Clockwork inflation

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A B S T R A C T

We investigate the recently proposed clockwork mechanism delivering light degrees of freedom with suppressed interactions and show, with various examples, that it can be efficiently implemented in inflationary scenarios to generate flat inflaton potentials and small density perturbations without fine-tunings. We also study the clockwork graviton in de Sitter and, interestingly, we find that the corresponding clockwork charge is site-dependent. As a consequence, the amount of tensor modes is generically suppressed with respect to the standard cases where the clockwork set-up is not adopted. This point can be made a virtue in resurrecting models of inflation which were supposed to be ruled out because of the excessive amount of tensor modes from inflation.

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1. Introduction

The clockwork mechanism [1,2] allows to explain the presence of light degrees of freedom with highly suppressed interactions in theories where there are no small parameters to start with. A general theory of the clockwork mechanism valid for scalars, fermions, gauge bosons, and gravitons has been recently proposed in Ref. [3]. Let us briefly show how it operates for scalars and consider a theory endowed with a global $U(1)^{N+1}$ spontaneously broken at the scale $f$. The degrees of freedom at energies smaller than $f$ are the $N + 1$ Goldstone bosons $\pi_i$

$$U_i(x) = e^{i N_i(x)/f}, \quad i = 0, \ldots, N.$$  \hfill (1.1)

The $\pi_i$ fields transform by a phase under the corresponding Abelian factor $U(1)$. Suppose now that the low-energy description of the theory is described by the Lagrangian

$$\mathcal{L} = \frac{f^2}{2} N \sum_{i=0}^{N} \bar{\psi}_i U_i^\dagger \gamma^\mu U_i \psi + \frac{m^2 f^2}{2} \sum_{i=0}^{N-1} (U_i^\dagger U_{i+1}) + \text{h.c.}$$

$$= \frac{1}{2} \sum_{i=0}^{N} (\partial \pi_i)^2 + \frac{m^2}{2} \sum_{i=0}^{N-1} (\pi_i - q \pi_{i+1})^2 + \mathcal{O}(\pi^4)$$

where the presence of the explicit mass terms breaks softly the symmetry $U(1)^{N+1}$ down to a single $U(1)$. The square mass matrix is given by

$$M_\pi^2 = m^2 \begin{pmatrix} 1 & -q & 0 & \cdots & 0 \\ -q & 1 + q^2 & -q & \cdots & 0 \\ 0 & -q & 1 + q^2 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 + q^2 - q \\ 0 & 0 & 0 & \cdots & -q \\ 0 & 0 & 0 & \cdots & q^2 \end{pmatrix}.$$  \hfill (1.2)

In the mass eigenstate basis $\phi_i$ ($i = 0, \ldots, N$)

$$\pi = O \varphi, \quad O^T M_\pi^2 O = \text{diag}(m_{\phi_0}^2, \ldots, m_{\phi_N}^2),$$  \hfill (1.4)

where $O$ is a real orthogonal matrix, the eigenvalues are given by

$$m_{\phi_0}^2 = 0, \quad m_{\phi_k}^2 = \lambda_k m^2,$$

$$\lambda_k = q^2 + 1 - 2q \cos \frac{k\pi}{N+1}, \quad k = 1, \ldots, N.$$  \hfill (1.5)

The elements of the rotation matrix $O$ are given by

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\[ O_{i0} = \frac{\mathcal{N}_0}{q^4}, \quad O_{ik} = \frac{\mathcal{N}_k}{q^4} \left[ q \sin \frac{ik\pi}{N+1} - \sin \frac{(i+1)k\pi}{N+1} \right]. \]

and

\[ \mathcal{N}_0 = \sqrt[2]{\frac{q^2 - 1}{q^2 - q^2 - 2\pi}}, \quad \mathcal{N}_k = \sqrt[2]{\frac{2}{(N+1)\lambda_k}}. \]

The key point of the clockwork mechanism is now that the massless eigenstate \( \phi_0 \) is coupled to the rest of the fields in the theory with a coupling which is suppressed by \( O_{i0} \sim q^{-4} \). In particular, if the rest of the degrees of freedom in the matter sector couples only to the \( N \)-th pion \( \pi_N \), the state \( \phi_0 \) couples to them with a suppressed coupling scaling like \( q^{-N} \). If \( N \) is large and \( q > 1 \), then the coupling is efficiently suppressed.

In the case in which the number of copies is very large, it has been pointed out that there exists also a five-dimensional continuum limit of the clockwork mechanism [3]. It is achieved by introducing a dilaton field \( S \) in a five-dimensional braneworld with the fifth dimension compactified on \( S_1/\mathbb{Z}_2 \). The corresponding action reads

\[
S = \int d^4x \sqrt{-g} \left[ \frac{M_5^2}{2} \left( R - \frac{1}{3} \partial_M S \partial^M S + 4k^2 e^{-\frac{S}{2}} \right) - \frac{e^{-\frac{S}{2}}}{\sqrt{g}} \left( \delta(y) V_0 + \delta(y - \pi R) V_\pi \right) \right],
\]  

where \( k \) characterizes the negative vacuum energy in the bulk, \( R \) is the radius of the fifth dimension, \( M_5 \) is the fundamental scale in the bulk, and \( V_0 \) and \( V_\pi \) are tensions on the brane satisfying the relation \( V_0 = -V_\pi = -4kM_5^2 \). The corresponding metric is found to be

\[
ds^2 = e^{\frac{S}{2}} \left( dx^2 + \eta_{\mu \nu} dx^\mu dx^\nu \right),
\]  

with \( \eta_{\mu \nu} \) the flat Minkowski metric. In this picture hierarchies are produced on the \( y = \pi R \) brane and the discrete suppression factor \( q^{-N} \) is replaced in the continuum with \( e^{-k \pi R} \).

