BENCHMARKING THE BEAM LONGITUDINAL DYNAMICS CODE 
BLOND

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Abstract
The relatively recent Beam Longitudinal Dynamics code BLonD has already been applied to a wide range of studies for all present CERN synchrotrons. Its application area ranges from studies of RF manipulations, over single and multi-bunch interactions with impedance, to the action of feedback loops and RF noise. In this paper, we present benchmarks and comparisons with measurements, theory, or other codes, which have increased greatly the trust in the code. Tests related to bunch-to-bucket transfer, feedback loops, diffusion due to noise injection, as well as collective effects, are presented.

INTRODUCTION
The CERN BLonD code [1] has been developed by a group of developers for about two years now. Despite this short development time, it can model single or multi-bunch longitudinal dynamics in multi-harmonic RF systems, with beam-based feedback loops, RF phase noise injection, and (multi-turn) intensity effects due to machine impedance and/or space charge. Similarly vast is the range of applications the code has been used for already. Benchmarks against theoretical predictions, comparisons with measurement data, and other particle tracking codes have greatly increased trust in the code, and thus also trust in predictions given by BLonD for beam behaviour after upgrades or in future machines. In this paper, we present a few selected benchmark and comparison studies.

BENCHMARKS & COMPARISONS

Bunch-to-Bucket Transfer
A good test case for RF manipulations is the bunch-to-bucket transfer between the PS and the SPS. After arrival to flat top in the PS, the bunch distribution can be reconstructed using phase-space tomography [2] in a stationary RF bucket at 10 MHz. Bunches are then split twice into two bunches each, using the harmonics 20 MHz and 40 MHz. Then, the bunches are rotated in phase space using a sudden increase in the 40 MHz and 80 MHz voltages. These short bunches are subsequently extracted and injected into the SPS buckets, which are dominated by 200 MHz and have a higher-order component of 800 MHz in bunch-shortening mode. Starting from the reconstructed distributions, we have compared the bunch length at PS extraction and the transmission after the start of the SPS ramp between measurements, simulations using BLonD, and simulations performed in 2012 [3] with the former ESME [4] code. The results using two different RF voltages for the bunch rotation are summarised in Fig. 1.

Simulations with both codes reproduce reasonably well the measurement data and the observation that, using a higher rotation voltage, a better transmission and shorter bunch length can be achieved. The discrepancy between simulations and measurements is attributed to the fact that simulations use a single reconstructed particle distribution, while measurements are averaged over many bunches and many cycles with non-negligible bunch-to-bunch and shot-to-shot variations. Comparing results obtained with BLonD and ESME starting from the same bunch distribution, the values obtained for transmission and bunch length agree within 1-2 %, see Table 1. This small difference is probably due to the ESME tracking being based on real time and the BLonD tracking based on number of turns, which can result in ±1 turn difference in extraction time between ESME vs. BLonD, and given the rotation, the bunch length (and with it the transmission) is very sensitive to extraction time.

Feedback Loops
A unique feature of BLonD is the ‘tailor-made’ modelling of beam-based feedback loops for the CERN synchrotrons. Amongst others, an exact model of the LHC beam phase...
loop and synchronisation loop has been implemented and benchmarked against measurements. The phase loop is used to centre the RF bucket w.r.t. the bunch, damp common mode oscillations, and requires therefore a fast action. The synchronisation loop is used to counteract longer-term frequency drifts and to make the RF frequency track the momentum ramp. The same feedback gain settings of 1/(5 turns) for the phase loop and 1/(50 turns) for the synchronisation loop have been used both in simulations and in measurements.

To benchmark against this analytical formula, five different representations of the phase noise, generated with different random number generator seeds were applied during $10^5$ turns to the same bi-gaussian distribution with an initial 4-sigma bunch length of $\tau(0) = 0.4$ ns, with some noise-free tracking before and after, as shown in Fig. 3. LHC injection parameters have been used for the simulation with a bucket length of 2.5 ns. Both in the simulations and the analytic formula, a flat noise spectrum covering the full range of synchrotron frequencies was used. The results for different spectral densities are summarised in Table 2. The error becomes larger as the spectral density is increased. For all cases, however, an excellent agreement between particle simulations and theory is found.

