Recent results on tetra- and penta-quark candidates at LHCb

Tomasz Skwarnicki

On behalf of the LHCb collaboration at
Outline

- $\Lambda_b \rightarrow J/\psi \ p \ K^- \ (P_c(4450)^+ \rightarrow J/\psi \ p, \ P_c(4380)^+ \rightarrow J/\psi \ p)$ vs $B^0 \rightarrow \psi' \ \pi^+ K^- \ (Z_c(4430)^+ \rightarrow \psi' \ \pi^+)$ analyzed with two approaches:
  - Amplitude analysis (LHCb-PAPER-2015-029, LHCb-PAPER-2014-014)
  - Model independent approach based on angular moments (LHCb-PAPER-2015-038, LHCb-PAPER-2016-009)
- $\Lambda_b \rightarrow J/\psi \ p \ \pi^- \ (P_c(4380,4450)^+ \rightarrow J/\psi \ p, \ Z_c(4200)^+ \rightarrow J/\psi \ \pi^+)$
  - Amplitude analysis preliminary! (LHCb-PAPER-2016-015)
- $B^+ \rightarrow J/\psi \ \phi \ K^+ \ (X(4140,4274,\ldots)^+ \rightarrow J/\psi \ \phi$ and other states)
  - Amplitude analysis preliminary! (LHCb-PAPER-2016-018, LHCb-PAPER-2016-019)

$\psi', J/\psi \rightarrow \mu^+\mu^-, \ \phi \rightarrow K^+K^-$
LHCb: first dedicated b,c detector at hadronic collider

- Advantages over $e^+e^-$ B-factories (Belle, BaBar):
  - ~1000x larger b production rate, b decays mostly to c
  - produce b-baryons at the same time as B-mesons
  - long visible lifetime of b-hadrons (no backgrounds from the other b-hadron)

- Advantages over ATLAS, CMS, CDF, D0:
  - RICH detectors for $\pi/K/p$ discrimination (smaller backgrounds)
  - Small event size allows large trigger bandwidth (up to 5 kHz in Run I); all devoted to flavor physics
LHCb data samples (3 fb$^{-1}$)

- More than a factor of 10 better statistics than at the B factories, at smaller background
- Very comparable signal statistics and bkg levels between the B and $\Lambda_b$ data samples

\[ B^0 \rightarrow \psi' \pi^+ K^- \]

<table>
<thead>
<tr>
<th>$m_{\psi'\pi K}$ [MeV]</th>
<th>LHCB</th>
<th>Signal (25,176±174)</th>
<th>Bkg (4.1%)</th>
<th>Belle: 7.8%</th>
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<tbody>
<tr>
<td>5250 - 5300</td>
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</table>

<table>
<thead>
<tr>
<th>$m_{J/\psi p K}$ [MeV]</th>
<th>LHCB</th>
<th>Signal (26,007±166)</th>
<th>Bkg (5.4%)</th>
<th>Belle: 2,010±50</th>
<th>BaBar: 2,021±53</th>
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<td>5500 - 5700</td>
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</table>

PRL 112, 222002 (2014)

$B^0 \rightarrow \psi' \pi^+ K^-$

$\sigma_m = 5.4$ MeV

PRL 115, 07201 (2015)

$\Lambda_b \rightarrow J/\psi p K^-$

$\sigma_m = 7.5$ MeV
$B^0 \rightarrow \psi' \pi^+ K^-$

Is it a reflection of interfering $K^*$'s $\rightarrow \pi^+ K^-$?

Proper amplitude analysis necessary to check.
$\Lambda_b^0 \rightarrow J/\psi pK^-$: unexpected structure in $m_{J/\psi p}$

$\Lambda(1520)$ and other $\Lambda^*$'s $\rightarrow pK^-$

- Unexpected, narrow peak in $m_{J/\psi p}$

Is it a reflection of interfering $\Lambda^*$'s $\rightarrow pK^-$? Proper amplitude analysis necessary to check
Full amplitude analyses

For the best sensitivity and to avoid dependence of efficiency on fit model, use all degrees of freedom in the decay.

4D maximum likelihood fit

$$\Omega \equiv (\theta_{K^*}, \theta_{\psi}, \Delta \phi_{\psi,K^*})$$

$$\text{PDF}(m_{K\pi/Kp}, \Omega) = |\text{MatrixEle}(m_{K\pi/Kp}, \Omega | J_R^P, M_R, \Gamma_R, H_R)|^2 \times \text{eff}(m_{K\pi/Kp}, \Omega) + \text{PDF}_{\text{bkg}}(m_{K\pi/Kp}, \Omega)$$

Fixed to known values. $M_R, \Gamma_R$ varied within errors for systematics.

1-3 independent complex helicity couplings $H_R$ per $K^*$ resonance

6D maximum likelihood fit

$$\Omega \equiv (\theta_{\Lambda_b}, \Delta \phi_{\Lambda^*, \Lambda_b}, \theta_{\psi}, \Delta \phi_{\psi, \Lambda_b})$$

4-6 independent complex helicity couplings $H_R$ per $\Lambda^*$ resonance

- Dealing with baryons results in larger number of helicity couplings per resonance to determine from data (nuisance parameters)
Model of conventional resonances

### Well established states from PDG

<table>
<thead>
<tr>
<th>State</th>
<th>$J^P$</th>
<th>$M_0$ (MeV)</th>
<th>$\Gamma_0$ (MeV)</th>
<th># of complex couplings</th>
<th>Red.</th>
<th>Ext.</th>
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</thead>
<tbody>
<tr>
<td>NR</td>
<td>$0^+$</td>
<td>—</td>
<td>—</td>
<td>1</td>
<td>1</td>
<td>1</td>
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<tr>
<td>$K^*(800)^0$</td>
<td>$0^+$</td>
<td>682</td>
<td>547</td>
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<td>1</td>
<td>1</td>
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<tr>
<td>$K^*(892)^0$</td>
<td>$0^+$</td>
<td>896</td>
<td>49</td>
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<tr>
<td>$K^*(1410)^0$</td>
<td>$1^-$</td>
<td>1414</td>
<td>232</td>
<td>3</td>
<td>3</td>
<td>3</td>
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<tr>
<td>$K^*(1430)^0$</td>
<td>$0^+$</td>
<td>1425</td>
<td>270</td>
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<tr>
<td>$K^*(1430)^0_2$</td>
<td>$2^+$</td>
<td>1432</td>
<td>109</td>
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<tr>
<td>$K^*(1680)^0$</td>
<td>$1^-$</td>
<td>1717</td>
<td>322</td>
<td>3</td>
<td>3</td>
<td>3</td>
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<tr>
<td>$K^*_3(1780)^0$</td>
<td>$3^-$</td>
<td>1776</td>
<td>159</td>
<td>0</td>
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</tr>
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</table>