The goal of this note is to show that the clockwork mechanism can be adopted in inflationary theories to efficiently generate flat inflaton potentials sustaining a de Sitter phase as well as small masses and couplings to match the small observed scalar perturbations. We will present in section 2 various examples of such ability from the four-dimensional discrete perspective. In section 3 we will study the phenomenon of inflation from the five-dimensional continuum perspective and show that the amount of clockwork producing small masses/couplings depends on the Hubble rate during inflation. Maybe more interestingly, in section 4 we show that, within the clockwork set-up, the clockwork charges of the gravitons are site-dependent and the amount of tensor modes generated during inflation is suppressed with respect to the standard scenario due to the fact that tensor modes are intrinsically bulk degrees of freedom. Section 5 contains our conclusions.

2. Clockwork inflation: the four-dimensional discrete perspective

In this section we show how to exploit the clockwork theory in inflation. The clockwork set-up is suitable to get either small masses (compared to the fundamental mass scale of the problem) or small couplings (compared to couplings of order unity). This is exactly what is needed during inflation in order to get the right amount of density fluctuations. The comoving curvature perturbation \( \zeta \) in the flat gauge is [4]

\[
\zeta = \left( \frac{H}{\dot{\phi}} \right) \delta \phi \sim \left( \frac{H}{M_{\text{pl}} \sqrt{\epsilon}} \right),
\]  

where the subscript * indicates that quantities should be computed at the epoch of Hubble radius exit for the comoving scale \( k = aH \). \( \phi \) is the inflaton field, \( H \) is the Hubble rate during inflation, \( M_{\text{pl}} \) is the reduced Planck mass, and one has to remember that observables scales in our current universe correspond to the last 60 e-folds or so before the end of inflation. Dots indicate differentiation with respect to time and

\[
\epsilon = -\frac{\dot{H}}{H^2} \simeq \frac{M_{\text{pl}}^2}{2} \left( \frac{V'}{V} \right)^2
\]  

is one of the slow-roll parameters. The observed perturbations are matched if

\[
\left( \frac{V^{3/2}}{M_{\text{pl}}^2 V} \right)^* \simeq 5.3 \cdot 10^{-4}.
\]  

2.1. Large field models of inflation

To illustrate the advantages of the clockwork set-up in producing flat potential during inflation, let us consider the class of large field models of inflation. The simplest model of inflation is given by a linear potential

\[
V(\phi) = m^2 \phi,
\]  

Of course, this is the potential during inflation and additional terms are supposed to be there in order to describe the dynamics after inflation, mainly reheating. For instance a simple modification of the potential above will be \( V(\phi) = m^2 (\phi^2 + a M^2)^{1/2} \) where \( a \) is a constant and which gives the potential above for \( \phi \gg m \), but has the right minimum at \( \phi = 0 \).

We know that the slow-roll conditions are attained when \( \phi \gg M_{\text{pl}} \) and that the density perturbations are given by [4]

\[
\zeta \sim \left( \frac{m}{M_{\text{pl}}} \right)^{3/2} \sim 10^{-5},
\]  

for \( m \sim 10^{15} \text{ GeV} \ll M_{\text{pl}} \). In the clockwork scenario, we can assume that there are \( N + 1 \) copies of the inflaton fields and the potential is

\[
V(\pi_1, \cdots, \pi_N) = \frac{M_2^2}{2} \sum_{i=0}^{N-1} (\pi_i - q \pi_{i+1})^2 + M_2^2 \pi_N,
\]  

where \( M_2 \ll M_1 \) (say smaller by a factor of 10), but both close to the fundamental scale. The first piece of the potential is invariant under a shift symmetry

\[
\pi_i \to \pi_i + \frac{1}{q^i},
\]  

which is broken by the last term in the potential. Upon diagonalization of the mass matrix (which is not altered by the presence of the linear term) and going to energy (to be identified with the Hubble rate \( H \)) much smaller than \( M_2 \), the lightest mass eigenstate state \( \phi_0 \) will have a potential

\[
V(\phi_0) = \frac{M_2^2}{q^N} \phi_0.
\]  

Taking for instance \( M_2 \sim 10^{-1} M_{\text{pl}}, q = 2 \), we need \( N \sim 20 \) copies to match the observed level of perturbations. We should also note that possible one-loop contributions from the matter sector to the
Inflaton potential are suppressed, at least by a factor of $q^{-N}/16\pi^2$, and therefore such contributions are fully under control.

Let us also stress that the vacuum expectation value of the inflaton plays the role of the order parameter for the breaking of the residual shift symmetry of the clockwork. Therefore, the scale $M_2$, as we imposed above, needs to be smaller than the clockwork scale $M_1$. In this respect, the implementation of the clockwork mechanism needs a hierarchy of scales, even though not a fine-tuned one. This applies as well to the other models of inflation we will discuss later on.

Another issue the clockwork can be useful for in large field models is the one of super-Planckian field excursion. This was already noticed in Ref. [2]. If the scalar perturbations are ascribable to only one scalar degree of freedom, then

$$\frac{\Delta \Phi}{M_{\text{pl}}} \approx \left(\frac{r}{2 \times 10^{-2}}\right)^{1/2},$$

(2.9)

where $r$ is the so-called tensor-to-scalar ratio. A future detection of gravitational waves requires in general variation of the Planck scale. This would pose a problem as slow-roll models of inflation disregard the possible presence of higher-order operators with powers of $(\phi/M_{\text{pl}})$. However, super-Planckian field excursions can be mimicked while preserving the regime of renormalizable four-dimensional field theory by the clockwork mechanism. Indeed, the slow-roll of the inflaton field over super-Planckian field values corresponds to a clockwork of phase-rotations of the $N + 1$ copies of fields $\pi_i$ with a $U(1)^{N+1}$ global symmetry whose effective decay constant is amplified with respect to the original one by a factor $q^N$.

