDIX
\[ X(b) = jZ_1 \tan \left( k_1(x_1 + \frac{\lambda_1}{2} - b) \right) = jZ_1 \tan \left( k_1(x_1 - b) \right) \]  \hspace{1cm} (A6)

According to eq. (A5) this transforms at port \( a \) to
\[ X(a) = \frac{Z_2}{Z_1} X(b) = jsZ_2 \tan \left( k_1(x_1 - b) \right) \]  \hspace{1cm} (A7)

and at \( x_2 \)
\[ X(x_2) = Z_2 \frac{X(a) + jZ_2 \tan (k_2(a - x_2))}{Z_2 + jX(a) \tan (k_2(a - x_2))} \]  \hspace{1cm} (A8)

But the voltage node at \( x_2 \) implies that \( X(x_2) = 0 \), so that finally
\[ \tan \left( k_2(x_2 - a) \right) = s \tan \left( k_1(x_1 - b) \right) \]  \hspace{1cm} (A9)

If we now substitute \( \eta = k_2x_2 - k_1x_1 \), the \( s = 1 \) solution has \( \eta = k_2a - k_1b \). For \( s = 1 \), \( \eta \) oscillates symmetrically about this value. It is therefore convenient to further substitute \( \Delta = \eta - (k_2a - k_1b) \) and \( \zeta = k_1(x_1 - b) \). Then, \( \text{eq. (A9)} \) can be rewritten
\[ F(\Delta, \zeta) = \tan(\Delta + \zeta) - s \tan(\zeta) = 0 \]  \hspace{1cm} (A10)

Extrema of \( F(\zeta) \) must simultaneously satisfy
\[ F(\Delta, \zeta) = 0 \quad \text{and} \quad \frac{\partial}{\partial \zeta} F = 0 \]  \hspace{1cm} (A11)

Differentiating and using \( \cos^2 \theta = 1 + \tan^2 \theta \), we get
\[ \frac{1 + s^2 \tan^2 \zeta_e}{1 + \tan^2 \zeta_e} = s \quad \text{or} \quad \tan \zeta_e = \frac{1}{\sqrt{s}} \]  \hspace{1cm} (A12)

where the subscript \( e \) denotes the value at the extrema. Using \( F(\Delta, \zeta) = 0 \) and some manipulation, we arrive at
\[ s = \left[ \frac{1 + \sin \Delta_e}{\cos \Delta_e} \right]^2 \]  \hspace{1cm} (A13)