**Total # of free parameters**: 28 (Red.) 34 (Ext.)

### No high-$M_0$ high-$J^P$

<table>
<thead>
<tr>
<th>State</th>
<th>$J^P$</th>
<th>$M_0$ (MeV)</th>
<th>$\Gamma_0$ (MeV)</th>
<th># of complex couplings</th>
<th>Red.</th>
<th>Ext.</th>
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</thead>
<tbody>
<tr>
<td>$\Lambda(1405)$</td>
<td>$1/2^-$</td>
<td>1405</td>
<td>50</td>
<td>3</td>
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<td>$\Lambda(1520)$</td>
<td>$3/2^-$</td>
<td>1520</td>
<td>16</td>
<td>5</td>
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<td>6</td>
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<tr>
<td>$\Lambda(1600)$</td>
<td>$1/2^+$</td>
<td>1600</td>
<td>150</td>
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<td>4</td>
<td>4</td>
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<td>$\Lambda(1670)$</td>
<td>$1/2^-$</td>
<td>1670</td>
<td>35</td>
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<td>$\Lambda(1690)$</td>
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<td>$\Lambda(1800)$</td>
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<td>4</td>
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<tr>
<td>$\Lambda(1810)$</td>
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<td>1810</td>
<td>150</td>
<td>3</td>
<td>4</td>
<td>4</td>
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<tr>
<td>$\Lambda(1820)$</td>
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<td>$\Lambda(1890)$</td>
<td>$3/2^+$</td>
<td>1890</td>
<td>100</td>
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<td>6</td>
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<tr>
<td>$\Lambda(2100)$</td>
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<td>6</td>
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<tr>
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<td>$\Lambda(2350)$</td>
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<td>$\Lambda(2585)$</td>
<td>$5/2^-$</td>
<td>2585</td>
<td>200</td>
<td>0</td>
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</table>

**Total # of free parameters**: 64 (Red.) 146 (Ext.)

- Dealing with baryons results in more than doubling of known states to include...
Fits with conventional resonances only

- Cannot describe the data with the conventional resonances alone
Fits including exotic hadrons

- The models based on well established conventional resonances describe these projections well (without or with exotics):
  - They dominate the rate
  - If exotics present (as shown above) they spread across wide range of these masses
Fitting decay angles important for resolving overlapping resonances

- They greatly increase discrimination power between resonances of different $J^P$
- Without using full decay phase-space difficult to do efficiency correction correctly
Exotic hadrons

**B^0 \rightarrow \psi' \pi^+K^-** 

\[ \Lambda_b^0 \rightarrow J/\psi \ pK^- \] (Red.)

- **J^P=1^+** at 9.7σ incl. syst. (in Belle at 3.4σ)

- Best fit has $J^P=(3/2^-, 5/2^+)$, also $(3/2^+, 5/2^-)$ & $(5/2^-, 3/2^-)$ are preferred. $(5/2^-, 3/2^+)$ cannot be ruled out within systematics
Argand diagrams: exotic hadron amplitudes without Breit-Wigner assumption

Exotic hadron amplitudes for $6 \frac{m_{\psi'/\pi}}{m_{J/\psi p}}$ bins near the peak mass (all other model parameters fitted simultaneously)

Good evidence for resonant character

Large errors

Such studies make exotic hadron amplitude model-independent, but the results are still dependent on the model of conventional hadrons. Simultaneous PWA of the latter is not possible since exotics reflect into variables characterizing conventional hadrons.

However, we can assume exotics are not present and test for their presence in model-independent way - next 6 slides.
Rectangular Dalitz plane: variables of conventional hadrons

- For fixed $m_{K\pi/Kp}$ there is one-to-one relation between $m_{\psi\pi/\psi p}$ and $\cos^{0K^*/L^*}$

\[ B^0 \rightarrow \psi'\pi^+K^- \]

PRD 92, 112009 (2015)
LHCb-PAPER-2015-038

(效率修正)

\[ \Lambda_b^0 \rightarrow J/\psi pK^- \]

arXiv:1604.05708
LHCb-PAPER-2016-009
Legendre moments

\[ \frac{dN}{d \cos \theta} = \sum_{l=0}^{l_{\text{max}}} \left\langle P_l^U \right\rangle P_l(\cos \theta) \quad \theta = \theta_{K^*} \quad \text{or} \quad \theta_{\Lambda^*} \]

\[ \left\langle P_l^U \right\rangle = \int_{-1}^{+1} \frac{dN}{d \cos \theta} P_l(\cos \theta) d \cos \theta \propto \sum_{i=1}^{n_{\text{events}}} \frac{1}{\varepsilon_i} P_l(\cos \theta_i) \]

Decomposition into \( <P_l> \) corresponds to decomposition into “frequencies”

With \( l_{\text{max}} \to \infty \) can reproduce any \( \frac{dN}{d \cos \theta} \)

Smooth cos\( \theta \) structures
produce low rank moments

\[ K^*/\Lambda^* \] can contribute only to low-rank moments

\[ l_{\text{max}} = J_1 + J_2 \] for interfering resonances

In \( K^*/\Lambda^* \)-only hypothesis (\( H_0 \)) \[ l_{\text{max}} = 2J_{\text{max}} \]

\( J_{\text{max}} \) is the highest spin of \( K^*/\Lambda^* \) resonance possible

Reflections of exotic hadrons can contribute to low and high rank moments:

- Detecting non-zero moments above \( 2J_{\text{max}} \) signals presence of exotics
- The narrower the peak the higher the \( 2J_{\text{max}} \) required. The sensitivity is better for narrower exotic hadrons.
- Exotic hadron contributions spread over wide range of \( m_{K^*}/m_{Kp} \). An effective way of testing \( H_0 \) is to aggregate the information about \( \cos \theta_{K^*/Kp} \) moments in a function of \( m_{\psi'}/m_{J/\psi'} \).
Setting highest rank of Legendre moments

The sensitivity of the method improves by considering $l_{\text{max}}(m_{K\pi}/m_{Kp}) = 2 J_{\text{max}}(m_{K\pi}/m_{Kp})$ dependence:

- it can be set from known $K^*/\Lambda^*$ resonances, quark model predictions as a guide

Much fewer known states than predicted!