### 2.2. Hybrid models of inflation

To illustrate the advantage of the clockwork mechanism in terms of efficiently producing small couplings during inflation, let us consider the hybrid model of inflation [6,7] with $N + 1$ copies of the fields $\pi_i$ and extra field $\Phi$

$$V(\pi_1, \ldots, \pi_N, \Phi) = \frac{M_{\text{pl}}^2}{2} \sum_{i=0}^{N-1} (\pi_i - q \pi_{i+1})^2 + V_0 \left(1 - \frac{\Phi}{M}\right) + \frac{1}{4} \lambda \pi^2 \Phi^2 + \cdots.$$  

(2.10)

The dots represent one or more additional terms, which give the potential a minimum at which it vanishes but play no role during inflation. By performing the standard diagonalization of the clockwork mechanism for the mass squared term in Eq. (2.10) and working at energies much smaller than the Hubble rate $H$, the potential (2.10) for the lightest eigenstate $\phi_0$ is reduced to

$$V(\phi_0, \Phi) = V_0 \left(1 - \frac{\Phi}{M}\right) + \frac{\lambda_0}{4} \phi_0^2 + \cdots, \quad \lambda_0 = \frac{\lambda}{q^N}. $$

(2.11)

For suitable choices of the parameters, inflation takes place with the field $\Phi$ held at the instantaneous minimum, leading to a potential

$$V(\phi_0) = V_0 \left(1 - \frac{V_0}{\lambda^2 M^2 \phi_0^2}\right). $$

(2.12)

Imposing the condition (2.3) gives [4]

$$10^{-12} \sim \frac{\lambda_0 V_0^{1/2} M}{M_{\text{pl}}},$$

(2.13)

and one could explain a small coupling $\lambda_0$ with the clockwork mechanism even though all the other mass scales in the problem are of the order of the Planck scale.

### 2.3. Small field inflation

In small field models of inflation the problem is to have a flat enough potential close to the origin

$$V = V_0 - \frac{1}{2} m^2 \phi^2 + \cdots.$$  

(2.14)

As the spectral index of the scalar perturbations is given by [4]

$$n = 1 - 2 M_{\text{pl}}^2 m^2/V_0 \sim 0.04,$$

one needs $m^2 \sim 10^{-11} H^2$ to be in agreement with the observations. To produce a potential suitable for small field inflation, in the clockwork scenario it is enough to couple the pion $\pi_N$ to fermions charged under some strong group. Below the confinement scale the lightest mode acquires a potential of the form used in natural inflation [8]

$$V(\phi_0) = \Lambda^4 \cos \frac{\phi_0}{q^N f}, $$

(2.15)

in such a way that, expanding around the maximum of the potential we obtain

$$V_0 = \Lambda^4, \quad m^2 = \frac{\Lambda^4}{q^2 N f^2} $$

(2.16)

and the condition $m^2 \ll H^2$ requires

$$f^2 \gg \frac{M_{\text{pl}}^2}{q^2 N}. $$

(2.17)

This condition can be easily satisfied if $f \ll M_{\text{pl}}$ even for moderate values of $N$. Furthermore, the normalization of the density perturbations (2.3) imposes

$$\frac{V_0^{1/2}}{M_{\text{pl}}^2} \simeq 5.4 \times 10^{-4} \frac{1 - n}{2} \frac{N_c}{N} \frac{\phi_0}{M_{\text{pl}}}, $$

(2.18)

where $N_c$ is the number of e-folds till the end of inflation and $\phi_0$ is the value of the inflaton field $\phi_0$ when inflation ends. Since the typical scale for $\phi_0$ is $q^N f \gg M_{\text{pl}}$, one sees that the clockwork can allow sizeable $\Lambda$. In this set-up inflation will end when the slow-roll conditions are violated and the $\phi_0$ field rolls to its minimum and reheating will take place by coupling the inflaton field to, for instance fermions, through a coupling $\partial_\mu \phi_0 \bar{\psi} \gamma^\mu \gamma_5 \psi$.

### 2.4. Starobinsky inflation

Another illustrative example of the efficiency of the clockwork mechanism is provided by the so-called Starobinsky model of inflation [12]. Following Ref. [3], we consider $N$ copies of linearized GR to which we add the Starobinsky-like action

$$S = \frac{M_{\text{pl}}^2}{2} \int d^4x \sqrt{-g} \mathcal{R}(g_{N \mu\nu}) + \frac{1}{2\alpha^2} \int d^4x \sqrt{-g} R^2(g_{N \mu\nu}), $$

(2.19)

where we have added a quadratic curvature term for the $N$-th term whose strength is parametrized by a dimensionless parameter $\alpha$. It is known that in $(R + R^2)$-theory, there is a massive scalar mode on top of the gravitons which can be uncovered with the usual methods, leading to the action.

