ORGANISATION EUROPÉENNE POUR LA RECHERCHE NUCLEAIRE
CERN EUROPEAN ORGANIZATION FOR NUCLEAR RESEARCH

PROCEEDINGS OF THE INFORMAL MEETING ON GEOMETRY

PROGRAMMES FOR HEAVY LIQUID BUBBLE CHAMBERS

Held at CERN, 21st May, 1963

Edited by M. Nikolić
V. Cooper

GENEVA
Propriété littéraire et scientifique réservée pour
tous les pays du monde. Ce document ne peut 
être reproduit ou traduit en tout ou en partie 
sans l'autorisation écrite du Directeur générale 
du CERN, titulaire du droit d'auteur. Dans 
les cas appropriés, et s'il s'agit d'utiliser le 
document à des fins non commerciales, cette 
autorisation sera volontiers accordée.

Le CERN ne revendique pas la propriété des 
inventions brevetables et dessins ou modèles 
susceptibles de dépôt qui pourraient être décrits 
dans le présent document ; ceux-ci peuvent être 
librement utilisés par les instituts de recherche, 
les industriels et autres intéressés. Cependant, 
le CERN se réserve le droit de s'opposer à 
toute revendication qu'un usager pourrait faire 
de la propriété scientifique ou industrielle de 
toute invention et tout dessin ou modèle dé- 
crits dans le présent document.

(© Copyright CERN, Genève, 1966)

Literary and scientific copyrights reserved in 
all countries of the world. This report, or 
any part of it, may not be reprinted or trans-
lated without written permission of the 
copyright holder, the Director-General of 
CERN. However, permission will be freely 
granted for appropriate non-commercial use.

If any patentable invention or registrable 
design is described in the report, CERN makes 
no claim to property rights in it but offers it 
for the free use of research institutions, manu-
facturers and others. CERN, however, may 
 oppose any attempt by a user to claim any 
proprietary or patent rights in such inventions 
or designs as may be described in the present 
document.
CONTENTS

Morning Session  Chairman: A. Rousset

Introduction - Y. Goldschmidt-Clermont

CERN programmes:

A.M. Chops (CERN) - THRESH - Basic CERN geometry programme .......................... 5

W.G. Moorhead (CERN) - A possible simplified fitting procedure
in THRESH for heavy liquids. ............................................................................. 39

B. Ronne (CERN) - CERN experience in using THRESH for heavy
liquid chambers. .............................................................................................. 47

Secretaries: J. Howie, O. Skjeggestad.

Programmes from other laboratories:

M. di Corato (Milan) - The geometry programme for heavy liquid in
use at the University of Milan. ....................................................................... 53

J. Hennessy (Ecole Polytechnique, Collège de France) - The geometry
programme Bulle 0 of the Ecole Polytechnique and experience in
its use. ........................................................................................................... 59

J. Sparrow (Rutherford Laboratory) - The heavy liquid geometry programme
in use at the Rutherford Laboratory. .............................................................. 93

A.E. Werbrouck (Turin) - RANGE programme. ............................................. 119

Secretaries: G. Myatt, K. Soop.

Afternoon Session  Chairman: G. Tomasini

M. Huybrechts (Laboratory for High-Energy Nuclear Physics in
Brussels) - A new geometry programme. .................................................... 147

Electron measurements:

L. Behr (Ecole Polytechnique) - Mesure par courbure de l'énergie des
electrons dans les chambres à liquides lourds. ........................................... 157

H. Burneister (CERN) - Measurement of the total track length of
electron showers using a new first approximation in THRESH. ..................... 167

O. Czyżewski (CERN) - Measurement of γ-ray energy in a Xe bubble
chamber. ......................................................................................................... 173

Secretaries: D. Cundy, N. Paty

General discussion under the chairmanship of Y. Goldschmidt-Clermont. .... 185

Secretaries: E. Fett, B. Ronne

List of Participants ....................................................................................... 199
MEETING ON THE ANALYSIS OF PHOTOGRAPHS OF
HEAVY LIQUID BUBBLE CHAMBERS

INTRODUCTION

The computers and their programs are today a part of the equipment of the experimental physicists working in the field of high energies, particularly with bubble chambers. The constant efforts on the development of the methods of analysis of bubble chamber photographs are of the same nature as the efforts made to perfect other aspects of the experimental apparatus; better instruments open the way to better experiments.

The geometry programs for the analysis of photographs of heavy liquid chambers, which are the subject of the present day of discussion, have been developed over several years. They have been used for many experiments which have been successfully completed. Yet, the problem cannot be considered as solved, instead the field is very dynamic. Physicists have learned by experience where lay the limitations to their present methods, where the programs of computation give results of insufficient reliability, where improvements are needed to increase the accuracy.

The programs available today are perhaps more satisfactory for the hydrogen bubble chambers than for the heavy liquid chambers, which present difficulties of their own. These are connected with the higher density and the shorter radiation length, giving rise to a higher rate of energy loss, to more single and multiple scatterings,
to sudden losses of energy by radiation. Yet the fundamental physical processes are the same for both types of chambers and it may well be that some of the advances made in solving the problems which appear today as sore points for the heavy liquids will be of later use also for hydrogen. Other problems are connected with the methods of measurement, which are also very similar for all types of bubble chambers.

The bubble chamber experiments are constantly increasing in statistical accuracy: the detailed investigation of high energy phenomena requires increasing numbers of events. The systematic errors which may be introduced by the methods of analysis must be correspondingly kept in check, approximations which are now satisfactory may become progressively too coarse, several proposed improvements which appear superfluous today may become essential tomorrow. These remarks apply not only to the determination of the quantities measured - such as angles, ranges, radii of curvature - but also to the evaluation of the errors of measurements. A precise evaluation of these experimental errors is crucial for the correct interpretation of the results of the fitting procedure used in the kinematics programmes.

These problems will become even more important in the near future, when the more automatic methods of measurement, such as the HPD, will enable the physicists to increase by an order of magnitude the number of events analysed in their experiments.

The present day of discussion between experts of many laboratories working in this field gives them an excellent opportunity to compare methods and to exchange ideas. A prompt publication of the papers and of the discussions will give access to this useful material to those who were unable to attend the meeting. The
papers and discussions are thus less expected to present achievements and results than, instead, to provide a basis for further thought, development and progress.

It is a pleasure to acknowledge the work of Dr. Nikolic, ably assisted by Mrs. Cooper, who organised the meeting, and of the scientific secretaries who collected the papers and wrote down the discussions for prompt publication.

Y. Goldschmidt-Clermont.
THRESH - BASIC CORN GEOMETRY PROGRAMME

by

Miss A.M. Chops

General description of THRESH for heavy liquid chambers

THRESH was written for the geometrical reconstruction of events in hydrogen, so that we can assume that tracks are very close to helices.

However, before any helix-fitting is done, points are reconstructed in space, taking into account the refraction of the light rays in the different media. These general principles can also be used for heavy liquid events.

We could consider THRESH as divided into two parts:
- the first one consisting in reconstructing points in space,
- the second one, consisting of the helix fit through these points.

I will describe THRESH as it is used up to now, thus with the helix fit, but for heavy liquid this helix fit could be replaced by a more adequate method, as the one proposed by M. Huybrechts.

The first part has also to be improved to give more accurate reconstructed points in space. This was not imperative
for tracks in hydrogen, because these reconstructed points are only used to find a first approximation of the helix.

Improvements which are proposed, are

1) the choice of the 2 best views is made for each measurement in turn, as prescribed by H. Burmeister;

2) a secondary order interpolation to find corresponding points in space, as prescribed by W.G. Moorhead.

A very general flow of the THRESH for heavy liquid is shown on page 35, while a more detailed flow, with the characteristics of the different programmes now used, is given on page 37.
Introduction

The programme described here is the IBM 709 version of the Mercury programme, described by W.G. Moorhead in (ref. 5) to which I shall make frequent reference.

The original translation of the Mercury programme into Fortran was done at Saclay, as part of a collaboration. The Saclay programme is called GAP 2 (ref 4), and has been considerably modified for CERN needs, to become THRESH.

For several months a new set of programmes (ref.1) has been used in the IEP group to analyse bubble chamber events. These programmes are REAP, THRESH, GRIND and COOK.

REAP is a Ferranti-Mercury programme which reads the paper tapes coming from the measuring machines, orders the data in reconstruction lists, and gives an output in BCD onto a magnetic tape. The reading and sorting part of this programme consists of the input programme of G.R. Macleod (ref.2). The way in which the measurements have to be done and conventions for labelling points and tracks are given in the report (ref.2). The magnetic tape, being REAP output, contains for each event a set of BCD records.

THRESH reads this magnetic tape, one event at a time, and does the geometrical reconstruction in space of each event. THRESH is a programme written for IBM 709, the calculations being written in FORTRAN, and the routines dealing with output on tapes in FAP (ref.3). This report handles the way in which the event is reconstructed in space. More details about the programme will be found in the manual.

GRIND and COOK are finally the programmes which analyse physically the events (ref. 5).
General description of THRESH

The geometrical reconstruction in space of an event is made, using measurements on a maximum of 4 stereoscopic views.

It is assumed that the bubble chamber is in a uniform magnetic field, perpendicular to the front glass, so that the charged particles can be considered to describe helices. For each track, THRESH finds the parameters of this helix, and its location in the bubble chamber with respect to an orthogonal axis system (see fig 1) of which the x – y plane is defined by the position of fiducial marks. The measurements on the different views are all transformed to a common reference system which is defined by the apparent positions of the fiducial marks on the back of the front glass. The data needed to make up this reference system e.g. camera co-ordinates, refractive indices and thicknesses of the different media, are kept in a "TITLE 1", and are relevant for a whole series of events, i.e. an experiment. "TITLE 1" for the different experiments are kept on a magnetic tape; the "General Data tape" from which THRESH selects the TITLE 1 appropriate for the events being reconstructed.

The measurements of a whole event are read from the output tape from REAP, or any other tape which gives this data in the same format, and the reconstruction is made. In general outline, the method is followed which was used in the previously used geometry programme (ref. 6). The least squares helix fitting procedure is still used, but elsewhere some modifications have been made, of which the most important is a different method of finding the first approximation of the helices.

A flow diagram gives the sequence in which the calculations are done in THRESH. More details about the programme itself can be found in the THRESH manual, in which future changes will be noted.
I. The reference system

THRESH starts by reading the serial number of the event to be processed, and checks if the TITLE 1 which has to be used for this event is already into store. If it was not, it reads the correct TITLE 1 and does the following calculations, which are used for all the events, using this TITLE 1.

1) Sines and cosines of the angle between the x-axis and the line joining each pair of cameras.

2) The apparent positions $(F', G')$ (see fig. 1) of the fiducial marks on the plane $z = 0$. The formulas given in CERN 60-33 (p.33) are used.

When these calculations are finished the complete event is read in and stored.

---

Fig. 1

$F, Q, R$ are the co-ordinates of the camera \( \) given in TITLE 1

$F, G, H$ the co-ordinates of a fiducial mark \( \) (here on the back glass)

$F', G'$ co-ordinates of the apparent position of that fiducial mark.

The axis system is: $z = 0$ in the back of the frontglass

$z$ is positive towards the cameras.
II. Coefficients of transformation to the reference plane $z = 0$

For each view in turn we find the coefficients, which describe the transformation of IEP measurements on a photograph, to the reference plane $z = 0$. We consider the most general transformation

$$ F' = \alpha_1 + \alpha_2 F + \alpha_3 G \quad \text{(II.1)} $$
$$ G' = \alpha_4 + \alpha_5 F + \alpha_6 G $$

where $F, G$ are the IEP measurements, $F', G'$ the apparent positions on $z = 0$ plane.

Equations (II.1) are solved for each view, by least squares to give $\alpha_1$, using the $(F, G)$ of all the measured fiducial marks in the view. We have already calculated independently $(F', G')$ for these points, as described in the previous section.

The method of least squares is used to give an estimate of the quality of the measurements, because by back substitution (in II.1) we find again $F', G'$, which are compared with the values found in the previous section (the tolerance is given in the TITLE 1). We obtain a test on the constancy of the film position when some fiducials are measured before and some after the rest of the view.

To find 6 independent $\alpha_i$ by least squares, we need at least 4 measured fiducial marks. However, when only 3 fiducials are measured, the programme will also find the transformation coefficients, but then we assume two more conditions

$$ \alpha_2 = \pm \alpha_6 $$
$$ \alpha_3 = \pm \alpha_5 \quad \text{(II.2)} $$
which mean that the co-ordinate system on the photograph and the system in the chamber defined by the TITLE I have to be orthogonal, and the magnification to go from one system to the other has to be the same in $x$ and $y$ direction. The upper signs are used when the systems are either both right-handed or both left-handed - the lower signs are used when one system is right-handed and the other left-handed.

Using conditions (II.2), we can again do the back-substitution and have an idea of the quality of the measurements.

When only two fiducials are measured in a view, it would still be possible to find the transformation coefficients with conditions (II.2), but in the TITLE should be mentioned which pair of signs is needed.

III. Reconstruction of labelled points

A light-ray between a point in space and its measurement on a photograph is described in the chamber by the equations

$$ x = \frac{F_x}{x} + G_x $$  \hspace{1cm} (III.1)  \\
$$ y = \frac{F_y}{y} + G_y $$  \hspace{1cm} (III.2)

The formulas which give the coefficients $F_x, G_x, F_y, G_y$ are given in CERN 60-53 (p.31 (I.3), (I.4)). We call the line, which is described by (III.1) and (III.2), a reconstruction line. For each measurement we can find such a 'reconstruction line'.

If a point is measured in at least two views, its co-ordinates in space $(x, y, z)$ are found as the intersection of its reconstruction lines. Solving a system of equations (III.1), (III.2) by least squares, using all the measurements for this point, we find $x, y, z$ with their standard errors $\Delta x, \Delta y, \Delta z$. 
If \((\Delta x + \Delta y + \Delta z)\) is greater than a given value, given in the TITLE 1, the result is rejected and the reconstruction is tried again using one view less. If none of the possible combinations of two views give a good reconstruction (i.e., \(\Sigma \Delta > \) tolerance), the point is only reconstructed if it is the beginning point of a track, since we need the co-ordinates of the beginning point to make a helix fit to the track. We proceed then in the following way:

We ignore the measurements given for this point, and instead we use the first measurement on the track in one view for which we find the co-ordinates in space \((x, y, z)\), by the method of near correspondings points (see Appendix A) using the two first measurements on the track in another view. This method is also used when the beginning point of a track is not measured as a labelled point.

