SHiP sensitivity to Heavy Neutral Leptons

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Abstract

This note describes in detail the Heavy Neutral Lepton sensitivity studies presented in the Technical Proposal and related addenda.
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1 Introduction

This note describes in detail the procedure used to estimate the sensitivity of the SHiP experiment in the parameter space of theories with Heavy Neutral Leptons. Section 1.1 introduces the Neutrino Minimal Standard Model as a case study, and Appendix A lists the formulas used in the computations.

The goal of the SHiP experiment is to suppress all backgrounds to a total level of less than 0.1 expected events in 5 years of operation. Section 2.3 discusses in detail the case of background induced by Standard Model neutrino interactions.

The sensitivity plots presented in Section 2.4 assume a level of background of 0.1 events for a foreseen total exposure of $2 \times 10^{20}$ proton-target collisions. This way, they can be interpreted in two ways. If no event is observed, the plots represent the area of the parameter space that SHiP can rule out at 90% C.L. (confidence level). On the other hand, those plots also represent the 3σ evidence contour if 2 candidate events are observed.

Section 3, finally, discusses the synergy between HNL searches at SHiP and at possible future colliders.

This note presents the studies performed until the publication of the Technical Proposal [1] and related addenda [2].

1.1 The $\nu$MSM

The “Neutrino Minimal Standard Model” ($\nu$MSM) is a minimalistic model that attempt at explaining the pattern of neutrino masses, dark matter and the matter-antimatter asymmetry observed in the present Universe by introducing three Heavy Neutral Leptons (HNLs) [3, 4]. These states are SU(2) × U(1) singlets, insensitive to the electroweak interaction (thus the sterile nickname). Therefore, the SM lagrangian would extend to:

$$L = L_{SM} + \bar{N}_i i\not{\partial}N_i - f_{\alpha\alpha} \Phi \bar{N}_i L_{\alpha} - \frac{M_i}{2} \bar{N}_i \gamma^5 N_i + h.c.$$ (1)

where $\Phi$ and $L_{\alpha}(\alpha = e, \mu, \tau)$ are respectively the Higgs and lepton doublets, $f$ is a matrix of Yukawa couplings and $M_i$ is a Majorana mass term. $N_i$ represents the sterile neutrino fields.

As the Majorana masses are assumed to be of the order of the electroweak scale or below, the model can only be consistent with the neutrino experiments if the Yukawa couplings are very small, $f_i^2 \sim \mathcal{O}(m_\nu M_i/v^2)$, where $m_\nu$ are the masses of active neutrinos and $v = 174$ GeV is the vacuum expectation value of the Higgs field [4]. Hereinafter, as often in literature, the mixing couplings $f_i^2$ will be referred to as $U_\alpha^2$, with $\alpha = e, \mu, \tau$.

One of the new states, $N_1$, is a dark matter candidate with lifetime possibly greater than the age of the Universe, and a mass of $\mathcal{O}$(keV) [6]. This hypothesis can be verified by looking for a monochromatic line coming from the decay $N_1 \rightarrow \nu\gamma$. Recently a possible hint of such decay was reported in [7, 8], however it is not clear yet if this signal could be attributed to other astrophysical sources. The other two HNLs are degenerate in mass, with $m_N$ in the MeV-GeV range, and are responsible for the baryon-antibaryon asymmetry.
of the Universe through a process of leptogenesis made possible by their lepton number violating Majorana mass term [4, 9–11]. The two heavy states also allow for the observed pattern of neutrino masses through the type I see-saw mechanism, first introduced in the context of Grand Unified Theories [12–15], made possible by the coexistence of Dirac and Majorana mass terms.

![Figure 1: The SM foresees massless and only left-handed neutrinos. In the $\nu$MSM, three right-handed counterparts $N_1$, $N_2$ and $N_3$ are added to the particle content of the SM [5].](image1)

The phenomenology of sterile neutrino production was described in [16–20]. HNLs can be produced in decays where a SM neutrino is replaced by an HNL through kinetic mixing. They then decay to SM particles by mixing again with a SM neutrino. These massive neutrino states can decay to a variety of final states through the emission of a $W^\pm$ or $Z^0$ boson (see Figure 2). Branching ratios for the production and decay of HNLs can be

![Figure 2: Decay of a HNL through mixing with a SM neutrino.](image2)
obtained from the Dirac neutrino case [21] as shown in [17] and reported in Appendix A.

1.2 Heavy Neutral Leptons model parameters

The free parameters for any model with HNLs are four: the three mixing parameters $U_{e2}^2$, $U_{\mu2}^2$, $U_{\tau2}^2$, and the HNL mass $m_N$. Usually the total coupling to the SM $U_{\text{tot}}^2 = \sum_i U_i^2$ is used to present the results, since this is the important parameter from the cosmological point of view. Moreover, the interpretation of the limits in the parameter space depends on the hierarchy of the active neutrino masses. In this work, only the total coupling $U^2$ is left free to vary over the whole parameter space. The relative strength of the HNL coupling to the three SM flavours are fixed according to five scenarios (models), conforming to existing theoretical works: [10, 17]

1. $U_{e2}^2 : U_{\mu2}^2 : U_{\tau2}^2 \sim 52 : 1 : 1$, inverted hierarchy [17]
2. $U_{e2}^2 : U_{\mu2}^2 : U_{\tau2}^2 \sim 1 : 16 : 3.8$, normal hierarchy [17]
3. $U_{e2}^2 : U_{\mu2}^2 : U_{\tau2}^2 \sim 0.061 : 1 : 4.3$, normal hierarchy [17]
4. $U_{e2}^2 : U_{\mu2}^2 : U_{\tau2}^2 \sim 48 : 1 : 1$, inverted hierarchy [10]
5. $U_{e2}^2 : U_{\mu2}^2 : U_{\tau2}^2 \sim 1 : 11 : 11$, normal hierarchy [10]

The sensitivity limits obtained for the above scenarios are shown and discussed in Section 2.4.

In order to extract SHiP’s sensitivity, two techniques are combined. The official experiment software FAIRSHIP is used to precisely evaluate the final state acceptance and the reconstruction efficiency, and to devise a signal selection strategy. FAIRSHIP has the advantage of containing a detailed description of the material and the geometry of the detector (GEANT4 [22]), as well as a realistic description of the magnetic fields. A fast Monte Carlo simulation is used to estimate the signal yield as a function of the position in the HNL parameter space. Scenario II, with a total coupling to the SM of $U^2 = 9.3 \cdot 10^{-9}$ and a HNL mass of 1 GeV/c$^2$, was chosen to investigate SHiP’s acceptance in detail in order to tune the fast simulation, and will be referred to as benchmark scenario throughout this section.

