SCALING RELATIONS BETWEEN FLUCTUATIONS AND CORRELATIONS IN MULTIPARTICLE PRODUCTION

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ABSTRACT

Using the general properties of intermittency models to describe fluctuations and correlations in multiparticle production, one derives a scaling relation between factorial moments and factorial correlators. Both a 3-dimensional intermittency simulation and a direct comparison of data in hadron-hadron scattering give a satisfactory fulfilment of the scaling relation. We also show that this is not the case for conventional one-dimensional short-range correlations.

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As was shown some time ago\textsuperscript{1,2}, it is possible in high energy multiproduction studies to define observables which are in general free from the effect of purely statistical fluctuations. For instance, one can introduce factorial moments\textsuperscript{3} $F_p$ and factorial correlators\textsuperscript{4} $F_{p,q}$ as follows:

$$
F_p(\delta) = \frac{k_m(k_m-1)\ldots(k_m-p+1)}{(k_m)^p},
$$

$$
F_{p,q}(\delta,D) = \frac{k_m(k_m-1)\ldots(k_m-p+1)k_n(k_n-1)\ldots(k_n-q+1)}{(k_m)^p(k_n)^q},
$$

where the bins $m$ and $n$ are considered as fixed, with bin size $\delta$ and separation distance $D$ in the considered variable, say rapidity or rapidity-azimuth. The averaging $\langle \cdots \rangle$ is taken over events.

For intermittent-like fluctuations one has the behaviour\textsuperscript{1,2}

$$
\langle F_p \rangle \propto \delta^p, \\
\langle C_{p,q} \rangle = \frac{\langle F_{p,q} \rangle}{\langle F_p \rangle \langle F_q \rangle} \propto D^{p+q},
$$

where $C_{p,q}$ is $\propto \delta^{p+q} - \delta^p - \delta^q$ are fixed (intermittency) indices characterising the scaling behaviour of the moments and correlators. Note the important prediction\textsuperscript{3} that the observables $\langle C_{p,q} \rangle$ are expected not to depend on the individual bin size $\delta$, at least for $\delta$ smaller than some limiting value $\delta_0$.

Indeed, there exist already data on factorial moments\textsuperscript{3} and correlators\textsuperscript{4} in meson-nucleon reactions at 200 GeV/c momentum (NA22 collaboration), with which one could confront the predictions (2). While the factorial moments follow satisfactorily the intermittency prediction, the correlators seem to give a more ambiguous answer, namely

i) the bin size independence of the correlators is well fulfilled

ii) the power law behaviour in the distance $D$ is at best approximate

iii) the relation between intermittency indices is far from expected, since $C_{p,q} \propto \delta^{p+q} - \delta^p - \delta^q$ (cf. Ref. [4]).

The mystery deepens further when one considers, for instance, the conventional model of short-range rapidity correlations\textsuperscript{5}, which happens to get satisfactory results for factorial moments for the same reactions and meets difficulties when describing correlators.\textsuperscript{6}

In this paper, one shows that this (apparent) mystery can be solved by looking for more general scaling relations between factorial moments and correlators, which are valid even after projection onto a subsymmetric plane-space. Indeed, it was recently shown for factorial moments\textsuperscript{3} that projection deteriorates the prediction (2) for moments $F_p$, leading to an attenuation — if not suppression — of the intermittency signal at high resolution. It is only in the original dimension in which intermittency is exact that relations (2) are valid.

One of the purposes of this paper is to show that there are relations between correlators and moments which persist after projection, or better, are projection-independent.

