A STUDY OF DISTORTION EFFECTS IN GEOMETRICAL RECONSTRUCTION OF EVENTS IN THE CERN 2 m HYDROGEN BUBBLE CHAMBER

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1. INTRODUCTION

During the early stages of bubble chamber physics, when bubble chambers were small in size (≈ 1/2 m) geometrical reconstruction of events was done using directly measured geometrical and optical parameters. In making this reconstruction lens distortions and tilt of the film plane with respect to the reference plane of the chamber were neglected. With the advent of big bubble chambers such as the British National Hydrogen Bubble Chamber (1.5 m) and the CERN 2 m Hydrogen Bubble Chamber one could no longer neglect these distortion effects (1,2). Further it was realized that the geometrical and optical parameters were difficult to measure directly with sufficient accuracy. To solve these problems bubble chamber calibration programs were developed (3) to fit geometrical and optical parameters and to obtain distortion coefficients from measurements on pictures of known fiducials. One of the most frustrating and time consuming work is to demonstrate the adequacy of the parameters and distortion coefficients obtained from these programs.

The present study is based on the work done with the CERN 2 m HBC. In making this investigation the program CONGEN (CONstant GENERator) (4) has been used. A short description of the relevant information regarding the chamber is given in section 2. The general principles of the program CONGEN are outlined in section 3. Section 4 contains a detailed discussion of distortion effects present in the chamber. Several approaches (past and present) used to correct for these distortion effects are described and the merits and demerits of these approaches have been discussed. The results from the present analysis are presented in section 5.

2. TECHNICAL DETAILS REGARDING THE CERN 2 m HBC

In order to simplify subsequent discussion a short description of the chamber is given here. A detailed report on the geometrical and optical parameters is available in the form of a CERN internal report (5).

Fig. 1 gives the horizontally symmetric section of the chamber showing the main body of the chamber together with the positions of the flashes and the cameras. There are two large windows of the chamber. The one facing the
flashes is called the flash window and the other facing the cameras is called the camera window. Each camera looks at the chamber through a set of three small glass windows (one in the cold tank and two in the vacuum tank). All windows are made of borosilicate crown glass (Schott BK 7). The recommended value of the refractive index is

\[ n = 1.5239 - 0.0003, \quad T = 26^\circ K \text{ and } \lambda = 4450 \text{ Å} \]

The dimensions of the large windows are: length \( \approx 2170 \text{ mm} \), width \( \approx 771 \text{ mm} \) and thickness \( \approx 171 \text{ mm} \) at \( 20^\circ \text{C} \). The total thickness of each set of small windows is \( \approx 75 \text{ mm} \) at \( 20^\circ \text{C} \) (6).

The fiducials are etched on three different window faces: the outer face of the camera window (plane 1), the inner face of the camera window (plane 2) and the inner face of the flash window (plane 3). Fig. 2 shows the arrangement of fiducials on these three planes. Some of these fiducials are not seen on the films. They are used for alignment only. There are a few others which are so faint in one or more views that they cannot be used for reconstruction.

The cameras are mounted on the camera plate shown in Fig. 1. Their arrangement on the camera plate is shown in Fig. 3. All positions are referred to a coordinate system given by the plane 2, called the reference plane. Each window plane with fiducial marks and the camera plate has its own coordinate system. Their positions relative to the reference plane are measured under working conditions of the chamber with an alignment telescope.

The fiducials of each plane have been measured with a precision of 50 \( \mu \text{m} \). It is not clear if this stated value is a good estimate of the measurement standard deviation or is just a reasonable upper limit (7). The inner faces of both large windows are parallel to each other within \( 0.2 \pm 0.1 \text{ mrad} \). The best value of the chamber depth is given as \( 504.5 \pm 0.4 \text{ mm} \) at \( 26^\circ \text{K} \).

The objectives are of the type: Schneider Symmar 5.6/180 working at f/32. The objectives have a pin-cushion distortion which is shown in Fig. 4 as a function of field angles. The film-backs are rigidly mounted on the camera plate. The films are sucked on to the film-backs by applying vacuum through a grid of fine holes in the film-backs (8).
There are a number of other interesting informations regarding the chamber which are summarized below.

a. The geometrical distance from the principal planes of the objectives (object side) to the reference plane is 2430 ± 5 mm.

b. The optic axis of the objectives are perpendicular to the reference plane within 0.5 mrad.

c. The film planes are parallel to the reference plane within 0.5 mrad.

d. The random light deflections for the large camera window due to internal stresses, lack of surface flatness and non-uniform refractive index are estimated to be smaller than 0.02 ± 0.01 mrad upto 30° angle of incidence. The camera window used during 1966-69 had a wedge angle of 0.06 ± 0.01 mrad which gives a uniform vertical deflection of 0.03 ± 0.01 mrad upwards.

e. The recommended value of the refractive index of liquid hydrogen under normal working conditions (expanded pressure = 2 kg/cm², T = 26°K) and for λ = 4450 Å is given as n = 1.1005 ± 0.0003.

f. The spectrum of the flash lamps is reduced to a narrow band by interference filter: Balzers Filtrasflex K 2 at 4450 Å (band width at 30 % of the peak transmission from 4300 Å to 4650 Å).