2 Estimation of the HNL sensitivity

The number of sterile neutrinos that are detectable by SHiP in the nominal data taking period depends on their production rate and the corresponding experimental acceptance. It is given by:

$$n(HNL) = N(\text{p.o.t}) \times \chi(pp \to HNL) \times P_{\text{vtx}} \times A_{\text{tot}}(HNL \to \text{visible})$$  \hspace{1cm} (2)

[1], where
• $N(p.o.t) = 2 \times 10^{20}$ is the number of proton on target collisions expected in five years of SHiP operation at nominal conditions.

• $\chi(pp \to HNL)$ is the total production rate of sterile neutrinos per proton-target interaction. It is equal to:

$$\chi(pp \to HNL) = 2 \times [\chi(pp \to c\bar{c}) \times Br(c \to HNL) + \chi(pp \to b\bar{b}) \times Br(b \to HNL)] \times U_{tot}^2$$

(3)

where $\chi(pp \to c\bar{c}) = 1.7 \times 10^{-3}$ and $\chi(pp \to b\bar{b}) = 1.6 \times 10^{-7}$ are the production fractions of $c$- and $b$-mesons for a 400 GeV proton beam colliding on a Molybdenum target. Sterile neutrinos are mainly produced in $D_s$ meson decays, but $B_s$ mesons also contribute and they are the only source of sterile neutrinos for masses above 2 GeV. The fractions of heavy-meson decays into sterile neutrinos $Br(c \to HNL)$ and $Br(b \to HNL)$ take into account all the dominant kinematically allowed decay channels of $D_s$ and $B_s$ mesons into sterile neutrinos:

$$D \to K\ell + HNL$$
$$D_s \to \ell + HNL$$
$$D_s \to \tau \nu_\tau$$ followed by $\tau \to \ell \nu + HNL$ or $\tau \to \pi + HNL$

$$B \to \ell + HNL$$
$$B \to D\ell + HNL$$
$$B_s \to D_s \ell + HNL$$

(4)

The widths of these channels are parametrised as shown in Appendix A according to [17], as a function of the sterile neutrino mass and couplings. Other decays with smaller branching ratios represent a small correction and therefore they are not included. The factor two is added to take into account the fact that each of the quarks in the pair can hadronise individually and result in the production of an HNL.

• $P_{vtx}$ is the probability that the decay vertex of a sterile neutrino of given mass and couplings is located inside the SHiP fiducial volume. Its estimation is presented in detail in the following.

• $A_{tot}(HNL \to visible)$ is the experimental acceptance of all the visible final states, i.e. the fraction of sterile neutrinos decaying in SHiP fiducial volume that result in a detectable final state. It is defined as:

$$A_{tot}(HNL \to visible) = \sum_{i=\text{visible channel}} Br(HNL \to i) \times A(i)$$

(5)

where the index $i$ runs over final states with two charged particles. The estimation of the geometrical acceptance $A(i)$ is explained in detail in the following.
2.0.1 Geometrical acceptance

The software package Pythia8 [23] is used to build a sample of charm and beauty mesons produced in the SHiP target. For every meson, all of the kinematically allowed decays into massive sterile neutrinos from Equation 4 are simulated using the ROOT TGenPhaseSpace class.

A two-dimensional binned Probability Density Function (PDF) for the momentum $p$ and polar angle $\theta$ of sterile neutrinos produced in the proton-target interaction is obtained with the fast simulation. The PDF is built from simulated events, weighted with the branching ratio of the meson decay in which the HNL is produced. This PDF corresponds to the four-momentum spectrum of sterile neutrinos detectable by SHiP.

The sterile neutrino lifetime is estimated as the sum of the widths of its main decay channels:

$$
\begin{align*}
HNL &\rightarrow 3\nu \\
HNL &\rightarrow \pi^0\nu \\
HNL &\rightarrow \pi^\pm\ell \\
HNL &\rightarrow \rho^0\nu \\
HNL &\rightarrow \rho^\pm\ell \\
HNL &\rightarrow \ell^+\ell^-\nu
\end{align*}
$$

The branching ratios for these channels are computed according to the formulas in Appendix A [17] and shown in Figure 3 as a function of the HNL mass. The formulas in Ref. [17] for the decay of the HNL into mesons are valid up to $m_{HNL} \sim 1 \text{ GeV}/c^2$. If the sterile neutrino mass is much larger than the QCD scale, $m_{HNL} \gg \lambda_{QCD}$, the two quarks from $HNL \rightarrow q\bar{q}\nu$ and $HNL \rightarrow q\bar{q}\ell$ decays tend to hadronize individually. For masses in the region of $1 - 5 \text{ GeV}/c^2$, the inclusive $HNL \rightarrow qq\nu$ decay width is extrapolated from the parametrisation of $HNL \rightarrow \ell^+\ell^-\nu$ with appropriate corrections.

![Figure 3: Sterile neutrino branching ratios as a function of its mass for the benchmark scenario [1].](image-url)

For every bin of the PDF, the probability that an HNL with the corresponding
four-momentum decays within the acceptance of SHiP is computed as

$$P_{vtx}(p, \theta \mid m_{HNL}, U_2^f) = \int_{SHiP} \frac{e^{-l/\gamma c t}}{\gamma c t} dl.$$  \hspace{1cm} (7)

Hence, a new PDF is built, in which the content of each bin of the four-momentum spectrum is weighted according to Eqn. 7. The new spectrum is further corrected with a geometrical factor $A(\theta)$, selecting only sterile neutrinos in the angular acceptance of a cylinder strictly containing the SHiP vacuum vessel.

Figure 4: Binned probability density function in momentum and polar angle for the benchmark scenario, for $m_N = 1$ GeV (left) and for $m_N = 3$ GeV (right).

The resulting weighted spectrum represents the distribution in $(p, \theta)$ of sterile neutrinos that are detectable at SHiP. An example is shown in Figure 4 for the benchmark scenario. Its integral over the four-momentum spectrum represents the total probability that an HNL of given mass and couplings produces a vertex inside the fiducial decay volume:

$$P_{vtx}(m_{HNL}, U_2^f) = \int P_{vtx}(p, \theta \mid m_{HNL}, U_2^f) \, dp \, d\theta$$  \hspace{1cm} (8)

In the benchmark scenario, this probability corresponds to roughly $4.5 \times 10^{-5} \ (0.96 \times 10^{-2})$ for HNLs of mass 1 GeV (3 GeV), which PDF is shown in Figure 4.

2.0.2 Final state acceptance

The visible fraction of sterile neutrinos $A_{tot}(HNL \rightarrow \text{visible})$ is a function of the branching ratio and of the final state acceptance of the visible HNL decay channels (see Equation 5). All the decay channels providing two charged particles in the final state are considered detectable. Decays such as $HNL \rightarrow \rho^0 \nu$ followed by $\rho^0 \rightarrow \pi^+ \pi^-$ are also included. However, final states with one charged and one neutral pion are conservatively considered not reconstructable [1].