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1. In this respect, the clockwork mechanism applied to natural inflation is reminiscent of the aligned mechanism [9–11]. However, alignment does require a hierarchy of parameters which is naturally achieved through clockwork.
\[ S = \frac{M^2_N}{2} \int d^4x \sqrt{-g} \left( R(g_{N \mu \nu}) + \frac{1}{2} (\partial \phi_N)^2 - \frac{3}{4} \alpha^2 \epsilon^N \left( 1 - e^{-\sqrt{\frac{1}{2} \epsilon g_{N \mu \nu}}} \right)^2 \right). \]  

(2.20)

For this theory, as shown in Ref. [3], there is a massless graviton with a corresponding Planck mass, which in the large \( N \)-limit is

\[ M^2_{pl} = q^N M^2_N. \]  

(2.21)

Therefore, the action for the massless graviton and the scalar becomes

\[ S = \int d^4x \sqrt{-g} \left\{ \frac{M^2_{pl}}{2} R - \frac{1}{2} (\partial \phi_N)^2 - \frac{3}{4} \alpha^2 \epsilon^N \left( 1 - e^{-\sqrt{\frac{1}{2} \epsilon g_{N \mu \nu}}} \right)^2 \right\}. \]  

(2.22)

During inflation \( \phi_N \) takes large values and the dynamics is dominated by the vacuum energy

\[ V = \frac{3}{4} \alpha^2 M^4_{pl} q^{-4N}. \]  

(2.23)

In order to get the correct normalization (2.3) one needs \( q^{-2N} = 10^{-5} \) for \( q = O(1) \). For instance, for \( q = 2 \), the moderate value \( N = 8 \) is required. In addition, the tensor-to-scalar ratio is \( r = (12q^{-2N}/N^2) \). Hence, the tensor modes are suppressed by an extra factor of \( q^{-2N} = 10^{-5} \), leaving to no room for tensor modes in the clockwork Starobinsky inflation model.

2.5. The clockwork and the generation of perturbations from light fields other than the inflaton

Even though the inflationary paradigm is by itself quite elegant and simple, the mechanism giving rise to the adiabatic cosmological perturbations is far from being established. It is fair to say that we do not know at present what is the source of the scalar perturbations during inflation, the inflaton field itself or some other field. The total curvature perturbation \( \zeta \) might not be a constant (in time) on super-Hubble scales and changing on arbitrarily large scales due to a non-adiabatic pressure perturbation which may be due to extra scalar degrees of freedom.

For instance, in the curvaton mechanism [13,14] the curvature perturbation is generated from an initial isocurvature perturbation associated to the quantum fluctuations of a light scalar field \( \sigma \), the curvaton, whose energy density is not dominant during inflation. The curvaton isocurvature perturbation becomes the adiabatic one once the curvaton decays into radiation after the end of inflation.

During inflation a flat spectrum is produced in the curvaton field

\[ \delta \sigma = \left( \frac{H}{2\pi} \right)_\sigma. \]  

(2.24)

After inflation, the curvaton field starts oscillating during some radiation-dominated era, causing its energy density to increase and converting the initial isocurvature into curvature perturbation \( \zeta \). The curvaton mechanism works as long as the curvaton can be quantum mechanically excited during inflation. This requires that the mass \( m \) of the curvaton field is much smaller than the Hubble rate during inflation, \( m \ll H \). This poses a problem as non-renormalizable couplings between the inflaton and the curvaton are expected to generate \( O(H^2) \) correction to the mass squared of the curvaton. For instance, in supergravity one has corrections to the Kähler potential of the curvaton of the form

\[ \delta K \supset \int d^2 \theta \frac{\partial \phi}{M_{pl}} \sigma^T \sigma \supset O(1) H^2 \sigma^T \sigma, \]  

(2.25)

where \( \phi \) is the inflaton field. The clockwork provides a possible solution to this problem. Suppose that there are \( N + 1 \) copies of curvatons, let us call them again \( \pi_i \) with potential

\[ V(\pi_1, \ldots, \pi_N) = \frac{M^2_{pl}}{2} \sum_{i=0}^{N-1} (\pi_i - q \pi_{i+1})^2. \]  

(2.26)

This potential has the usual shift symmetry \( \pi_i \rightarrow \pi_i + 1/q \). Like in the construction of Ref. [2], this shift symmetry is a manifestation of the fact that the \( \pi_i \)'s are indeed pseudo Nambu–Goldstone bosons of a \( U(1)^{N+1} \) global symmetry. Since gravity is expected to break such a global symmetry, one expects corrections to the mass of the form

\[ \sum_{i,j=0}^{N} c_{ij} H^2 \pi_i \pi_j = c_{NN} H^2 \pi_N^2 + \cdots, \]  

(2.27)

where the \( c_{ij} \) are \( O(1) \) coefficients and we have supposed that the vacuum energy is located at the \( N \)-th site. Upon diagonalizing the mass matrix (2.26) one finds that the lightest eigenstate \( \sigma_0 \) receives corrections to its mass squared suppressed at least by \( 1/q^N \ll 1 \), which is enough to obtain a light curvaton during inflation.

3. Clockwork inflation: the five-dimensional continuum perspective

In this section we investigate the clockwork inflationary scenario from the continuum limit point of view. We imagine that the vacuum energy driving inflation is located at one of the two branes and, as a result, each fifth dimensional section is inflating with constant Hubble rate. Let us start with the five-dimensional action [3,15]

\[ S = \int d^5x \sqrt{-\tilde{g}} \left\{ \frac{M^2_5}{2} \left( \frac{1}{3} \partial M \partial M + 4k^2 e^{-\frac{i}{2} \delta} \right) + L_{\text{bulk}} \right\} + S_{\text{brane}}, \]  

(3.1)

where

\[ S_{\text{brane}} = -\sum_{\phi} \int d^4x \sqrt{\tilde{h}} \left\{ \frac{M^2_5}{2} [K] + e^{-\frac{i}{2} \delta} L_{\text{brane}} \right\}. \]  

(3.2)

\( L_{\text{bulk}} \) is a bulk matter action, \( L_{\text{brane}} \) is the brane action and \( [K] \) is the jounce of the trace of the extrinsic curvature

\[ K_{\mu \nu} = \partial_\mu h^N \partial_\nu h^N. \]  