g. The integral thermal contraction of the glass BK 7 when cooled from room temperature to 25°K is: [L(293°K) - L(25°K)] / L(293°K) = (1.28 ± 0.04) · 10⁻³

h. The integral thermal contraction of stainless steel is:

\[
\frac{[L(293°K) - L(26°K)]}{L(293°K)} = (2.96 ± 0.04) \cdot 10^{-3}
\]

i. As the chamber has dark-field-illumination, the light refracted through the bubble forms a diffraction image. The size of the image depends only on the wave length of light and the relative aperture. For the normal operation (relative aperture f/32) the bubble diameter on the film is:

\[1.22 \cdot \lambda \cdot 32 = 17.4 \mu \text{m}\]. The bubble diameter in the chamber is \(\approx 500 \mu \text{m}\). Increasing the size of bubble in the chamber does not change image size but increases the darkness of the image. (Actually the bubble acts as two point sources which are not resolved.)
These are the relevant technical data about the chamber with their accuracies and tolerances. It is clear that many of these parameters are difficult to measure directly with sufficient accuracy: Therefore it is very desirable to parameterize the geometrical and optical configuration of the chamber and the camera system in such a way that one can optimize the sensitive parameters. It is the aim of a chamber calibration program like CONGEN to obtain the values of these parameters which allow reconstruction of tracks and space points in an unbiased and accurate way.

3. GENERAL PRINCIPLES OF CONGEN

The program uses known information regarding the chamber and measurements of corresponding points (fiducials, recognizable single bubbles etc.) in a set of pictures and performs a least mean squares fit to give optimized values of geometrical parameters and distortion coefficients with errors as well as a large amount of other output which can be used for trouble shooting and for evaluating the adequacy of the parameters.

The chi-squared which is minimized by the program is made up of three components:

a. Consistency between the measurements in the three views.

b. Deviation of the geometrical parameters from their initial values if assigned apriori errors.

c. Constraints placed on the spatial coordinates of certain of the measured points (fiducial marks).

The contribution to the chi-squared from (a) is formed as follows:

1. The apparent positions of the fiducial marks are calculated using current values of the geometrical parameters.

2. The distortion coefficients are fitted from the apparent positions of the fiducials and the measurements of the fiducials in each view of each picture separately. The distortion coefficients for each view are then averaged over all pictures and the average values are used subsequently.

3. All measurements are transformed to the reference plane by a 6 parameters linear transformation and are corrected for distortion using the
coordinates found in (2) to obtain \((x_m , y_m)\).

4. Use the three possible pairs of cameras, three sets of coordinates in space are calculated for each measured point \((x_i , y_i , z_i)\). These are then averaged to find \((x_a , y_a , z_a)\).

5. These average coordinates are reprojected to find the resulting apparent positions \((x_r , y_r)\).

6. The contribution to chi-squared is obtained from the discrepancy between the reformed measurements and the reprojected coordinates scaled by the measurement error \(\varepsilon_m\):

\[
X^2 = \sum \frac{\left[(x_r - x_m)^2 + (y_r - y_m)^2\right]/\varepsilon^2_m}{\varepsilon^2_m}
\]  (1)

Each point contributes three degrees of freedom. Contribution from source (b) is added to the chi-squared obtained above. The contribution from source (c) is obtained from the known apparent values of the fiducials \((x_A , y_A)\) and and the calculated values \((x_m , y_m)\) where the error in the known position of the fiducial \(x_A\) is taken into account.

\[
X^2 = \sum \frac{\left[(x_A - x_m)^2 + (y_A - y_m)^2\right]/\varepsilon^2_A + \varepsilon^2_m}{\varepsilon^2_m}
\]  (2)

In the above procedure an important assumption has been made. It is assumed that measurements of the positions of the fiducials in space are so much more trustworthy and unbiased than any other data that these positions are treated as fixed. The variations of the projected positions in various views is only due to changes in the values of the geometrical parameters. This is referred to as fixed fiducial approach.

The program handles three views of any chamber in which (apart from distortion corrections) the film planes are parallel to each other and to all glass surfaces between the camera lens and the sensitive liquid. The following thirteen parameters can be fitted by the program.

1. Thickness of the glass between the camera lens and the chamber liquid.
2. Index of refraction of the glass.
3. Index of refraction of the chamber liquid.
4. The relative orientation of the flash window relative to the camera window.

5. The coordinates of the three cameras. The camera coordinates are fitted in a cylindrical coordinate system. A circle is passed through the x and y coordinates of the three cameras. The center and the radius of the camera circle, the angle of the first camera relative to the x-axis, the angle between camera 1 and camera 2 and the angle between camera 1 and camera 3 are the six camera coordinates. These together with their z-coordinates make the nine coordinates. In this way it is very easy to take care of systematic errors in telescope readings. It is possible to select one or a few sensitive parameters which are likely to be poorly known.

The program is controlled by a series of run cards and data cards. Their choice is based on the long experience in handling such problems. The program fits various sets of distortion coefficients or keeps some fixed and fits others depending on the choice of run card. The chi-squared converges to within 0.1 of its final value (≈ 1/3 standard deviation) in one iteration.

4. DISTORTION EFFECTS IN THE CERN 2 m HBC

The main sources of distortion in the CERN 2 m HBC are the following:

(a) Radial lens distortion of the camera objectives.
(b) Tilt of the film planes with respect to the reference plane.
(c) Non-linear stretching of the film and/or incomplete sucking of the film on to the film-backs.

The first two effects are consistently the same from frame to frame and can be corrected for by fitting a set of distortion coefficients, but the third effect could vary from frame to frame and from camera to camera.

Besides the above mentioned sources there are two other sources of distortion.