For every detectable and kinematically allowed channel, a sample of events is simulated using ROOT TGenPhaseSpace. The four-momentum of the decaying HNL is sampled
from the weighted spectrum described in Section 2.0.1. The position of the decay vertex is sampled from an exponential distribution with the sterile neutrino lifetime as parameter. The daughter tracks are propagated through the spectrometer magnetic field until they cross the exit lid of the vacuum vessel. The effect of the magnetic field is simulated as a momentum kick along the $y$ axis at the $z$ position of the centre of the magnet.

A detailed event selection is put in place by means of the FairSHIP framework and is described in Section 2.1. In the fast Monte Carlo, simulated events are considered reconstructable if:

- the vertex is within the fiducial volume of the vacuum vessel, that is, it is located at least $5\,\text{m}$ downstream of the entrance lid and upstream of the first spectrometer straw tracker station. This corresponds to the requirement that the HNL daughters do not cross the straw veto tagger, used offline to suppress the background originating from neutrino interactions in the material upstream of the vessel.
- The daughter tracks are contained within the vacuum vessel at the $z$ of the centre of the spectrometer magnet.
- The daughter tracks are contained within the vacuum vessel at the $z$ position of the exit lid.

The selection ensures that the tracks that exit from the fiducial volume and re-enter as a result of the magnetic kick are discarded. Overall, depending on the HNL mass, the daughter tracks are in acceptance in 20%-50% of the cases, as shown in Figure 5.

The final state acceptance is computed as

$$A(i) = \frac{\# \text{ reconstructable}}{\# \text{ simulated}}. \quad (9)$$

The intrinsic uncertainty of the toy Montecarlo on the total HNL acceptance, given by the product $P_{\text{vtx}} \times A(i)$, was found to be of the order of 30% relative to a single simulation test. This explains in part the fluctuations shown in Table 1 (more details are given in Section 2.1).
2.1 HNLs in the full SHiP simulation

The official SHiP computing framework, named FairShip [1], is based on FairROOT [24], based in turn on the ROOT package [25]. Detectors and other material are defined with the ROOT TGeo classes; particle transport and detector response is simulated through Geant4 [22], while track reconstruction makes use of Genfit [26]. Events are produced using the following generators: Pythia8 [23] for the proton-target collision, Genie for inelastic neutrino interactions, and Pythia6 [27] for inelastic muon scattering.

The physics parameters of Pythia8 were modified to allow the generation of HNLs. To generate signal, all the leptonic and semi-leptonic decays of $D^\pm$, $D^0$, $D_s$, $\Lambda_c$ and of the $\tau$ lepton were set to include an HNL in place of the SM neutrino. The HNL decay table is dynamically produced on the basis of the HNL mass and couplings: the kinematically available decay channels are activated, with their amplitudes computed as described in Appendix A, and used to determine the HNL lifetime. It is possible to activate and deactivate selected HNL decay channels in order to study the corresponding final states. The full list of possible final states is analogous to that of the toy simulation (Eqn. 6):

\[
\begin{align*}
\text{HNL} & \rightarrow 3\nu \\
\text{HNL} & \rightarrow \pi^0\nu_\alpha \quad \text{with} \; \alpha = e, \mu, \tau \\
\text{HNL} & \rightarrow \pi^\pm\ell \quad \text{with} \; \ell = e, \mu \\
\text{HNL} & \rightarrow \rho^0\nu_\alpha \quad \text{with} \; \alpha = e, \mu, \tau \\
\text{HNL} & \rightarrow \rho^\pm\ell \quad \text{with} \; \ell = e, \mu \\
\text{HNL} & \rightarrow \ell^+\ell^-\nu_\alpha \quad \text{with} \; \ell, \alpha = e, \mu, \tau \; \text{and} \; \alpha \neq \ell \\
\text{HNL} & \rightarrow \ell^+\ell'^-\nu_\ell \quad \text{with} \; \ell, \ell' = e, \mu, \tau \; \text{and} \; \ell' \neq \ell
\end{align*}
\] (10)

In FairShip, the $P_{\text{vtx}}$ factor in Equation 2 corresponds to weights applied to each generated HNL to account for the probability that such particle leaves a decay vertex inside the SHiP fiducial volume. The weights are computed as in Eqn. 7. The total geometrical acceptance $P_{\text{vtx}} \times A$ is equal to the ratio of the sum of the weights of HNL candidates satisfying geometrical selection criteria, divided by the total number of HNLs generated. The error on the acceptance is calculated by generating various independent HNL signal samples, and by taking the root-mean-square deviation of the computed acceptances as error. It is found to be of order $\sim 10\%$. The following loose selection criteria were used to estimate the total acceptance with FairShip:

- the HNL decay vertex is located between the straw veto tagger and the exit lid of the vacuum vessel;
- the $x, y$ position of the sterile neutrino decay vertex is inside the elliptical fiducial volume ($r_x = 250$ cm, $r_y = 500$ cm);
- both HNL daughters leave a signal in one of the straw stations before the magnet (1 or 2) and in station 4 after the magnet. These hits are within the elliptical fiducial volume ($r_x = 250$ cm, $r_y = 500$ cm);
• 150 MeV of energy are deposited in the ECAL (only for \( HNL \rightarrow \mu \pi \) and \( HNL \rightarrow e \nu \nu \));

• muons from HNL decays leave a signal in the first two muon stations (only for \( HNL \rightarrow \mu \pi \) and \( HNL \rightarrow \mu \mu \nu \)).

No reconstruction nor selection were applied at this time [1].

2.1.1 Comparisons of signal acceptances between the toy MC and FairShip

Comparisons between the toy MC and the full simulation are shown in Table 1, for the \( N \rightarrow \mu \pi \) decay channel and for HNL masses ranging from 0.3 GeV/c\(^2\) to 1.1 GeV/c\(^2\) and various couplings, and in Figure 6. The test observable in Table 1 is the total detector acceptance \( A \equiv A_{\text{tot}}(HNL \rightarrow \text{visible}) \times P_{\text{vtx}} \). The results show good agreement between the toy and the full simulation. The signal acceptance results obtained with the FairShip software and the toy Monte Carlo, without applying further offline selection criteria, are in agreement within errors. The signal acceptance for the benchmark scenario in the \( \pi \mu \) channel is found to be \( A = (5.8 \pm 1.8) \times 10^{-6} \) for the toy Monte Carlo and \( A = (5.6 \pm 0.6) \times 10^{-6} \) for FairShip.

Figure 6 compares the sterile neutrino momentum and polar angle distributions obtained with FairShip and with the toy Monte Carlo for the benchmark scenario. A good level of agreement is observed for the momentum distribution. Even if the two procedures result in slightly different polar angle spectra, a systematic overestimation of the acceptance will be prevented by means of an acceptance correction factor. Such factor is computed as the ratio between the acceptance computed with FairShip and the acceptance computed with the toy. This factor is used in the sensitivity studies described in this chapter in order to provide a more realistic estimate of the detector acceptance, together with the reconstruction and selection efficiencies. In the toy Monte Carlo, all the kinematically allowed decay channels of the \( D_s \) and \( B_s \) mesons are included in the simulation and contribute to the sterile neutrino spectrum with relative contributions that vary according to the sterile neutrino couplings and mass. In the FairShip software, only the charm contribution was initially included, and the mass/coupling dependence of the branching ratios was neglected (see Section 2.1). To achieve a fair comparison between the two techniques, only sterile neutrinos coming from \( D_s \) mesons were used, both in the toy Monte Carlo and in FairShip, to produce Table 1 and Figure 6.

