Probing Anomalous \( WW \gamma \) and \( WWZ \) Couplings with Polarized Electron Beam at the LHeC and FCC-Ep Collider

I. Turk Cakir, A. Senol, A. T. Tasci, O. Cakir

Abstract—We study the anomalous \( WW \gamma \) and \( WWZ \) couplings by calculating total cross sections of two processes at the LHeC with electron beam energy \( E_e=140 \) GeV and the proton beam energy \( E_p=7 \) TeV, and at the FCC-ep collider with the polarized electron beam energy \( E_e=80 \) GeV and the proton beam energy \( E_p=50 \) TeV. At the LHeC with electron beam polarization, we obtain the results for the difference of upper and lower bounds as \( (0.975, 0.118) \) and \( (0.285, 0.009) \) for the anomalous \( (\Delta \kappa_\gamma, \lambda_\gamma) \) and \( (\Delta \kappa_Z, \lambda_Z) \) couplings, respectively. For FCC-ep collider, these bounds are obtained as \( (1.101, 0.065) \) and \( (0.320, 0.002) \) at an integrated luminosity of \( \mathcal{L}_{\text{int}}=100 \) fb\(^{-1}\).

Keywords—Anomalous Couplings, Future Circular Collider, Large Hadron electron Collider, \( W \)-boson and \( Z \)-boson.

I. INTRODUCTION

The \( SU(2) \times U(1) \) gauge symmetry of the Standard Model (SM) results in the triple gauge boson interactions. A precise determination of the trilinear gauge boson couplings is necessary to test the validity of the SM and the presence of new physics up to a high energy scale. Since the tree-level couplings of the \( WW \gamma \) and \( WWZ \) vertices are fixed by the SM, any deviations from their SM values would indicate the new physics beyond the SM. The photoproduction of the \( W \) and \( Z \) bosons through triple gauge boson interactions in the lepton-hadron colliders HERA+LC and in the Large Hadron electron Collider (LHeC) has been studied theoretically in the papers [1]-[3] and [4], respectively. An investigation of the LHeC to probe anomalous \( WW \gamma \) coupling has been presented in [5], [6].

The present bounds on the anomalous \( WW \gamma \) and \( WWZ \) couplings are provided by the LEP [7], Tevatron [8], [9] and LHC [10], [11] experiments. Recently, the ATLAS [10], [11] and CMS [12], [13] Collaborations have established updated constraints on the anomalous \( WW \gamma \) and \( WWZ \) couplings from the SM and \( WZ \) production processes. The results from ATLAS and CMS experiments based on two-parameter analysis of the anomalous couplings are given in Table I.

In this work, we investigate the \( ep \rightarrow e \gamma X \) and \( ep \rightarrow e \gamma X \) processes with anomalous \( WW \gamma \) and \( WWZ \) couplings at the high energy electron-proton collider LHeC and FCC-ep (Future Circular Collider-electron proton) collider [14]. LHeC is considered to be realised by accelerating electrons 140 GeV and colliding them with the 7 TeV protons. We take into account the energies of the FCC-ep as 80 GeV for electron beam and 50 TeV for proton beam. We also consider the possibility of the electron beam polarization at LHeC [15] and FCC-ep which extends the sensitivity to anomalous triple gauge boson couplings.

II. ANOMALOUS COUPLINGS

The \( WW \gamma \) and \( WWZ \) interaction vertices are described by an effective Lagrangian with the coupling constants \( g_{WW\gamma} \) and \( g_{WWZ} \) and dimensionless parameter pairs (\( \Delta \kappa_\gamma, \lambda_\gamma \)) and (\( \Delta \kappa_Z, \lambda_Z \)).

\[
\mathcal{L} = ig_{WW\gamma}[g_1(W_{\mu\nu\rho}W^{\rho} - W_{\mu\nu}W_{\sigma}A^\sigma) + \kappa_1 W_{\mu\nu}W_{\sigma}A^\sigma + \frac{\Delta \kappa_\gamma}{m_W^2}W_{\mu\nu}^\gamma A^{\mu\nu}] + ig_{WWZ}[g_1(W_{\mu\nu}\gamma^Z - W_{\mu\nu}\gamma^Z) + \kappa_1 W_{\mu\nu}^Z A^{\mu\nu} + \frac{\Delta \kappa_Z}{m_W^2}W_{\mu\nu}^Z A^{\mu\nu}] 
\]

where \( g_{WW\gamma} = g \sin \theta_W \) and \( g_{WWZ} = g \cos \theta_W \). In general these vertices involve six C and P conserving couplings [16]. However, the electromagnetic gauge invariance requires that \( g_1 \) is 1. The anomalous couplings are defined as \( \kappa_V = 1 + \Delta \kappa_V \) where \( V = \gamma, Z \) and \( g_1^V = 1 + \Delta g_1^V \). The field strength tensors for the \( W \)-boson, \( Z \)-boson and photon, respectively.

