Uncertainties and noise in coupling measurements at the Large Hadron Collider

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Geneva, 01.09.2016
1 Introduction

In the past, an increase of the coupling was observed in the BBQ data but not from the AC-dipole measurements [Maclean, 2014]. We want to check the hypothesis that noise causes this behaviour and for this, look at the example of noise induced by the Landau octupoles (sec.2). This report presents a second computational method for the coupling from turn-by-turn data and conclusions on the influence of noise on the measurements as well as an estimation of the uncertainties on the coupling which, based on these observations, is necessary (sec.3). We validate this estimation using simulations of multi-BPM data (sec.3.1).

Throughout the presented work, we use several established and new formulae, techniques and results, to be found in [Franchi et al., 2014], [Persson and Tomás, 2014] and [Maclean et al., 2016].

2 Influence of octupole noise on the measured coupling

For the beam optics in the LHC various magnetic systems are installed, including the so-called Landau octupoles which act on the focusing of the beam. Depending on the operation of these magnets, a change in the measured coupling was observed referring to the established method of determining $C^-$ which is stored in the timber database. This change was thought to be non-physical as it would require large displacements and additional rotation of the octupoles to reach the observed magnitude. Additionally, measurements with the AC-dipole were inconsistent with the change of coupling. Because of this, the measurement is expected to deliver wrong results meaning a coupling which is not dominated by the physical coupling but by unwanted effects. To make sure that no errors appear in the computation of $C^-$, a second method to calculate it is applied and compared to the established one. The used formula is [Franchi et al., 2014]

$$|C^-| = 2|Q_x - Q_y| \arctan \left( \frac{A_{01}B_{10}}{A_{10}B_{01}} \right)$$

with tunes $Q_{x,y}$ and amplitudes $A_{10}$ (hor., main), $A_{01}$ (hor., secon), $B_{01}$ (ver., main) and $B_{10}$ (ver., sec).

The comparison is shown in fig.1, demonstrating a good agreement between both methods concerning the moving average. Therefore the drop in coupling is proven not to be a flaw in the computation but results from the measured data itself. Looking at the determined coupling as well as the tunes that are found in the analysis for the main and secondary lines, it is obvious that for a non-vanishing octupole power current $I_{MO}$ the data contains much more deviations, i.e. noise. Based on these observations we expect noise in the raw data to be the source of the change in the coupling. Following this idea, the coupling indeed would be as small as measured in the second half of the time window but the enhanced noise level would form lines in the spectrum which are wrongly identified as secondary lines by the algorithms. A high noise intensity in this way results in a bigger coupling than the physically correct value.

To validate this hypothesis, we perform simulations with different noise levels. The motivation for this approach is easy to see with eq.(1): As approximation we may take the linear term of $\arctan(x)$, $x$, and assume $\frac{A_{01}}{A_{10}} \approx \frac{B_{01}}{B_{10}}$. Now, compare the coupling with and without noise $\sigma_n$ for exemplary values, e.g.

$$\frac{A_{01}}{A_{10}} = 0.1 \quad \Rightarrow \quad \frac{A_{01} + \sigma_n}{A_{10} + \sigma_n} = \frac{0.3}{1.2} = 0.25 \quad \Rightarrow \quad \frac{C^-_{noise}}{C^-} \approx \frac{0.25}{0.1} = 2.5.$$
Figure 1: Coupling determined by both methods in a time window with a shutdown of the Landau octupoles. A clear drop of $C^-$ can be observed for both techniques. More gaps, i.e. failures of the routine are visible for high noise level/octupole current.

We see an increase of the coupling by the factor 2.5 for realistic values of the coupled line (0.1) and noise (0.2). By this example, it is clearly seen that noise can cause the increase of coupling that was observed with the BBQ data. To confirm the approximation, we run simulations generating a signal with a noise component of certain amplitude and perform the analysis on this data.

Although assuming a very simple form of the signal and using white noise (see (2)), the influence of the noise on the measured coupling indeed can be confirmed. The generated signal is

\[
X = \sin(2\pi \cdot Q_x t) + c \sin(2\pi \cdot Q_y t) + N_x \\
Y = \sin(2\pi \cdot Q_y t) + c \sin(2\pi \cdot Q_x t) + N_y
\]

with \( p(N) = \frac{1}{\sqrt{2\pi D^2}} e^{-\frac{N^2}{2D^2}} \),

where \( X \) and \( Y \) are the simulated raw data values in the horizontal and vertical plane respectively, \( c \) denotes the coupled amplitude and \( N_{x,y} \) describes the noise that is generated for the artificial signal. \( p(N) \) is the probability distribution for a value of \( N \) for the noise and has a width of \( D \), determining the scale of the noise amplitude. The noise is generated independently but with the same parameters for both planes.