Two results are derived in this paper, taking as an example the quantities $F_2$ and $F_{1,1} = C_{1,1}$. First, one establishes that the one-dimensional projection of an intermittent 3-dimensional model\textsuperscript{7} gives a satisfactory description of $F_2$ and $C_{1,1}$ of the NA22 data in the range $\delta \leq D \leq 1$ in rapidity. Second, we derive the following result:

$$\text{If a correlator } C_{1,1}(D,\delta) \text{ is effectively independent of } \delta \text{ in the range } \delta \leq D \leq \delta_0, \text{ then the relation}

C_{1,1}(D) = 2F_2(2D) - F_2(D)

\text{is valid.}$$

Both the 3-dimensional simulation and hadron-hadron data (for $\delta_0 = 1$) are shown to verify relation (3). One-dimensional "conventional" short-range correlations do not verify analytically the same relation, as will be shown at the end. Relations like (3) can be generalized to higher-rank moments and correlators $F_q$ and $C_{p,q}$ and, in a modified form, to distances $D$ larger than $\delta_0$.

In order to derive relation (3) one is first led to consider the particular case of adjacent bins $m$ and $n$ in formulae (1). Using the trivial equality $k_mk_n = \frac{1}{2}(k_m+k_n)(k_m+k_n-1)-\frac{1}{2}k_m(k_m-1)-\frac{1}{2}k_n(k_n-1)$, one gets by averaging and correctly normalizing

$$F_{1,1}(D,\delta = D) = 2F_2(2D) - F_2(D),$$

which for values of $D$ smaller than $\delta_0$ do not depend on the chosen bin size by hypothesis (for intermittent-like fluctuations). One thus gets formula (3) which can be directly confronted with data (cf. Fig. 1).

Let us now see what happens for a truly intermittent model in 3-dimensional momentum space. We will use the singular two-particle distribution function ansatz\textsuperscript{8}

$$\rho_2(y_1, y_2) = \frac{e^{-[(y_1+y_2)]^\alpha}}{[(p_1 - p_2)^2 + m^2]^\alpha},$$

to calculate the factorial moment $F_2$ and the factorial correlator $F_{1,1}$. It is clear that while the singularity plays an important role when calculating $F_2$, it will not be directly felt in the determination of $F_{1,1}$ except for neighbouring bins. The reason is that in the latter case the singularity is felt only at the common point of the two bins, while in the computation of $F_2$ it will show up anywhere in the interval of integration — that is, whenever $p_1 = p_2$. Therefore one expects intuitively $F_2 \gg F_{1,1}$.

Due to the form of $\rho_2(y_1, y_2)$, an analytical calculation of $F_{1,1}$ and $F_2$ is possible. Thus $F_{1,1}$ has been determined by Monte-Carlo methods as in Ref. [8] and the results are shown in Fig. 2. As predicted $F_2 \gg F_{1,1}$ and furthermore for $\delta > 1$ the NA22 data is well described by ansatz (4) with the same parameters used to fit $F_2$ — thus showing the consistency of the approach. On the other hand, relation (3) is satisfied exactly for all values of $\delta$ when using (4), which is a consequence of the presence of a truly intermittent behaviour at all scales. As we will now see, the presence of a scale $\delta_0$ beyond which bin-size invariance of correlators is lost alters things substantially.

It is in fact interesting to confront formula (3) with models that explicitly break the bin-size invariance of correlators, at least analytically. For instance, in the short-range correlation model of Ref. [5] one gets the following relations\textsuperscript{8,9}

$$F_2(\delta) = 1 + e^{\frac{\alpha-1 + e^{\frac{-\alpha}{\alpha^2/2}}}{\alpha}}.$$
and also

\[ F_{1,4}(D, \delta) = 1 + c + \gamma \left( \frac{e^{\alpha} - 1}{\alpha \gamma} \right) e^{-D/\xi} \]  

(6)

with \( \alpha = \delta / \xi \), \( \xi \) denoting the correlation length, and \( c, \gamma \) are parameters. One may check from (5) and (6) that

\[ F_{1,4}(D, \delta = D) = 2F_3(2D) - F_3(D) 
= 1 + c + \gamma \left( \frac{1 - e^{-D/\xi}}{D/\xi} \right)^2 \]  

(7)

while

\[ F_{1,4}(D, \delta = 0) = 1 + c + \gamma e^{-D/\xi} \]  

(8)

Expressions (7) and (8) are parametrically different, and numerically coincide only for \( D/\xi \ll 1 \). Note, however, that short-range correlations expressed in more than one dimension could lead to better results once projected onto one dimension. Further work is needed to clearly distinguish between the models. In general, one gets from (7) and (8) \( F_{1,4}(D, \delta = 0) > F_{1,4}(D, D) \). This is a reflection of the non-scale-invariance of the model, which is quite distinct from its satisfactory description of projected factorial moments.