(d) Turbulence and temperature gradient in the chamber liquid.
(e) Random light deflections due to internal stress, lack of surface flatness and non-uniform refractive index of the camera window and other small windows.
In view of the tolerances given in section 2, the magnitudes of these effects are expected to be small. The main systematic effects are from sources (a) and (b). A number of approaches have been used to correct for them.

4.1 RADIAL LENS DISTORTION OF THE CAMERA OBJECTIVES

The specification given by CERN for the camera objectives was that the radial lens distortion should be less than 1°/mm at 20° field angle (9). The computed and measured radial lens distortions are shown in Fig. 4. It is seen that the specifications are well satisfied. For some cameras the maximum field angle is ≈ 25° which means a displacement of 1.6 mm in the chamber system.

4.2 TILT OF THE FILM PLANES WITH RESPECT TO THE REFERENCE PLANE

It was mentioned in section 2 that the film planes are parallel to the reference plane within 0.5 mrad. This is considered to be a reasonable upper limit (10). In the extreme case this angle could be up to 2 mrad.

4.3 CORRECTION FOR DISTORTION EFFECTS

In earlier attempts to correct for these distortion effects the following set of 6 distortion coefficients was fitted to the measurements of fiducial marks (2).

\[
\begin{align*}
\begin{bmatrix} X' \\ Y' \end{bmatrix} &= \begin{bmatrix} 1 + S \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix}, \\
\end{align*}
\]

where

\[
S = \frac{\tilde{z}_1}{D} + \frac{\tilde{z}_2}{D^2} + \frac{\tilde{z}_3}{D^3} + \frac{\tilde{z}_4}{D^4} + \frac{\tilde{z}_5}{D^5} + \frac{\tilde{z}_6}{D^6}.
\]

In these expressions \(X\) and \(Y\) are uncorrected coordinates transformed to the optic axis of the camera, \(X'\) and \(Y'\) are the corrected coordinates also in the optic axis system and \(D\) is the distance of the cameras from the reference plane \((Z=0)\). The coefficients \(\tilde{z}_1\) and \(\tilde{z}_2\) give the film plane tilt and the coefficients \(\tilde{z}_3\), \(\tilde{z}_5\) and \(\tilde{z}_6\) give the radial lens distortion. The possibility of \(\tilde{z}_4\) and \(\tilde{z}_6\) being different and inclusion of \(\tilde{z}_3\) is done to account for non-rotational arrangements and other, not obvious, effects. Using these distortion coefficients a significant improvement was observed in the reconstruction of events. But there was appreciable bending of the reference plane in the \(Z\)-direction.
and fairly large residuals for reconstructed tracks (11). It was therefore tried to fit an additional (empirical) parameter which could take into account the remaining distortions (12) as follows.

\[
X' = \left[ 1 + S \right] X - X
\]

\[
Y' = \left[ 1 + S \right] Y + \frac{5}{7} X
\]

Only one additional coefficient was introduced as a) particles are mainly travelling in the x-direction which is four times as long as the y-direction and b) with too many coefficients the fit gets weaker.

However, it was observed that even after fitting 7 distortion coefficients for each camera the reconstruction was far from satisfactory. Two main defects were noticed.

1. The dip angle of the beam track was found to vary continuously along the length of the chamber.
2. For some cameras the magnitudes of distortion coefficients was unphysical. For example the values of \( \beta_2 \) corresponded to film plane tilt of 1 - 14 mrad. In view of the tolerance given in section 2 these values are quite unphysical. Similarly the value of \( \beta_4 \) and \( \beta_5 \) were 2.5 times the values expected from the curves shown in Fig. 4.

In view of these difficulties, which were quite serious for the present experiment (the beam momentum being 19 GeV/c), a fresh investigation of the whole problem was undertaken.

In programs like PYTHON(3) the entire fitting procedure is based on the reconstruction of fiducials. The fit is in no way influenced by the reconstruction within the chamber. In contrast to this CONGEN has the possibility of using measurements of corresponding points to fit geometrical parameters. These corresponding points could be interaction vertices, origin of delta electrons, decay vertices etc. In the present investigation recognizable single bubbles on beam tracks were measured as corresponding points. Compared to interaction vertices, origin of delta electrons etc., the single bubbles can be measured more accurately. If many such single bubbles (upto 40 bubbles in each picture).
selected over the entire length of the beam track, are measured then reconstruction of these single bubbles provides a useful and sensitive indication of the effect of fitting geometrical parameters on reconstruction in the active region of the chamber.

It may appear that measurement of recognizable single bubbles is liable to many errors and thus is not a very useful approach. But it was found after a little training that single bubbles could be recognized and measured satisfactorily. Fig. 5 shows a comparison of \( \chi^2 \)-distribution (obtained from the consistency between the measurements in the three views, see section 3) for the fiducials and the single bubbles. It is clear that the single bubbles are measured as accurately as the fiducials. There can be other objections to the use of single bubbles to judge the reconstruction. For example if one wants to use the distribution of single bubbles along the beam track to say something regarding the shape of the reconstructed beam track then the following objections can be raised.

1. The bubbles could move significantly due to turbulence and temperature gradient in the chamber liquid.
2. Variation in the refractive index of the chamber liquid could change relative positions of the bubbles.
3. The beam particle could undergo scattering, which should not be considered as distortion effect.

Some of these objections could be real, but it will be shown later that the distribution of single bubbles does represent the reconstruction within the chamber.