The one-loop corrections to the \( WW \gamma \) and \( WWZ \) vertices within the framework of the SM have been studied in [17]-[19]. These corrections to the \( \Delta \kappa \) and \( \lambda \) have been found to be of the order of \( 10^{-2} \) and \( 10^{-3} \), respectively. The values of the

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**Table I**

The available 95% C.L. two-parameter bounds on anomalous couplings (\( \Delta \kappa_\gamma, \lambda_\gamma \)) and (\( \Delta \kappa_Z, \lambda_Z \)) from the ATLAS and CMS experiments.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>ATLAS (upper-lower)</th>
<th>CMS (upper-lower)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Delta \kappa_\gamma )</td>
<td>-0.420, 0.480</td>
<td>-0.250, 0.250</td>
</tr>
<tr>
<td>( \lambda_\gamma )</td>
<td>-0.068, 0.062</td>
<td>-0.050, 0.042</td>
</tr>
<tr>
<td>( \Delta \kappa_Z )</td>
<td>-0.045, 0.045</td>
<td>-0.160, 0.180</td>
</tr>
<tr>
<td>( \lambda_Z )</td>
<td>-0.063, 0.063</td>
<td>-0.055, 0.055</td>
</tr>
</tbody>
</table>

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couplings $\kappa^2 = \kappa Z^2 = 1$ and $\lambda^2 = \lambda Z^2 = 0$ correspond to the case of the SM. Since unitarity restricts the $WW\gamma$ and $WWZ$ couplings to their SM values at very high energies, the triple gauge couplings are modified as $\Delta \kappa (q^2) = \Delta \kappa(0)(1 + q^2 / \Lambda^2)^2$ and $\lambda (q^2) = \lambda(0)(1 + q^2 / \Lambda^2)^2$ where $V = \gamma, Z$. The $q^2$ is the square of momentum transfer into the process and $\Lambda$ is the new physics energy scale. The $\Delta \kappa(0)$ and $\lambda(0)$ are the values of the anomalous couplings at $q^2 = 0$. We assume the values of the anomalous couplings remain approximate constant in the interested energy scale ($\Lambda^2 > q^2$). We take $\Delta \kappa_V$ and $\lambda_V$ as free parameters in the considered range and find the bounds on these couplings effectively. For the numerical calculations, we have implemented interactions terms in the CalcHEP [20].

Fig. 1 Representative Feynman diagrams for subprocess $e+e^-\rightarrow e+\gamma'$

Fig. 2 Representative Feynman diagrams for subprocess $e+e^-\rightarrow e+Zq'$

III. PRODUCTION CROSS SECTIONS FOR LHEC

According to the effective Lagrangian, the anomalous vertices for triple gauge interactions $WW\gamma$ and $WWZ$ are presented in the Feynman graphs as shown in Figs. 1 and 2. In order to calculate the cross sections for the process $e+e^-\rightarrow e+\gamma'X$ and $e+e^-\rightarrow e+gZX$, we apply the transverse momentum cut on photon and jet as $p_T^\gamma > 50$ GeV, $p_T^Z > 20$ GeV; missing transverse momentum cut $p_T^m > 20$ GeV, pseudorapidity cuts $|\eta_{\gamma, Z}| < 3.5$; a cone radius cut between photons and jets $\Delta R_{\gamma, Z} > 1.5$. Using these cuts and the parton distribution functions of CTEQ6L [21], the total cross sections of the process $e+e^-\rightarrow e+\gamma'X$ as a function of anomalous couplings $\Delta \kappa_V$ and $\lambda_V$ for $E_e = 140$ GeV with electron beam polarizations $P_e = 0.8$ and $P_e = 0$ are presented in Figs. 3 and 4. In Figs. 5 and 6, the total cross sections of the $e+e^-\rightarrow e+gZX$ process are given for the same energy. It is clear from these figures that the polarization ($P_e = 0.8$) enhances the cross sections according to the unpolarized case.
plane for the integrated luminosities of 10 fb⁻¹ and 100 fb⁻¹ at electron beam energy \( E_e = 140 \text{ GeV} \) with polarization \( P_e = 0.8 \)

The difference of the upper and lower bounds on the anomalous couplings \( \Delta \kappa_V \) and \( \lambda_V \) (where \( V = \gamma, Z \)) can be written as

\[
\delta \Delta \kappa_V = \Delta \kappa_V^{\text{upper}} - \Delta \kappa_V^{\text{lower}}, \delta \lambda_V = \lambda_V^{\text{upper}} - \lambda_V^{\text{lower}}
\]

The current limits on anomalous couplings and the difference of the upper and lower bounds for electron beam energies of 140 GeV with integrated luminosities \( L_{int} = 10 \text{ fb}^{-1} \) and \( 100 \text{ fb}^{-1} \) at LHeC with the unpolarized (polarized) electron beam are given in Table II. We have obtained two-parameter limits on \( \delta \kappa_Z \) and \( \delta \lambda_Z \) which can be compared to the ATLAS and CMS results. However, the limits on \( \delta \kappa_Z \) is found to be much more sensitive than the current limits.