If a labelled point is measured only in one view, this point will again be reconstructed only if it is a beginning point of a track; the method of corresponding points is used again but using the given measurement.

IV. Reconstruction of tracks

For each track the following steps are followed:

1) Some checks are made on the measurements of the track on each photograph.

2) The coefficients \(F_x, G_x, F_y, G_y\) of the reconstruction lines are found for each measurement.

3) The points in space are found, corresponding with all the measurements of one view.

4) A helix is fitted through these points, and its parameters used as a first approximation.
5) Errors are calculated using the first approximation.
6) The best helix is found by least squares using the measurements on all views.

1. Checks made on the photographs

a) The measurements $X_1, Y_1$ of the track are arranged by considering the distance of these measurements to the measurement of the apex, (or the first point on the track if the apex was not measured in that view.) The measurements are put in order of increasing distances from the beginning point. If two measurements at the same distance from the beginning point, only the first will be kept.

b) We define a new orthogonal axis system $(X_T, Y_T)$ so that $X_T$ passes through the first and last measurement on the track. Therefore we rotate the $X$ axis through an angle

$$\arctan \frac{Y_N - Y_1}{X_N - X_1}$$

In the system $(X_T, Y_T)$, a circle is fitted to the measurements by least squares.

$$a_0 + a_1 X_T + a_2 (X_T^2 + Y_T^2) = Y_T \quad (IV. 1, 1)$$

If the measurement which is at the greatest orthogonal distance away from this circle, is further away than the tolerance in fringes (given in TITLE 1), this measurement is rejected and the circle refitted. If a second bad measurement is found, the view will not be used to reconstruct the track.
2. **Reconstruction line for each measurement**

For each measurement-pair on the track we then find the coefficients $F_x', G_x', F_y', G_y'$ which describe the light-ray

\[
\begin{align*}
  x &= F_x'z + G_x' \quad \text{(IV.2,1)} \\
  y &= F_y'z + G_y' \quad \text{(IV.2,2)}
\end{align*}
\]

They are found as in CERN 60-33 (p.31 (13), (14)) as mentioned in Section III.

At this point we check if the beginning point of the track was reconstructed. If it was not found as a labelled point, we find it by the method of near corresponding points, as described in Section III.

3. **Corresponding points on the helix**

To find the first approximation of the track in space we use the method of near corresponding points, as described in Appendix A.

We have first to choose the two best views, and we will find the points in space which correspond with the measurements in view $\alpha$.

a) View $\alpha$ is chosen to be the one on which we see the track most nearly as an orthogonal projection, i.e. that in which the average value of $F_x'^2 + F_y'^2$ over all measurements is smallest.

b) For view $\beta$ we choose the one, such that the line joining the two cameras $\alpha$ and $\beta$ makes the greatest angle with the tangent to the track at the beginning point.

* For a very curved track it is better to choose view $\beta$ again for each measurement.*
We write this condition as:

\[ \sin (\phi - \frac{\theta}{a_1, b_1}) \text{ is maximum.} \]

where \( \phi \) is the angle between the x-axis and the tangent to the track at the considered point. We find an approximation to this angle \( \phi \) by fitting a circle which passes through \((A, B, C)\) \((A, B, C \text{ being the co-ordinates of the beginning point})\), and the intersections \((x_i, y_i)\) of all the light-rays describing the measurements in view \(\alpha\), with the plane \(z = C\).

\[(x - A)^2 + (y - B)^2 + \lambda_1 (x - A) + \lambda_2 (y - B) = 0 \quad (IV. \ 3,1)\]

The centre of this circle is \((A - \frac{\lambda_1}{2}, B - \frac{\lambda_2}{2})\)

therefore

\[\tan \phi = -\frac{x - (A - \frac{\lambda_1}{2})}{y - (B - \frac{\lambda_2}{2})} \quad (IV. \ 3,2)\]

For each measurement on view \(\alpha\), we must find the two nearest corresponding measurements on view \(\beta\). Between these two we find by linear interpolation the reconstruction line in view \(\beta\) for the point whose measurement we consider in view \(\alpha\). In this way we find a set of points \(x_i, y_i, z_i\) with which we find a first approximation to the helix.
4. **First approximation of the helix**

The helix which is fitted is described by:

\[ x' = \rho \cos \theta - 1 \]  \hspace{1cm} (IV. 4,1)

\[ y' = \rho \sin \theta \]  \hspace{1cm} (IV. 4,2)

\[ z' = \rho \theta \tan \alpha \]  \hspace{1cm} (IV. 4,3)

The axis system \((x', y', z')\) is the original system \((x, y, z)\) rotated through an angle \(\beta\) about the \(z\)-axis and translated to a new origin \((A, B, C)\). (See fig. 2).

The parameters which we have to find are:

- \(\rho\) = the radius of the helix
- \(\tan \alpha\) = tangent of the dip angle
- \(\beta\) = the azimuthal angle of the beginning point, considering the \(x\)-axis as zero
- \(A, B, C\) the co-ordinates of the beginning point of the track, and which are considered as known in first approximation, calculated in Section III.

\(-\rho\) and \(\beta\) are found by fitting a circle by least squares through the projections of the points \((x_i, y_i, z_i)\) on the plane \(z = 0\).

\[ (x_i - A)^2 + (y_i - B)^2 + \lambda_1 (x_i - A) + \lambda_2 (y_i - B) = 0 \]
thus

\[ \rho = \frac{1}{2} \sqrt{\lambda_1^2 + \lambda_2^2} \]  

(IV. 4,4)

\[ \beta = \arctan \frac{\lambda_2}{\lambda_1} \]  

(IV. 4,5)

To find an approximation for \( \tan \alpha \), we have to assign an azimuthal angle \( \theta_i \) to each reconstructed point in space

\[ \theta_i = \arctan \frac{y_i - (b - \frac{\lambda_2}{2})}{x_i - (a - \frac{\lambda_1}{2})} - \beta \]  

(IV. 4,6)

For angle \( \theta_i \) smaller than 0.02 we use instead

\[ \theta_i = \frac{(y_i - b) \cos \beta - (x_i - a) \sin \beta}{\rho} \]  

(IV. 4,7)

This gives us for \( \tan \alpha \) using (IV. 4,3)

\[ \tan \alpha = \frac{\Sigma_i z_i \theta_i}{\rho \Sigma_i \theta_i^2} \]  

(IV. 4,8)

where \( \Sigma_i \) is taken over all points found in space.

6650/p
5. Errors on first approximation

We can estimate errors on the parameters of the track in first approximation. These will be given as results in the case that the next step in fitting a helix (IV. 6) to the track does not give satisfactory results.

We can calculate variances \( \sigma_{\lambda_1}^2, \sigma_{\lambda_2}^2 \) and co-variance \( \sigma_{\lambda_1,\lambda_2} \) upon \( \lambda_1, \lambda_2 \) the two parameters of the fitted circle from which we deduced \( \rho \) and \( \beta \). Since \( \rho \) and \( \beta \) are functions of \( (\lambda_1, \lambda_2) \) we can use the relation

\[
\sigma_f^2 = \left( \frac{\partial f}{\partial \lambda_1} \right)^2 \sigma_{\lambda_1}^2 + \left( \frac{\partial f}{\partial \lambda_2} \right)^2 \sigma_{\lambda_2}^2 + 2 \frac{\partial f}{\partial \lambda_1} \cdot \frac{\partial f}{\partial \lambda_2} \sigma_{\lambda_1 \lambda_2}
\]

and for co-variances

\[
\sigma_{f_1 f_2}^2 = \frac{\partial f_1}{\partial \lambda_1} \cdot \frac{\partial f_2}{\partial \lambda_1} \sigma_{\lambda_1}^2 + \frac{\partial f_1}{\partial \lambda_2} \cdot \frac{\partial f_2}{\partial \lambda_2} \sigma_{\lambda_2}^2 + \frac{\partial f_1}{\partial \lambda_1} \cdot \frac{\partial f_2}{\partial \lambda_2} \sigma_{\lambda_1 \lambda_2}
\]

Using these formulas upon (IV. 4,4) and (IV. 4,5), we obtain

\[
\Delta \rho = \sigma_{\rho} = \frac{1}{2 \rho} \left[ \lambda_1^2 \sigma_{\lambda_1}^2 + \lambda_2^2 \sigma_{\lambda_2}^2 + 2 \lambda_1 \lambda_2 \sigma_{\lambda_1 \lambda_2} \right]^{\frac{1}{2}}
\]

\[
\Delta \beta = \sigma_{\beta} = \frac{1}{4 \rho^2} \left[ \lambda_1^2 \sigma_{\lambda_2}^2 + \lambda_2^2 \sigma_{\lambda_1}^2 - 2 \lambda_1 \lambda_2 \sigma_{\lambda_1 \lambda_2} \right]^{\frac{1}{2}}
\]

\[
C_{\rho, \beta} = \sigma_{\rho \beta} = \frac{1}{16 \rho^3} \left( - \lambda_1 \lambda_2 \sigma_{\lambda_1}^2 + (\lambda_1^2 - \lambda_2^2) \sigma_{\lambda_1 \lambda_2}^2 + \lambda_1 \lambda_2 \sigma_{\lambda_2}^2 \right)
\]

For \( \Delta \tan \alpha \) we use

\[
\Delta \tan \alpha = \frac{\Sigma (z_i - \rho \theta_i \tan \alpha_i)^2}{\rho^2 \Sigma \theta_i^2 (N - 1)}
\]
6. Final least squares fit of the helix

We found an approximate value for \( \rho, \tan \alpha, \beta, A, B, C. \) For each measurement we have also its reconstruction line (IV. 2). The final fit consists in finding small corrections to the coefficients of the helix so that the equations (IV. 2,1 and IV. 2,2) and the equations of the helix (IV. 4, 1 to 3) are simultaneously satisfied. The method is described in CERN 60-33 (section 5.4). To apply this method we have to associate an azimuthal angle \( \theta_i \) with each measurement \( i. \)

\[ \theta_i \text{ is given by} \]

\[ \theta_i = \arctan \frac{y_i - (B - \rho \sin \beta)}{x_i - (A - \rho \cos \beta)} - \beta \quad \text{(IV. 6,1)} \]

in which \( \rho, \beta, A, B \) are the results from the first approximation. \( x_i, y_i \) are found with formulas (IV. 2,1 and 2) in which we start for the first measurement with \( z_1 = C. \) So we find \( \delta_1 \) with (IV. 6,1) and hence a better \( z_1 \) with (IV. 4,3), and thus a better \( \theta_1. \) We are thus doing the process twice, the first time using the \( z \) value of the previous measurement.

This last fit of the helix is an iterative process which converges in normal cases. However, if this process does not converge, we use the first approximation (Section IV. 4) as a satisfactory result.

**Straight tracks**

The assumption is made that a track which should be reconstructed as straight, has for its second label a letter which is further in the alphabet than a letter given in TITLE 1.

6650/p
For these tracks the radius $\rho$ is fixed to a large value which is given in Table 1.

The calculations followed are the same as those for curved tracks except for the following:

1. for finding view $\beta$ (Section IV. 3), we take for $\Phi$

$$\Phi = \frac{\Sigma Y_1}{\Sigma X_1}$$

2. for the first approximation we have

$$\beta = -\frac{\Sigma (x_1 - A)}{\Sigma (y_1 - B)}$$

and $$\theta_i = \frac{\sqrt{(x_1 - A)^2 + (y_1 - B)^2}}{\rho}$$

3. in the least squares fit (Section IV. 6), the corrections to $\beta$, $\tan \alpha$, $A$, $B$, $C$ are found in the same way as for curved tracks, but $\Delta \rho$ is not found, since $\rho$ is fixed to a constant value.
V. Final remarks

1. About errors calculated in THRESH

The errors given for the parameters of the helices and points are the standard errors and co-variances resulting from the least squares method as described in (App. III of ref. 6). Work is being done to find an estimate of the external errors (ref. 7 and 8).

**FIG. 2**
APPENDIX A.

Method of near corresponding points

This method is used to find the co-ordinates \((x_i, y_i, z_i)\) for the measurements on a view \(\alpha\) of a track.

Given the measurement \((X^\alpha, Y^\alpha)\) defining \(F_x^\alpha, G_x^\alpha, F_y^\alpha, G_y^\alpha\) in the equations of the reconstruction line

\[
x = F_x^\alpha z + G_x^\alpha
\]
\[
y = F_y^\alpha z + G_y^\alpha
\]

The method consists in finding by interpolation between all the values

\[
F_x^\beta, G_x^\beta, F_y^\beta, G_y^\beta
\]

\(x_i, y_i\) for the measurements in an other view \(\beta\), a set of values \(F_x^\beta, G_x^\beta, F_y^\beta, G_y^\beta\) such that

\[
x = F_x^\beta z + G_x^\beta
\]
\[
y = F_y^\beta z + G_y^\beta
\]

intersects the line (5) in space.
The condition for intersection of lines (5) and (6) can be written as

\[
\phi \equiv \begin{vmatrix} F^\alpha_x - F^\beta_x & F^\alpha_y - F^\beta_y \\ G^\alpha_x - G^\beta_x & G^\alpha_y - G^\beta_y \end{vmatrix} = 0
\quad (7)
\]

The function \( \phi \) is evaluated for the measurements on view \( \beta \), in succession until there is a change in sign between two successive measurements \( i, i + 1 \). A linear variation of \( F \)'s and \( G \)'s is assumed between the two measurements \( i, i + 1 \). Thus we can write

\[
F^\beta_x = F^\beta_{x_1} + \lambda (F^\beta_{x_{i+1}} - F^\beta_{x_1}) \quad \text{and similarly for}
\]

\[
F^\beta_y, G^\beta_x, G^\beta_y.
\]

Hence we can write (7) as

\[
\phi (\lambda) = 0
\]

We find \( \lambda \) which satisfies this equation by successive approximations, starting with \( \lambda_0 = 0 \) and \( \lambda_1 = 1 \). We get a better value \( \lambda_2 \)

\[
\lambda_2 = \frac{\lambda_0 \phi (\lambda_1) - \lambda_1 \phi (\lambda_0)}{\phi (\lambda_1) - \phi (\lambda_0)}
\]

6650/p
which replaces $\lambda_0$ or $\lambda_1$, depending on whether $\phi(\lambda_0)$ has the same sign as $\phi(\lambda_0)$ or $\phi(\lambda_1)$. Having found $\lambda$, $F_x^\beta$, $G_x^\beta$, $F_y^\beta$, $G_y^\beta$ are calculated and hence, from eq.(5)

$$z = - \frac{G_x^\alpha - G_x^\beta}{F_x^\alpha - F_x^\beta} = - \frac{G_y^\alpha - G_y^\beta}{F_y^\alpha - F_y^\beta}$$

(8)

One of the fractions in eq.(6) can takes the form $\frac{G_y^\alpha - G_y^\beta}{F_y^\alpha - F_y^\beta}$, entirely due to choice of co-ordinate axis. In this case $z$ is taken as the value of the other fraction of eq.(8). Finally we find $x$ and $y$ by the formulas (5).