In the new investigation a drastic departure was undertaken. Instead of fitting 24 distortion coefficients for the three cameras, it was assumed that, in the first approximation, the film plane tilts relative to the reference plane are negligible. Only the radial lens distortion of the camera objectives was taken into account. For this the value computed by the manufacturers, shown in Fig. 4, was used. In spring 65, when the investigation was undertaken, the measured values shown in Fig. 4 were not available.
Thus using measurements of the fiducials and the recognizable single bubbles on beam tracks and by correcting for known radial lens distortion (it was assumed to be the same for all cameras) the geometrical parameters were fitted. This fit is referred to as the fit with known radial lens distortion. For comparison the geometrical parameters were also fitted where the distortion coefficients were used as given by (4). This fit is referred to as the fit with seven distortion coefficients.

4.4 COMPARISON OF THE TWO FITTING PROCEDURES

Fig. 6 shows the reconstruction in X-Z plane for the single bubbles on a beam track and the fiducials (which are on the reference plane) in the same picture. No distortion correction has been made in this case. The fiducials are expected to lie at \( Z = 0 \). But a strong variation is observed which is due to pin-cushion distortion of the objectives. A similar variation is observed in the distribution of single bubbles along the beam track. It could be argued that the variation is due to scattering of the beam track. However, the same behavior i.e. bending on the same side at the same location is observed for a number of beam tracks in other pictures, where the fiducials also show the same feature. This rules out the scattering argument. The other argument based on turbulence, temperature gradient could not explain the observed effect, more so the similarity of the two forms. Thus the variation in the distribution of single bubbles from a straight line is mainly due to distortion effects (13).

The seven distortion coefficient fit was made on the same data. The reconstruction in this case is shown in Fig. 7. A radical improvement in the reconstruction of fiducials is observed. They now lie very close to the expected positions. This is quite natural as the distortion coefficients are fitted to the fiducials. However, the reconstruction of single bubbles is poor, specially at the two ends of the chamber. This is further substantiated by the comparison shown in Fig. 5. The chi-squared from consistency of measurements (see section 3) is shown for the fit with no distortion coefficients vs. the corresponding values for the fit with seven distortion coefficients. As described in section 3 the smallness of this \( \chi^2 \) implies that the three sets of space coordinates obtained by using the three possible pairs of cameras lie close to each
other i.e. they are consistent with each other. This does not say anything as to where they lie in relation to their expected position.

Fig. 8 shows that the effect of fitting seven distortion coefficients is to improve this $\chi^2$ for the fiducials, but for the single bubbles the $\chi^2$ is blown up. This means that after fitting the seven distortion coefficients the space reconstruction within the chamber obtained by using the three pairs of cameras is less consistent with each other. Thus it was suspected that the net effect of fitting the seven distortion coefficients was to improve the reconstruction of fiducials at the cost of the reconstruction within the chamber. This may partly be due to the reason that the coefficients are partly correlated and also due to the fact that the coefficients related to distortion along Y-axis are poorly determined as the chamber is four times shorter in width compared to the length.

On the same data a fit with known radial lens distortion was obtained. The reconstruction in this case is shown in Fig. 9. Compared to Fig. 8 there is a significant improvement both for the fiducials as well as for the single bubbles. However, there is a residual effect for both which could be removed by scaling the radial lens distortion (keeping the same form). A comparison similar to Fig. 8 showed that there was no similar increase in the $\chi^2$ from consistency both for fiducials as well as for the single bubbles. This is shown in Fig. 10.

This indicates that one can obtain a somewhat better reconstruction within the chamber liquid by using only known radial lens distortion. To get a title for THRESH the fiducial marks and recognizable single bubbles on a beam track were measured in a number of pictures. CONGEN can fit geometrical parameters using input data from 5 pictures. In every picture 14 fiducials and about 40 single bubbles were measured. A simultaneous fit of geometrical parameters giving the camera positions was made using input data from 5 pictures. The measurements were corrected for known radial lens distortion.

The THRESH title obtained in this way (called the new title) was compared with the title obtained from CERN with 7 distortion coefficients fit. Four prong events in a roll (roll no. 755) were processed with these two titles where everything else was kept the same. Fig. 11 shows a comparison of residuals for
tracks obtained from the two runs. It is seen that the residuals obtained using the new title are more often smaller, thus substantiating the argument that the new title gave a better reconstruction. A similar comparison was made in GRIND by processing those events which were reconstructed in THRESH with the two titles. Fig. 12 shows a comparison of $\chi^2$-probability for 4 constraints fits in the two cases. Once again it is observed that the events when reconstructed using the new title gave better $\chi^2$-probability. The above discussion can be summarized as follows:

1. The reconstruction within the chamber obtained with the new title was in general better than the one obtained by using the title with 7 distortion coefficients.

2. The title did not include unphysical distortion coefficients. The only distortion correction used was the computed radial lens distortion. The measured values were not available in Spring '68.

3. The reconstruction of beam tracks shows that the possible variation of dip angle of the beam tracks along the length of the chamber is $\lesssim 1$ mrad. On a single track such a variation could be due to Coulomb scattering. As noted before a systematic study of this effect requires measurements of a large number of tracks.