- Because the $J/\psi$ mass is smaller than $\psi'$ mass, must allow for higher excitations in the $\Lambda_b^0 \to J/\psi pK$ analysis, higher $l_{\text{max}}$
Test the hypothesis \((H_0)\) that the data contain only conventional hadrons.

Form a model of the data implementing this hypothesis:

\[
PDF(m_{K\pi/Kp}, \cos\theta_{K^*/\Lambda^*} | H_0) = F(m_{K\pi/Kp}) \cdot F(\cos\theta_{K^*/\Lambda^*} | m_{K\pi/Kp})
\]

\[
F(\cos \theta_{K^*/\Lambda^*} | m_{K\pi/Kp}) = \sum_{l=0}^{l_{\text{max}}(m_{Kp})} \langle P^U_l \rangle(m_{K\pi/Kp}) \cdot P_l(\cos \theta_{K^*/\Lambda^*})
\]
Test $H_0$ model on $m_{\psi'\pi/p}$ distribution

$$\text{PDF}(m_{\psi'\pi/p}|H_0) = \int dm_{K\pi/Kp} \text{PDF}(m_{K\pi/Kp}, \cos \theta_{K^*/\Lambda^*}(m_{\psi'\pi/p}) | H_0)$$

$$B^0 \rightarrow \psi' \pi^+ K^-$$

BaBar did not have enough statistics to see $Z(4430)$ this way.

Negative results like this impossible to interpret without amplitude analysis since $Z-K^*$ interfere!

BaBar PRD 79, 112001 (2009)

LHCb PRD 92, 112009 (2015)

LHCb data inconsistent with $K^*$ contributions alone

BaBar PRD 79, 112001 (2009)

LHCb data inconsistent with $\Lambda^*$ contributions alone

This model independent proof of the presence of exotic hadron contributions is especially important for the $\Lambda_b$ data, because of the difficulties in construction of a complete model of $\Lambda$ excitations.
Rejection of $H_0$ can be quantified

Test variable:

(quasi) log-likelihood-ratio

$$\Delta(-2\ln L) \equiv -2 \sum_{i=1}^{n_{\text{events}}} \ln \frac{\text{PDF}(m_{\psi\pi}|H_0)}{\text{PDF}(m_{\psi\pi}|H_1)}$$

$$\Delta(-2\ln L) \equiv -2 \sum_{i=1}^{n_{\text{events}}} \ln \frac{\text{PDF}(m_{J/\psi p}|H_0)/I_{H_0}}{\text{PDF}(m_{J/\psi p}|H_1)/I_{H_1}}$$

$H_0: l_{max}(m_{K\pi}) \quad H_1: l_{max}^{H1} = 30$

$$I_{H} = \int \text{PDF}(m_{J/\psi p}|H) \varepsilon(m_{J/\psi p}) \, dm_{J/\psi p}$$

$H_0: l_{max}(m_{KP}) \quad H_1: l_{max}^{H1} = 31$

This variable tests a significance of moments between $l_{max}(m_{K\pi/Kp})$ and $l_{max}^{H1}$

PDF( $\Delta(-2\ln L)|H_0$ ) $B^0 \rightarrow \psi' \pi^+K^-$

PDF( $\Delta(-2\ln L)|H_0$ ) $\Lambda_b \rightarrow J/\psi \, p \, K^-$

However, this approach cannot characterize exotics – amplitude analysis is still necessary.
Cabibbo suppressed vs favored $\Lambda_b$ decays

PRL 115, 07201 (2015)

$\Lambda_b \rightarrow J/\psi \, p \, K^-$

LHCb-PAPER-2015-029

26,007 ± 166 signal events

$\sigma_m$ = 7.5 MeV

5.4% bkg

$\Lambda_b^0$ signal range

sideband

$\Lambda_b \rightarrow J/\psi \, p \, K^-$

LHCb preliminary

1,885 ± 50 signal events

~18% bkg

$\Lambda_b \rightarrow J/\psi \, p \, \pi^-$

LHCb-PAPER-2016-015 (in preparation)

Data

Fit

Signal

$\Lambda_b^0 \rightarrow J/\psi p K^-$

Background

- More than a factor of 10 lower signal statistics in $\Lambda_b \rightarrow J/\psi \, p \, \pi^-$ analysis than in $\Lambda_b \rightarrow J/\psi \, p \, K^-$
- Relative background fraction higher by more than a factor of 3
$\Lambda_b^0 \to J/\psi p\pi^-$: Cabibbo suppressed

$N(1535)$ and other $N^*$'s $\to p\pi^-$

- No obvious structure in $m_{J/\psi p}$

Statistics is low.
Proper amplitude analysis necessary to check for consistency with Cabibbo favored case.
Model of conventional N* resonances

Better established states from PDG

<table>
<thead>
<tr>
<th>State</th>
<th>J/P</th>
<th>M₀ (MeV)</th>
<th>Γ₀ (MeV)</th>
<th># of complex couplings</th>
<th>Only significant states limit</th>
<th>All states limit</th>
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<tbody>
<tr>
<td>NR pπ</td>
<td>1/2−</td>
<td>-</td>
<td>-</td>
<td>4</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>N(1440)</td>
<td>1/2+</td>
<td>1430</td>
<td>350</td>
<td>3</td>
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<td>N(1520)</td>
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<td>115</td>
<td>3</td>
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<tr>
<td>N(1535)</td>
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<td>4</td>
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<td>N(1650)</td>
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<td>N(1710)</td>
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<td>N(1875)</td>
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<td>N(2570)</td>
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<td>250</td>
<td>0</td>
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</tr>
</tbody>
</table>

| Total # of free parameters | 40 | 106 |

- Use reduced model for central values, extended for systematics and significance of exotic contributions
- Because of insufficient statistics forced to neglect higher orbital angular momenta for most of the N* states
- Almost as many free parameters in the fit as in $\Lambda_b \rightarrow J/\psi \ p \ K^-$ with 14 times smaller statistics and 3 times higher relative bkg
Exotic hadron contributions to $\Lambda_b \rightarrow J/\psi \ p \ \pi^-$