2.2 Reconstruction efficiency and offline selection

In this section the offline selection of HNL is studied. The main background, consisting of SM neutrinos interacting in the vicinity of the decay volume, is used as case study to optimize the selection criteria. The remaining background, mainly coming from cosmic muons or residual muons from the proton-target interaction, is sub-dominant. Two-charged particles events due to cosmic rays interacting in the material will be reduced to a negligible level by the liquid scintillator tagger and by the requirement of a reasonable impact parameter with respect to the target. Combinatorial background will also be
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Table 1: Comparison of signal acceptances for the $HNL \rightarrow \pi\mu$, $\mu\mu\nu$ (in the benchmark scenario) and $ee\nu$ (in a scenario with couplings $O(10^{-6})$ and $m_N = 100$ MeV/$c^2$) decay channels using the toy Monte Carlo and FairSHIP. The first four columns specify the position in the HNL parameter space. The fifth column identifies the scenario among those presented in this section. The sixth column is the result obtained with the toy Monte Carlo simulation, and the seventh column is the FairSHIP result. Finally, the last column shows the ratio between the two results. Most ratios are compatible with unity within the uncertainty. The oscillations are due to the fact that the HNL physics is represented only partially in the FairSHIP software, and to the poor statistical significance obtainable with the toy [1].

identified, thanks to the dedicated timing detector. Signal-like events due to the decay

Figure 6: Sterile neutrino momentum (left) and polar angle (right) distributions obtained in FairSHIP (red) and in the toy Monte Carlo (blue) [1].
of a $V^0$ produced in the interaction of a neutrino with the upstream material are also efficiently vetoed thanks to a system of several tagging detectors [1].

Figures 7-9 show distributions of $ndf$ (number of degrees of freedom), $\chi^2/ndf$, distance of closest approach of the daughter tracks, $z$ position of the decay vertex, reconstructed candidate mass and reconstructed mass for events with $\chi^2/ndf < 5$ and $ndf > 15$, and impact parameter to the target. These observables are shown for $HNL \rightarrow \mu \pi$, $HNL \rightarrow \mu \mu \nu$ and $HNL \rightarrow e e \nu$ decays, respectively, and superimposed to the same observables for neutrino-induced background [1].

The main criterion to design the selection is the goal to achieve an estimate of less than one background event in the whole data taking period, keeping an high efficiency for both the fully reconstructed (mainly $HNL \rightarrow \pi \mu$) and the partially reconstructed signal ($HNL \rightarrow \ell \ell \nu$). The following four kinds of selection criteria have been devised:

- **Track multiplicity:**
  - “1 HNL candidate”: only one candidate is reconstructed, i.e. the event features only two charged tracks.

- **Fiducial cuts:**
  - “Vtx in fiducial vol.”: the vertex is located in the fiducial volume, at least 20 cm downstream of the straw veto, and at least 20 cm upstream of the first tracker station (our longitudinal vertex resolution is $\sigma_z \sim 9$ cm). It is contained in the elliptical shape of the vessel, with a 1 cm tolerance at the border (our transversal vertex resolution is $\sigma_{x,y} \sim 0.3$ cm).
  - “Tracks in fiducial vol.”: the tracks forming the HNL candidate are fully contained in the vessel, with a 1 cm tolerance at the border.

- **Track quality cuts:**
  - “Event reconstructed”: the track fit converged for both daughter tracks. FAIR-SHIP only creates a reconstructed HNL candidate if two reconstructed tracks of opposite charge create a vertex with a maximum closest-approach distance of 30 cm.
  - “N.d.f. > 25”: number of degrees of freedom > 25 (this ensures a sufficient amount of hits in each tracking station. Tracks not crossing all the 4 tracking stations are not reconstructed).
  - “DOCA < 1 cm”: distance of closest approach < 1 cm: the average for signal events is 3.6 mm.
  - “$\chi^2/ndf < 5$”: the reduced chi-square of the track fit is less than 5 for both daughters.

- **Background-suppressing cuts:**
Figure 7: $\chi^2/ndf$, $ndf$, distance of closest approach of the daughter tracks, $z$ position of the decay vertex, reconstructed candidate mass and reconstructed mass for events with $\chi^2/ndf < 5$ and $ndf > 15$, and impact parameter to the target distributions for 2-track signal candidates in the $HNL \rightarrow \mu\pi$ channel (solid black line). The red line represents neutrino-induced reconstructed background events [1].
Figure 8: $\chi^2$/ndf, ndf, distance of closest approach of the daughter tracks, $z$ position of the decay vertex, reconstructed candidate mass and reconstructed mass for events with $\chi^2$/ndf < 5 and ndf > 15, and impact parameter to the target distributions for 2-track signal candidates in the $HNL \rightarrow \mu\mu\nu$ channel (solid black line). The red line represents neutrino-induced reconstructed background events [1].
Figure 9: $\chi^2/\text{ndf}$, ndf, distance of closest approach of the daughter tracks, z position of the decay vertex, reconstructed candidate mass and reconstructed mass for events with $\chi^2/\text{ndf} < 5$ and ndf > 15, and impact parameter to the target distributions for 2-track signal candidates in the $\text{HNL} \to eev$ channel (solid black line). The red line represents neutrino-induced reconstructed background events [1].
- daughters track momentum $> 1.5$ GeV: this helps suppressing the combinatorial background.

- “IP < 0.1 m”: impact parameter to the target $< 10$ cm: the average for fully reconstructed signal events is 1.65 cm. This cut is released to 2.5 m for partially reconstructed final states.

- “Event not vetoed”: it corresponds to the online selection, i.e. no activity in any of the VETO detectors: upstream veto, straw veto, liquid scintillator and the $\nu_{\tau}$ RPC muon spectrometer itself.

- Particle identification:
  - for final states including one (two) muons, the efficiency of a selection based on the presence of one (two) muon track(s) in the first two muon stations is evaluated.

The number of sterile neutrino candidates selected after the different requirements is given in Tables 2 to 4 for signal samples, and in Table 5 for sample of neutrino-induced background events (see Section 2.3). The online selection is applied as last selection stage in order to increase the statistical significance of the selection efficiencies on the neutrino background. The efficiency of each cut is computed with respect to the preceding cut. In particular, Table 6 shows the effect of the online selection alone on the neutrino background.