It must be noted that in obtaining this title it has been assumed that there are no significant tilts of the film planes relative to the reference plane. This is only a simplifying assumption and need not be true. However, the title in its simple form gave reasonably good reconstruction and was therefore used in the major part of the four prong experiment.
5. RESULTS FROM THE PRESENT ANALYSIS

During 1969 about 400 000 pictures were taken with the 19 GeV/c proton beam to make a detailed study of strange particle production in proton-proton interactions. To obtain THRESH title for these films a new investigation of distortion effects was once again undertaken. The aim of this investigation was to extend the earlier investigation with a view to study the following aspects:

(a) Need of fitting film plane tilt coefficients
(b) The so-called roll to roll and frame to frame variation of distortion
(c) Need to take into account any other distortion effect.

5.1 PROCEDURE FOR OBTAINING THE DATA

For this purpose a set of 11 fiducials on the front glass was measured in about 100 pictures which were spread over 3 rolls. Fig. 13 shows the positions of these fiducials. Each view in a picture was measured twice and a picture was considered acceptable only if the two measurements of each fiducial in each view agreed within 3 µm on the film. The pictures were measured on EXETRA machines, manufactured by SOPELEM, Paris. For the accepted pictures a mean value of the two measurements was taken.

To optimize geometrical parameters it is preferable to have fiducials on more than one plane. However, the fiducials on the flash window (plane 3) were not used. The main reasons for this are the following.

1. The relative positions of the two windows are poorly known.

2. Due to turbulence in liquid hydrogen and variation of refractive index of liquid hydrogen these fiducials are less well defined as compared to the fiducials on the reference plane. This is particularly true for the top fiducials where shifts \( \geq 200 \) µm have been observed \(^{14}\). This is due to streamers of cold liquid coming down from the heat exchanger.

Measurements of single bubbles on beam tracks as corresponding points provide the information at a plane other than the reference plane. Their space reconstruction is very sensitive to the camera positions. The \( \chi^2 \) from consistency (see section 3) blows up for small changes in camera positions.
5.2 RECONSTRUCTION WITH KNOWN RADIAL LENS DISTORTION

Using initial values of geometrical parameters and after correcting for the computed radial lens distortion the apparent values of the fiducials were computed. A THRESH type linear transformation was used to obtain measured apparent positions:

\[
X_m = \alpha_1 X + \alpha_2 Y + \alpha_3 \\
Y_m = \alpha_4 X + \alpha_4 Y + \alpha_4
\]

where \( X \) and \( Y \) are the measured values from SOM. To check the consistency of these measured apparent positions a \( \chi^2 \) was computed as follows:

\[
\chi^2 = \frac{(X_m - X_a)^2 + (Y_m - Y_a)^2}{\varepsilon_m^2 + \varepsilon_a^2}
\]

where \( X_a \) and \( Y_a \) are the computed apparent positions, \( \varepsilon_m \) is the measuring error (this error is taken as 4 \( \mu m \) in the film plane or 50 \( \mu m \) in the chamber coordinate system) and \( \varepsilon_a \) is the error in the known positions of the fiducials (this error is given as 50 \( \mu m \) in Ref. 5). As there are 11 fiducials measured in each view and six parameters are obtained to give the transformation (5) the \( \chi^2 \) expected for each picture is \( 3(11 \cdot 2 - 6) = 48 \). Fig. 14 shows a plot of \( \sum \chi^2 \) for all the frames.

A \( \chi^2 \)-distribution with 48 degrees of freedom is near gaussian with a \( \sigma \) of 9.3. The observed distribution peaks at \( \approx 65 \) and has a tail on the higher side. It may be argued that the shift in the peak is due to underestimating the errors. However, the assigned errors are somewhat higher already (for example the measurement error is taken as 50 \( \mu m \) which from other investigations appears to be too high). There are two rolls (roll no. 493 and 513) in which all measurements give too high \( \chi^2 \). It is likely that for these rolls some view has bad sucking down to the film-backs. For roll no. 513 there is indication that view 1 and 3 are bad whereas view 2 is normal.

Thus the main conclusion to be drawn from Fig. 14 is that there is some other
source of systematic error which has not been taken into account. One such source could be the tilts of the film planes for the three cameras relative to the reference plane.

5.3 INTRODUCTION OF FILM PLANE TILT COEFFICIENTS

As mentioned in section 2 these planes are parallel to the reference plane within 0.5 mr. To correct for this type of distortion effect the following two coefficients were fitted.

$$ (X', Y') = \left[ 1 + \beta_1 \frac{X}{D} + \beta_2 \frac{Y}{D} \right] (X, Y) $$

(7)

In fitting these coefficients a constant radial lens distortion was introduced whose value was taken as same for the three lenses. The coefficients were averaged over 5 frames and this average value was used in computing $\chi^2$ corresponding to (6). In this case the expected $\chi^2$ is $5(3 \cdot 2 - 6) - 3 \cdot 2 = 234 \pm 21.6$. A distribution of the observed $\chi^2$ is shown in Fig. 15. It is seen that this $\chi^2$-distribution is consistent with the estimated errors. Thus it is felt that the radial lens distortion and the film plane tilt are the two sources of distortion which can account for the observed distortion.