### Table 2: Diffusion Benchmark for Short Bunches showing simulated vs. predicted bunch length after $10^5$ turns

<table>
<thead>
<tr>
<th>$S^\text{DS}_\psi$ [$\text{rad}^2/\text{Hz}$]</th>
<th>$4 \times \text{r.m.s. bunch length } \tau$ [ns]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>BLonD</td>
</tr>
<tr>
<td>$1 \times 10^{-8}$</td>
<td>0.418 ± 0.001</td>
</tr>
<tr>
<td>$2 \times 10^{-8}$</td>
<td>0.439 ± 0.004</td>
</tr>
<tr>
<td>$4 \times 10^{-8}$</td>
<td>0.461 ± 0.006</td>
</tr>
<tr>
<td>$1 \times 10^{-7}$</td>
<td>0.549 ± 0.012</td>
</tr>
</tbody>
</table>

**Collective Effects**

A range of benchmarks has been performed also related to collective effects caused by machine impedance or space charge; two of them are presented here. The first one is

$$
\sigma_\psi(t_{\text{diff}}) = \sqrt{\sigma_\psi^2(0) + \frac{1}{2} \omega_{s,0}^2 S^\text{DS}_\psi(\omega_{s,0}) t_{\text{diff}}},
$$

where $\omega_{s,0}$ is the angular synchrotron frequency at the centre of the bunch, $S^\text{DS}_\psi$ the double-sided spectral density of the phase noise, and $t_{\text{diff}}$ the diffusion time.

Diffusion

RF phase modulation or noise injection are widely-applied methods to shape the bunch profile [5] or create controlled emittance blow-up [6]. Diffusion due to (phase) noise injection can be modelled analytically [7] and in the case of bunches that are short compared to the bucket length, the evolution of the r.m.s. bunch length $\sigma_\psi$ (in radians) in a stationary RF bucket can be expressed as:
the evaluation of the synchrotron frequency shift due to potential-well distortion, for a bunch in steady state. Several methods to compute the synchrotron frequency are compared in Fig. 4, showing excellent agreement: particle tracking using BLonD (dots), semi-analytical formula derived from the Hamiltonian with intensity effects (full lines), and the incoherent synchrotron frequency shift from potential-well distortion [8] (dashed line).

![Figure 4: Synchrotron frequency distribution using the SPS impedance model at different intensities (low to high: blue, green, red).](image)

The same method was also used for a code-to-code comparison against PyORBIT in the PS Booster [9], where the dominant impedance source is space charge, as well as for beam measurements of the SPS impedance [10].

The second benchmark is to test the ability of BLonD to simulate instabilities. In this benchmark, the intensity threshold of loss of Landau damping for the dipole mode as a function of emittance was studied. Simulations using the LHC impedance model were compared to measurements in the LHC and the analytical scaling [11] for an effective reactive impedance of $\Im Z/nE = 0.09 \Omega$ [12]:

$$N_{th} \propto \frac{e^{5/2}}{\Im Z/nE^{5/4}V^{1/4}},$$

where $N_{th}$ is the intensity threshold, $e$ is the longitudinal emittance, $E$ the particle energy, and $V$ the RF voltage. Also this benchmark results in good agreement, see Fig. 5.

As collective effects can also be simulated for multi-bunch and multi-turn wakes, further benchmarks are foreseen to test the full capabilities of the code.

### CONCLUSIONS

BLonD is a recently developed beam dynamics particle tracking code with diverse features. It has been used for a wide range of applications in synchrotrons in- and outside of CERN. Extensive benchmarks and comparison studies have been performed to test the various features of the code. RF manipulations, beam-based feedback loops and diffusion, as well as collective effects have been studied. Studies were performed either as benchmarks against theoretical predictions, comparisons with measurement data, and/or other simulation codes. The excellent agreement found in all these studies have greatly increased the trust in BLonD.

### ACKNOWLEDGEMENTS

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### REFERENCES