(3.3)

defined in terms of the normal to the branes \( \Sigma_0 \) located at positions \( x_0 \) and the projection tensor \( h_\mu^N \). Phenomenological aspects of this theory have been explored in [15–17]. The field equations that follow from the action (3.1) are

\[ R_{MN} - \frac{1}{2} \delta_{MN} R = \frac{1}{M^2_5} T_{MN}, \]  

(3.4)

\[ \nabla^2 S - 4k^2 e^{-\frac{i}{2} \delta} = 0, \]  

(3.5)

supplemented by the Israel matching conditions for the branes \( \Sigma_\alpha \).
\[ [\mathcal{K}_{\mu\nu}]_{\Sigma_a} = \frac{1}{M_5^2} \left( T^{(a)}_{\mu\nu} - \frac{1}{3} g_{\mu\nu} T^{(a)} \right) \] \[ \Sigma_a, \]

\[ [n^M \partial_M \mathcal{S}]_{\Sigma_a} = \frac{3}{M_5^2} \frac{\partial}{\partial S} \left( e^{-\frac{1}{2} S} \mathcal{L}_{\text{brane}} \right) \] \[ \Sigma_a, \] \hspace{1cm} (3.6) \]

and \( T^{(a)}_{\mu\nu} \) is the four-dimensional brane energy-momentum tensor. We are looking now for solutions of the form

\[ ds^2 = e^{2\sigma(y)} \left( dy^2 + ds_4^2 \right), \quad S = S(y), \] \hspace{1cm} (3.7) \]

where

\[ ds_4^2 = g_{\mu\nu} dx\mu dx\nu = \frac{1}{H^2 \eta^2} (-d\eta^2 + dx^2). \] \hspace{1cm} (3.8) \]

is a four-dimensional de Sitter metric. The equations of motion are then

\[ 36(\sigma''^2 - H^2) - S'' = 12k^2 e^{-\frac{1}{2} S + 2\sigma}, \] \hspace{1cm} (3.9) \]

\[ \sigma'' - \sigma' + H^2 + \frac{1}{9} \sigma'' = 0, \] \hspace{1cm} (3.10) \]

\[ S'' + 3\sigma'S' = 4k^2 e^{-\frac{1}{2} S + 2\sigma}. \] \hspace{1cm} (3.11) \]

In addition, the Israel matching conditions along the branes \( \Sigma_a \) located at \( y = y_a \) with tension \( \mathcal{L}^{(a)}_{\text{brane}} = \mathcal{V}_a \) for the metric (3.7) give

\[ [\sigma']_{\Sigma_a} = -\frac{1}{6M_5^2} e^{\sigma_a - \frac{1}{2} S_a} \mathcal{V}_a, \]

\[ [S']_{\Sigma_a} = -\frac{1}{2M_5^2} e^{\sigma_a - \frac{1}{2} S_a} \mathcal{V}_a. \] \hspace{1cm} (3.12) \]

where \( \sigma_a = \sigma(y_a), \mathcal{S}_a = S(y_a). \) The scalar field equation (3.11) is not independent as it is connected to the Bianchi identity. So only Eqs. (3.9) and (3.10) are independent. Eliminating \( S'' \) we get the system of equations

\[ 3\sigma'' + 9\sigma' - 9H^2 = 4k^2 e^{-\frac{1}{2} S + 2\sigma}, \] \hspace{1cm} (3.13) \]

\[ \sigma'' - \sigma' + H^2 + \frac{1}{9} \sigma'' = 0. \] \hspace{1cm} (3.14) \]

When \( H = 0 \), the solution to Eqs. (3.13) and (3.14) is the linear dilaton solution

\[ \sigma = \sigma_0 = \frac{2k}{3} y, \quad S = S_0 = 2ky, \] \hspace{1cm} (3.15) \]

where the boundary condition \( \sigma_0(0) = S_0(0) = 0 \) has been assumed. However, the solution of Eqs. (3.13) and (3.14) for non-zero \( H \) is not easy to be found. Nevertheless, by using Eq. (3.13) we can express the scalar \( S \) in terms of the warp factor as

\[ e^{-\frac{1}{2} S} = \frac{3}{4k^2} e^{-2\sigma} \left( \sigma'' + 3\sigma'r' - 3H^2 \right). \] \hspace{1cm} (3.16) \]

By taking the derivative of Eq. (3.14) and comparing with Eq. (3.13), we can completely decouple the scalar \( S \) and we find that \( \sigma \) satisfies the third order equation

\[ \left[ 2\sigma' \left( 3H^2 - 3\sigma'r' + 2\sigma'' \right) + \sigma'''' \right]^2 + 4 \left( H^2 - \sigma'r' + \sigma'' \right) \left( 3H^2 - 3\sigma'r' - \sigma'' \right)^2 = 0, \] \hspace{1cm} (3.17) \]

Although we were not able to find an exact solution to Eq. (3.17) we can try to find a solution perturbatively in \( H^2 \). For this, we may write

\[ \sigma \approx \sigma_0 + H^2 \sigma_1 \] \hspace{1cm} (3.18) \]

and treat \( \sigma_1 \) as a first order perturbation. We then find that \( \sigma_1 \) satisfies

\[ \sigma'''' + 2k\sigma'' - 2k = 0, \] \hspace{1cm} (3.19) \]

and therefore,

\[ \sigma_1 = \frac{1}{2} y^2 + C_1 e^{-2ky} + C_3 y + C_2, \] \hspace{1cm} (3.20) \]

where \( C_{1,2,3} \) are integration constants. In addition, we find that \( S \) is given by

\[ S \approx 2ky + \frac{3}{2} H^2 \left[ y^2 + 5C_1 e^{-2ky} + y \left( 2C_3 - \frac{3}{k} \right) \right] + 2C_2 - \frac{3C_3}{k} + \frac{3}{2k^2}, \] \hspace{1cm} (3.21) \]