**TABLE II**

The 95% C.L. Current Limits on the Anomalous Couplings and the Difference of the Upper and Lower Bounds for Electron Beam Energy of \( E_e = 140 \text{ GeV} \) with \( L_{int} = 100 \text{ fb}^{-1} \) for Polarized and Unpolarized Electron Beam

<table>
<thead>
<tr>
<th>( P_e )</th>
<th>( \Delta \kappa_Z )</th>
<th>( \delta \Delta \kappa_Z )</th>
<th>( \lambda_Z )</th>
<th>( \delta \lambda_Z )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.8</td>
<td>-0.182,0.793</td>
<td>0.975</td>
<td>-0.039, 0.079</td>
<td>0.118</td>
</tr>
<tr>
<td>0</td>
<td>0.192,0.798</td>
<td>0.990</td>
<td>-0.041, 0.081</td>
<td>0.122</td>
</tr>
<tr>
<td>0.8</td>
<td>0.251,0.844</td>
<td>1.095</td>
<td>-0.047, 0.086</td>
<td>0.133</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( P_e )</th>
<th>( \Delta \kappa_Z )</th>
<th>( \delta \Delta \kappa_Z )</th>
<th>( \lambda_Z )</th>
<th>( \delta \lambda_Z )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.8</td>
<td>-0.143,0.142</td>
<td>0.285</td>
<td>-0.001, 0.008</td>
<td>0.009</td>
</tr>
<tr>
<td>0</td>
<td>0.273,0.089</td>
<td>0.362</td>
<td>-0.003, 0.009</td>
<td>0.012</td>
</tr>
<tr>
<td>0.8</td>
<td>0.253,0.215</td>
<td>0.468</td>
<td>-0.004, 0.010</td>
<td>0.014</td>
</tr>
</tbody>
</table>

**V. PRODUCTION CROSS SECTIONS FOR FCC-EP**

For calculate the cross sections for the process \( ep \rightarrow e X \) and \( ep \rightarrow e X \), we apply the transverse momentum cut on photon and jet as \( p_T^X > 20 \text{ GeV} \) and \( p_T^J > 20 \text{ GeV} \); missing transverse momentum cut \( p_T^M > 20 \text{ GeV} \); pseudorapidity cuts \( \eta_{Y,J} \) in the range of between -5 and 0; Using these cuts and the parton distribution functions of CTEQ6M [14], the total cross sections of the process \( ep \rightarrow e q X \) as a function of anomalous couplings \( \Delta \kappa_Z \) and \( \lambda_Z \) for \( E_e = 80 \text{ GeV} \) with \( P_e = 0.8 \) and
without \((P_e=0)\) electron beam polarization are presented in Figs. 9 and 10. It is clear from these figures that the polarization \((P_e=-0.8)\) enhances the cross sections according to the unpolarized case.

The cross sections depending on anomalous couplings \(\Delta \kappa_Z\) and \(\lambda_Z\) of the process \(ep \to e\gamma X\) at \(E_e=80\) GeV with \(P_e= \pm 0.8\) and without \((P_e=0)\) electron beam polarization are presented in Figs. 11 and 12.

\[\begin{align*}
\text{Fig. 9} & \quad \text{The cross section depending on anomalous coupling } \Delta \kappa_\gamma \text{ of the process } ep \to e\gamma X \text{ at } E_e=80\text{ GeV for different electron beam polarizations} \\
\text{Fig. 10} & \quad \text{The cross section depending on anomalous } \lambda_\gamma \text{ coupling of the process } ep \to e\gamma X \text{ for } E_e=80\text{ GeV} \\
\text{Fig. 11} & \quad \text{The cross section depending on anomalous } \Delta \kappa_Z \text{ coupling of the process } ep \to e\gamma X \text{ for } E_e=80\text{ GeV} \\
\text{Fig. 12} & \quad \text{The cross section depending on anomalous } \lambda_Z \text{ coupling of the process } ep \to e\gamma X \text{ for } E_e=80\text{ GeV}
\end{align*}\]

VI. ANALYSIS FOR FCC-EP

The contour plots of anomalous couplings in \(\Delta \kappa_\gamma - \lambda_\gamma\) plane for the integrated luminosities of 10 fb\(^{-1}\) and 100 fb\(^{-1}\) at electron beam energies \(E_e=80\) GeV are given in Fig. 13. For the process \(ep \to e\gamma X\), we make analysis of the signal and backgrounds when \(Z\) decays leptonically, \(Z \to l^+ l^-\) where \(l=e, \mu\). The contour plots of anomalous couplings in \(\Delta \kappa_Z - \lambda_Z\) plane for the integrated luminosities of 10 fb\(^{-1}\) and 100 fb\(^{-1}\) at electron beam energies of \(E_e=80\) GeV are presented in Fig. 14.