- Open ended search for exotic hadrons in $\Lambda_b \rightarrow J/\psi \ p \ \pi^-$ with the present statistics is not possible

- Test data for presence of previously observed states:
  - $P_c(4380)^+, P_c(4450)^+ \rightarrow J/\psi \ p$ observed by LHCb in $\Lambda_b \rightarrow J/\psi \ p \ K^-$
    - Masses, widths, $P_c^+$- decay helicity couplings fixed (fit only their production couplings – 4 free parameters). Varied within the errors for the systematics.
    - Fix $J^P$ assignments to (3/2-, 5/2+). Use other possible assignments for the systematics.
    - Significance determined including all systematic effects.
  - $Z_c(4200)^+ \rightarrow J/\psi \ \pi^+$ observed by Belle in $B^0 \rightarrow J/\psi \ \pi^+ \ K^-$
    - Mass, width fixed. Varied for the systematics.
    - Helicity couplings are free (10 fit parameters).
    - $J^P=1^+$ well determined.
Amplitude fits to $\Lambda_b \rightarrow J/\psi \ p \ \pi^-$

- Significance of $P_c(4380)^+, P_c(4450)^+, Z_c(4200)^-$ taken together is $3.1\sigma$

- Evidence for exotic hadron contributions to $\Lambda_b \rightarrow J/\psi \ p \ \pi^-$!
Results for $\Lambda_b \to J/\psi p \pi^-$

- Significance of $P_c(4380)^+, P_c(4450)^+, Z_c(4200)^-$ taken together is $3.1\sigma$ (including systematic uncertainty) – evidence for exotic hadrons.

- Individual exotic hadron contributions are not significant. For example, significance of $P_c(4380)^+$ plus $P_c(4450)^+$ is $<1.7\sigma$ - no independent confirmation of the $P_c^+$ states (it increases to an evidence level, $3.3\sigma$, if assume production of $Z_c(4200)^-$ is negligible).

<table>
<thead>
<tr>
<th>State</th>
<th>Fit fraction (%)</th>
<th>$BR(\Lambda_b\to P_c^+\pi)/BR(\Lambda_b\to P_c^+K^-)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Z_c(4200)^-$</td>
<td>$7.7 \pm 2.8^{+3.4}_{-4.0}$</td>
<td>---</td>
</tr>
<tr>
<td>$P_c(4380)^+$</td>
<td>$5.1 \pm 1.5^{+2.1}_{-1.6}$</td>
<td>$0.050 \pm 0.016^{+0.020}_{-0.016} \pm 0.025$</td>
</tr>
<tr>
<td>$P_c(4450)^+$</td>
<td>$1.6^{+0.8}<em>{-0.6}^{+0.6}</em>{-0.5}$</td>
<td>$0.033^{+0.016}<em>{-0.014}^{+0.011}</em>{-0.009} \pm 0.009$</td>
</tr>
</tbody>
</table>

Expected if the additional internal W emission diagram negligible:

The $\Lambda_b \to J/\psi p \pi^-$ data are consistent with the presence of $P_c(4380)^+, P_c(4450)^+$ at the level expected from $\Lambda_b \to J/\psi p K^-$ measurement and Cabibbo suppression.
Confusing experimental situation concerning $X \rightarrow J/\psi \phi$ states

- Some experiments saw narrow $X(4140)$ [i.e. $Y(4140)$], some didn’t.
- Possibly 2$^{nd}$ $J/\psi \phi$ structure in B decays, $X(4274)$, but seen at inconsistent mass. No published claim of its significance.
- Possibly $X(4351)$ state seen in $\gamma \gamma$ collisions
LHCb B\(^+\) → J/\(\psi\) φ K\(^+\) data samples (3 fb\(^{-1}\))

B\(^+\) → J/\(\psi\) φ K\(^+\)

• Statistically, the most powerful B → J/\(\psi\) φ K sample analyzed so far

**LHCb-PAPER-2016-018**

**LHCb-PAPER-2016-019**

In preparation

Use sidebands to subtract background
B⁺ → J/ψ φ K⁺

LHCb preliminary

Are these reflections of interfering K⁺ → φK⁻ ?
Proper amplitude analysis necessary to check

Kaon excitations
LHCb vs CMS data

- Compare $m_{J/\psi\phi}$ to the CMS data (the previous best sample).
- Non-B background subtracted, corrected for signal efficiency.

- LHCb data more precise.
- Qualitative agreement over the full mass range.

Used publicly available CMS background-free distribution and CMS efficiency dependence on $m_{J/\psi\phi}$

LHCb efficiency corrections via 6D parameterization of efficiency in all dimensions of the decay phase-space.

Normalized to the same area.

The vertical scale is arbitrary.
Amplitude analysis needed

- Amplitude analysis is needed to demonstrate that the \( X \to J/\psi \phi \) peaks are not due to reflections of interfering kaon excitations ("K*") decaying to \( \phi K^+ \)
  - Smoothness of \( m_{\phi K} \) spectrum does not mean that there are no kaon excitations in the data. The narrowest known K* state in the relevant mass range is 150 MeV. Many overlapping resonances expected. Only analysis of the masses in correlation with the decay angles can disentangle them.