This method is also used to find the co-ordinates of the beginning of a track as mentioned in Section III.
APPENDIX B

Details on the existing programme in March 1963

I give here a short description of what each subroutine does:

Main programme: contains print instructions at beginning of run reading call cards and select cards.

NOF: is a FAP routine which reads into an array NO (18) all the possible combinations in groups of 2 and 3 of the integers 1, 2, 3, 4. This to be able to retry reconstruction of points with different views.

EVENT 2: is the reading routine for an event.

GI 2: reads into store the TITLE 1 required for the event.

TITRE: does calculations I, to set up a reference system.

GEOM 2: does calculations II.

MC: is called by different routines to construct the normal system of linear equations in the method of least squares.

BRER: is a routine to solve a system of linear equations.

ERREUR: finds the error matrix of the unknowns of the linear system.

GEOM 6: reconstructs the points in space, if at least 2 views are given.
FGXGXY : finds the coefficients of the reconstruction lines (III, IV.2).

GEOM D : does calculations IV.1.

ARRAN : does calculation IV.1, c.

GEOM E : finds an approximation for the apex of a track when it was not found by GEOM G.

POICOS : finds the co-ordinates of a point in space, knowing two near corresponding measurements on View β.

LAMB 12 : finds the parameters λ₁, λ₂ of the circle IV.3, l.

GEOM G : finds the near corresponding measurements on View β.

GEOM H : finds IV.6.1.

LSHELX : does the final least squares helix fit.

GEOM PR : is the BCD output routine.

SHUNT : places the results in order to go into Grind.

OP : FAP output routine
    entries (INTOP (OPCHK (OP

MXBQU : an entry into MXPACK : a FAP programme handling with matrices.
    This entry is to solve a linear system.
Faults and rejections

A whole event will only be rejected if:

1) no call cards are given for the experiment
2) the required TITLE 1 is not on the General data tape
3) the output tape of REAP is bad
4) less than two views are available to do the reconstruction - a view can be rejected if:
   a) not enough (<3) fiducials are measured
   b) a fiducial mark is badly measured

A labelled point can have:

1) not enough measurements
2) not enough good views for the point
3) errors on the co-ordinates too large

A track will be reconstructed if at least two good views are available.

A view can be rejected for different reasons:

1) reasons giving fault 1 or 2
2) more than 50 points measured on the track
3) less than 3 points
4) less than 3 points after rejecting a point (IV.1, b)
5) more than 1 point cut of tolerance (IV.1. b)  
6) the distance of the apex to the circle (IV.1.1) is greater than the tolerance, and more than one view is given for the apex  
7) the radius of circle (IV.1.1) is < 0

When a track is not reconstructed its parameters are put to zero, and its radius put to a negative big number (10 x max. meas. radius in TITLE 1).
REFERENCES

1. DD/IED/61/37 New IEP programmes with library facilities. G.R. Macleod
2. CERN 60-11 Input programme for measurements of track chamber photographs. G.R. Macleod
3. DD/EXP/63/1 The tape routines for THRESH, GRIND and COOK. R. Lorkin
4. Programme GAPZ, Saclay
5. GRIND manual
6. CERN 60-33 A programme for the geometrical reconstruction of curved tracks in a bubble chamber. W.G. Moorhead
7. DD/IEP/61/34 Tentative to define a "good measurement" and realistic error matrix. E. Fett, L. Montanet
8. DD/IEP/61/30 Note on errors on reconstructed tracks in bubble chambers. W.G. Moorhead
DISCUSSION FOLLOWING THE TALK OF MISS A. M. CNOPS

Werbrouck: What is the approximate reconstruction time in THRUSH for a track about 50 cm long and measured on 3 views?

CNops: One event consisting of 4 - 5 tracks uses 20 - 30 seconds of IBM 709 time, including input and output. The greatest part of this time is taken by the reconstruction.

Glasser: What is the main reason why tracks fail to converge in THRUSH?

CNops: The main reason is because the first approximation is bad, and this could be due to the wrong choice of views.

Goldschmidt-Clermont: For about 50% of the cases where a track does not converge, we seem to have a good first approximation.
Data Input

Transformation coefficient to reference system

Reconstruction of labelled points

Checks on photographs

for each track

Choice of two best views for track

Corresponding points in space

Curve fitted

Output

Burmeister

Moorhead

Huybrechts
<table>
<thead>
<tr>
<th>Column 1</th>
<th>Column 2</th>
<th>Column 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data 1</td>
<td>Data 2</td>
<td>Data 3</td>
</tr>
<tr>
<td>Data 4</td>
<td>Data 5</td>
<td>Data 6</td>
</tr>
</tbody>
</table>

### Table Explanation

<table>
<thead>
<tr>
<th>Description</th>
<th>Details</th>
</tr>
</thead>
<tbody>
<tr>
<td>Column 1</td>
<td>Detailed data and statistics.</td>
</tr>
<tr>
<td>Column 2</td>
<td>Additional context and notes.</td>
</tr>
<tr>
<td>Column 3</td>
<td>Summary of key results.</td>
</tr>
</tbody>
</table>

**Notes:***

- Data 1 is derived from experimental results. |
- Data 2 correlates closely with theoretical predictions. |
- Data 6 requires further validation through additional experiments.
A POSSIBLE SIMPLIFIED FITTING PROCEDURE IN THRESH

FOR HEAVY LIQUIDS

by

W. G. Moorhead

The least squares helix fit in space carried out in THRESH is not suitable for heavy liquids because the particles are deflected so much, due to multiple scattering and energy loss, that they do not follow helices. In practice, there are too many cases where this fit does not converge.

One might stop at what is now the first approximation. But, referring back to the discussion following the talk of Miss Gnops, concerning the first approximation in THRESH, there are three reasons why this might not be adequate as a final answer.

1) Only a linear interpolation is used to find the corresponding points.

2) The circle which is fitted through the projections of the corresponding points is forced to pass through a definite point (i.e. the beginning point of the track), which is really too big a restriction.

3) Also the track ceases to be a circle after a certain distance. An optimum length ought to be used for fitting this circle.

The FOC programme of Berkeley was originally written for heavy liquids. It is proposed to take some of its features and incorporate them in a special version of THRESH for heavy liquids.
FOG is part of the FOG-CLOUDY-FAIR bubble chamber data processing system designed by Howard White. What is stated here can be found in reports on that system, in particular the FOG, and CLOUDY-FAIR manuals (LRL memo. P 101) and an LRL report (The Fog, Cloudy and Fair programmes for bubble chamber data reduction by H.S. White et al).

First of all FOG finds "near corresponding points" in space by a second-order interpolation from one view to another. Optimum lengths are then used for finding the radius (by a parabola fit) and the azimuthal and dip angles.

The FOG method of finding near corresponding points is described in an appendix to this paper.

For measurement of the radius of curvature a maximum arc length of 60° is taken. The track is rotated in the xy plane into a new $\xi \eta$ system in which the $\xi$-axis lies along the chord. The parabola $\eta = a \xi^2 + b \xi + c$ is fitted. It is stated in ref. 2 (see above) that the correction to the radius of this parabola \( R_p = \frac{1}{2} a \), in order to give the equivalent circle radius \( R_c \) is

\[
R_c = R_p \left[ 1 + \frac{1}{16} \left( \frac{L}{R_p} \right)^2 \right]
\]

and that this factor remains near unity if a limit of 60° of arc is chosen.

The optimum length for finding the azimuthal angle $\beta$ is given by

\[
L_{\text{opt}}^\beta = 0.165 \left( X_0 \right)^{1/3} \left( \frac{E}{100 \text{eV}} \right)^{2/3} \left( \frac{\epsilon}{25} \right)^{1/3}
\]

where $X_0$ is the radiation length of the medium, $\epsilon$ is the "RMS error of the end points" and the other symbols have their usual meanings.
For the dip angle:

\[ L_{\text{opt}}^\beta = \left( \frac{\varepsilon_2}{\varepsilon_x} \right)^\frac{2}{5} L_{\text{opt}}^\beta \]

where \( \varepsilon_2, \varepsilon_x \) are the respective errors in \( z, x \).

The method of finding the azimuthal angle is as follows. One takes the next measured point \( B \), beyond \( L_{\text{opt}}^\beta \) and calculates \( \beta_1 \). By fitting a circle of radius \( R \) through \( A \) and \( B \), \( \beta_2 \) is found. \( \beta = \beta_1 \times \beta_2 \).

![Diagram](image)

**Fig. 1**

Huybrechts will describe a weighting method of arriving at optimum lengths. 

If these optimum length fitting procedures are used in THRESH, it is felt necessary that THRESH should provide more accurate space points than those based on the linear interpolation between successive measurements which are at present found and used for finding a first approximate helix.

Each linearly interpolated fit could be improved as follows:

6650/p
The second order interpolation can be carried out on the standard reference plane, i.e. the back of the front glass.

Suppose that a "corresponding point" \( A \) with coordinates \( G_{x}^{II}, G_{y}^{II} \) (and associated \( F_{x}^{II}, F_{y}^{II} \)) has been found on view 2 by linear interpolation between points \( i, i+1 \). A circle, or parabola, may now be drawn through points \( i, i+1, i+2 \) on view 2. It is required to find the point \( B \) on this curve, near \( A \) which will also give a corresponding point, i.e. intersect the same light ray from the other view.

As a first approximation \( B \) may be the intersection of the curve with the perpendicular to the chord through \( A, B \) will have coordinates \( G_{x}^{II}, G_{y}^{II} \). From these \( F_{x}^{II}, F_{y}^{II} \) are easily found.

Then linear interpolation may be used between \( i \) and \( B \) or \( B \) and \( i+1 \) and the procedure repeated until the difference between two successive \( z \)-values is sufficiently small.
1. All distances perpendicular to the front glass are replaced by $\sum \frac{d_i}{\mu_i}$ where $d_i$ is the path through the medium of refractive index $\mu_i$. Distances $\rho$ of the measured points on the film from the intersection of the film plane with the camera optic axis are found and corrected by

$$\rho' = \rho - a\rho^3$$

to give a third order optical term. 
"a" is a constant. It is dependent on the depth of the point measured in the chamber, but at first the centre of the chamber is assumed.
To this order and with this assumption the optical system may now be replaced by conical systems of rays passing through each camera nodal point.

2. The measured points may now be projected to a plane such as the back of the front glass.

When two views have been found for interpolation (such that the line joining the cameras makes the greatest angle with the track) the points are referred to a common axis system. This axis system is along and perpendicular to the line joining the feet of the optic axes in this plane.

3. Then, given a point in view 1, one has merely to interpolate in the points of view 2 to find an interpolated point with the same Y value. x, y, z in space are then easily found by similar triangles.

4. Now that an approximate Z value is known, the value of "c" used above can be improved and iteration used.
DISCUSSION FOLLOWING THE TALK OF W. G. MOORHEAD

Werbrouck: I would like to comment that in FOG only one iteration is necessary. This is of importance for the users of small computers.

Hennessy: The method of using an optimum length was invented by Taft, and has been used in Brookhaven over the last two years.
CERN EXPERIENCE IN USING THRESH FOR HEAVY LIQUID CHAMBERS

by

B. Ronne

PROGRAMMES USED BY THE HLBC GROUP AT CERN

The following chain of programmes is now used.

\[\text{REAP} \rightarrow \text{THRESH} \rightarrow \text{JOIN} \rightarrow \text{GRIND}\]

REAP reads the paper tape from the measuring machines and prepares the data in a suitable form for input to THRESH. REAP gives an output in BCD onto a magnetic tape. Many checks are made in REAP. Some types of measuring errors are thus detected and there is, furthermore, a control that the same label is not used twice in the same photograph or that a label has not been used without giving a photograph number.

REAP works in most cases very well in our experiment. We have recognised only one minor fault. It cannot treat 13 tracks. When it happens to be 13 tracks in an event, we have therefore to measure one additional dummy track.

A description of how REAP is constructed and how one has to label points and tracks is given in a report by MacLeod (CERN60-11).

THRESH is the geometrical reconstruction programme described by Miss A. M. Chops (page 5) and by W. G. Moorhead (page 39).

6650/p
JOIN  A particle stopping in heavy liquid is in general scattered so much that it is impossible to fit a long track to one helix. To get the range of such a track one has to divide it in several pieces and to fit each piece separately to a helix. These pieces are then connected to one track in JOIN. It may also be necessary in some cases to measure the track of a high energy particle in many pieces and to add up the information from the different pieces to get a good momentum determination.

The following example illustrates what is calculated in JOIN. In Fig. 1 is shown a track measured in 4 pieces, (3 curved and 1 straight). 

\[ \text{ANGLE gives for each piece} \]

the curvature (\( \rho \)), dip angle (\( \lambda \)) and azimuth angle (\( \phi \)) all quantities with errors.

JOIN calculates

a) The range AD with errors,

b) \( \lambda \) and \( \phi \) at production (A),

\( \lambda \) and \( \phi \) at decay point (D), if we have a \( V^+ \),

c) The mean momentum \( \langle p \rangle \) with errors,

d) \( L^S \), which is the length, measured from A, which corresponds to the mean momentum.

JOIN starts to combine the first two pieces to one track. This new track is then combined with the third piece to one track, and so on. In the case of two curved pieces we have for instance
\[
\begin{align*}
\frac{1}{\Phi_{AI}} &= \frac{1}{\Phi_{AI}} W_{AI} \times \frac{1}{\Phi_{IJ}} W_{IJ} \\
L_{AI}^S &= \frac{L_{AI}}{2} W_{AI} \times \left( L_{AI} + \frac{L_{IJ}}{2} \right) W_{IJ} \\
W_i &= \frac{L_i \cos^2 \lambda_i}{L_{AI} \cos^2 \lambda_{AI} + L_{IJ} \cos^2 \lambda_{IJ}} \quad i = AI, IJ
\end{align*}
\]

The weights \((W_i)\) are thus calculated from the multiple scattering errors in the measured momenta, and the internal errors in \(\rho_{AI}\) and \(\rho_{IJ}\) are not used. In the present version of JOIN we have assumed implicitly that the decrease of momentum due to ionization along the track is linear. For high momentum tracks this is a good approximation; for stopping tracks the momentum determined from range is far more precise than that from curvature.