It is quite natural to extend the previous discussion to higher-rank correlators and moments. This can be obtained using similar relations between adjacent bin multiplicities \( k_m k_n \) and \( (k_m + k_n)^{p+v} \) plus lower degree polynomials. Less obvious are the generalizations of relation (3) to distances \( D > \delta_0 \); one could in this case consider the relation between next-to-nearest neighbouring bins \( m \) and \( n \) separated by bin \( l \) all with bin-size equal to \( \delta_0 \). From the identity

\[ k_m k_n = \frac{1}{2} (k_m + k_l + k_n)(k_m + k_l + k_n - 1) - \frac{1}{2} (k_m + k_l)(k_m + k_l - 1) \]
\[ - \frac{1}{2} k_n (k_n - 1) - k_l k_n \]

one gets (for \( \delta < \delta_0 \))

\[ F_{1,4}(D = 2\delta_0) = \frac{3}{2} F_3(3\delta_0) - 4F_3(2\delta_0) + \frac{1}{2} F_3(\delta_0) \]  

(9)

For even higher distances analogous formulae can be derived. The scale invariance is now expressed differently from relation (3), valid only for \( D < \delta_0 \); that is, scale invariance with respect to bin-size remains valid (provided \( \delta < \delta_0 \)) but the relation with the second moment changes with the number of “fundamental” lengths \( \delta_0 \) to be considered. Could \( \delta_0 \) be taken as the maximal size of “intermittent” clusters? The question remains to be answered.

On the other hand, it is clear in Fig. 2 that ansatz (4) gives a very good description of experimental data on \( F_3 \) for quite large intervals of rapidity. Could this mean that the approach developed by Sax and Sarcevic and the intermittency approach are just two faces of the same phenomenon? Clearly, only a thorough study of factorial moments and correlators also for large intervals in rapidity will enable us to answer this question unambiguously in the near future.

As a final comment, let us discuss the interesting comparison between “conventional” short-range correlations and intermittence-like ones generated by fluctuating patterns; contrary to what one would expect, intermittency models do not contradict the “conventional” picture of short-range rapidity correlations, provided one takes into account the projection effects. It is only by considering higher-dimensional moments and/or the relation with correlators exhibited in the present paper that one expects to find a difference. In some sense, one could consider that 3-dimensional intermittency – as implemented, for instance, in the model of Ref. [8] – gives a kind of description of the short-range rapidity correlation in terms of a scale-invariant, higher-dimensional correlation function.\(^7\, 8\) A more general question remains then: how can one derive such a fractal behaviour from some theory, for which a probable candidate is QCD? Before entering this difficult problem it is necessary to know all the relevant data. Bin-bin factorial correlators are, as shown here, privileged quantities in this context. Therefore, a simultaneous experimental measurement of factorial correlators and inelastic events, allowing by comparison of their behaviour, a much better analysis of the physics underlying the correlations between produced particles. We urge experimentalists to measure these observables in various reactions.

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REFERENCES

4- V.V. Aivazyan et al. (NA22 Coll.), Nucl. Phys. 332/90.
FIGURE CAPTIONS

Fig. 1: Comparison between NA22 experimental results for factorial moment $F_2$ and correlator $F_{1,1}$ (open circles and squares, respectively) and relation (3) (black triangles).

Fig. 2: Comparison between NA22 experimental results for factorial moment $F_2$ (open circles) and correlator $F_{1,1}$ (open squares) and simulation based on ansatz (4) (black circles and squares). The parameters used for the simulation are the same as in Ref. [8].