It is straightforward to verify that \( \sigma = \sigma_0 + H^2 \sigma_1 \) and \( S \) in Eq. (3.21) indeed satisfies the equations (3.13) and (3.14) to leading order in \( H^2 \). With a compact fifth dimension and two branes at \( y = 0 \) and \( y = \pi R \) with brane action

\[ S_{\text{brane}} = \int d^4x \int_0^{\pi R} dy \sqrt{g} e^{-\frac{1}{2} S} \left[ \delta(y) V_0 + \delta(y - \pi) V_\pi \right]. \] \hspace{1cm} (3.22) \]

we find from Eq. (3.12) that the discontinuities of \( \sigma, S \) should satisfy

\[ 3[\sigma']_{0,\pi R} = [S']_{0,\pi R}. \] \hspace{1cm} (3.23) \]

The solutions that satisfy (3.23) have \( C_1 = 0 \) and if we take \( C_2 = 0 \) and \( C_3 = 1/2k \) so that \( \sigma(0) = S(0) = 0 \), we get

\[ \sigma = \frac{2k}{3} |y| + \frac{1}{2} H^2 \left( y^2 + \frac{1}{k} |y| \right), \] \hspace{1cm} (3.24) \]

\[ S = 2k|y| + \frac{3}{2} H^2 \left( y^2 + \frac{1}{k} |y| - \frac{3}{k} y \right). \] \hspace{1cm} (3.25) \]

Note that if \( S_\pi = S(\pi R) \), we have from Eq. (3.25)

\[ S_\pi = 2k\pi R + \frac{3}{2} H^2 \left( k^2 \pi^2 R^2 - \frac{2}{k} \pi R \right), \] \hspace{1cm} (3.26) \]

which fixes the radion field \( R \) in terms of the boundary value of the dilaton \( S \) at \( y = \pi R \). The latter can be determined by a potential for example of the form

\[ U(S) = \frac{M_5^2}{2} (S - S_\pi)^2. \] \hspace{1cm} (3.27) \]

Such a potential fixes the boundary value of the dilaton and consequently the value of \( R \). In addition, the tensions \( V_0 \) and \( V_\pi \) in Eq. (3.22) turn out to be

\[ V_0(H) = -\left( 4k + \frac{3H^2}{k} \right) M_5^3, \]

\[ V_\pi(H) = \left( 4k + \frac{3H^2}{k} \right) e^{-\sigma(\pi R) + S(\pi R)/3} M_5^3. \] \hspace{1cm} (3.28) \]

At leading order in \( H^2/k^2 \), the extra vacuum energy driving inflation in the \( y = \pi R \) brane \(^2\) is given by

\(^2\) We are adopting the so-called \( \pi \)-frame here, where the SM resides in the \( y = \pi R \) brane, as opposed to the 0-frame adopted in [3] for the continuum clockwork. The two frames are equivalent, and there is a well-defined transformation that connects the two frames.
\[ U_\pi = e^{4\sigma(\pi R)/3} \left( V_\pi(H) - V_\pi(0) \right) = \frac{3H^2}{k} e^{2k\pi R M^3_{pl}}. \]

We may write the last equation as
\[ H^2 = \frac{U_\pi}{3M^2_{pl}}, \tag{3.30} \]
which is just the standard Friedmann law on the brane expected with a stabilized dilaton to lowest order in \( H^2 \). In (3.30) we have defined the four-dimensional Planck mass in flat space-time as
\[ M^2_{pl} = M_5^3 \frac{e^{2k\pi R}}{k}, \tag{3.31} \]
which coincides with the one found by dimensional reduction of the five-dimensional action reported in [3]. If the vacuum energy driving inflation comes from the \( y = \pi R \) brane, the scalar perturbations have the same behavior at leading order in the slow-roll parameters as in the four-dimensional case [18] and we expect that no detectable signature remains from the non-trivial geometry in the bulk.\(^3\) The reason is the following. Differently from the tensor modes, which are genuinely five-dimensional free fields quantized in the bulk, scalar metric perturbations are generated by the brane scalar field which is quantized on the brane and is four-dimensional in all regimes. The coupling between the inflaton field and the metric concerns the long-wavelength limit for which the metric evolves as in the four-dimensional theory. Moreover, such a coupling is localized on the brane, and no signature remains from the warped geometry in the bulk. Corrections to this result are of the order of \(-HR^2 = e^{(HR)^2}\) as metric perturbations probe the extra dimension on times scales \(-H^2/H\). The same logic is not valid for tensor modes, as we now proceed to discuss.

4. Tensor modes in clockwork inflation

In this section we study the behavior of the tensor modes during a de Sitter stage in the clockwork set-up. The goal is to show that tensor modes are suppressed in this scenario with respect to the standard case, the reason being that the tensor modes feel the bulk by full strength and that during the de Sitter stage the latter is more warped.

We start from the five-dimensional continuum perspective. The advantage of using tensor modes to probe the clockwork mechanism is that there is a dependence only on the geometry, not on the microscopic models of inflation and stabilization of the extra fifth dimension.

4.1. Tensor modes: the five-dimensional continuum perspective

As shown in Refs. [18,20], a massless tensor mode is produced in braneworld scenarios of inflation with the amplitude \( H/M_{pl} \). The point is that the effective Planck mass during inflation for a curved de Sitter braneworld differs from that of the flat brane at low energies, due to a dependence on the Hubble rate \( H \). Thus, the amplitude of tensor modes from inflation in a clockwork scenario is usually different from a standard period of inflation without clockwork.