The difference of the upper and lower bounds on the anomalous couplings \(\Delta \kappa_V\) and \(\lambda_V\) (where \(V=\gamma, Z\)) can be written as

\[
\delta \Delta \kappa_V = \Delta \kappa_V^{\text{upper}} - \Delta \kappa_V^{\text{lower}}, \delta \lambda_V = \lambda_V^{\text{upper}} - \lambda_V^{\text{lower}}
\]

The current limits on anomalous couplings and the difference of the upper and lower bounds for electron beam energies of \(E_e=80\) GeV with integrated luminosities 100 fb\(^{-1}\) at FCC-ep with the unpolarized (polarized) electron beam are given in Table III. We have obtained two-parameter limits on \(\delta \Delta \kappa_V\) and \(\delta \lambda_V\) which can be compared to the ATLAS and CMS results. However, the current limits on \(\delta \lambda_Z\) is found to be much more sensitive at the FCC-ep.

<table>
<thead>
<tr>
<th>TABLE III</th>
</tr>
</thead>
<tbody>
<tr>
<td>THE 95% C.L. CURRENT LIMITS ON THE ANOMALOUS COUPLINGS AND THE DIFFERENCE OF THE UPPER AND LOWER BOUNDS FOR ELECTRON BEAM ENERGY OF (E_e=80) GEV WITH (L_{\text{int}}=100) FB(^{-1}) FOR POLARIZED ELECTRON BEAM</td>
</tr>
<tr>
<td>(P_e) &amp; (\Delta \kappa_\gamma) &amp; (\delta \Delta \kappa_\gamma) &amp; (\lambda_\gamma) &amp; (\delta \lambda_\gamma) &amp; (\delta \lambda_Z)</td>
</tr>
<tr>
<td>(0.8) &amp; -0.1001 &amp; 1.101 &amp; -0.0260 &amp; 0.039 &amp; 0.0650</td>
</tr>
<tr>
<td>(-0.8) &amp; -0.0193 &amp; 0.320 &amp; -0.0011 &amp; 0.0012 &amp; 0.0023</td>
</tr>
</tbody>
</table>
Fig. 13 Two dimensional 95% C.L contour plot anomalous couplings in the $\lambda_{\gamma}^{\gamma} - \Delta \kappa_{\gamma}$ plane for the integrated luminosity of 10 fb$^{-1}$ and 100 fb$^{-1}$ at electron beam energy $E_{e}=80$ GeV with polarization $P_{e}=-0.8$

Fig. 14 Two-dimensional 95% C.L contour plot of anomalous couplings in the $\lambda_{Z}^{Z} - \Delta \kappa_{Z}$ plane for the integrated luminosity of 10 fb$^{-1}$ and 100 fb$^{-1}$ at electron beam energy $E_{e}=80$ GeV with polarization

VII. CONCLUSION

The $WW\gamma$ and $WWZ$ anomalous interactions through the processes $ep\rightarrow q\gamma X$ and $ep\rightarrow qZ X$ can be studied independently at the LHeC and FCC-ep. We obtain two-parameter accessible ranges of triple gauge boson anomalous couplings at LHeC and FCC-ep with the polarized electron beam at the energies $E_{e}=140$ GeV and $E_{p}=7$ TeV, and $E_{e}=80$ GeV and $E_{p}=50$ TeV, respectively. Our limits compare with the results from two-parameter analysis given by ATLAS and CMS Collaborations [10]-[13]. We find that the sensitivities to anomalous couplings $\Delta \kappa_{\gamma}^{\gamma}$ ($V=\gamma, Z$) will be of the order of $10^{-4}$, which is an order of magnitude larger than the SM loop level sensitivity of $10^{-2}$, however a measurement of these couplings above $10^{-2}$ would offer a possible new physics signal. We conclude that the anomalous couplings $\lambda_{\gamma}^{\gamma}$ and $\lambda_{Z}^{Z}$ can be well constrained with the sensitivity of the order of $10^{-2}$ and $10^{-3}$ at the FCC-ep with polarized electron beam. The LHeC and FCC-ep could give complementary information about anomalous couplings compared to Tevatron and LHC.

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