JOIN works sufficiently well, but it may, of course, still be improved in one or more details. We have earlier had much trouble because the given errors in ranges have been too small. This fault is now eliminated.

GRIND is the kinematic programme used at CERN.

EXPERIENCES IN USING THRESH

THRESH has been used by us for about 6 months, mainly in the analysis of \(\Lambda_{\rho}\)'s and \(E^-\)'s from the T8 experiment. An ordinary \(E^-\)-event is shown in Fig. 2. We have 7 tracks per event. Some of these are divided in pieces so that the average number of measured pieces per event is 10. All events are measured twice.

6650/p
We have looked at the results from 150 \( \Xi^- \) measurements, i.e. 1500 tracks or pieces which have passed through THRESH. Excluded are all events where RMAP has found errors or where RMAP does not work well (13 pieces). Of the measured tracks, about 600 were taken as straight and 900 as curved. We find that the fraction of non-converging tracks is 2.9 \( \pm \) 0.7\% for straight tracks, 2.7 \( \pm \) 0.6\% for curved and 2.8 \( \pm \) 0.4\% for the total number of tracks. The probability that a track, which has not converged in the first measurement, will not converge in the second measurement also, is 20 \( \pm \) 5\%. Only in one case we have had to remeasure a track 3 times before it converged. Of course, these results were obtained only after months of experience in recognizing kinks which would have caused non-convergence.

From the figures above we find that only about 10\% of all \( \Xi^- \)-events (10 pieces measured twice) have had to be remeasured because tracks do not converge in THRESH. A much greater fraction of all events (about 40\%) have, however, been remeasured, mainly because of JOIN and GRIND troubles and to a minor extent because of preparation errors.

There exists now a programme (COMBINE), which can collect good tracks from different measurements and form a good total event. One may here, for instance, take the momentum of a particle from one measurement and the angles from another.
In Table 1, we give some typical internal errors given in THRESH. The measurements are performed on a SOM-machine.

<table>
<thead>
<tr>
<th>Measured quantity</th>
<th>Percentile</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>25th</td>
</tr>
<tr>
<td>$\Delta x$, $\Delta y$ mm</td>
<td>0.09</td>
</tr>
<tr>
<td>$\Delta z$ mm</td>
<td>0.50</td>
</tr>
<tr>
<td>$\Delta \phi$ degrees</td>
<td>0.15</td>
</tr>
<tr>
<td>$\Delta \lambda$ &quot;</td>
<td>0.40</td>
</tr>
</tbody>
</table>

Table 1.
DISCUSSION FOLLOWING THE TALK OF B. RONNE

Hennessy: How do you succeed in putting together all the pieces measured between kinks on a track? Do you make mass assumptions to take into account energy loss?

Ronne: We do not take into account energy loss.

Hesten: You showed the frequency of tracks that did not converge in THRESH for 1500 measured pieces. How many "real tracks" does this correspond to?

Ronne: A little more than 1000 tracks.

Rousset: I would like to comment that the length of measured pieces usually is shorter than the optimum length.

Sparrow: Do you measure a kink as a corresponding point?

Ronne: Yes, we try to do this as much as possible.
THE GEOMETRY PROGRAMME FOR HEAVY LIQUID IN USE

AT THE UNIVERSITY OF MILAN

by

M. di Corato

(University of Milan)

The programmes we use in Milan are Thresh for geometry and Gap for kinematics.

Thresh has not been changed from the original version, except for input and output.

Gap has been very much modified, essentially for reconstruction of gammas and analysis of radiative decays. Some of the modifications concern geometrical reconstruction of tracks. Our version of Gap is called Gap 990. The most important "geometrical" modifications are two:

(a) in order to measure a track AK it is not necessary to measure many points on the track, as in the normal way. If the physicist decides on the scanning table that the track is sufficiently straight, it is possible to measure only the initial point and the final point as normal points AA and KK. Then the track AK is "measured" as a dummy track. That means that it is not measured at all: we only punch the labels and the mass, if known (in Gap 990 there is the possibility of assigning a fixed mass to the tracks). This track will not be reconstructed by Thresh, because it is a track with no measured points (it will be assigned fault number 8). Gap will examine all the tracks that have failed in Thresh. If their second label belongs to a
particular class (class 18 of title 5) and if their initial and final points have been successfully reconstructed as points, then Gap will substitute to the wrong track AK the straight track connecting point A to point K. Sometimes one may prefer to try to measure a track as a normal track, but, if the reconstruction fails, it may be judged enough to have the two-points track. In that case the track is measured normally. If it is successfully reconstructed, then it is a normal track; if it fails, the two-point track is calculated as in the case of the dummy track. As I have said, this is done only if the second label belongs to class 18: for instance, it is done for the track AK, not for the track AG.

(b) The second difference from the hydrogen version of Gap is the possibility of breaking a track in two or more segments. The procedure is essentially described in (2). Also for partial tracks it is possible to use the procedure of dummy tracks. A track measured in many segments is given the angles of the first segment; the range is the sum of the ranges; the curvature is a weighted mean value of the curvatures obtained in all the segments where they were measured; the point to which the curvature measurement refers is a weighted mean of the partial measurements points.

Before passing our events to Gap we look at the printed results of Thresh and, if it is the case, we remeasure tracks that have failed. It is not necessary to remeasure all the event, because we have all the results punched on cards, and we can substitute the wrong tracks. Sometimes an intermediate track fails because the apex is not the same point on the 3 views. In that case it may not be necessary to remeasure the track, but it may be enough to discard the bad view. So what we do is to duplicate the event and to try the 3 different combinations of views. Frequently one combination is successful.

In order to give an idea of how things go in practice, I
give a table of the results of a small sample of our measurements, according to track length \( l \) and dip \( \lambda \)

<table>
<thead>
<tr>
<th></th>
<th>( 0 &lt; \lambda &lt; 20^\circ )</th>
<th>( 20^\circ &lt; \lambda &lt; 45^\circ )</th>
<th>( \lambda &gt; 45^\circ )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( l &gt; 10 \text{ cm} )</td>
<td>41 O.K.</td>
<td>14 O.K.</td>
<td>4 O.K.</td>
</tr>
<tr>
<td></td>
<td>1 no</td>
<td>3 no</td>
<td></td>
</tr>
<tr>
<td>( 5 \text{ cm} &lt; \lambda &lt; 10 \text{ cm} )</td>
<td>6 O.K.</td>
<td>3 O.K.</td>
<td>3 O.K.</td>
</tr>
<tr>
<td></td>
<td>3 no</td>
<td>3 no</td>
<td>1 no</td>
</tr>
<tr>
<td>( l &lt; 5 \text{ cm} )</td>
<td>7 O.K.</td>
<td>5 O.K.</td>
<td>2 O.K.</td>
</tr>
<tr>
<td></td>
<td>1 no</td>
<td>3 no</td>
<td>1 no</td>
</tr>
</tbody>
</table>

These results refer to partial tracks. Some of them are 2-points tracks. After second measurement all tracks but three were successfully reconstructed.

The liquid in the chamber is a mixture propane-freon of density 0.9 and radiation length 22 cm. The beam consists of K particles of 800 MeV/c.

Those measurements require a very long time to be performed because very frequently it is necessary to find intermediate corresponding points. An event with one primary, one K-zero and one or two gammas takes about one hour. Measurements were faster by a factor 2, and results better, in a previous experiment with incoming momentum 6 GeV/c and density 0.5.

In order to make measurements faster, especially in very high density mixtures, it would be certainly preferable to make the
machine itself look for corresponding intermediate points, and
measure a track in partial segments only in one view. We did not
do that because we preferred not to modify the reconstruction
method but to make the necessary changes on the reconstructed
points and segments of tracks before entering the kinematic pro-
gramme.
References


Henneisy: I see that you have used the Paris programme and the CERN programme. Have you measured events with both and compared the results?

M. di Corato: We have used the Paris programme with our previous mixture only, because we have not now the use of a Bull machine. Yes, these agree.

Henneisy: That's something!
THE GEOMETRY PROGRAMME BULLE 0 OF THE ECOLE POLYTECHNIQUE

AND EXPERIENCE IN ITS USE

by

J. Hennessy

(Ecole Polytechnique, Collège de France)

SUMMARY:
I  General
II  Arithmetic and Comments
III  Input/output formats
IV  Experience

1. GENERAL

The purpose of this programme is to provide a measurement of the momentum and directions (with errors) of bubble-chamber tracks measured by an apparatus of the IEP type (these IEP's can be of the "cartesian" or "bipolar" types). Tracks are measured on two views, and are given one or more mass-assignments.

The method used is as follows:

1) Compute the four parameters of a linear transformation which changes the coordinates of points measured on the film (by a "cartesian-type" IEP *) to coordinates linked to the chamber. As many of these sets of parameters are computed as there are views used in the event.

2) Transform the coordinates of the track points by means of the appropriate sets of parameters. One thus obtains two curves on the front window. The first to be computed is known as the "principal" curve, the other as the "auxiliary" curve.

3) Transform the curves thus obtained into a system of coordinates linked to the pair of cameras used for that particular track. Distinguish between "2 point tracks" ** and "other tracks".

4) Fit the two curves to a pair of parabolas. Reject bad points.

---

* For "bipolar-type IEP's, see Appendix A.
** For 2 points tracks see § 19).
5) Compute on "auxiliary" curve the coordinates of the points corresponding to those measured on the "principal" curve.

6) Compute in space the point whose projections are the two points thus obtained.

7) Reverse of § 3).

8) Check that points thus obtained are inside chamber. If not, reject them.

9) Fit cylindrical projection in front window to parabola. Reject bad points. Compute curvature of parabola and dip angle $\Phi$ of entire curve, also its length $2L$.

10) Compute magnetic field in middle of curve and at two points symmetrically placed towards both ends. Compute weighted average of these three fields, and momentum at central point of curve, $\vec{p}$.

11) Using mass assignment, for electron tracks, go to 20), for non-electron tracks, reduce momentum and length to comparable values for proton. Check that $1/2$ reduced length $\leq$ residual range for reduced $\vec{p}$.

12) a) It is. Then compute momentum loss (gain) at end (s) of track and reverse reduction § 11).

b) It isn't. Then suppose measurement of length is more accurate than measurement of curvature. Suppose therefore track stops at one end and compute momentum from range at the other. Reverse reduction

13) Compute $p\Phi$ at end (s) of track, and "optimum length" giving a minimum value to the quadratic sum of the "scanning" (in 1/L) and "scattering" (in $\sqrt{L}$ ) errors on the track directions $^*$.

$^*$ Actually the optimum length for the azimuth angle is not the same as for the dip angle. The length calculated in 13) is intermediate between the two.
Use as auxiliary point for angle measurements the one closest to the "optimum length" from the end of the track, but further away if possible (point M).

14) Compute dip angle $\Phi$, between end and point M (projected in m).

15) Use as first approximation of azimuth $\Theta$ the direction of the chord joining end to m, as second approximation the tangent to the circle passing at end and m, and whose radius is such that it gives the correct momentum. Correct this angle for energy loss (third approximation).

16) Compute error on sagitta, on $\Theta$, on $\Phi$, and print results.

17) Eventually repeat from §11) for other end of track other mass assignments.

18) Repeat from §2) for next track.

19) For 2-points tracks, compute space points corresponding to two ends and two views, then $\Theta$ and $\Phi$ and errors, print results and go to §18).

20) Electron computation. Go back to §18).

2. ARITHMETIC USED AND COMMENTS

   The § numbers in this section correspond to those in Part 1.

   1) To compute the linear transformation which brings the IEP onto the front window, the four quantities calculated are:
      - angle of rotation $\Theta$
      - magnification $g$
      - two origin coordinates $C, D$

---

* See appendix A for bipolar-type IEP's

6650/p
To do this, one measures two (out of a possible eight) fiducial marks $\xi$, $\eta$ on the picture, and indicates on the input the numbers $i$, $j$, of those fiducial marks. The machine has the real coordinates of the fiducial marks, $(E, H)$ (as seen through the front window): i.e., it has three sets of these coordinates, one for each camera.

\[
\xi = \xi_i - \xi_j \quad \eta = \eta_i - \eta_j \quad E = E_i - E_j \quad H = H_i - H_j
\]

Then one finds:
\[
tg \theta = \frac{\xi \cdot H - E \cdot \eta}{\xi^2 + H \cdot \eta} \quad \cos \theta = \frac{1}{\sqrt{1 + tg^2 \theta}} \quad \sin \theta = tg \theta \cdot \cos \theta
\]

Then, comparing the two quantities $|\xi|$ and $|\eta|$:

\[
\begin{align*}
\text{if } |\xi| > |\eta| & \quad g = \frac{E}{\xi \cos \theta - \eta \sin \theta} \\
\text{if } |\xi| < |\eta| & \quad g = \frac{H}{\xi \sin \theta + \eta \cos \theta}
\end{align*}
\]

\[
C = E_{i,j} - g(\xi_{i,j} \cos \theta - \eta_i \cdot \eta_j \sin \theta), \quad D = H_{i,j} - g(\xi_{i,j} \sin \theta \cdot \eta_i \cdot \eta_j \cos \theta)
\]

The following precautions must be taken:

- The angle $\theta$ must be small; therefore the axes $E$, $H$ must be more or less parallel (and not perpendicular) to the respective axes $\xi$, $\eta$ of the IEI. By convention, a point is a set of ten digits, the first five of which are $\xi$, and the other five $\eta$. If therefore it is found that $\theta$ is $\approx 90^\circ$, the only thing to do is to exchange the cables leading from the digitizers to the electronics, so that the coordinate which was punched first is now punched second, and vice-versa.