Using a sufficient sample size of 500 files, we may compare the coupling for two noise levels which is shown in fig.2.

We therefore may conclude that the current methods of determining the coupling are sensitive to noise in the raw data and in conclusion the uncertainty on the computed values must be estimated using the noise level.
Figure 2: The determined coupling for 500 samples of two noise levels each at $c = 0.1$. The drop which was observed for the octupole depowering clearly was reproduced.

3 Using noise parameters for the coupling analysis

For this section, we want to introduce two estimations which use the noise parameters that are calculated by drive. On one hand it is important to achieve an estimation of the uncertainties for the determined coupling based on the standard deviation of the noise. On the other hand we want to provide a limit on the secondary line based on the noise average level for cases in which the coupled amplitude is not found by drive. This is a suggestion discussed earlier which can be found in [Persson et al., 2016].

The input file Drive.inp contains two frequency intervals which are used by drive for the noise calculations. The amplitude of all lines in the recognized spectrum which lie in these windows are used to determine the average noise level and the corresponding standard deviation.

For the first estimation the following procedure is applied: The uncertainty on the amplitudes output by drive are assumed to be comparable to the standard deviation of the noise lines, $\sigma_x$ for the horizontal and $\sigma_y$ for the vertical spectrum. Using error propagation, the uncertainty of the coupling is computed via

$$\sigma_{C^-} = \frac{|Q_x - Q_y|}{A_{10}B_{01} + A_{01}B_{10}} \sqrt{\frac{A_{01}B_{10}}{A_{10}B_{01}}} \left(\sigma_x A_{10} B_{10}^2 + (A_{01} \sigma_y)^2 + \left(\frac{\sigma_x A_{01} B_{10}}{A_{10}}\right)^2 + \left(\frac{\sigma_y A_{01} B_{10}}{B_{01}}\right)^2\right).$$

(3)

Note that the presented methods are restricted to an error on the measured amplitudes and do not take into account uncertainties on the tunes. We therefore expect a slight underestimation of $\sigma_{C^-}$. 

3
The error on the coupling not only is necessary to estimate the precision of the measurements but also will be used for a weighted average for the overall resulting value of $C^-$ (see below).

The second usage of the noise parameters is fairly straightforward: For a vanishing secondary line we may assume that drive neglects the coupled amplitude in comparison to the other found lines, i.e. noise in the data. In conclusion, we can set an upper limit on the strength of the coupled line by using the average noise level detected by drive.

The weight used for the weighted averages is not the error itself, i.e. $\sigma_{C^-}$, but the inverse variation, $\sigma^{-2}$ such that we have

$$C^- = \sum_{i=0}^{N_{\text{BPM}}} \frac{C_i^- \cdot e^{i(\varphi_x - \varphi_y)}}{\sigma_i^2} \cdot \left(\sum_{i=0}^{N_{\text{BPM}}} \frac{1}{\sigma_i^2}\right)^{-1}$$

(4)

where

$$C_i^- = \sum_{j=0}^{N_{\text{files}}} \frac{C_{ij}^- \cdot \sigma_{ij}^2}{\sigma_i^2}$$

and

$$\sigma_i = \left(\sum_{j=0}^{N_{\text{files}}} \frac{1}{\sigma_{ij}^2}\right)^{-\frac{1}{2}}$$

(5)

with the number of files $N_{\text{files}}$ and of BPMs $N_{\text{BPM}}$, the individual coupling values $C_{ij}$ and corresponding uncertainties $\sigma_{ij}$. The phase advance $\varphi_x - \varphi_y$, determined from the phases of the main line in each plane, corrects for the different positions of the BPMs. Therefore the averaging of the complex $C^- / f_{1001}$ is possible without all values cancelling out because of different phases. The results of this computation are written to the `getcouple.out` file which contains the couplings as well as the phases for all BPMs.

The following details should be noted:

- On eq. (5): In case of multiple files analysis, the gained results are averaged over the various files for each BPM before saving to `getcouple.out` and processing further to get the overall results. The uncertainty for this first file-averaging is propagated on the same way as the one for the overall result, providing consistency (averaging twice separately having the same impact on the error as one overall averaging process) but not being proven to be reasonable.

- For the averaging processes described above, an important fact is that $C^-$ is complex-valued. This results in a “reduced” absolute value for the average when comparing to the single-BPM values. The sensitivity in this behaviour of phase advances and the critically high impact of noise based on it are further discussed in sec.3.1. However, the routines of this work use the method discussed in [Persson and Tomás, 2014] including the phase advance factor $\exp[i2\pi(\varphi_x - \varphi_y)]$ and all results given in the output files or in this report are referring to $|C^-|$.