<table>
<thead>
<tr>
<th>Selection</th>
<th>Entries</th>
<th>Acceptance</th>
<th>Selection efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Event reconstructed</td>
<td>4471</td>
<td>$6.43 \times 10^{-6}$</td>
<td>-</td>
</tr>
<tr>
<td>1 HNL candidate</td>
<td>4386</td>
<td>$6.27 \times 10^{-6}$</td>
<td>97.6 %</td>
</tr>
<tr>
<td>Vtx in fiducial vol.</td>
<td>3777</td>
<td>$5.37 \times 10^{-6}$</td>
<td>85.7 %</td>
</tr>
<tr>
<td>Tracks in fiducial vol.</td>
<td>3508</td>
<td>$4.77 \times 10^{-6}$</td>
<td>88.8 %</td>
</tr>
<tr>
<td>N.d.f. &gt; 25</td>
<td>3345</td>
<td>$4.45 \times 10^{-6}$</td>
<td>93.2 %</td>
</tr>
<tr>
<td>DOCA &lt; 1 cm</td>
<td>3161</td>
<td>$4.15 \times 10^{-6}$</td>
<td>93.3 %</td>
</tr>
<tr>
<td>$\chi^2$/N.d.f. &lt; 5</td>
<td>3161</td>
<td>$4.15 \times 10^{-6}$</td>
<td>100.0 %</td>
</tr>
<tr>
<td>Daughters $P &gt; 1$ GeV</td>
<td>3160</td>
<td>$4.15 \times 10^{-6}$</td>
<td>99.9 %</td>
</tr>
<tr>
<td>IP &lt; 0.1 m</td>
<td>3137</td>
<td>$4.11 \times 10^{-6}$</td>
<td>99.1 %</td>
</tr>
<tr>
<td>Event not vetoed</td>
<td>2969</td>
<td>$3.91 \times 10^{-6}$</td>
<td>95.1 %</td>
</tr>
<tr>
<td>1 muon in 1st muon station</td>
<td>2955</td>
<td>$3.89 \times 10^{-6}$</td>
<td>99.4 %</td>
</tr>
<tr>
<td>1 muon in 2nd muon station</td>
<td>2916</td>
<td>$3.82 \times 10^{-6}$</td>
<td>98.2 %</td>
</tr>
</tbody>
</table>

Table 2: Effect of the offline selection on $HNL \rightarrow \pi\mu$

The resulting acceptance after all selection criteria are applied is compared with the raw acceptance computed with the toy Monte Carlo. The ratio between these two values is applied as efficiency factor to the toy simulation in order to take into account the
reconstruction and selection efficiencies when providing estimates for the SHiP sensitivity to sterile neutrinos.

### 2.3 Background studies

The principal background to the hidden particle decay signal originates from the inelastic scattering of neutrinos and muons in the vicinity of the detector, producing long-lived neutral mesons. Another source of background are random combinations of tracks from the residual muon flux, or other charged particles from inelastic interactions in the proximity, which enter the decay volume and together mimic signal events. Cosmic muons can
Table 5: Effect of the offline selection on neutrino-induced background

<table>
<thead>
<tr>
<th>Selection</th>
<th>Entries</th>
<th>Events / 5 years</th>
<th>Selection efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Event reconstructed</td>
<td>79547</td>
<td>2.11 \times 10^4</td>
<td>-</td>
</tr>
<tr>
<td>1 HNL candidate</td>
<td>60469</td>
<td>1.67 \times 10^4</td>
<td>79.1 %</td>
</tr>
<tr>
<td>Vtx in fiducial vol.</td>
<td>13687</td>
<td>3.61 \times 10^4</td>
<td>21.6 %</td>
</tr>
<tr>
<td>Tracks in fiducial vol.</td>
<td>13291</td>
<td>3.5 \times 10^3</td>
<td>97.0 %</td>
</tr>
<tr>
<td>N.d.f. &gt; 25</td>
<td>7064</td>
<td>1.65 \times 10^2</td>
<td>47.1 %</td>
</tr>
<tr>
<td>DOCA &lt; 1 cm</td>
<td>752</td>
<td>228</td>
<td>13.9 %</td>
</tr>
<tr>
<td>$\chi^2$/N.d.f. &lt; 5</td>
<td>751</td>
<td>228</td>
<td>99.9 %</td>
</tr>
<tr>
<td>Daughters $P &gt; 1$ GeV</td>
<td>519</td>
<td>137</td>
<td>60.2 %</td>
</tr>
<tr>
<td>IP &lt; 0.1 m (2.5 m)</td>
<td>1 (265)</td>
<td>0.139 (47)</td>
<td>0.1 % (34.2 %)</td>
</tr>
<tr>
<td>Event not vetoed</td>
<td>0</td>
<td>0</td>
<td>0.0 %</td>
</tr>
</tbody>
</table>

Table 6: Effect of the online selection on neutrino-induced background

<table>
<thead>
<tr>
<th>Selection</th>
<th>Entries</th>
<th>Events / 5 years</th>
<th>Selection efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Event reconstructed</td>
<td>79547</td>
<td>2.11 \times 10^4</td>
<td>-</td>
</tr>
<tr>
<td>Event not vetoed</td>
<td>215</td>
<td>64.2</td>
<td>0.3 %</td>
</tr>
</tbody>
</table>

Careful studies using the full SHiP Monte Carlo simulation indicate that a level of background of 0.1 events for 5 years of data taking is achievable, thanks to the redundant system of veto detectors [1, 2].

The flux of neutrinos is estimated to be $1.0 \times 10^{11}$ neutrinos per spill. Background studies were performed with an energy spectrum ranging from 2 GeV to about 100 GeV\(^1\). Neutrinos are mainly produced in decays of pions and kaons produced at the SHiP target. A large sample of neutrino interactions with the detector material was simulated, corresponding to the amount of neutrino interactions expected in five years of SHiP operation. Pythia8 [23] was used to generate proton-target interaction events that were then processed by Geant4 [22] to simulate the transport of particles through the experiment material. Neutrino interactions are simulated with Genie [29].

Neutrino interactions were found to take place mainly in the muon magnetic spectrometer of the tau neutrino detector, in the entrance window and the surrounding walls of the vacuum vessel. The probability that neutrinos interact with the residual gas inside the decay volume is negligible. Overall we expect about $10^7$ neutrino interactions in 5 years; about $10^4$ such events have two tracks of opposite charge reconstructed in the HS spectrometer as potential signal candidates [1]. The topology of these events is such

\(^{1}\)Neutrinos with energy lower than 2 GeV have negligible impact on the background studies, since they mostly interact through elastic scattering.
that the relatively loose signal selection criteria introduced in Section 2.2, together with the online preselection operated by the veto detectors, allow to reject the totality of the simulated background-induced candidate events: the interaction products do not point at the target, do not have a reconstructed vertex inside the decay volume, and have very poor track quality. This is true, in general, for all background sources [28].