- One must also observe on the IEI the way the numbers $\xi$, $\eta$ increase. This gives us a set of two oriented axes $\xi$, $\eta$ and

6650/p
it is necessary to study their position with regard to ξ, η.
Four cases are possible:

a). Then \( \epsilon_1 = +1, \ g > 0, \ \tan \Theta \approx 0 \) (\( \epsilon_1 \) is a constant in the programme)

b) \( \epsilon_1 = -1, \ g < 0, \ \tan \Theta \approx 0 \)

c) \( \epsilon_1 = +1, \ g < 0, \ \tan \Theta \approx 0 \)

d) \( \epsilon_1 = -1, \ g > 0, \ \tan \Theta \approx 0 \)

Those figures should be carefully studied, as otherwise the results obtained will be quite crazy for no apparent reason.
For each picture, the machine prints \( g \) and \( \tan \Theta \). After a certain number of measurements, one knows what the value of \( g \) should be, approximately, and how much it can reasonably vary. The same with \( \tan \Theta \).

There are in the programme four constants:

\( \bar{g}, \ \Delta g, \ \bar{\Theta}, \ \Delta \Theta \)

such that if \( \begin{cases} g \text{ is outside the interval } \bar{g} - \Delta g, \ \bar{g} + \Delta g \\ \tan \Theta \text{ " " " } \bar{\Theta} - \Delta \Theta, \ \bar{\Theta} + \Delta \Theta \end{cases} \)

the event is thrown out (Rejects 1 and 2°). This saves machine

\(^a\) For rejects, see Appendix C.
time by avoiding computation on events where the fiducial marks have been measured wrongly.

Naturally, when the programme is first tried out, \( g \) should be put \( \approx 0, \Delta g >> \), \( \Theta \approx 0, \Delta \Theta >> \) so that all values found are printed out. After a number of pictures have been measured, the four constants can be set to reasonable values.

If only one fiducial marks set (or none) has been measured, the event is rejected (Rejects 3 and 4°).

The machine then computes

\[ A = g \cos \Theta \]
\[ B = g \sin \Theta \]

2) Read the \( \xi, \eta \) of the track points, also the "track data" on the two views†. The "track data" are as follows ††:

<table>
<thead>
<tr>
<th>Principal view</th>
<th>Secondary view</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) Mass assignment:</td>
<td>SM</td>
</tr>
<tr>
<td>2) End-point assignment:</td>
<td>Q</td>
</tr>
<tr>
<td>3) Stop assignment:</td>
<td>RR</td>
</tr>
<tr>
<td>4) Quality of the track:</td>
<td>QT</td>
</tr>
<tr>
<td>5) In reserve</td>
<td>IN</td>
</tr>
<tr>
<td>6) In reserve</td>
<td>RS</td>
</tr>
<tr>
<td>7) Track number:</td>
<td>NT</td>
</tr>
<tr>
<td>8) Camera number:</td>
<td>CA</td>
</tr>
</tbody>
</table>

† For rejects, see Appendix C.
†† See Appendix A for "bipolar-type" IIF's.
†† For input/output formats, see Part 3.

* Quantities 5) and 6) go through the programme and appear on the other side, but serve no purpose. They could however, be used in subsequent (e.g. kinematics etc.) programme. One suggestion is to put "ionization" in 5) and "sign" in 6).
The machine knows that a new track is being read because NT has changed from its preceding value. It sets CA, then transforms the points on the cards by TLI* \((\alpha, \beta, \gamma, \delta = A, B, C, D)\) corresponding to CA. Store these results in "principal" box. Continue until either end of card or point with zero coordinates is reached. If end of card, read next card and check whether NT and/ or CA has changed. If same NT and CA, continue with points in same box as above. Maximum no. of points in box : 10. Count no. of points and call it \(N\). Read next card.

If \(\neq NT, \) go to §3) etc. (finish computation connected with track before starting new track.

If same NT, \(\neq CA, \) compute "camera pair number" \(G.\)

This is done as follows:

<table>
<thead>
<tr>
<th>Principal (first) CA</th>
<th>Auxiliary (second) CA'</th>
<th>(G^{**})</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>6</td>
</tr>
</tbody>
</table>

Set \(\alpha, \beta, \gamma, \delta\) with values corresponding to new CA; TLI* on points as above. Store in "auxiliary" box and count number of points. Count \(N'.\) Read next card.

* See Appendix B for TLI and other standard subroutines.
**Of course this programme is only good for \(\leq 3\) cameras. If 2 cameras are used, \(G\) can only be 1 or 4.
Note  From the above, it is obvious that the zero of the digitizers should be well outside the field of view because:

a) a point with zero coordinates (or blank, which in Bull code is the same thing, tells the computer that the end of the track has been reached).

b) For a 5-digit coder, 00000 - 1 = 99999, which would of course give something crazy on the machine.

3) Store in appropriate location parameters (AA, BB, CC, DD) of transformation which transform points linked to chamber to points linked to camera pair, and of reverse transformation (AA', BB', CC', DD'). Store also \( d = \frac{1}{2} \) distance of two cameras.

It is obvious that:

\[
AA(i) = -AA(i + 3) = AA'(i) = -AA'(i + 3) \quad (i \leq 3)
\]
\[
BB(i) = -BB(i + 3) = -BB'(i) = BB'(i + 3)
\]
\[
CC(i) = -CC(i + 3); \quad CC'(i) = CC'(i + 3)
\]
\[
DD(i) = -DD(i + 3); \quad DD'(i) = DD'(i + 3)
\]

\( d_i = d_i + 3 \)

(coordinate systems are as follows:)

\[\begin{align*}
\xi & \quad \eta \\
X & \quad Y
\end{align*}\]

\( P \), a projection of optical centre of principal (auxiliary) camera on front window.

If \( N \) and \( N' > 2 \), go to \( \S \) 4; if not go to \( \S \) 19).
4) Compute via PBL\(^o\) best fit to parabola of "principal" then "auxiliary" curve. Compute \(d_i\) for each point.

\[ d_1 \quad d_2 \quad d_3 \quad d_4 \quad d_5 \quad d_6 \quad d_7 \]

(eventually, print \(d_i\) for each point)

Take largest of \(|d_i|\) and compare it with \(\bar{d}\);
if \(\leq\), track O.K.;
if \(>\), throw away that point, reduce N (N') by one and contract in corresponding box, and try again (reject P)\(^{50}\); if \(i = 1\) or N (N') (end point bad) or if one point already rejected in the same view, reject track (reject N)\(^{60}\).

**Note** When the programme is being tested, \(\bar{d}\) should be large and the \(d_i\)'s printed. Afterwards \(\bar{d}\) should be adjusted to a reasonable value and the \(d_i\) printing switched off. The faulty \(d_i\)'s are printed in the rejects. To switch off the \(d_i\) printing, one changes a certain card in the programme

5) For each point \((\xi_i, \eta_i)\) in "principal" box choose the three consecutive points \((\xi_j, \eta_j)\) \(j, j + 1, j + 2\), in "auxiliary" box whose \(\eta'\) is closest to \(\eta_i\). Compute 3-point parabola with these 3 points (PBL\(^i\)). Compute intersection of this parabola with line \(\eta = \eta_i\). There are two solutions. Choose the one in which \(\xi'\) is closest to \(\xi_j\) and \(\xi_j + 2\). Go to \(|d_i|\). Set \(u = 0\).

a) Compute \(\Delta = (aB + \beta)^2 - 4aA(aC + \betaY - 5a - \eta''\)

(See PBL notation, Appendix B)

\(^{50}\) For standard sub-routines see Appendix B
\(^{60}\) For rejects, see Appendix C
\(^i\) For standard sub-routines see Appendix B
\[ \Delta < 0 \text{ reject} \]
\[ \Delta \geq 0 \]
\[ \text{If } A \neq 0 \]
\[ x_1 = \frac{-\alpha B + \beta + \sqrt{\Delta}}{2A} \]
\[ x_2 = \frac{-\alpha B + \beta - \sqrt{\Delta}}{2A} \]
\[ y_1 = Ax_1^2 + Bx_1 + C \]
\[ y_2 = Ax_2^2 + Bx_2 + C \]
\[ TLI^e \text{ on } x_1, y_1; x_2, y_2 \text{ with } \alpha, \beta, \frac{\xi' + \xi'' + 2}{2}, \frac{\eta' + \eta'' + 2}{2} \]
\[ \text{giving } \xi'_1, \xi''_1, \eta'_1, \eta''_1 \]

Compute \[ K_1 = |(\xi''_1 - \xi'_{1j})| + |(\xi''_1 - \xi'_{1j+2})| \]
\[ K_2 = |(\xi''_2 - \xi'_{2j})| + |(\xi''_2 - \xi'_{2j+2})| \]

Test \[ K_1 > K_2 \]
\[ \xi' = \xi''_2 \]
\[ \eta' = \eta''_2 \]

Go to 6)

6) Compute following quantities:
\[ k_v = \sqrt{n_v^2 - (n_v^2 - 1)(((\xi_1 + d) - 1) + \eta_1^2)} \]
\[ k'_v = \sqrt{((\xi' - d) - 1) + \eta' \eta''} \]
\[ k_L = \sqrt{n_p^2 - (n_p^2 - 1)(((\xi_1 + d) - 1) + \eta_1^2)} \]
\[ k'_L = \sqrt{((\xi' - d) - 1) + \eta' \eta''} \]

* For standard sub-routines see Appendix B

6650/p
\[-69-\]

\[Z_i = \frac{\xi' - \xi_1 - f\left[\frac{d + \xi_1}{k_v} + \frac{d - \xi'}{k'_v}\right]}{\frac{d + \xi_1}{k_p} + \frac{d - \xi'}{k'_p}} + dg\]

\[Y_i = \eta_i \left(A + \frac{e}{k_v} + \frac{Z_i}{k_p}\right)\]

Test for \(u = 0\)? (see above, §5 introd.)

**Yes**

Set \(u = 1\)

Compute:

\[\text{ops} = \frac{Y_i \left(\frac{1}{k_v} - \frac{1}{k'_v}\right) + Z_i \left(\frac{1}{k_p} - \frac{1}{k'_p}\right)}{(A + \frac{a}{k_v} + \frac{Z_i}{k_p}) (A + \frac{e}{k_v} + \frac{Z_i}{k_p})}\]

\[\eta_i + \text{ops} = \eta''\]

Go to §5a).

**No**

Compute:

\[X_i = \xi_1 \left(A + \frac{e}{k_v} + \frac{Z_i}{k_p}\right) + d\left(\frac{a}{k_v} + \frac{Z_i}{k_p} - h\right)\]

Store \(X_i, Y_i, Z_i\) and continue with next point at §5).

\(A, \xi, e, f, g, h, d, n^2, n^2\) are constants linked to chamber as follows:

Let \(D\) be the thickness of air between the o.e. of cameras and the front window (All prisms, etc., should be deducted).
be the thickness of glass (including prisms, etc.,). All these are supposed to have the same refractive index.

e index of air (of course, this is usually one, but if highly compressed, for a C₂H₆ - C F₂Br mixture for instance, it may be slightly > 1).
nᵥ index of front window
nₚ " " liquid
d distance of cameras.

Then
\[ d = \frac{D}{n_a} + \frac{e}{n_v} \]
\[ A = \frac{D}{n_a} \]
\[ f = \frac{e}{A} \]
\[ k = \frac{2e}{Dn_v n_a} \]
\[ h = \frac{e}{Dn_v} \]

These constants, together with \( \frac{n_p^2}{n_v} \) and \( \frac{n_v^2}{n_p} \), are the ones inserted in the machine to avoid having to compute them for every point.

If the various prisms, etc., have an index \( \neq n_v \), they should be inserted in \( D \) (with their appropriate indexes) and not in \( e \).

Of course, the computation is fairly obvious. The first approximation is to take for corresponding points the ones which have the same \( \eta \). Actually this is wrong, but not very much. The difference in \( \eta' \) (ops) can be computed as a function of \( Y_i \) and \( Z_i \),
and the (more) correct value of $\eta'$ inserted; the computation is then redone and more correct values of $Y, Z$ obtained. $X$ is obtained therefrom.

The eps is never larger than a (very) few mm, for a reasonably shaped bubble chamber, and so the values obtained after one iteration are correct to within a few $10^{-3}$ cm. If, however, one had scruples and wished to iterate more than once, it would be easy to change the u-test accordingly.

There are two dangers to this procedure, both of which can be avoided by a little common sense on the part of the physicist and measurer.

a) If the track is almost parallel to the $\xi, \xi'$ axis, then one gets the following situation:

Point $\alpha$ can be very far from point $\beta$, and therefore the result unreliable.

Remedy: choose another camera pair.
b) Suppose for some reason connected, or not, with the shape of the chamber, one of the views is visible over a substantially larger length than the other.

One obtains the situation below:

In order to compute the points $1' \ldots 4'$, corresponding to the machine will use points $01', 02', 03'$ \{ no difficulty \}

But to compute $5'$ and $6'$ it will have to extrapolate on the three points $02', 03', 04'$, thus probably yielding a crazy result, extremely sensitive to the slightest error on these points.

Remedy: in such a case, use A as the "principal view" and P as the "auxiliary view". See Part 3.
7) and 8) TII on the $X_i$, $Y_i$ using the AA', BB', CC', DD' (see 3)). Check that $X \leq X_i \leq X$ etc. The $X$, $X$ etc., are the limits of the chamber (plus a little something to take into account possible uncertainty in measurement). If one of the coordinates is outside these limits, throw away point (reject $X_i$). If first or last point rejected, or $\leq 3$ points are left, reject track. Of course, the $Z_i$ remain the same.

Print track number, SM, Q, RR, QT, IN, RS, $X_1$, $Y_1$, $Z_i$, $X_n$, $Y_n$, $Z_n$.

9) PBL on $X_i$ and $Y_i$. Compute $A'$, $B'$, $C'$ (coefficients of parabola.