Fig. 16 shows a distribution of the values of $\beta_1$ and $\beta_2$ for each camera. The spread in the values of $\beta_2$ is much larger than the spread in the values of $\beta_1$. This can be understood in terms of the fact that in X-direction the fiducials are distributed over longer distances than in Y-direction. The spread is consistent with the error calculated from least mean square fit. It is seen that the value of $\beta_1$ is significant in camera I and the value $\beta_2$ is significant for camera III. The other $\beta$'s are much smaller. The value of $\beta_1$ in camera I corresponds to a tilt of about 1.1 mrad and the value of $\beta_2$ in camera III corresponds to a tilt of about 2.5 mrad. Taking the value of 0.5 mrad given in section 2, as the error in parallelism these two tilts are quite significant. It must be remarked that the value of $\beta_2$ in camera III is the effective film plane tilt which does not imply that the physical plane is tilted by this amount. But the combined effect of physical tilt together with the effect of bad sucking could account for the fitted value of $\beta_2(III)$. 
Consistency check on the spread of these coefficients was made by measuring frame a number of times and then computing the spread. The spread was to be in agreement with the values obtained from least mean square fit. Shows that for most of the frames there is no observable variation in the rotation coefficients. However, in rolls 493 and 513 the values are much higher.

EFFECT OF FITTING LENS DISTORTION COEFFICIENTS

In above discussion the radial lens distortion for the cameras was kept and was taken same for the three cameras. It was thought worthwhile to study the effect of fitting this along with the other two coefficients. For this case the following three coefficients were fitted.

\[ \frac{X'}{Y'} = (1 + \beta_1 \frac{X}{D} + \beta_2 \frac{Y}{D} + \beta_3 \frac{X^2 + Y^2}{D^2}) \frac{X}{Y} \]  

17 shows a distribution of the values of \( \beta_3 \) for the three cameras. The values indicate the values of computed and measured radial lens distortion for these camera lenses. The mean values of \( \beta_3 \) for camera I and camera III are in agreement with the computed value whereas the value of \( \beta_3 \) for camera II is much larger than the computed or measured value. It was mentioned in section 2 that the results for different azimuthal angles and different objectives are equal within errors. However, a reexamination of the measurement report (13) shows that the objective for camera II does have significantly higher distortion coefficient. Fig. 18 shows a comparison of the computed values with the values obtained from this report for field angle of 20°. Agreement is reasonably good. This shows that a better reconstruction could be expected if the radial lens distortion was allowed to be different for different cameras. The expected \( \chi^2 \) in this case is 5.3(11 - 2 - 6) - 3 - 3 = 81 = 21.5. In place of the expected reduction in \( \chi^2 \) by 3 the observed reductions are about 35 which is very significant. This implies that the radial lens distortion could be taken different.
5.5 OPTIMIZING GEOMETRICAL PARAMETERS

In order to optimize geometrical parameters 5 frames were measured in roll 482 where the measurements were found to be representative of the sample. Besides the 11 fiducials about 40 corresponding points along a beam track were measured in each frame.

Eleven geometrical parameters were fitted together (9 for the camera positions, refractive index of the glass plate, refractive index for liquid hydrogen). The thickness of glass plate was not fitted as it is strongly correlated to the refractive index of the glass plate. Further the angle between front and back windows was not varied as the back fiducials were not measured. The following two types of titles were prepared.

1. Using a constant value for the lens distortion coefficient, $\beta_3$ (as obtained from the measurements of these lenses), the coefficients $\beta_1$ and $\beta_2$ were fitted. This title is referred to as 2 distortion coefficients fit.

2. Fitting all the three $\beta$ coefficients. This title is referred to as 3 distortion coefficients fit.

In order to judge which of these two titles should be used the following comparisons were made.

(a) Reconstruction in the X-Y plane.

From the reconstruction of corresponding points (single bubbles) on the beam tracks several independent and nearly independent sets of 3 points were chosen. Nearly independent set means that one point was common. Using these sets of points the radius of beam track was calculated for different X-locations in the chamber. The mid-points of these track segments were grouped in 10 cm bins and the average values were calculated. The error assigned to the radius was as in GRIND with a $\sigma$ of 40 $\mu$m. Fig. 19 shows the variation of the radius of the beam track segments with the location of the mid-points in the chamber. The data has been taken from the 2 distortion coefficient fit. The dotted line indicates the expected radius. 36.71 m. for 18.1 GeV/c protons (the field value has been taken as 17.345 kG). The effect of the field variation in the
chamber is to decrease the radius at the ends by 0.3 percent which means about a mm in the Figure. The effect of energy loss along a length 1 m is only about 25 MeV. This is again less than a mm in the Figure. The points are distributed on both sides of this expected value. There is a weak trend that the radius is lower in the middle of the chamber. However on the present statistics the trend is not significant. Similar plot for the 3 distortion coefficient fit is shown in Fig. 20. Here also nearly the same behaviour is observed. Considering the errors the two titles are equivalent in this respect.

(b) Reconstruction in the X-Z plane.

Fig. 21 shows the reconstruction of beam track and the fiducials in the X-Z plane for one of the frames (no. 4820250). For the beam track there is no significant variation of dip angle. The observed variation is about 1 mr. This may very well be a real variation due to Coulomb scattering. Reconstruction of fiducials is shown in the lower part of the Figure. The reconstruction of the reference plane is flat within errors, taken as 200 μm. Here also there is nothing to prefer between the two titles. Similar behaviour is observed in the other 4 frames.

In each X-location about 5 single bubbles were measured as close to each other as possible. So the variation in Z is a reflection of the measuring error. To get an estimate of this error each group of points was symmetrically super-imposed on each other. The resulting distribution is shown in Fig. 22. This distribution has a σ of about 150 μm, which corresponds to a σ of ~40 μm in the X-Y plane. Which is in agreement with view mentioned in section 5.2 that the estimated x₁ of 50 μm is some what higher.