- All previous analyses performed naïve 1D mass fits to \( m_{J/\psi \phi} \)
  - Ad hoc assumptions about kaon contributions (e.g. 3-body phase-space distribution, incoherent)
  - No sensitivity to \( J^{PC} \) of \( X \) structures

Perform first amplitude analysis of \( B^+ \to J/\psi \phi K^+ \)

\[
\sum \equiv (\theta_{K^*}, \theta_\psi, \Delta \phi_\psi, K^*, \Theta_\phi, \Delta \phi_{K^*, \phi})
\]

\[
|\text{MatrixEle}(m_{K\phi}, \sum | J^P_R, M_R, \Gamma_R, H_R)|^2
\]

6D maximum likelihood fit

1-4 independent \textbf{complex} helicity couplings \( H_R \) per \( K^* \) resonance
Model of conventional K* resonances

- All K* states (except 0^{++}) between kinematic boundaries are allowed to decay to φK but may not have been seen in experiment because previous searches are typically old scattering experiments with low statistics at high masses.

- All known excited states in this mass range are broad: Γ~150-400 MeV.

- Guidance from quark model was used to inform choices for K* sector.

- Try both known and unknown K* states.

- No constraints placed on mass or width parameters (fits don’t depend on predictions or previous measurements).

- Take K* contributions greater than ~2σ significance.
Amplitude fits with kaon excitations only

- Fits without exotic contributions were tried:
  - Example: two $2P_{1+}$, two $2D_{1-}$, and one of $1^3F_{3+}$, $1^3D_{1-}$, $3^3S_{1-}$, $3^1S_0^-$, $2^3P_{2+}$, $1^3F_{2+}$, $1^3D_{3-}$, $1^3F_{4+}$. Contained 104 free parameters.

- Further $K^*$ additions, including states not predicted by the quark model, does not change the conclusion that non-$K^*$ contributions are needed to adequately describe all distributions.
We considered adding possible exotic $X \rightarrow J/\psi \phi$ and $Z^+ \rightarrow J/\psi K^+$ states as well as removing insignificant or implausible ($\Gamma > 1000$ or $< 100$ MeV) conventional $K^{*+} \rightarrow \phi K^+$ states leading us to a default model.

- Only $X$ states give very significant improvements in fit qualities over the models with $K^*$'s alone.
- The default fit model is shown here.
Fitted angles

- Fit quality is good in all fitted variables
K* results

Default model: 6 K* resonances (of 4 different J^P) + 1 NR φK

Our results are given by the red points with error bars.

Excellent agreement between our results and both theory and previous experiments (see backup slide for numerical results).

Godfrey-Isgur, PRD 32, 189 (1985)

Established

Unconfirmed

High spin states (3-4) not observed but also expected to be suppressed by orbital angular momentum barrier in B decay.

Possible mixing of 3S with 1D state.
X(1++)

• Significant X(4140) 8.4σ,
  – mass consistent with the previous measurements, but the width substantially larger
  – $J^{PC}=1^{++}$ determined at 5.7σ including systematic errors

• Significant X(4274) 6.0σ,
  – Consistent with the unpublished CDF results. First significant claim for this structure.
  – $J^{PC}=1^{++}$ determined at 5.8σ including systematic errors
• Significant structures at higher masses, best described by two new $0^{++}$ resonances $X(4500), X(4700)$:
  – Significances of $6.1\sigma$, $5.6\sigma$
  – $J^{PC}=0^{++}$ determined at $4.0\sigma$, $4.5\sigma$, respectively
X exotics as $D_s^{(*)}\overline{D}_s^{(*)}$ cusps?

Breit-Wigner Model

Cusps Model

\[ BW(m|M_0, \Gamma_0) = \frac{1}{M_0^2 - m^2 - iM_0\Gamma(m)} \]

\[ \Pi(m|\beta_0) = \int \frac{d^3q}{(2\pi)^3} \frac{e^{-2q^2/\beta_0^2}}{m - M_A - M_B - \frac{q^2}{2\mu_{AB}} + ic} \]

Cusp model by
E. S. Swanson, arXiv:1504.07952
(see also PRD91, 034009 (2015))
Argand diagram: Breit-Wigner vs cusp

Peak amplitude

Different shape and phase running with mass

BW

CUSP

Above threshold

Below threshold

Peak amplitude
Cusps

- Cusp peaks at the sum of masses of the virtual narrow-$D_{sX}^(*)$ pairs.
- Width of cusp in Swanson model is controlled with a free parameter ($\beta_0$).
- $J^P$ of cusp determined by $J^P$'s of virtual $D_s$ pairs (cusps occur in S-wave).

\[ \times 10^{-3} \]

![Graph showing all combinations of narrow $D_s$ excitations with $J^P$ values and $m_{J/\psi\phi}$ axis ranging from 0 to 4.7 GeV.]

(\(\beta_0 = 0.3\) GeV)

Is $X(4140)$ a $D_s^+D_s^{*-}$ cusp?
Right $J^P=1^+$

Is $X(4274)$ a $D_s^+D_{s0}^{*-}$ cusp?
Wrong $J^P=0^-$
Is $X(4140)$ a $D_s^+D_s^{*-}$ cusp?

- The cusp is preferred by 1.6-3$\sigma$ over the Breit-Wigner amplitude for $X(4140)$ from the fit likelihood ratio.
- No success in describing any other $J/\psi\phi$ mass structures as a cusp.

\[ \beta_0 = 297 \pm 20 \text{ MeV} \]

vs 300 MeV used by Swanson.
Theoretical interpretations of X(4140), X(4274)

Molecular models

- The determination of the quantum numbers of X(4140) as $J^{PC}=1^{++}$ rules out many interpretations. Namely, $0^{++}$ or $2^{++}$ $D_s^*D_s^*$ molecules. The large width is also not expected for true molecular bound states.

- However, X(4140) may be a $1^{++}$ $D_sD_s^*$ cusp (form of rescattering)

Hybrid models

- Hybrid charmonium states proposed for X(4140) would have $J^{PC}=1^{-+}$. Thus they are also ruled out.

Tightly-bound tetraquark models

- There are tetraquark models which predict states with $J^{PC}=0^{-+}, 1^{-+}$ or $0^{++}, 2^{++}$ near X(4140); these can be ruled out.

- A tetraquark model implemented by Stancu [JP G37, 075017 (2010), arXiv:0906.2485] correctly assigns $1^{++}$ to X(4140) and predicts a second $1^{++}$ state at a mass not much higher than X(4274)

- A Lattice calculation by Padmananth et al [PRD92, 034501 (2015)], based on a diquark tetraquark model, found no evidence for a $1^{++}$ tetraquark below 4.2 GeV
Summary

• We have demonstrated that exotic hadron contributions are present in $B^0 \rightarrow \psi' \pi^+ K^-$ and $\Lambda_b \rightarrow J/\psi \ p K^-$ decays with the model independent approach.