Let us indeed consider the equation of motion of the graviton field in the continuum clockwork scenario starting from the factorized metric (3.7). By decomposing the five-dimensional tensor mode as

\[ h_{MN}(y, \chi^\mu) = e^{-\frac{1}{2}\sigma(y)} h_{M}^{(m)}(y), \tag{4.1} \]

where \( m \) is the eigenvalue corresponding to the Kaluza–Klein modes and the transverse-traceless conditions are imposed on \( Q_{\mu\nu}^{(m)}(\chi^\mu) \), one can show that the eigenvalue problem can be written as [18,20]

\[ -\mathcal{D}_y \mathcal{D}_\chi h_m(y) = m^2 h_m(y), \]

\[ \mathcal{D}_y = \partial_y + \frac{3}{2} \sigma'(y) = \partial_y + \frac{2k}{3} \theta(y) + \frac{H^2}{k^2} \left( y + 2k\theta(y)e^{-2ky} \right). \tag{4.2} \]

From this equation one immediately reads off the presence of the zero mode graviton with wavefunction

\[ h_0(y) = e^{\frac{2}{3}\sigma(y)}. \tag{4.3} \]

Furthermore, one can show that there is a mass gap for the other Kaluza–Klein modes of at least \( 3H^2/2 \) in the mass squared. The key point is that during inflation the effective four-dimensional Planck mass as seen by the massless tensor mode is different from the one in flat space since the function \( \sigma(y) \) in de Sitter, as shown in the previous section,

\[ \sigma(y) = \frac{2k}{3} |y| + \frac{1}{2} H^2 \left( y^2 + \frac{1}{k} |y| \right) + O\left( H^4 R^4 \right) \tag{4.4} \]

is modified from the flat case. It is easy to see that the warp factor \( e^{\sigma(y)} \) is convex and an increasing function of \( H^2 \). Therefore, the four-dimensional \( M_{pl} \) defined as

\[ \left( M_{pl}^{4D} \right)^2 = 2M_5^3 \int_0^\pi R dy e^{3\sigma} = M_5^3 \left[ e^{2k\pi R} \left( 1 + \frac{3}{2} H^2 R^2 \right) - 1 \right]. \tag{4.5} \]

is an increasing function of \( H^2 \), see Fig. 1. As the amount of tensor modes is proportional to \( (H/M_{pl}^{4D})^2 \), this means that in the clockwork scenario the amount of tensor modes is reduced with respect to the traditional case. This is a negative point? Not necessarily so. There are many models of inflation which has been ruled out by the recent Planck data [21] as they produce a too large amount of tensor modes. Examples are the chaotic large field model of inflation \( \lambda \phi^4 \) [22] and power-law inflation [23]. By embedding these
models into the clockwork theory, these models become allowed again by current data.

4.2. Tensor modes: the four-dimensional discrete perspective

Let us analyze now the clockwork graviton in de Sitter space from the four-dimensional perspective. In particular we assume \( N + 1 \) copies of metrics \( g^{\mu\nu}_i \) describing \( N + 1 \) copies of general relativity with their associated \( N + 1 \) massless gravitons and Planck mass \( M_i \). The gravitons \( h^{\mu\nu}_i \) are fluctuations around the de Sitter metric \( g^{(dS)}_{\mu\nu} \) such that \( g_{\mu\nu} = g^{(dS)}_{\mu\nu} + h_{\mu\nu}/M_i^2 \). Clockworking will break the \( N + 1 \) diffeomorphisms to a single diffeomorphic invariance corresponding to a single massless graviton. The clockwork dynamics will then be described by the following action

\[
S = \int d^4x \sqrt{-g} \left\{-\frac{1}{4} \sum_{i=0}^{N} h^{\mu\nu}_i \left( -\Box + 2H^2 \right) h_{\mu\nu} - \frac{m^2}{4} \sum_{i=0}^{N-1} \left( h_{\mu\nu}^{(i)} - q_i + h^{\mu\nu}_i \right)^2 \right\},
\]

(4.6)

where \( h^{\mu\nu}_i \) is transverse traceless \( \nabla_{\mu} h^{\mu\nu}_i = 0 \), \( \gamma_{\mu\nu} h^{\mu\nu}_i = 0 \). For \( q_{i+1} = 0 \), the action above describes \( N + 1 \) massive gravitons on de Sitter in transverse traceless gauge [24,25]. Note that we have allow for different \( q's \) in the mass term \( (q_1 \neq q_2 \ldots \neq q_N) \) since the de Sitter metric is not flat. We will motivate this choice by deconstructing the five-dimensional space where we will see that this choice is necessary when the background is not flat in general. The clockwork theory described by the action (4.6) is invariant now under the transformation

\[
h^{\mu\nu}_i \rightarrow h^{\mu\nu}_i + \frac{1}{Q_i} \left( \nabla^{\mu} \xi^{\nu} + \nabla^{\nu} \xi^{\mu} \right), \quad Q_i = \prod_{j=1}^{i} q_j = q_1 \cdots q_i.
\]

(4.7)

where \( \xi^{\mu} \) is a vector in de Sitter. Therefore, we expect a massless graviton in the spectrum, the existence of which can be verified by diagonalizing the mass matrix

\[
M_{h}^2 = m^2 \begin{pmatrix}
1 & -q_1 & 0 & \cdots & 0 \\
-q_1 & 1 + q_1^2 & -q_2 & \cdots & 0 \\
0 & -q_2 & 1 + q_2^2 & \cdots & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & 0 & \cdots & 1 + q_{N-1}^2 \\
0 & 0 & 0 & \cdots & -q_N & q_N^2
\end{pmatrix}
\]