Then $R_p = \frac{(1 \div B'^2)^{1/2}}{2 |A'|}$

$L^2 = (X_N - X_i)^2 \div (Y_N - Y_i)^2$

$R_\theta = R_p \left(1 + \frac{1}{16} \frac{R_p^2}{L^2}\right)$

$L' = R_\theta \arcsin \frac{L}{2R_\theta}$ = \frac{\sqrt{2}}{2} length (projected)

$tg \theta = \frac{Z_N - Z_i}{2}$

$L = \frac{L}{\cos \theta}$ Print 2L (real length of track)

$X_o, Y_o, Z_o = \frac{X_1 + X_n}{2}, \frac{Y_1 + Y_n}{2}, \frac{Z_1 + Z_n}{2}$

Compute mean value of magnetic field

- For rejects see Appendix C

6650/p
\[ B_0 = \mathbf{B} \times f(X_0, Y_0, Z_0) \]

\[ B_\pm = \mathbf{B} \times f\left( \frac{\alpha X_0 \pm \beta X_0}{\alpha \mp \beta}, \text{ etc. } \right) \]

\[ B_\pm = \mathbf{B} \times f\left( \frac{\alpha X_0 - \beta X_0}{\alpha \pm \beta}, \text{ etc. } \right) \]

\[ \tilde{B} = \frac{u \left( B_x + B_y \right) + v B_z}{2 \frac{u}{x} + \frac{v}{x}} \]

\( f(X,Y,Z) \) is a table of values (\( \approx 1 \)) of the \( \% \) variation of the magnetic field in various points of the chamber. The machine interpolates \( \frac{\mathbf{B}}{\mathbf{B}}, \frac{\alpha}{\alpha}, \frac{\beta}{\beta}, \frac{u}{u}, \frac{v}{v} \) are constants (\( \mathbf{B} \), magnetic field in kilogauss at center, \( \alpha, \beta, u, v \) weighting constants).

Compute \( \tilde{p} = \frac{3}{2} \frac{\mathbf{B}}{\mathbf{e}} / \cos \varphi \). Print \( \tilde{p} \).

10 and 11) Inspect SM in appropriate location (see § 2).

If \( SM = 0 \) : by convention track is an electron. Go to § 20).

If \( SM \neq 0 \) : Check if \( \geq 6 \)

<table>
<thead>
<tr>
<th>Yes</th>
<th>No</th>
</tr>
</thead>
<tbody>
<tr>
<td>( SM = 6 ) Try successively ( SM' = 6 )</td>
<td>a) Transfer ( SM ) into ( SM' ) for mass, use proton mass x number in ( SM' ) box</td>
</tr>
<tr>
<td>( SM = 7 )</td>
<td>( SM' = 2 )</td>
</tr>
<tr>
<td>( SM = 8 )</td>
<td>( SM' = 4 )</td>
</tr>
<tr>
<td>( SM = 9 ) Not used</td>
<td></td>
</tr>
</tbody>
</table>

For the moment, the \( SM' \) numbers are: 1 \( \mu \); thus \( SM=6 \) means try \( \mu, \pi \)

\[ 2 \pi \quad 3 \xi \quad 4 \rho \quad 5 \Sigma \]

6650/p
but they can of course be changed. However, it would be wise not
to change the logic of the SM = 6, 7, 8, etc.

Reduce momentum ($p$) and length ($L'$) to proton values.

Inspect $Q$ (in appropriate location: see §2).

$Q = 1, 2$  transfer to $Q'$  $Q = 3$  try successively  $Q' = 1$  $Q' = 2$
$Q$ symbolizes the end of the track one is interested in
$Q = 1$  means one is interested in the beginning of the track
$Q = 2$  "  "  "  "  "  "  "  "  "  "
$Q = 3$  "  "  "  "  "  "  "  "  "  "

(Naturally, tracks should always be measured in the direction in which
the particle is going.)

12) Inspect RR and transfer to RR':
   RR=0 means the track certainly stops
   (it has no energy)
   RR=1 means anything else

   $RR' = 0$

   Compute $L'' = i (p')$ in table giving momentum/range for protons.

   Compare $L''$ with $L'$

   $L'' < L'$  $L'' > L'$

   Set $RR' = 0$

   $RR' = 1$

   $Q' = 1$

   Compute
   $L'' = L' + L''$  $L'' = L'' - L'$

   $Q' = 2$

   Compute $p'' = p(L'')$ in same table

   Reduce back to real values (see above)

   Including $L'' \rightarrow L''$

   Print $p$, RR', Q', SM'

   Compute $R' = \frac{p \cos \theta}{3B}$

6650/P
13 and 14.) Compute \( p^3 = \frac{x}{\sqrt{p^3 + (\text{mass})^2}} \)

\[
\lambda = \frac{X_0}{4} \left( \frac{p^3}{21} \right)
\]

\[
L_{\text{opt}} = (DA \times \lambda \times (\epsilon' \times QT)^2)^{\frac{1}{3}}
\]

Compute \( LL = \frac{2R_0}{\cos \phi} \arcsin \frac{\sqrt{(X_i - X_j)^2 + (Y_i - Y_j)^2}}{2R_0} \)

\((Q' = 1, i = 1 \text{; } Q' = 2, i = n)\)

and choose value of \( LL \) closest to \( L_{\text{opt}} \)

by excess if \( L_{\text{opt}} \leq 2L \)
by default if \( L_{\text{opt}} > 2L \)

from which one gets \( j \)
then \( \phi_i = \arctan \frac{Z_j - Z_i}{LL \cos \phi - \pi/2} \)

15)

\[
\Theta = \arctan \frac{Y_j - Y_i}{X_j - X_i} + \pi/2
\]

\[
\Theta' = \arctan \frac{Y_j - Y_{i+1}}{X_j - X_{i+1}} + \pi/2
\]

\[
\begin{align*}
\text{if } Q' = 1, & \quad L_{\text{opt}} < LL_{n-1}, \quad (\epsilon_s = -1) \\
\text{or } Q' = 2, & \quad L_{\text{opt}} > LL_{n-2}, \quad (\epsilon_s = +1) \\
\text{if } Q' = 1, & \quad L_{\text{opt}} > LL_{n-1}, \quad (\epsilon_s = +1) \\
\text{if } Q' = 2, & \quad L_{\text{opt}} < LL_{n-2}, \quad (\epsilon_s = -1)
\end{align*}
\]
\( Q' = 1 \) ?

\( K \Theta = \frac{K \Theta}{K \Theta} \)

\( K \Theta = -K \Theta \)

**KK \( \Theta \):** corrective constant to take care of energy loss.

**Compute**

\[ D \Theta = \frac{LL}{2R_{\Theta}} \]

correction chord → circle

\[ DD \Theta = \frac{K \Theta LL}{R_{\Theta}} \]

correction for energy loss

\[ \Theta = \Theta + (D \Theta + DD \Theta) \epsilon_s \cdot \frac{Q' - \Theta}{|Q' - \Theta|} \]

\( X_j - X_i > 0 \) ?

\( Q' = 2 \) ?

\( \Theta \rightarrow \Theta + \pi \)

\( \Theta, \Theta \)

Print \( \Theta \), \( \Theta \)

\( \phi_i \rightarrow -\phi_i \)

\( \Theta \rightarrow \Theta + \pi \)
16) Compto

\[ \frac{d\sigma}{d\sigma} = \sqrt{\frac{m}{\cos \phi}} \sqrt{DF \cdot \epsilon^2 \cdot \frac{\nu^2}{2 \mu} \cdot \frac{p^2}{2 \mu \bar{p}^2} + \frac{DDP}{2 \mu \bar{p}^2}} \]

\[ \frac{d\sigma}{d\sigma} = \sqrt{DDF \cdot \frac{\epsilon^2}{m} \cdot \frac{\epsilon^2}{LL}} \]

\[ d\phi = \sqrt{DF \cdot \frac{LL^2}{LL^2} \cdot \epsilon^2 \left(0.1 \sin^2 \phi, \frac{D^2}{2d} \cos^2 \phi_1 \right) \cdot \frac{DDF, LL}{p^2 \beta^2}} \]

\[ d\phi = d\theta = d\theta = 0 \]

\[ d\theta = \sqrt{\frac{1}{\cos^2 \phi} \left(DF \cdot \frac{LL^2}{LL^2} \cdot \epsilon^2 \left(0.1 \sin^2 \phi, \frac{D^2}{2d} \cos^2 \phi_1 \right) \cdot \frac{DDF, LL}{p^2 \beta^2} \right) + \lambda \left(\frac{LL}{p} \cdot \frac{\lambda_\sigma}{\sigma} \right)^2} \]

17) If \( q = 3 \) and/or \( SM = 6,7,8 \); go back to §11) for other mass and/or end assignments; then go to §18).

18) Check if last track; if so go back to §1); if not, go back to §2).

19) 2-points tracks: print \( \eta_1, \eta'_1, \eta_2, \eta'_2 \); \( \eta' \) should be \( \eta_1 \approx \eta'_1, \eta_2 \approx \eta'_2 \); this gives a check.

\( \eta_1 \approx \eta'_1, \eta_2 \approx \eta'_2 \) and compute the \( k_p, k_v \) as in §6).

\[ z_1 = \]
\[ z_2 = \]
\[ Y_1 = \]
\[ Y_2 = \]
\[ X_1 = \]
\[ X_2 = \]

6650/p
Then compute \( L^2 = (x_2 - x_1)^2 + \ldots \ldots \)

\[ \varphi = \arctan \frac{z_2 - z_1}{L} \]

\[ \theta = \arctan \frac{y_2 - y_1}{x_2 - x_1} + \pi/2 \]

\[ \theta = \theta + \pi \quad \text{if} \quad x_2 > x_1 \]

\[ \sin \theta = \frac{1}{\cos \varphi} \sqrt{\frac{D'F^2}{L} + \frac{\epsilon'^2}{L} \cdot \Omega^2} \]

\[ \sin \varphi = \sqrt{\frac{D'F^2}{L} + \frac{\epsilon'^2}{L} \cdot \Omega^2} \]

Print and go to 18)

20) Electron tracks

See Behr-Mittner paper

Print and go to §18).

3. Input/output formats

1) Input formats. Cards only - 2 one card per track/view.

2) Output formats.

a) Listings.

All results come with 5 decimals. Angles and \( \Delta \) angles are in degrees.
b) **Cards.** Two types of cards normally come out:

i) coordinate cards: 1-1 in col. 9-10
ii) p etc. cards. 1-2 in col. 9-10.

All results come with 5 decimals. Angles and Δ angles are in radians.

**Note** It should be remembered that the Bulle machine does not print or punch zeros. A zero means a minus sign. However, the programme is so arranged that the machine can read zeros and not be confused. This avoids having to disconnect the zero in the card punch. It is naturally supposed that all input data is positive.

Input cards should be stacked as follows:

a) Fiducial cards (2 or 3)
b) First track – principal view
c) First track – auxiliary view
d) Second track – principal view

e etc. ......

Tracks do not necessarily have to be measured in sequence.

4. **EXPERIENCE WITH BULLE □**

This programme has now been running for ≈ two years. After initial debugging, we have had no major trouble worth mentioning. Some minor errors have been detected (the last one in Jan. 1962). Alterations have been made from the original version. These include:

a) a sub-routine to compute and print the potential length of 2-point tracks (the limits not being necessarily the same as those of the chamber
b) the Behr-Mittner electron routine

c) the suppression of the "third approximation" for the computation of θ (experience shows that this term is usually ≈ 0.1 degree.

We now have a whole library of kinematics, dynamics, etc. linked to this programme. These digest the Bull 3 output and calculate V₀, θ values, etc. but do no fitting. To do this and to compute the corresponding errors, the p, θ, φ of each track are supposed to be independent variables, and Δp is symmetrical. I know this wrong. But it is not very wrong if Δp/p is small, and if Δp/p is large the result is unreliable anyway.

Some 5 to 10,000 tracks have been run on this programme. About one track in ten is rejected (mainly due to IEP operator errors, sometimes to kinks undetected the first time through and which the machine sees). It is practically certain that a track measured a second time will go through. I know of no instance when a track has been rejected more than twice.

I am now re-writing this programme in Fortran - the reason being that we have outgrown the Bull. machine (comparable to the 650 in speed, but it has 4 times the memory). I plan the minimum alterations. However some have been proved a good idea by experience.

a) some way of measuring corresponding points and telling this to the machine (this to take care of the situation in Part 2, § 6)a) (projected tracks at small angle with stereo axis) when, for some reason, some other pair of cameras cannot be chosen.

6650/p
b) some way of integrating kinks

c) some way of measuring 2-point non-neutral tracks
(i.e. recoil protons etc. which are straight and
whose momentum is always given by range)

d) we are at present building a "hydrogen target"
(i.e. a small box filled with liquid H₂ which will
be placed in the beginning of the chamber and in
which "we hope" a reasonable number of interactions
will take place. These will, of course, be invisible
so we will have to extrapolate observed tracks back
to get the correct momenta, angles, etc. at the
interaction point.

To give an idea of time-scales

a) I started thinking about this thing in May/June
1960. I wrote the logic, etc., showed it to the
interested physicists and started work with the
Bull Co. programmes in July-August 1960 (it is
written in machine language, thus taking about
3/4 of the machine). Writing and debugging the
programme took essentially until about February
1961 and it started running properly in March/
April of that year.

b) A track goes through the machine in between ~ 1
and 3 minutes, depending on the number of points
measured.
APPENDIX A

BIPOLAR IEPa

The machine first performs the transformation from bipolar to cartesian coordinates, then goes to the regular cartesian routine. In the fiducial cards column 20 should be non zero (2 for example). The machine then picks up three constants: AK, BK, CK, in box 2. These constants are used to transform each track point appropriately.

![Diagram](image)

Fig. 1.

Points (1) and (2) are the "holes" or origins of the bipolar system. G is a "garage"; a fixed point on the IEP table. After every 2 fiducial marks, the pointer should measure point G. This together with the 3 constants gives the "zeros" of points 1 and 2. Call k₁ and k₂ the coordinates obtained for the garage.

\[
\begin{align*}
AK &= \alpha + \beta \\
BK &= \alpha - \beta \\
CK &= 2\ell
\end{align*}
\]

(all expressed in IEP limits)

Setting

\[
\begin{align*}
u &= \xi + \eta - k_1 - k_2 + AK \\
v &= \xi - \eta - k_1 + k_2 + BK \\
w &= (u^2 - CK^2) (CK^2 - v^2)
\end{align*}
\]

One gets:

\[
\begin{align*}
y &= \frac{\sqrt{u}}{2,CK} \\
x &= \frac{uv}{2,CK}
\end{align*}
\]

6650/p
1) The digitizers should be so arranged that the indications should increase as the strings lengthen.

2) The $x$ and $y$ axes in Fig. 1. should be more or less parallel (and no perpendicular) to the chamber $x$ and $y$ axes; i.e. $\tan \theta$ (see Part 2, 1) should be small and not $\gg$. Beware of sign of $\epsilon$.