At the level of online selection, the requirement of having at least one veto detector with a positive response, together with a loose requirement on the pointing of the interaction products to the target, rejects about 99.7% of tracks coming from neutrino interactions (see Table 6). If no online selection was applied, the signal selection criteria introduced in Section 2.2, with an impact parameter lower than 10 cm (2.5 m) with respect to the proton target, allow the rejection of 99.99% (99.77%) of the reconstructed neutrino-induced candidates (see Table 5). Figures 7-9 show a comparison of the distributions of the observables used in the offline selection for the HNL signal and the neutrino background. The set of selections applied is highly redundant and can be trimmed down to study specific channels.

2.4 Sensitivity to HNLs

The signal acceptance decreases to $A = (4.4 \pm 1.8) \times 10^{-6}$ at the reconstruction level, following the offline selections. This factor is also included in the toy Monte Carlo.

Having validated the toy Monte Carlo results against those of the full simulation (Section 2.1), the toy can be used to assess the sensitivity contours in the sterile neutrino mass-couplings parameter space. The toy Monte Carlo technique has the advantage of being computationally faster and easier to configure with respect to a change of coordinates in the HNL parameter space. It can also be used to completely determine the expected number of signal events in five years of SHiP operation, because it provides algorithms to estimate both the rate of sterile neutrinos produced at the target and the acceptance to the sterile neutrino decay products. On the other hand, the accuracy of the result is of the order of 30%. Fluctuations due to fine tunings of the selection criteria would be subdominant with respect to statistic fluctuations. Therefore, following the analysis of Tables 2-4, averaging over the reconstruction and selection efficiencies of the different final states, we apply a correction factor of

$$f_{\text{reco}} = \begin{cases} 40\% & \text{if } m_N < 2m_\mu \\ 60\% & \text{otherwise} \end{cases} \quad (11)$$

to the final result of the toy simulation.

The SHiP sensitivity, evaluated for the five scenarios introduced in Section 1.2, is shown in the five plots of Figure 10, respectively. The figures also show the variation of the $\nu$MSM parameter space for different relative strengths of the couplings to the three SM flavours. Regions of large coupling and mass (“BAU”) are greyed out because a HNL with those parameters would not suffice to explain the level of matter-antimatter asymmetry in the Universe. Observations of Big Bang nucleosynthesis would find a natural explanation in the $\nu$MSM if the two massive HNLs lie in the region on the right of the “BBN” curve.
Figure 10: Sensitivity contours in the parameter space of the $\nu$MSM for scenarios I-IV introduced in Section 1.2. They can be interpreted as 90% C.L. exclusion limits if no event will be observed with a level of background of 0.1 events in 5 years, and as 3$\sigma$ discovery potential if two events would be observed. Finally, couplings that are too low would not allow the SM neutrino masses to be generated through the seesaw mechanism ("Seesaw" curve in the plots). The exclusion limits set by previous experiments are also shown in the plots [10, 17].

Figure 11: Variation of the sensitivity contours for scenarios II (left) and IV (right) as a function of the background estimates. The solid blue curve represents the 90% C.L. upper limit assuming 0.1 background events in $2 \times 10^{20}$ proton-target collisions. The dashed blue curve assumes 10 background events. The dotted blue curve assumes a systematic uncertainty of 60% on the level of background, i.e. $10 \pm 6$ background events [2, 28]. Finally, Figure 11 shows the impact on the sensitivity of a higher level of background
in SHiP. Assuming 10 background events in the nominal data taking period (a factor 100 larger than the expected 0.1 background events), and even with a systematic uncertainty of 60%, marginally affects the HNL sensitivity, compared to the significant improvement on the limits from previous experiments. Moreover, the estimates shown in Figure 11 do not take into account that the invariant mass can be used as additional selection criteria, once an hypothesis on the HNL mass is made.

3 HNL search at future circular colliders

Heavy right-handed neutrinos can also be searched for at high luminosity lepton colliders, such as the Future $e^+e^-$ Circular Collider (FCC-ee), currently being studied within the “Future Circular Collider Study” project at CERN [30]. The machine being studied would fit in a 100 km tunnel and would be able to address centre-of-mass energies in the 90-350 GeV range, allowing precision tests of the Standard Model and accurate measurements of the characteristics of the Higgs boson, with as ultimate goal a 100 TeV proton collider that would fit in the same tunnel. Luminosity studies show that such $e^+e^-$ machine, operated at the centre-of-mass energy corresponding to the $Z$ resonance, could produce $10^{12}$ to $10^{13}$ $Z$ bosons per year with the “crab-waist” scheme, and thus allow to investigate extremely rare decays [31–33].

The portion of the $\nu$MSM parameter space accessible for the SHiP experiment was described in Section 2.4. In [34] we extend the mass range up to the mass of the $W$ boson. For HNL masses $m_{HNL} \gtrsim m_W$, the rate of interactions is enhanced due to the now kinematically allowed decay channel $HNL \rightarrow \ell W$, leading to stronger constraints on the mixing parameter resulting from baryon-antibaryon asymmetry (BAU) [35]. The resulting parameter space (Figure 12) is bound on all sides, due to the intersection of the BAU and seesaw constraints.

A review of possible methods to perform HNL searches at future $e^+e^-$ colliders is given

Figure 12: Interesting domains in the mass-coupling parameter space of heavy neutrinos and current experimental limits, for normal and inverted hierarchy of the left-handed neutrino masses [34].
Hints of the existence of sterile neutrinos can be found in the discrepancy between the measured number of neutrino families – the ratio of the $Z$ invisible width to its leptonic decay width – and that of the SM lepton flavours. The former, $N_\nu = 2.9840 \pm 0.0082$ [36], appears to be about two standard deviations lower than three, and such a deficit could be compatible with the presence of sterile neutrinos. However, for small mixing angles between sterile and active neutrinos, as those predicted by all models trying to explain the BAU, the most efficient way to look for sterile neutrinos at a lepton collider is to operate it as a $Z$ factory.

HNLs can be produced in $Z \to \nu \bar{\nu}$ decays with a SM neutrino kinematically mixing to an HNL, therefore producing $Z \to \nu N$. At very small couplings, the lifetime of the HNL becomes substantial, giving the possibility to suppress background arising from $W^*W^*$, $Z^*Z^*$ and $Z^*\gamma^*$ processes with the requirement of a displaced secondary vertex.