• Using amplitude analysis we have confirmed $Z_c(4430)^+ \rightarrow \psi' \pi^+$ in $B^0 \rightarrow \psi' \pi^+ K^-$ and demonstrated its resonant character with Argand diagram.

• Using amplitude analysis we have observed two pentaquark $P_c(4450)^+,P_c(4380)^+ \rightarrow J/\psi \ p$ candidates in $\Lambda_b \rightarrow J/\psi \ p K^-$.

• Using amplitude analysis we have found $3.1\sigma$ evidence for exotic hadron contributions in $\Lambda_b \rightarrow J/\psi \ p \ \pi^+$, but confusion between $P_c(4450)^+,P_c(4380)^+$ and $Z_c(4200)^- \rightarrow J/\psi \ \pi^-$ contributions prevents establishing either pentaquark or $Z_c(4200)^-$ in these decays. We have demonstrated that the $\Lambda_b \rightarrow J/\psi \ p \ \pi^-$ data are consistent with the $P_c(4450)^+,P_c(4380)^+$ rate measured in $\Lambda_b \rightarrow J/\psi \ p K^-$ and Cabibbo suppression.

• The first full amplitude analysis of $B^+ \rightarrow J/\psi \ \phi \ K^+$ decays has been performed. The data cannot be described by a model that contains only excited kaon states decaying into $\phi \ K^+$ and four $J/\psi \ \phi$ structures are observed, each with significance over $5\sigma$. The quantum numbers of these structures are determined with significance of at least $4\sigma$. The lightest is best described as a $D_s^+D_s^{*-}$ cusp, but a resonant interpretation is also possible with mass consistent with, but width much larger than, previous measurements of the claimed $X(4140)$ state. We have also contributed to kaon spectroscopy for higher-mass excitations.
BACKUP SLIDES
Confusing experimental situation concerning $X \rightarrow J/\psi \phi$ states

### $X(4140)$ summary

<table>
<thead>
<tr>
<th>Year</th>
<th>Experiment luminosity</th>
<th>Ref</th>
<th>$B \rightarrow J/\psi \phi K$ statistics</th>
<th>mass [MeV]</th>
<th>$X(4140)$ peak width [MeV]</th>
<th>sign.</th>
<th>fraction</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td>2008</td>
<td>CDF 2.7 fb$^{-1}$</td>
<td>PRL 102,242002</td>
<td>58 ± 10</td>
<td>4143.0 ± 2.9 ± 1.2</td>
<td>11.7 $^{+8.3}_{-5.6}$ ± 3.7</td>
<td>3.8σ</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2009</td>
<td>Belle</td>
<td>LP2009 (unpub.)</td>
<td>325 ± 21</td>
<td>4143.0 fixed</td>
<td>11.7 fixed</td>
<td>1.9σ</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2011</td>
<td>CDF 6.0 fb$^{-1}$</td>
<td>arXiv:1101.6058 (unpub.)</td>
<td>115 ± 12</td>
<td>4143.4 $^{+2.9}_{-3.0}$ ± 0.6</td>
<td>15.3 $^{+10.4}_{-6.1}$ ± 2.5</td>
<td>5.0σ</td>
<td>14.9 ± 3.9 ± 2.4</td>
<td></td>
</tr>
<tr>
<td>2011</td>
<td>LHCb 0.37 fb$^{-1}$</td>
<td>PRD85, 091103</td>
<td>346 ± 20</td>
<td>4143.4 fixed</td>
<td>15.3 fixed</td>
<td>1.4σ</td>
<td>&lt; 7 @ 90%CL</td>
<td></td>
</tr>
<tr>
<td>2013</td>
<td>CMS 5.2 fb$^{-1}$</td>
<td>PL, B734, 261</td>
<td>2480 ± 160</td>
<td>4148.0 ± 2.4 ± 6.3</td>
<td>28 $^{+15}_{-11}$ ± 19</td>
<td>5.0σ</td>
<td>10 ± 3 (stat.)</td>
<td></td>
</tr>
<tr>
<td>2013</td>
<td>D0 10.4 fb$^{-1}$</td>
<td>PRD89, 012004</td>
<td>215 ± 37</td>
<td>4159.0 ± 4.3 ± 6.6</td>
<td>19.9 ± 12.6 $^{+1.0}_{-8.0}$</td>
<td>3.1σ</td>
<td>21 ± 8 ± 4</td>
<td></td>
</tr>
<tr>
<td>2014</td>
<td>BaBar 422 fb$^{-1}$</td>
<td>PRD91, 012003</td>
<td>189 ± 14</td>
<td>4143.4 fixed</td>
<td>15.3 fixed</td>
<td>1.6σ</td>
<td>&lt; 13.3 @ 90%CL</td>
<td></td>
</tr>
</tbody>
</table>

Average

| $p \bar{p} \rightarrow J/\psi \phi...$ | 4152.5 ± 1.7 $^{+6.2}_{-5.4}$ | 16.3 ± 5.6 ± 11.4 | 4.7σ (5.7σ) |