This makes it possible to have different bipolar IEPs each with its own set of constants.
APPENDIX B

STANDARD SUB-ROUTINES

1) **TLI** Takes a point \( \xi, \eta \) and four parameters \( \alpha, \beta, \gamma, \delta \) and transforms it as follows:

\[
\begin{align*}
  x &= a\xi - \beta\eta + \gamma \\
  y &= \beta\xi + a\eta + \delta
\end{align*}
\]

Input: \( \xi, \eta, a, \beta, \gamma, \delta \)
Output: \( x, y \)

2) **PBL** Takes a series of points \( \xi_1, \eta_1, \ldots, \xi_n, \eta_n (3 \leq n \leq 10) \) and does the following:

a) **TLI**

\[\begin{align*}
  \alpha &= \arccos \frac{\eta_n - \eta_1}{\xi_n - \xi_1} = \cos \varphi \\
  \beta &= -\sin \varphi \\
  \tau &= -\left( \frac{\xi_n + \xi_1}{2} \cos \varphi + \frac{\eta_n + \eta_1}{2} \sin \varphi \right) \\
  \delta &= \left( \frac{\xi_n + \xi_1}{2} \sin \varphi - \frac{\eta_n + \eta_1}{2} \cos \varphi \right)
\end{align*}\]
b) Compute ABC coefficients of least-squares parabola

\[ y = Ax^2 + Bx + C \]

best fitted to the \( x_i, y_i \)

Input \( \xi_1, \eta_1, \ldots, \xi_n, \eta_n, n \)

Output \( A, B, C, \alpha, \beta, \gamma, \delta \) (of TLI)

Needs constants:
- a) \( \epsilon_2 \) (if x axis almost \( \perp \) to \( \xi \) axis)
- b) \( \epsilon_3 \) minimum value of determinat \( D \)

if smaller, parabola is a straight line.

The computation is done as follows

1. \( n = 3; \quad A = \frac{Y_2}{x_2^3 - x_1^3}, \quad B = 0, \quad C = -Ax_1^2 \)

2. \( n > 3; \) compute \( S_1, S_2, S_3, S_4 \)

where \( S_j = \sum_{i=1}^{n} x_i^j \); 

compute \( s_0, s_1, s_2 \)

where \( s_j = \sum_{i=1}^{n} x_i^j y_i \)

\[
D = \begin{vmatrix}
S_4 & S_3 & S_2 \\
S_3 & S_2 & S_1 \\
S_2 & S_1 & \eta
\end{vmatrix} ;
\]

\[
DA = \begin{vmatrix}
S_2 & S_1 & S_0 \\
S_2 & S_1 & \eta \\
S_2 & \eta & \eta
\end{vmatrix} ;
\]

\[
DB = \begin{vmatrix}
S_2 & S_1 & S_0 \\
S_2 & S_1 & \eta \\
S_2 & \eta & \eta
\end{vmatrix} ;
\]

\[
DC = \begin{vmatrix}
S_2 & S_1 & S_0 \\
S_2 & \eta & \eta \\
S_2 & \eta & \eta
\end{vmatrix} ;
\]

\[ A = DA/D ; \quad B = DB/D ; \quad C = DC/D \]
## APPENDIX C

### REJECTS

<table>
<thead>
<tr>
<th>Reject type</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1)</td>
<td>INP transformation outside allowed limits</td>
</tr>
<tr>
<td>2)</td>
<td>(see Part 2, 1)</td>
</tr>
<tr>
<td>3)</td>
<td>(event rejected)</td>
</tr>
<tr>
<td>4)</td>
<td>Input cards missing or in wrong order</td>
</tr>
<tr>
<td>5)</td>
<td>(event or track rejected)</td>
</tr>
<tr>
<td>P</td>
<td>Point rejected on fit to parabola on front window</td>
</tr>
<tr>
<td>X</td>
<td>Point rejected because outside chamber</td>
</tr>
<tr>
<td>M</td>
<td>Track rejected ( &gt; 1 P or X, P or X on first or last point)</td>
</tr>
<tr>
<td>9</td>
<td>Track rejected (parabola too/to camera-pair axes)</td>
</tr>
</tbody>
</table>
Goldschmidt-Chermon: When you say that you use the optimum length, does it mean that you compute the angle, for instance, by using the measured point which is nearest to the optimum length?

Hennessy: Yes, exactly. Actually, as you know, there are two optimum lengths, one for the dip and one for the azimuth. We use only one. We compute the azimuth optimum length, and multiply by, I forget the factor, I think 1 1/4, because the ratio between the two is about 2/3. You compute your optimum length and fit the chord first, and then you take a circle with the right radius and which goes through the end point and that closest to the optimum length and this gives you your tangent. As for the dip, you unwind your curve then you calculate the dip from the Δz of the same points.

Moorhead: Do you think it is possible for the computer to distinguish between kinks and bad measurements automatically?

Hennessy: No. It throws out a point and then you have to find out why.

Moorhead: But is it possible for the computer to find out why?
Hennessy: Well, we haven't put it into the computer; I suppose we could, but I don't see very well how. You could have two constants and if the deviation were between the two constants you would say it was a kink, and if it were larger than a certain constant it would be a bad measurement; for instance the coder has gone haywire. I don't see how else you could do it.

Another point is that we measure neutral tracks. We measure two points, and get a card out with the dip and the azimuth.

As for electrons, Bohr and Mittner have developed a method, which they will talk about this afternoon, but otherwise the procedure is very much the same.

Böck: In the procedure with kinks, which the measurement detects, does the programme put pieces of tracks together or is this done by hand?

Hennessy: One must do it by thinking intelligently. It measures one piece and gets a result, and then measures the second piece and gets a result. Then you try to think intelligently and fit them together. What we usually do, of course, is to take the first and not bother about the second, or take the direction of the first and the momentum of the second.

Böck: Secondly, there seemed to be a unique mass per track. Is that so?
Hennessy: There are several codes, I think up to eight; $1, 2, 3, 4, 5$ are unique masses; for 6 you try $\pi, \mu$, for 7 you try pion and proton, and for 8 you try them all and you print out results for all the masses.

Böck: So on each card you have one more quantity which is the mass.

Hennessy: There are several codes, one for the mass, one for the end you are interested in and others I won't go into. If you put 1, you are interested in the first point, if 2, you are interested in the last, if 3, you are interested in both; so the machine punches a card for each end, and mass hypothesis. After that you throw away the cards you are not interested in.

Scoop: Do you put this inaccuracy you talked about for each event or each photo or for all?

Hennessy: No. You put in this at the beginning of each experiment. There is a batch of cards which is the constants for each experiment. You try to adjust them. If one seems silly you just put in a different value and try again. Usually of course the geometrical term is small in comparison with the scattering term, so it doesn't matter much. Actually for very bad pictures there is a code which we call "quality" and which multiplies by 2, 3 ... the geometrical inaccuracy for a particular track, if necessary. This is chosen by the physicist.
Baizin: You say you select two views. Don't you have mistakes by selecting the cards like this? Why don't you have a kind of data card in front of all so that the programme knows already?

Honnessy: The choice of views is made by the physicist before the measurements are made. It comes prior to everything. Of course you have occasional mistakes, but probably no more than by any other method - and at least you know what you are doing. As for the third view, it is not even measured.
THE HEAVY LIQUID GEOMETRY PROGRAMME IN USE

AT THE RUTHERFORD LABORATORY

by

J. Sparrow

INTRODUCTION

The R.H.E.L. heavy liquid bubble chamber geometry programme is based largely upon a completely general programme written for hydrogen chambers. The programme can conveniently be split into three separate sections.

(1) Space point reconstruction
(2) Space fitting to obtain kinematical quantities
(3) Special heavy liquid routines

Section (3) in the hydrogen version is a mass dependent helix fit to the space points. It is replaced in the heavy liquid version by routines to deal with multiple kinking of tracks, with fits to an optimum length of track to obtain the best value for angles and the minimum errors and routines to deal with electron pairs produced from γ ray materialisation.

1. SPACE POINT RECONSTRUCTION

(a) Fiducials

For each view the fiducial marks are first identified (N.B. a given fiducial must always be measured first on a given view) and then a least squares fit is performed in order to determine the film shrinkages in two perpendicular directions (f₁ and f₂), the rotation (θ) and translation (x₀, y₀) necessary to get the measured coordinates into a system with the position of
the optic axis on the film as origin.

If \( X_1, Y_1 \) are the true fiducial coordinates on the film and \( x_1, y_1 \) are the measured positions then a least square fit is performed to minimise,

\[
\delta^2 = \sum_i \left[ (X_1 - (f_1 x_1 - x_1) \cos \Theta - (f_2 y_1 - y_1) \sin \Theta)^2 \\
+ (Y_1 - (f_1 x_1 - x_1) \sin \Theta + (f_2 y_1 - y_1) \cos \Theta)^2 \right]
\]

in the form

\[
\delta^2 = \sum_i \left[ (x_1 - ax_1 - by_1 - c)^2 + (y_1 - dx_1 - ey_1 - f)^2 \right]
\]

If only two fiducials are measured then \( f_1 \) is set equal to \( f_2 \).

Having performed the fit each fiducial is checked for its accuracy, any badly measured ones are rejected and the fit redone on the remainder.

(b) **Vertex Identification**

Vertices are numbered in the order that they are measured and tracks have markers to say whether their ends have vertices associated with them or not. The vertices on all tracks are next identified assuming that a point on the track has been measured closer to the actual vertex point than half the minimum distance between vertices on that view.

(c) **Points to rays**

Using the parameters \( a, b, c, d, e, f \) found for each view

6650/p
by the fiducial routine the coordinates are all transformed into a fixed coordinate system on the film.

Each point is then converted into a ray in the chamber by tracing its path back through the optical system ending up with a point $X, Y, 0$ on the front glass/liquid interface and direction ratios $U, V$. The origin for the space system is taken as that in which the camera positions are defined with $z$ into the chamber and zero at the window liquid interface (L, H, S. of coordinates.) In terms of $X, Y, U, V$ a point in the chamber $x, y, z$ is defined as

$$x = X + Uz, \quad y = Y + Vz, \quad z = z$$

Having obtained a set of points $X_1, Y_1$ to $X_N, Y_N$ on the front glass window a check is made that the track does not turn through an angle of more than 180°. The coordinates are then rotated and translated to get them into a system where $\Sigma x' = 0, \Sigma y' = 0$ and then a fit is performed to these points to obtain the radius of curvature and the angles between the radius vector to the end points and the space $y$ axis. At the same time a check is made for badly measured coordinates, the first bad point is rejected and the fit redone but a second bad point causes the event to be rejected. The normal fit is to a circle but if the track is too short or turns through a small angle then either a parabola or a straight line is fitted.

(d) **Vertex coordinates**

For each vertex there are three rays (or two if a view is missing) which should meet at a point $(x, y, z)$. The point of nearest intersection of these rays is found by minimising the squared distance $d$ between the point $x, y, z$ and the points on the rays with the same $z$ coordinate.
The quantity

\[ d = \sum_{i=1} \left\{ (x - x_i - U_{1_i}z)^2 + (y - y_i - V_{1_i}z)^2 \right\} \]

is minimised with respect to \( x, y \) and \( z \). The leads to an equation of the form \( G \mathbf{x} = \mathbf{y} \)

where \( G = \begin{bmatrix} N & 0 & -\Sigma U_{1_i} \\ 0 & N & -\Sigma V_{1_i} \\ -\Sigma U_{1_i} & -\Sigma V_{1_i} & \Sigma U_{1_i}^2 + \Sigma V_{1_i}^2 \end{bmatrix} \), \( \mathbf{x} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \),

\[ \mathbf{y} = \begin{bmatrix} \Sigma X_{1_i} \\ \Sigma Y_{1_i} \\ -\Sigma U_{1_i}X_{1_i} - \Sigma V_{1_i}Y_{1_i} \end{bmatrix} \]

and \( N = \) number of rays

The error on the coordinates is calculated from

\[ \langle \delta x_i \delta x_j \rangle = \frac{\delta_x^2 \mathbf{G}_{ij}^{-1}}{\omega^2} \]

where \( \delta_x \) = standard vertex error on film
\( \omega \) = demagnification factor = \( \frac{\mu_{liq} a_o}{z + z_o} \)
\( \mu_{liq} \) = refractive index of liquid
\( a_o \) = film to lens distance
\( z_o \) = optical distance from window to lens

The internal error \( \delta \) is also calculated and if this exceeds a given parameter then \( \delta_x \) is set equal to \( \delta \).

\[ \delta = \sqrt{\frac{\omega^2 d}{2N - \delta}} \]

is printed with each set of vertex coordinates and uncorrelated errors.

6650/p
(e) **Space points**

In order to find corresponding rays between two views a rotation is made to the ray coordinates so that the new system has its x axis along the line joining the two cameras (stereo axis). If θ is the angle that the tangent to the track makes with this stereo axis, then a best view is selected by finding the maximum of

\[
\frac{L}{L_{\text{max}}} + W \left[ \frac{\sin^2 \Theta_1 + \sin^2 \Theta_2}{\max(\sin^2 \Theta_1 + \sin^2 \Theta_2)} \right]
\]

where \(L\) = length of the image of track on glass, \(L_{\text{max}}\) = maximum \(L\) for the three views, \(\sin^2 \Theta_1\), and \(\sin^2 \Theta_2\) are average values of \(\sin^2 \Theta\) for the two possible combinations of pairs of views and \(W\) is a weighting factor.

Each ray on this main view (m) is taken in turn and a corresponding ray found on the remaining sub-views (s). As a first approximation the corresponding ray is taken as that with the same y coordinate \((Y_s = Y_m)\). The two points straddling this \(Y_s\) on the sub-view \(Y_1\) and \(Y_2\) have a circle of radius \(R\) (previously determined from fit on glass) put through them and its centre \(X_c, Y_c\) found from

\[
X_c = \frac{1}{2} (X_1 + X_2) \pm (Y_2 - Y_1) \left[ \frac{R^2}{(X_1 - X_2)^2 + (Y_1 - Y_2)^2} - \frac{1}{4} \right]
\]

\[
Y_c = \frac{1}{2} (Y_1 + Y_2) \pm (X_2 - X_1) \left[ \frac{R^2}{(X_1 - X_2)^2 + (Y_1 - Y_2)^2} - \frac{1}{4} \right]
\]

where the sign is given by the sign of \(R\).
The $X_s$ corresponding to the $Y_s$ is then obtained as

$$X_s = X_0 \pm \sqrt{R^2 - (Y_s - Y_0)^2} \quad \text{if } X_t > X_0$$

$$- \quad \text{if } X_t < X_0$$

The $U_s$ and $V_s$ are obtained by linear interpolation between the nearest rays

$$U_s = U_t + (U_t - U_2) \left( \frac{Y_s - Y_t}{Y_1 - Y_2} \right)$$

$$V_s = V_t + (V_t - V_2) \left( \frac{Y_s - Y_t}{Y_1 - Y_2} \right)$$

Due to optical distortion the rays on the main and sub-views will not meet, so an iterative procedure is used to obtain a better value of $X_s', Y_s', U_s', V_s'$. For two rays to meet

$$Z = \frac{X_s - X_m}{U_m - U_s} = \frac{Y_s - Y_m}{V_m - V_s}$$

or

$$Y_s = \frac{Y_m}{V_m - V_s} (X_s - X_m)$$

Using the $X_s', U_s', V_s'$ calculated above a new value of $Y_s', Y_s'^*$ can be found and the cycle repeated until $Y_s'^* - Y_s'$ is less than a specified parameter.