A method analogous to the one outlined in Section 2 was used to estimate the expected HNL yield at an hypothetical general purpose experiment at the FCC-ee. A simple detector with spherical symmetry and 100% reconstruction efficiency is assumed. All di-lepton final states $\ell^+\ell^-\nu$ are considered detectable. The expected HNL yield is computed as

$$n(HNL) = n(Z) \times \text{Br}(Z \to \nu\bar{\nu}) \times 2U_{tot}^2 \times \mathcal{P}_{vtx} \times \text{Br}(HNL \to \text{visible})$$

(12)

where $n(Z)$ is the integrated $Z$ yield and the factor 2 accounts for the fact that both neutrinos can mix to the HNL. The $\text{Br}(HNL \to \text{visible})$ factor is the total visible leptonic branching ratio of the HNL and includes, depending on the HNL mass, $e\nu$, $\mu\nu$, $e\mu\nu$, $e\tau\nu$, $\tau\tau\nu$, and $\mu\tau\nu$ final states. $\mathcal{P}_{vtx}$ is simply computed as:

$$\mathcal{P}_{vtx}(m_{HNL}, U_f^2) = \int_{r_{min}}^{r_{max}} e^{-l/\gamma_c \tau} \frac{1}{\gamma_c \tau} \, dl,$$

(13)

assuming that the detector has spherical symmetry. The integration boundaries $r_{min}$ and $r_{max}$ correspond to the minimum and maximum vertex displacement. If the accelerator operates at $\sqrt{s} = m_Z$, $Z$ bosons decay at rest and the HNL lifetime is boosted by a factor $\gamma = m_Z/2m_N + m_N/2m_Z$ [37].

Figure 13 compares the sensitivities of SHiP and of an hypothetical FCC-ee experiment in the parameter space of the $\nu$MSM, for two realistic FCC-ee configurations. The minimum and maximum displacements of the secondary vertex in FCC-ee depend on the characteristics of the tracking detectors of the hypothetical CMS-like experiment. For the first (second) FCC-ee configuration, an inner tracker with resolutions of 100 $\mu$m (1 mm) and an outer tracker with diameter of 1 m (5 m) have been considered. The production of $10^{12}$ ($10^{13}$) $Z$ bosons is assumed.

The SHiP experiment will be able to scan a large part of the parameter space below the $B$ meson mass. On the other hand, the results shown in Figure 13 show that heavier HNLs can be searched for at a future $Z$ factory. The synergy between SHiP and a future $Z$ factory would allow the exploration of most of the $\nu$MSM parameter space [37].
Figure 13: Physics reach in the HNL parameter space for SHiP and two realistic FCC-ee configurations for νMSM scenarios II (left) and IV (right). Previous searches are shown in green. Greyed-out areas represent the cosmological boundaries of the scenario [34, 37].
A HNL production and decay branching ratios

A.1 HNL production in two- and three-body meson decays

Let $H$ be a charm or beauty meson. Its leptonic decay into a sterile neutrino and a lepton of flavour $\alpha$ has the following branching ratio:

$$
\frac{d\text{Br} (H^+ \to l_\alpha^+ N)}{dE_N} = \frac{\tau_H}{8\pi} \frac{G_F^2 f_H^2 M_H M_N^2}{|V_H|^2 |U_\alpha|^2} \times \left( 1 - \frac{M_N^2}{M_H^2} + 2 \frac{M_l^2}{M_H^2} + \frac{M_l^2}{M_N^2} \left( 1 - \frac{M_l^2}{M_N^2} \right) \right) \times \sqrt{\left( 1 + \frac{M_N^2}{M_H^2} - \frac{M_l^2}{M_H^2} \right)^2 - 4 \frac{M_N^2}{M_H^2} \times \delta \left( E_N - \frac{M_H^2 - M_l^2 + M_N^2}{2M_H} \right)}
$$

(14)

where $\tau_H$ is the meson lifetime, $V_H$ is the CKM matrix element [38], and the meson decay constant $f_H$ can be taken from Table 7.

<table>
<thead>
<tr>
<th>$H$</th>
<th>$\pi^+$</th>
<th>$K^+$</th>
<th>$D^+$</th>
<th>$D_s$</th>
<th>$B^+$</th>
<th>$B_s$</th>
<th>$B_c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_H$ (MeV)</td>
<td>130</td>
<td>159.8</td>
<td>222.6</td>
<td>280.1</td>
<td>190</td>
<td>230</td>
<td>480</td>
</tr>
<tr>
<td>$V^\alpha_H$</td>
<td>$V_{ud}$</td>
<td>$V_{us}$</td>
<td>$V_{cd}$</td>
<td>$V_{cs}$</td>
<td>$V_{ub}$</td>
<td>$V_{as}$</td>
<td>$V_{cb}$</td>
</tr>
</tbody>
</table>

Table 7: Meson decay constants [38, 39] and CKM parameters involved in the decay diagram [17].

For the semileptonic decay into a sterile neutrino, a lepton of flavour $\alpha$, and a pseudoscalar meson $H'$, the branching ratio is:

$$
\frac{d\text{Br} (H \to H' l_\alpha N)}{dE_N} = \tau_H |U_\alpha|^2 |V_{HH'}|^2 G_F^2 \times \int dq^2 \left[ f_+^2(q^2) \times \left( q^2 \left( M_H^2 + M_{l_\alpha}^2 \right) - \left( M_N^2 - M_H^2 \right) \right) \right.
\left. + 2f_+(q^2)f_-(q^2) \left( M_N^2 \left( 2M_H^2 - 2M_{l_\alpha}^2 - 4E_N M_H - M_{l_\alpha}^2 + M_N^2 + q^2 \right) + M_{l_\alpha}^2 \left( 4E_N M_H + M_{l_\alpha}^2 - 2M_N^2 - q^2 \right) \right) \right.
\left. \times f_+^2(q^2) \left( 4E_N M_K + M_{l_\alpha}^2 - M_N^2 - q^2 \right) \left( 2M_K^2 - 2M_{l_\alpha}^2 - 4E_N M_K - M_{l_\alpha}^2 + M_N^2 + q^2 \right) \right.
\left. - \left( 2M_K^2 + 2M_{l_\alpha}^2 - q^2 \right) \left( q^2 - M_N^2 - M_{l_\alpha}^2 \right) \right]
$$

(15)

[17], where $q^2 = (p_l + p_N)^2$ is the momentum of the $l_\alpha N$ pair, $V_{HH'}$ is the corresponding CKM matrix element, and $f_+(q^2), f_-(q^2)$ are dimensionless hadronic form factors that can be found in literature [38].

Semileptonic decays with a vector boson in the final state are currently not considered, as they are subdominant with respect to the other channels.
A.2 HNL production in $\tau$ decays

Branching ratios of two-body decays of a $\tau$ lepton into a sterile neutrino and a scalar meson can be modeled with:

$$\frac{d\text{Br} (\tau \rightarrow HN)}{dE_N} = \frac{\tau_r |U_{\tau}|^2}{16\pi} G_F^2 |V_{H}|^2 f_H^2 M_\tau^3 \left[ \left( 1 - \frac{M_N^2}{M_\tau^2} \right)^2 - \frac{M_H^2}{M_\tau^2} \left( 1 + \frac{M_N^2}{M_\tau^2} \right) \right]$$

$$\times \sqrt{1 - \left( \frac{M_H - M_N}{M_\tau} \right)^2} \sqrt{1 - \left( \frac{M_H + M_N}{M_\tau} \right)^2} \times \delta \left( E_N - \frac{M_r^2 - M_H^2 + M_N^2}{2M_\tau} \right),$$

(16)

[17], where $\tau_r$ is the lifetime of the $\tau$, $V_H$ is the CKM matrix element, and the meson decay constant $f_H$ can be taken from Table 7.