These two comparisons were made to see if there is any significant shape distortion effect. However, to investigate the problem like variation of the beam momentum along the chamber a separate investigation is necessary.

(c) Reconstruction of individual points and change of geometrical parameters.

The goodness of the reconstruction of individual points is reflected in the χ² for the fit. The total χ² consists of the contributions from the constraints on
the fiducials, the single bubbles and the geometrical parameters. Table I gives
the expected $\chi^2$, the initial $\chi^2$ and the final $\chi^2$ after the fit for the two titles.
For the final $\chi^2$, each of the contribution is also given separately. It is seen
that the final $\chi^2$ for the 3 distortion coefficient title is significantly lower than
the corresponding value for the 2 distortion coefficient title. Further each
individual contribution from the three sources is lower. From these compari-
sions it is concluded that the reconstruction of fiducials as well as of the single
bubbles within the chamber is better for the title with 3 distortion coefficient
fit than for the title with 2 distortion coefficient fit and in doing so the geomet-
rical parameters are moved much less from their initial values. Thus it is
recommended that the title with 3 distortion coefficient fit be used in the anal-
ysis.

The main results of this investigation can be summarized as follows

1. In view of the limited number of the fiducials and their locations on the
windows it is not realistic to fit too many distortion coefficients.

2. In optimizing the geometrical parameters account should be taken of the
reconstruction within the chamber. The goodness of this reconstruction
should be the main concern of any chamber calibration method.

3. The main distortion effects are the radial lens distortion and the film
plane tilts. There is no evidence for non-radial lens distortion.

4. The three objectives do not have equal distortion. The objective for
camera II has larger distortion.

5. The effects like variation of measured beam dip along the length of the
chamber and variation of measured beam momentum in different parts of
the chamber are not significant in the present investigation. This indi-
cates that the residual distortion, the distortion remaining after correct-
ing for the two main effects, is not very large.

6. There is some evidence for bad rolls or part of bad rolls. Suitable tests
are planned to be included in TRESH to flag such frames or rolls and to
terminate further handling of these events.
ACKNOWLEDGEMENTS

I express my sincere thanks to Prof. Gösta Ekspong for his advice, constant encouragement and valuable criticism during the course of this work. He suggested several new approaches and improvements where none seemed to be possible. I am indebted to Prof. R. Plano for initiating me in this field. This work was started by him during his stay at the Institute of Physics, Stockholms University. I am also thankful to Dr. G. Kellner, Dr. J. Zoll, Dr. H.P. Reinhard and Dr. T. Ball for several informal discussions and useful information. Thanks are due to scanning staff in Stockholm for making careful measurements. This work has been done under the auspices of Swedish Atomic Research Council.
REFERENCES AND NOTES


4. CONGEN is written by Prof. R.J. Plano, Rutgers State University, New Brunswick, NJ 08903, USA. The present study was initiated by Prof. Plano during his visit to the Institute of Physics, University of Stockholm, Stockholm.

5. Optical Data of the CERN 2 m Bubble Chamber, CERN/D.Ph.II/200 69-1 (Unpublished).

6. In Spring 1970 the two large windows (the flash window and the camera window) have been replaced by new windows of a different type of glass with $n=1.5324$. The small windows will also be replaced next year. The new windows are to be of much higher quality, with regards to flatness etc., than the big windows.

7. The fiducials on these windows have been remeasured now. The errors are expected to have $\sigma=20\,\mu m$.

8. In March 1969 a new camera plate was installed, which is more rigid. The film-backs have also been changed. They are made by fusing together small quartz tubes (diameter $\approx 1\,mm$). This has the following advantages compared to the usual metal film-backs with drilled holes.
   a. The film-backs can be polished flat with optical precision.
   b. The film-backs stay flat.
   c. The total area of the holes is much higher, giving a faster and safer sucking down.

   However, the results presented in this report are based on metal film-backs with drilled holes.

9. Private information from Dr. H.F. Reinhard, T.C. Division, CERN.
10. Private information from Dr. T. Ball, CERN 2 m HBC.

11. For definition of residuals see THRESH MANUAL, T.C. Program Library, CERN (Unpublished).

12. Private communication from Dr. G. Kellner. The coefficient was proposed by Dr. D. Drijard.

13. When these large distortion effects are corrected, any remaining distortion effect may be due to other causes such as turbulence, variation of refractive index etc. To study if these indications are real distortions or only scattering of that track it is necessary to measure a large number of tracks. This is a suitable subject for future investigation.

14. Private information from Prof. G. Ekspong, presently at D.Ph.II, CERN.

15. Certificate of measurement of distortion on the camera objectives by Physikalisch-Technische Bundesanstalt. A copy of this report has been supplied to us by Mr. M. Dykes, CERN 2 m HBC.
<table>
<thead>
<tr>
<th>Title</th>
<th>$\chi^2$</th>
<th>$\chi^2$ (expected)</th>
<th>$\chi^2$ (initial)</th>
<th>$\chi^2$ (final)</th>
<th>$\chi^2$ (total)</th>
<th>$\chi^2$ (fiducials)</th>
<th>$\chi^2$ (bubbles)</th>
<th>$\chi^2$ (geom.param.)</th>
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FIGURE CAPTIONS

Fig. 1 The horizontally symmetric section of the chamber showing the main body of the chamber together with the positions of the flashes and the cameras.