### 3.1 Validation of the error estimation via noise

To validate the used error on the coupling, we carry out some simulations of multi-BPM raw data and analyze it with the new techniques for 1-BPM and 2-BPM-analysis described above, including the error estimation. From these simulations performed with various noise levels in the TBT data we gain information about the uncertainty of the coupling parameter $f_{1001}$, focusing on the absolute value which contributes to $C^-$. 
All data we analyze in this section is simulated with madx for injection optics beforehand, where we used the noise level as input parameter and always added the same value for the horizontal and vertical signal. The respective noise levels are given in the plots we will discuss.

At first, we look at the results for all single BPMs gained from datasets with different noise levels and compare the average coupling which was determined using the techniques described above to the single-BPM values. The single BPM values can be written as $|f_{1001,i}|$ while the shown average is the absolute value of the complex average including the phase advance, $|f_{1001} \cdot e^{i(\phi_x - \phi_y)}|$. [Persson and Tomás, 2014]. This is shown in fig.3 for the 1-BPM and in fig.4 for the 2-BPM method. The figure shows clearly the difference between the absolute value of the complex average and the real average of the absolute values, $|\bar{f}_{1001}|$. For the latter quantity, the phase advance has no influence as $|e^{i(\phi_x - \phi_y)}| = 1$. On these plots interesting main observations can be made:

- The 2-BPM method is much more sensitive for BPMs being sorted out because of a bad phase advance and therefore, the amount of missing BPMs in the plot is much higher (Note that the raw data for both plots is identical, especially the same amount of BPMs was used). As the noiseless dataset (blue) is entirely contained in the 2-BPM plot we may conclude that indeed noise is responsible for the failure in the analysis and that the phase advance has a big influence on the results.

- The errors on the single BPM values are bigger for the 1-BPM method which is based on the nature of the 2-BPM method taking into account the results of 2 BPMs at a time and thus reducing the single BPM error by means of a more stable result. For the 1-BPM method, the
error roughly describes the width of the single BPM values. However, this is not a necessity as the uncertainty between different BPMs for sure has more sources than the statistical error caused by noise. For the 2-BPM-case indeed the noise seems to have a bigger impact, causing higher deviations.

- The amplitude of the averaged $f_{1001}$ deviates from the average of the absolute values of $f_{1001}$, by 0.005 or less for the 1-BPM method and by at least 0.015 for the 2-BPM case. This behaviour is based on the partial cancelling of the single values that are averaged in the complex plane. It has been discussed and proven that nonetheless, the presented method delivers physical and correct results [Persson and Tomás, 2014]. Interestingly, the deviation from the average of $|f_{1001}|$, which is roughly identical for all noise levels, decreases for the 1-BPM method but increases for the 2-BPM routine.

We may conclude that the width of the coupling value distribution is described by the single-value-uncertainty for the 1-BPM but not for the 2-BPM case. High noise levels have an important influence on the 2-BPM routine referring to high losses of datapoints. Regarding the (weighted) average, we can observe the partial cancellation of the single results which plays an increasingly significant role for rising noise levels (2-BPM) or do show a very small impact, which even is reduced by noise (1-BPM). The latter behaviour is to be tested for real data and needs to be confirmed as reasonable.
4 Conclusion and outlook

We briefly want to summarize the results of the presented work.

Starting with two phenomena in the coupling measurements, the effect of noise on the established methods was demonstrated and confirmed with simulations. According to these results, enhanced noise levels cause significantly deviating results for the coupling which contradict direct measurements with the AC-dipole. The source of noise is irrelevant to this effect, the presented example was noise caused by octupoles.

To gain insights into the influence of noise on the coupling and information about the errors of the analysis, an estimation of uncertainties was introduced including a correction technique for bad data. These methods are built on the noise parameters as determined during the analysis. The estimation of the coupled line amplitude delivers reasonable outcomes and the error approximation was shown to give a good lower limit on the uncertainty of the coupling for the 1-BPM method. This was confirmed using a big amount of simulated data.

However, for the 2-BPM method the approximation on the error is not as good as in the 1-BPM case and the sensitivity regarding noise is much higher. This was shown by means of the high failure rate for increased noise levels and by obviously too small errorbars. Furthermore, the determined averaging technique is sensitive for the phases as well, yielding too small results for $f_{1001} \cdot e^{i(\phi_x - \phi_y)}$. For the 1-BPM analysis, the drop of coupling at a certain noise level is not fully understood although we were able to state an hypothesis about its source.

References