### $X(4274-4351)$ summary

<table>
<thead>
<tr>
<th>Year</th>
<th>Experiment luminosity</th>
<th>Ref</th>
<th>$B \rightarrow J/\psi \phi K$ statistics</th>
<th>mass [MeV]</th>
<th>X(4274-4351) peaks(s) width [MeV]</th>
<th>sign.</th>
<th>fraction</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td>2011</td>
<td>CDF 6.0 fb$^{-1}$</td>
<td>arXiv:1101.6058 (unpub.)</td>
<td>115 ± 12</td>
<td>4274.4 $^{+5.5}_{-0.7}$ ± 1.9</td>
<td>32.3 $^{+21.9}_{-15.3}$ ± 7.6</td>
<td>3.1σ</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2011</td>
<td>LHCb 0.37 fb$^{-1}$</td>
<td>PRD85, 091103</td>
<td>346 ± 20</td>
<td>4274.4 fixed</td>
<td>32.3 fixed</td>
<td>&lt; 8 @ 90%CL</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2013</td>
<td>CMS 5.2 fb$^{-1}$</td>
<td>PL, B734, 261</td>
<td>2480 ± 160</td>
<td>4313.8 ± 5.3 ± 7.3</td>
<td>38 $^{+30}_{-16}$ ± 16</td>
<td>3.0σ</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2013</td>
<td>D0 10.4 fb$^{-1}$</td>
<td>PRD89, 012004</td>
<td>215 ± 37</td>
<td>4328.5 ± 12.0</td>
<td>30 fixed</td>
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<tr>
<td>2014</td>
<td>BaBar 422 fb$^{-1}$</td>
<td>PRD91, 012003</td>
<td>189 ± 14</td>
<td>4274.4 fixed</td>
<td>32.3 fixed</td>
<td>1.2σ</td>
<td>&lt; 18.1 @ 90%CL</td>
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<tr>
<td>2010</td>
<td>Belle 825 fb$^{-1}$</td>
<td>PRL 104, 112004</td>
<td>$\gamma \gamma \rightarrow J/\psi \phi$</td>
<td>4350.6 $^{+4.6}_{-5.1}$ ± 0.7</td>
<td>13 $^{+18}_{-9}$ ± 4</td>
<td>3.2σ</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Amplitude fit results to $B^+ \rightarrow J/\psi \phi K^+$

**LHCb Preliminary!**

<table>
<thead>
<tr>
<th>Contribution or Ref.</th>
<th>$M_0$ MeV</th>
<th>$\Gamma_0$ MeV</th>
<th>F.F. %</th>
<th>$f_L$</th>
<th>$f_1$</th>
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<tbody>
<tr>
<td>$K^+(1^+)$</td>
<td>8.0σ</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>NR</td>
<td></td>
<td></td>
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<tr>
<td>$K^+(1^+)$</td>
<td>7.6σ</td>
<td>1793±59 +153/160</td>
<td>365±157 +135/215</td>
<td>12±10 +10/17</td>
<td>0.24 ± 0.21</td>
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<td>$2^1 P_1$</td>
<td>1900</td>
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<td></td>
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<tr>
<td>$K_1(1650)$</td>
<td>36</td>
<td>1650±50</td>
<td>150 ± 50</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$K'_{1^+}$</td>
<td>1.9σ</td>
<td>1968±65 +170/172</td>
<td>396±170 +174/178</td>
<td>23±20 +31/29</td>
<td>0.04 ± 0.08</td>
</tr>
<tr>
<td>$2^3 P_1$</td>
<td>1930</td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>$K^-(2^-)$</td>
<td>5.6σ</td>
<td>1777±35 +172/17</td>
<td>217±116 +22/15</td>
<td>0.64 ± 0.11</td>
<td>0.13 ± 0.13</td>
</tr>
<tr>
<td>$K^-(2^-)$</td>
<td>5.0σ</td>
<td>1780</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>$1^3 D_2$</td>
<td>1780</td>
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<tr>
<td>$K_2(1770)$</td>
<td>36</td>
<td>1773 ± 8</td>
<td>188 ± 14</td>
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<tr>
<td>$K'_{2^-}$</td>
<td>3.0σ</td>
<td>1853±27 +18/35</td>
<td>167±58 +83/72</td>
<td>0.53 ± 0.14</td>
<td>0.04 ± 0.08</td>
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<td>$1^3 D_2$</td>
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<td>$K_2(1820)$</td>
<td>36</td>
<td>1816±13</td>
<td>276 ± 35</td>
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<tr>
<td>$K^*(1^-)$</td>
<td>8.5σ</td>
<td>1722±20 +143/160</td>
<td>354±75 +140/181</td>
<td>6.7 ± 1.9 +14/4</td>
<td>0.82 ± 0.04</td>
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<tr>
<td>$K^*(1^-)$</td>
<td>8.5σ</td>
<td>1780</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>$1^3 D_1$</td>
<td>1780</td>
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<tr>
<td>$K^*(1860)$</td>
<td>36</td>
<td>1717±27</td>
<td>322 ± 110</td>
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</tr>
<tr>
<td>$K^*(2^-)$</td>
<td>5.4σ</td>
<td>2073±94 +285/240</td>
<td>678±311 +113/555</td>
<td>2.9 ± 0.8 +17/07</td>
<td>0.15 ± 0.06</td>
</tr>
<tr>
<td>$2^3 P_1$</td>
<td>1940</td>
<td></td>
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<tr>
<td>$K^*_2(1980)$</td>
<td>36</td>
<td>1973±26</td>
<td>373 ± 69</td>
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<tr>
<td>$K(0^-)$</td>
<td>3.5σ</td>
<td>1874±43 +99/115</td>
<td>168±90 +280/104</td>
<td>2.6 ± 1.1 +43/18</td>
<td>1.0</td>
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<td>$3^3 S_0$</td>
<td>2020</td>
<td></td>
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<tr>
<td>$K(1830)$</td>
<td>36</td>
<td>~1830</td>
<td>~250</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**All $X(1^+)$**

| $X(4140)$ | 8.4σ | 4146.5±4.5 +4.6/2.8 | 83±21 +21/14 | 13±3.2 +2.4/2.9 |
| $X(4140)$ | 8.4σ | 4146.9±2.3        | 17.8±6.8    |       |

**Table 1**

| $X(4274)$ | 6.0σ | 4273.3±8.3 +17/3.6 | 56±11 +18/11 | 7.1 ± 2.5 +35/24 |
| $X(4274)$ | 6.0σ | 4274.5±6.4 ±1.9 | 32 +12/15 +8 |       |
| $X(4500)$ | 6.1σ | 4506±11 +12/15 | 92±21 +21/20 | 6.6±2.4 +5.8/2.3 |
| $X(4700)$ | 5.6σ | 4704±10 +14/24 | 120±31 +32/23 | 12±5 +7/5 |
\( X(5568)^{\pm} \rightarrow B_s \pi^{\pm} \) from D0

- \( X(5568)^{\pm} \rightarrow B_s^0 \pi^{\pm} \) decay reported by D0 in February with a significance of 5.1\( \sigma \)