Fig. 2 The arrangement of fiducials on the outer face of the camera window (plane 1), the inner face of the camera window (plane 2) and the inner face of the flash window (plane 3).

Fig. 3 The arrangement of the cameras on the camera plate and the positions for telescope readings (These are used for finding the positions of the camera plate and the flash window relative to the reference plane).

Fig. 4 The computed and the measures radial lens distortion for the camera objectives as a function of field angles (from Ref. 5).

Fig. 5 Comparison of the $X^2$-distributions (obtained from the consistency between the measurements in the three views, see section 3) for the fiducials and the single bubbles.

Fig. 6 The reconstruction in X-Z plane for the single bubbles on a beam track and the fiducials (on the reference plane) in the same picture. No correction has been made for the radial lens distortion.

Fig. 7 The reconstruction in X-Z plane for the single bubbles on a beam track and the fiducials (on the reference plane) in the same picture as in Fig. 6. A seven distortion coefficient fit has been made.

Fig. 8 Scatter plot of the $X^2$ from consistency of measurements in the three views (see section 3) for the fit with no distortion coefficients versus the corresponding values for the fit with seven distortion coefficients.
Fig. 9  Reconstruction in X-Z plane for the single bubbles on a beam track and the fiducials (on the reference plane) in the same picture as in Fig. 6. The known radial lens distortion correction has been made.

Fig. 10  Scatter plot of the $\chi^2$ from consistency of measurements in the three views (see section 3) for the fit with no distortion coefficient versus the corresponding values for the fit with known radial lens distortion.

Fig. 11  Scatter plot of the residuals for tracks obtained in THRESH with the new title (using only known radial lens distortions) versus the corresponding values obtained with the CERN title (using seven distortion coefficients fit). Every thing else was kept the same in the two runs.

Fig. 12  Scatter plot of the $\chi^2$-probability for 4-constraints fits in a sample of four-prong events reconstructed in THRESH with the new title (using only known radial lens distortions) versus the corresponding values obtained with the reconstruction using the CERN title (seven distortion coefficients fit). Every thing else was kept the same in the two runs.

Fig. 13  The arrangement of the set of 11 fiducials on the reference plane used in the investigation described in section 5.

Fig. 14  The $\chi^2$-distribution for all the pictures obtained by correcting for the known radial lens distortion. The number of degrees of freedom is 48.

Fig. 15  The $\chi^2$-distribution obtained after correcting for the known radial lens distortion and then fitting film plane tilt coefficients. The coefficients were averaged over 5 pictures and this average value was used in computing the $\chi^2$, which is summed over the five frames.

Fig. 16  Distributions of the values of $\xi_1$ and $\xi_2$ for each of the three cameras. The spread in the values of $\xi_2$ is much larger than the spread in the values of $\xi_1$. This is due to the fact that in X-direction the fiducials are distributed over longer distances than in the Y-direction. Further the spreads are consistent with the errors calculated from least mean square fit.
Fig. 17 Distributions of the values of $\xi_3$ for the three cameras. The arrows indicate the values of the computed and the measured radial lens distortion as given in reference 5.

Fig. 18 The comparison of the fitted values of $\xi_3$ with the measured values obtained from reference 15 for field angle of $20^\circ$.

Fig. 19 The variation of the radius of the beam track segments with the location of the mid-points in the chamber. The error assigned to the radius was as in GRIND with a $f_o$ of $40 \mu m$. The data has been taken from the 2 distortion coefficients fit.

Fig. 20 The variation of the radius of the beam track segments with the location of the mid-points in the chamber. The error assigned to the radius was as in GRIND with a $f_o$ of $40 \mu m$. The data has been taken from the 3 distortion coefficients fit.

Fig. 21 The reconstruction in X-Z plane for the single bubbles on a beam track and the fiducials (on the reference plane) in the same picture.

Fig. 22 The distribution obtained by superimposing symmetrically the group of single bubbles measured at each X-location. The single bubbles were measured as close as possible at every X-location. So the spread in Z is due to measuring errors.
Fig. 2
Fig. 3

○ Camera objectives
※ Telescope positions
Fig. 4

Lens distortion $\frac{\Delta r}{r}$ at $f/32$ and $m=13.5$
Comparison of $\chi^2_c$ (consistency between measurements in the three views) for fiducials and single bubbles

Fig. 5
Frame no. 7600700

single bubbles on beam track

fiducials
Frame no. 7600700

single bubbles on beam track

fiducials
Fig. 8
Frame no. 7600700

single bubbles on beam track

Fig. 9
\( \chi^2 \) (from consistency, with known distortion coefficients)

\( \chi^2 \) (from consistency, without fitting distortion coefficients)

Fig. 10
Fig. 11
Comparison of $\chi^2$-probability for 4c fits

(role no. 715)

Fig. 12
No. of pictures

$\chi^2$ expected

Roll no 513

Roll no 493

Fig. 14
Fig. 16
Fig. 17
1. Fitted lens distortion
2. Measured lens distortion
Radius of beam track (m)

-60 -40 -20 0 20 40 60 80

X (cms)

R (19.1 GeV/c)

Fig. 19
Fig. 20

3 Distortion parameter Title

Radius of beamtracks (m)

R(19.1 GeV/c)
Frame no. 4820250

-2 Distortion parameter title
-3 Distortion parameter title

Reconstruction of corresponding points

Reconstruction of fiducials

Fig. 21
Fig. 2.2

Spread in Z direction (μm)

No. of points

-200 0 200

Fig. 22