(4.8)

It can easily be verified that the mass matrix \( M_{h}^2 \) has a zero eigenvalue corresponding exactly to the symmetry of Eq. (4.7). The action (4.6) can be written then as

\[
S = \int d^4x \sqrt{-g} \left\{-\frac{1}{4} \sum_{i=0}^{N} \tilde{h}^{\mu\nu}_i \left( -\Box + 2H^2 \right) \tilde{h}_{\mu\nu} - \frac{1}{4} \sum_{i=1}^{N} m_i^2 \tilde{h}_{\mu\nu}^{(i)} \right\},
\]

(4.9)

where \( m_i^2 \) are the non-zero eigenvalues of \( M^2 \) and

\[
\tilde{h}^{\mu\nu}_i = O^\dagger h^{\mu\nu}_i. \quad O^\dagger M^2 O = \text{diag}(m_0^2, m_1^2, \ldots, m_{N-1}^2).
\]

(4.10)

We see that the theory described a massless graviton \( h^{\mu\nu}_0 \) and \( N - 1 \) massive spin-2 states \( h^{\mu\nu}_i \), \( i = 1, \ldots, N - 1 \).

Let us now deconstruct the clockwork direction. The graviton fluctuations around a four-dimensional de Sitter background will be described by the action

\[
S = -\frac{1}{4} \int d^4x \int dy e^{2\sigma} \left[ \nabla_\lambda h^{\mu\nu} \nabla^\lambda h_{\mu\nu} \right. \\
&+ \partial_\mu h_{\nu\rho} \partial_\nu h_{\mu\rho} + 2H^2 h_{\mu\nu} h^{\mu\nu}\right],
\]

(4.11)

which after deconstruction turns out to be (after redefining \( h_{\mu\nu} \rightarrow e^{-\frac{1}{2}}h^{\mu\nu} \))

\[
S = -\frac{1}{4} \int d^4x \left[ \sum_{i=0}^{N} \left( \nabla_i h^{\mu\nu}_i \nabla^\lambda h_{\mu\nu} + 2H^2 h_{\mu\nu} h^{\mu\nu}_i \right) \\
+ \frac{1}{a^2} \sum_{i=0}^{N-1} e^{2\sigma(i)} \left( e^{-\frac{3}{2}(\sigma + (i+1))h^{\mu\nu}_{i+1} - e^{-\frac{3}{2}(\sigma + i))h^{\mu\nu}_i} \right)^2 \right].
\]

(4.12)

Simple manipulations lead to the following expression

\[
S = -\frac{1}{4} \int d^4x \left[ \sum_{i=0}^{N} \left( \nabla_i h^{\mu\nu}_i \nabla^\lambda h_{\mu\nu} + 2H^2 h_{\mu\nu} h^{\mu\nu}_i \right) \\
+ \frac{1}{a^2} \sum_{i=0}^{N-1} \left( h^{\mu\nu}_i - e^{-\frac{3}{2}(\sigma + (i+1))} h^{\mu\nu}_{i+1} \right)^2 \right].
\]

(4.13)

For the warp factor \( \sigma(y) \) given in Eq. (4.4), the deconstruction is carried out in the so-called 0-frame of the discrete clockwork, where the SM resides in the first site [3]. To go to the \( N \)-frame used here as well as in Ref. [3] where the Standard Model (SM) is localized at the \( N \)-th site in the discrete clockwork, we should change \( \sigma \rightarrow -\sigma \) so that \( q_1 \rightarrow 1/q_1 \). In this case we find that

\[
q_{i+1} = e^{\frac{1}{2}(\sigma(i+1) - \sigma(i))} \sim e^{\frac{1}{2} a^2 H^2 (1+2i)}.
\]

(4.14)

Hence, the charge increases in going from the first site to the \( N \)-th site, and (4.13) can be written as

\[
S = -\frac{1}{4} \int d^4x \left[ \sum_{i=0}^{N} \left( \nabla_i h^{\mu\nu}_i \nabla^\lambda h_{\mu\nu} + 2H^2 h_{\mu\nu} h^{\mu\nu}_i \right) \\
+ \frac{1}{a^2} \sum_{i=0}^{N-1} \left( h^{\mu\nu}_i - q_i + h^{\mu\nu}_{i+1} \right)^2 \right].
\]

(4.15)

This is identical to Eq. (4.6) which is motivated here as the deconstructed action along the clockwork direction. When the \( y = \text{const.} \) sections are flat, \( \sigma \) is a linear function of \( y \) as in Eq. (3.15) and hence we get that \( q_1 = q_2 = \ldots = e^{\frac{1}{2}a^2 H^2 (1+2i)} \). However, in the case in which the \( y = \text{const.} \) sections are not flat, and in particular de Sitter spaces like in the present setup, then we have that \( q_1 \neq q_2 \ldots \neq q_N \) and the charge in the \( N \)-th site is larger in de Sitter than it would be in Minkowski spacetime, explaining why the Planck scale is larger during inflation.

5. Conclusions

The clockwork is an ingenious mechanism to generate large mass/coupling hierarchies in theories where no small parameters are present to start with. In this paper we have offered an handful number of examples of how the clockwork set-up may help to construct inflationary models with no fine-tuning. Interestingly, clockwork inflation predicts an amount of tensor modes which is
smaller than in standard scenarios with no clockwork. While this result is bad news for current and future efforts in detecting tensor modes in the B-mode polarization of the CMB, it is certainly good news for inflation model builders as many models of inflation prematurely ruled out by Planck observations for their excessive tensor mode power spectrum, are now back to business.

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