- Signal implies large production rate within D0 acceptance

\[
\rho_{D0}^{X} = \frac{\sigma(p\bar{p} \rightarrow X + \text{anything}) \times \mathcal{B}(X \rightarrow B_s^0 \pi)}{\sigma(p\bar{p} \rightarrow B_s^0 + \text{anything})} \Bigg|_{\text{D0Acc.}}
\]

\[
= (8.6 \pm 1.9 \pm 1.4)\% 
\]

**Data**

- D0 Run II, 10.4 fb\(^{-1}\)
- 5,582 \pm 100 \( B_s \)

**Plots**

- D0 Run I, 10.4 fb\(^{-1}\)

- D0 Run II, 10.4 fb\(^{-1}\)

- \( \sqrt{\Delta\phi^2 + \Delta\eta^2} < 0.3 \)
No $X(5568)^{\pm} \rightarrow B_s \pi^{\pm}$ in LHCb data

- LHCb search first reported at Moriond
- Study based on large clean samples of $B_s^0$ decays
- (Right) no peak observed in $m(B_s^0 \pi)$ from $X(5568)$
- Upper limits set on production in the LHCb acceptance

$$\rho^{LHCb}_{X} \text{, } \begin{cases} 0.009 (0.010) & @ 90 \% \text{ CL} \\ 0.016 (0.018) & @ 90 \% \text{ CL} \end{cases} \quad \begin{cases} 5 \text{ GeV}/c \\ 10 \text{ GeV}/c \end{cases}$$

\[ B_s^0 \rightarrow D_s^+ \pi^- \]
66,300 ± 300 $B_s$

\[ B_s^0 \rightarrow J/\psi \phi \]
46,300 ± 200 $B_s$
Model independent analysis: $J/\psi K^-$

- Rule out the $\Lambda^*$-only hypothesis at $5.3\sigma$ (vs $9\sigma$ using $m_{J/\psi p}$)
- Points to exotic structures in $J/\psi p$ being more likely than in $J/\psi K$
Rectangular Dalitz plane

- For fixed $m_{K\pi}$ there is one-to-one relation between $m_{\psi\pi}$ and $\cos\theta_{K^*}$

\[
m_{\psi\pi}^2 = m_{\psi}^2 + m_{\pi}^2 + 2(E_{\psi}E_{\pi} + p_{\psi}p_{\pi}\cos\theta_{K^*})
\]

\[
p_{\psi}^2 = E_{\psi}^2 - m_{\psi}^2 \quad p_{\pi}^2 = E_{\pi}^2 - m_{\pi}^2
\]

\[
E_{\psi} = \frac{m_B^2 - m_{\psi}^2 - m_{K\pi}^2}{2m_{K\pi}} \quad E_{\pi} = \frac{m_{K\pi}^2 + m_{\pi}^2 - m_K^2}{2m_{K\pi}}
\]
Rectangular Dalitz plane

- For fixed $m_{Kp}$ there is one-to-one relation between $m_{J/\psi p}$ and $\cos \theta_{\Lambda^*}$

$$\Lambda_b^0 \rightarrow J/\psi pK^- \quad \text{(efficiency corrected)}$$

$$m_{J/\psi p}^2 = m_{J/\psi}^2 + m_p^2 + 2(E_\psi E_p + p_\psi p_p) \cos \theta_{\Lambda^*}$$

$$p_\psi^2 = E_\psi^2 - m_{J/\psi}^2 \quad p_p^2 = E_p^2 - m_p^2$$

$$E_\psi = \frac{m_{\Lambda_b}^2 - m_{J/\psi}^2 - m_{Kp}^2}{2m_{Kp}} \quad E_p = \frac{m_{Kp}^2 + m_p^2 - m_K^2}{2m_{Kp}}$$
Legendre Moments

Key idea:

$K^*/\Lambda^*$ can contribute only to low-rank moments

$$l_{\text{max}} = J_1 + J_2$$ for interfering resonances

In $K^*/\Lambda^*$-only hypothesis ($H_0$)

$$l_{\text{max}} = 2J_{\text{max}}$$

$J_{\text{max}}$ is the highest spin of $K^*/\Lambda^*$ resonance possible

Reflections of exotic hadrons can contribute to low and high rank moments:

- Detecting non-zero moments above $l_{\text{max}}$ signals presence of exotics

- The narrower the peak the higher the $l_{\text{max}}$ required. The sensitivity is better for narrower exotic hadrons.

- Exotic hadron contributions spread over wide range of $m_{K\pi}/m_{Kp}$. An effective way of testing $H_0$ is to aggregate the information about $\cos\theta_{K\pi/Kp}$ moments in a function of $m_{\psi'\pi}/m_{J/\psi'}$. 

Interfering $\Lambda^*$ resonances:

Asymmetric (P-violation in $\Lambda_b$ decays!) but relatively smooth $\cos\theta_{\Lambda^*}$ distributions

Two interfering $P_c^*$ resonances:

Reflection peak in $\cos\theta_{\Lambda^*}$ distributions, moving its position with $m_{Kp}$
Illustrations using amplitude models of $\Lambda_b^0 \rightarrow J/\psi pK$

Only exotic hadrons can contribute to excluded moments

The narrower the exotic hadron the better the sensitivity

Disclaimers:
- these are high statistics simulations to eliminate any statistical fluctuations (vertical scale is arbitrary)
- exotic hadron contributions are usually only a few % fit fractions, thus the amplitudes of the red curves is expected to be small in the real data
In preparation for quantitative test

- Creating $H_1$ hypothesis helps since exotic hadrons will generate higher moments than can be accommodated in $H_0$ ($\Lambda^*$-only hypothesis), but not very high moments:
  - Very high moments driven by statistical fluctuations
  - Looking for significance of moments with ranks just above $l_{max}(m_{K\pi})$ is more sensitive than looking at any rank moments above $l_{max}(m_{K\pi})$
The data vs amplitude simulations

\( F_t \) means PDF

- The data point falls in the region predicted by the full amplitude model (i.e. \( \Lambda^* s + 2P_c s \)) [speaks to the quality of the amplitude model]
- The sensitivity of the method depends dramatically on a \( P_c \) width; \( P_c(4380)^+ \) does not contribute much to the model independent result [know it from amplitude simulations]