Three-body $\tau$ leptonic decays can happen with the exchange of a $W^\pm$ boson in a time-like or space-like process. In the first case, there will be a $\nu_\tau$ in the final state; in the other case, the HNL will mix directly to the $\tau$ flavour. We use, respectively:

$$\frac{d\text{Br} (\tau \rightarrow \nu_\tau l_\alpha N)}{dE_N} = \frac{\tau_r |U_{\alpha}|^2}{2\pi^3} G_F^2 M_\tau^2 E_N \left( 1 + \frac{M_N^2 - M_l^2}{M_\tau^2} - \frac{2E_N}{M_\tau} \right)$$

$$\times \left( 1 - \frac{M_N^2}{M_\tau^2 + M_l^2 - 2E_N M_\tau} \right)^2 \sqrt{E_N^2 - M_N^2},$$

(17)

$$\frac{d\text{Br} (\tau \rightarrow \bar{\nu}_\alpha l_\alpha N)}{dE_N} = \frac{\tau_r |U_{\alpha}|^2}{4\pi^3} G_F^2 M_\tau^2 \left( 1 - \frac{M_N^2}{M_\tau^2 + M_l^2 - 2E_N M_\tau} \right)^2 \sqrt{E_N^2 - M_N^2}$$

$$\times \left[ (M_\tau - E_N) \left( 1 - \frac{M_l^2}{M_\tau^2 + M_N^2} \right) - \left( 1 - \frac{M_l^2}{M_\tau^2 + M_N^2} - 2E_N M_\tau \right) \right] \times \left( \frac{(M_\tau - E_N)^2}{M_\tau} + \frac{E_N^2 - M_N^2}{3M_\tau} \right).$$

(18)

[17].
A.3 HNL decay

Two-body decay modes of sterile neutrinos can be parametrized as:

\[ \Gamma (N \to \pi^0 \nu_\alpha) = \frac{|U_\alpha|^2}{32\pi} G_F^2 f_\pi^2 M_N^3 \left(1 - \frac{M_\pi^2}{M_N^2}\right)^2, \quad (19) \]

\[ \Gamma (N \to \pi^+ l^- \alpha) = \frac{|U_\alpha|^2}{16\pi} G_F^2 |V_{ud}|^2 f_\pi^2 M_N^3 \left[\left(1 - \frac{M_\pi^2}{M_N^2}\right)^2 - \frac{M_\pi^2}{M_N^2} \left(1 + \frac{M_\pi^2}{M_N^2}\right)\right] \times \sqrt{\left(1 - \frac{(M_\pi - M_l)^2}{M_N^2}\right) \left(1 - \frac{(M_\pi + M_l)^2}{M_N^2}\right)}, \quad (20) \]

\[ \Gamma (N \to \rho^0 \nu_\alpha) = \frac{|U_\alpha|^2}{16\pi} \frac{g_\rho^2}{M_\rho^2} G_F^2 M_N^3 \left(1 + 2 \frac{M_\rho^2}{M_N^2}\right) \left(1 - \frac{M_\rho^2}{M_N^2}\right)^2, \quad (22) \]

\[ \Gamma (N \to \rho^+ l^- \alpha) = \frac{|U_\alpha|^2}{8\pi} \frac{g_\rho^2}{M_\rho^2} G_F^2 |V_{ud}|^2 M_N^3 \left[\left(1 - \frac{M_\rho^2}{M_N^2}\right)^2 + \frac{M_\rho^2}{M_N^2} \left(1 + \frac{M_\rho^2 - 2M_\rho^2}{M_N^2}\right)\right] \times \sqrt{\left(1 - \frac{(M_\rho - M_l)^2}{M_N^2}\right) \left(1 - \frac{(M_\rho + M_l)^2}{M_N^2}\right)}, \quad (23) \]

\[ \times \sqrt{\left(1 - \frac{(M_\rho - M_l)^2}{M_N^2}\right) \left(1 - \frac{(M_\rho + M_l)^2}{M_N^2}\right)}, \quad (24) \]

where \( g_\rho = 0.102 \, \text{GeV}^2 \) [38].
For three-body decays into lepton flavours $\alpha$ and $\beta$ we have:

\[
\Gamma \left( N \to \sum_{\alpha,\beta} \nu_\alpha \bar{\nu}_\beta \nu_\beta \right) = \frac{G_F^2 M_N^5}{192\pi^3} \sum_{\alpha} |U_\alpha|^2 ,
\]

(25)

\[
\Gamma \left( N \to l^-_{\alpha \neq \beta} l^+_{\beta} \nu_\beta \right) = \frac{G_F^2 M_N^5}{192\pi^3} |U_\alpha|^2 \left( 1 - 8x_l^2 + 8x_l^6 - x_l^8 - 12x_l^4 \log x_l^2 \right)
\]

(26)

with \( x_l = \frac{\max[M_{l_\alpha}, M_{l_\beta}]}{M_N} \),

\[
\Gamma \left( N \to \nu_\alpha l^+_{l_\beta} l^-_{l_\beta} \right) = \frac{G_F^2 M_N^5}{192\pi^3} |U_\alpha|^2 \left[ (C_1(1 - \delta_{\alpha\beta}) + C_3\delta_{\alpha\beta}) \right.
\]

\[
\times \left( (1 - 14x_l^2 - 2x_l^4 - 12x_l^6) \sqrt{1 - 4x_l^2} + 12x_l^4 (x_l^4 - 1) L \right)
\]

+ 4 \left( C_2(1 - \delta_{\alpha\beta}) + C_4\delta_{\alpha\beta} \right)

\[
\times \left( x_l^2 (2 + 10x_l^2 - 12x_l^4) \sqrt{1 - 4x_l^2} + 6x_l^4 (1 - 2x_l^2 + 2x_l^4) L \right)
\]

(27)

with \( L = \log \left[ \frac{1 - 3x_l^2 - (1 - x_l^2) \sqrt{1 - 4x_l^2}}{x_l^2 \left( 1 + \sqrt{1 - 4x_l^2} \right)} \right] \), \( x_l \equiv \frac{M_l}{M_N} \), and

\[
C_1 = \frac{1}{4} \left( 1 - 4\sin^2 \theta_w + 8\sin^4 \theta_w \right) , \quad C_2 = \frac{1}{2} \sin^2 \theta_w (2\sin^2 \theta_w - 1) ,
\]

\[
C_3 = \frac{1}{4} \left( 1 + 4\sin^2 \theta_w + 8\sin^4 \theta_w \right) , \quad C_4 = \frac{1}{2} \sin^2 \theta_w (2\sin^2 \theta_w + 1) ,
\]

[17].
References