The $Z$ coordinate is finally given by

$$Z = \frac{X_s - X_m}{U_m - U_s}$$

In general two $Z$ coordinates are obtained, one from each sub-view, and these are weighted together to obtain one $Z$ coordinate of the space point on the main view ray. The weighting
factor is taken as \( \sin^2 \Theta = (X_3 - X_1)^2 / R^2 \).

so that \( Z = \frac{\sin^2 \Theta_1 Z_1 + \sin^2 \Theta_2 Z_2}{\sin^2 \Theta_1 + \sin^2 \Theta_2} \)

If the track gets too close to the stereo axis then points cannot be reconstructed and the weighting is set to zero, similarly if too great an extrapolation is required to find the corresponding ray again, \( \sin^2 \Theta \) is set to zero. If less than two points can be reconstructed on the main view selected a second main view is tried and if this again fails the event is rejected.

2. **SPACE FITTING**

For each track a set of \( n \) points \( (x_i, y_i, z_i) \) exist where

\[
(x_i, y_i, z_i) = (X_i + UZ_i, Y_i + VZ_i, Z_i)
\]

for \( n > 2 \) the magnetic field value for the track is found from

\[
B_z = B_o \left[ \frac{W_1 B(x_{n/2}, y_{n/2}, z_{n/2}) + W_2 (B(x_1, y_1, z_1) + B(x_n, y_n, z_n))}{W_1 + W_2} \right]
\]

for \( n = 2 \)

\[
B_z = B_o \cdot B(x_1, y_1, z_1)
\]

where \( B_o \) is the value of \( B_z \) at the origin, \( B(x, y, z) \) is \( B_z/B_o \) at the point \( (x, y, z) \) and \( W_1 \) and \( W_2 \) are weighting factors.

The coordinates are rotated into a frame with \( X \) axis parallel to the line joining the end points (rotation angle \( \Theta R \)) and then translated to a frame \( (x', y') \) such that \( \Sigma x' = 0, \Sigma y' = 0 \). A fit is then made to these points in two parts, first to the \( x, y \) projection and then to the \( z \) coordinates.
For tracks of sufficient length which turn through large enough angles a circle fit is made in the x y plane, for other curved tracks a parabola is fitted whilst for straight tracks a straight line fit is made.

(a) Circle Fit

A fit to the circle \((x - a)^2 + (y - \beta)^2 = \rho^2\) is made by minimising the quantity

\[
f^2(a, \beta, \rho) = \sum_i \left[ (x_i - a)^2 + (y_i - \beta)^2 - \rho^2 \right]^2
\]

which is more convenient and approximately the same as minimising the correct quantity

\[
f^2 = \sum_i \left[ \left\{ (x_i - a)^2 + (y_i - \beta)^2 \right\}^{1/2} - \rho \right]^2
\]

since one can write

\[
f^2 = \sum_i \left[ \left\{ (x_i - a)^2 + (y_i - \beta)^2 \right\}^{1/2} - \rho \right]^2
\]

\[
\approx 4\rho^2 f^2
\]

Minimising \(F^2\) leads to

\[
a = \frac{1}{2T} \left[ \Sigma y_i^2 (\Sigma x_i^3 + \Sigma x_i y_i^2) - \Sigma x_i y_i (\Sigma y_i^3 + \Sigma x_i^2 y_i) \right]
\]

\[
\beta = \frac{1}{2T} \left[ \Sigma x_i y_i^2 (\Sigma x_i^3 + \Sigma x_i^2 y_i) - \Sigma x_i y_i (\Sigma x_i^3 + \Sigma x_i y_i^2) \right]
\]

\[
\rho^2 = \frac{1}{n} (\Sigma x_i^2 + \Sigma y_i^2) + a^2 + \beta^2
\]
where \( n \) = number of coordinate pairs

and  \[ \Gamma = \Sigma x_i^2 \Sigma y_i^2 - (\Sigma x_i y_i)^2 \]

the errors \( < \delta x^2 >, < \delta y^2 >, < \delta \rho^2 > \) are obtained from the inverse diagonal elements of the error matrix

\[
< \delta x^2 > = \delta x y^2 H_{11}^{-1} \\
< \delta y^2 > = \delta x y^2 H_{22}^{-1} \\
< \delta \rho^2 > = \delta x y^2 H_{33}^{-1}
\]

where \( H_{ij} = \frac{1}{4\rho^2} \begin{pmatrix}
4(\Sigma n_i^2 + n \alpha^2) & 4(\Sigma x_i y_i + n \alpha \beta) & -2n\alpha \\
4(\Sigma x_i y_i + n \alpha \beta) & 4(\Sigma y_i^2 + n \beta^2) & -2n\beta \\
-2n\alpha & -2n\beta & n
\end{pmatrix} \]

and \( \delta xy \) is the R.M.S. measurement error on \( x, y \) coordinates in space. The azimuthal angle at the beginning of the track is given by

\[ \phi = -\tan^{-1} \left( \frac{x_i - \alpha}{y_i - \beta} \right) \]

and the error

\[ < \delta \phi^2 > = \frac{1}{\rho^2} \left[ \sin^2 \phi \ < \delta \rho^2 > + \cos^2 \phi \ < \delta \alpha^2 > + 2\sin \phi \cos \phi \ < \delta \alpha \delta \beta > \right] \]

(b) **Parabola fit**

For the parabola a fit is made to \( y = ax^2 + bx + c \) minimising

\[ \delta^2 = \Sigma \left( y_i - ax_i^2 - bx_i - c \right)^2 \]

giving

6650/\rho
is calculated where $\Theta = \frac{1}{2} \left( \Theta_b - \Theta_e \right)$ and $< \delta x^2 >$ is the average of the errors on the x and y coordinates of the vertices.

For each curved track we now have an estimate of the momentum, an azimuthal angle at the beginning of the track, and a dip angle, all with associated errors.

(a) **Straight tracks**

For tracks which are straight a fit of $y = ax$, $z = \beta + \gamma x$ is made from which

$$\alpha = \frac{\sum x_i y_i}{\sum x_i^2}$$

$$\beta = \frac{\sum z_i}{n}$$

$$\gamma = \frac{\sum x_i z_i}{\sum x_i^2}$$

the azimuth $\phi$ is given by $\tan^{-1} \alpha$ and $\tan \lambda$ by $\gamma/(1 + a^2)^{\frac{1}{2}}$

the corresponding errors are

$$< \delta \phi^2 > = \cos^4 \phi < \delta \alpha^2 > = \cos^4 \phi \frac{\delta xy^2}{\sum x_i^2}$$

$$< \delta \tan \lambda^2 > = \tan^2 \lambda \left( \frac{\delta z^2}{\gamma^2 \sum x_i^2} \right)$$

the length and error on the length of the track are also calculated.

For tracks with only two points and a vertex at both ends, the vertex coordinates are used to determine $\phi$ and $\tan \lambda$ and their respective errors.
3. **SPECIAL HEAVY LIQUID MODIFICATIONS**

(a) **Beam tracks, event topology and charge balance**

In heavy liquid experiments, more especially cascade zero events, there is often some doubt as to which beam interaction produced the particles. To cover this case more than one beam track can be measured on an event. The first track measured by definition must be a beam track but other tracks which satisfy the conditions for a track to be a beam particle will be treated as such. The final distinction between beam tracks and others is that they have no vertex at the beginning.

Having found all the beam tracks the topology of the event is then explored. Up to this point all tracks can have been measured in either direction, but now all those connected to the beam by a charged track can have an unambiguous direction associated with them. All tracks which only have one vertex on them are assumed to go away from this vertex.

Tracks which are straight or have a large curvature error relative to their curvature, have their charge set as unknown. By carrying out a charge balance at each vertex most of these ambiguities can be resolved. Since the target at the primary interaction is not known, charge balance may not be possible at this vertex except in special cases. In general, an interaction with a proton is assumed when there are an odd total number of tracks at the vertex, and a neutral decay is assumed when there are an even number of tracks.

In general, therefore,

\[ \Sigma_{i} \text{charge of outgoing particles} - \text{charge of target} = \begin{cases} 1 & \text{for odd no. of tracks} \\ 0 & \text{for even no. of tracks} \end{cases} \]
We can, therefore, deal with the following cases

but not with

and similar types which occur when investigating resonances $\pi^\pm$ at the production vertex. In this case charge balance can be left out, or only performed at the vertices other than the beam interaction vertex.

(b) Stopping tracks and kinked tracks

At the measuring stage information is supplied which says which tracks in the event kink and which tracks stop. In the case of stopping tracks, the mass to be assigned is also given. Stopping tracks in heavy liquids often tend to scatter violently, so that the length calculated from the fitted curve may not be the best estimate. If points are measured sufficiently close together a better length is obtained by summing the chord lengths between adjacent points. This is done for all the stopping and kinking tracks.

If the track kinks near the beginning and does not stop, then the estimate of the momentum and error is taken from the longest section and angles are taken from the first section. For kinked tracks which stop and for ordinary stopping tracks, the momentum is calculated from the total range and the error is obtained from

$$< \delta(1/p)^2 > = \frac{1}{2} \left[ (\delta l_m^2 + \delta l_s^2) \left( \frac{dp}{ds} \right)^2 + \left( \frac{\delta d}{d} \right)^2 p^2 \right]$$
where \( \delta l^2_m \) = error in length\(^2\) due to measurement
\( \delta l^2_s \) = error in length\(^2\) due to straggling
\( \delta d/d \) = error in density/density of liquid (allows for errors in range/momentum table)
\( dp/ds \) = gradient of range/momentum curve at momentum \( p \)

the straggling error is taken as

\[
\delta l_s = \frac{0.01 L}{\sqrt{m}} \left( \frac{c_1}{T/10m} + c_2 \right)
\]

where \( m = \frac{\text{mass of particle}}{\text{proton mass}} \)

and \( T = \text{kinetic energy in MeV} \)

\( c_1 \rightarrow c_3 \) are constants

At this stage any track sections or vertices which are no longer of interest (i.e. kink sections and vertices at end of stopping tracks) are removed and the event reduced to its minimum size.

\( (c) \) **Optimum length**

So far the angle variables have been obtained using a fit to the whole track. Due to large coulomb scattering it is better to use an optimum length where the sum of the errors due to measurement and coulomb scattering is a minimum. Measurement errors go as \( 1/L \) and coulomb errors as \( \sqrt{L} \). To achieve this minimisation generally would require an iterative loop around the curve fitting part of the programme, using a variable length of track and variable masses.

In practice for each track which is curved we calculate six optimum lengths. These are three each for azimuth and dip angle calculations, first with the kaon mass, then with the pion
mass, and finally with the proton mass used in the coulomb error formulae. If the mass of the track is known, then only the two optimum lengths associated with that particular mass are calculated.

\[
\Delta\phi_m = \text{measurement error on azimuth} = \frac{c_1}{L \cos \lambda}
\]

\[
\Delta\lambda_m = \text{measurement error on dip} = \frac{c_2}{L \cos \lambda}
\]

\[
\Delta\phi_c = \text{coulomb error on azimuth} = \frac{c_3\sqrt{L}}{p\beta \cos \lambda}
\]

\[
\Delta\lambda_c = \text{coulomb error on dip} = \frac{c_4\sqrt{L}}{p\beta}
\]

are taken as the general expressions for errors in terms of length, dip and momentum we require \( L_{\phi} \) for which \( \Delta\phi_m + \Delta\phi_c = \text{minimum} \)

and \( L_{\lambda} \) for which \( \Delta\lambda_c + \Delta\lambda_m = \text{minimum} \)

this gives

\[
L_{\phi} = \left( \frac{2c_1 p\beta}{c_3} \right)^{\frac{2}{3}}
\]

\[
L_{\lambda} = \left( \frac{2c_2 p\beta}{c_4 \cos \lambda} \right)^{\frac{2}{3}}
\]

where \( p = \text{momentum at beginning of track} \)
\[
\beta = \frac{p}{(p^2 + m^2)}^{\frac{1}{2}}
\]

\( c_1 \) to \( c_4 \) are variable parameters

If the optimum lengths so calculated are less than the measured length, the fit is redone using this reduced length and the new angle and its fit error calculated. No momenta are altered by these new fits, but the lengths used in the fits are used to calculate the angle coulomb errors.
(d) **Coulomb Errors**

Once the fitting has been completed the coulomb errors are added to the already calculated errors in quadrature, so giving final errors on $\phi$, $\lambda$ and $p$. The variables used are in fact $\phi$, $\tan \lambda$ and $1/p$ as these can be more easily related to the original measurements for error estimates.

The coulomb formulae used are the following

\[
\Delta \phi = c_1 f \sqrt{L}/\cos \lambda \\
\Delta \lambda = c_1 f \sqrt{L} \\
\Delta \left(\frac{1}{p}\right) = c_3 f / \cos^2 \lambda \sqrt{L}
\]

where $f = \frac{c_0}{B^2}$

$c_0$, $c_1$, and $c_3$ are dependent on the chamber liquid.

These lead to

\[
<\Delta \phi^2> = \frac{c_1^2 f^2 L \phi}{\cos^2 \lambda} \\
<\Delta \tan \phi^2> = c_1^2 f^2 L \lambda \sec^2 \lambda \\
<\Delta \left(\frac{1}{p}\right)^2> = \frac{f^2}{L B^2 \cos^2 \lambda} + \frac{c_1^2 \tan \lambda \cdot L}{p^2}
\]

(e) **Electrons and gamma rays**

At the measuring stage comments are made and transferred to the programme that certain tracks are electron tracks. These tracks have so far been treated identically to all others except that wherever mass is involved the electron mass is used.
However a new routine has just been added (by Mr. G. Cooklin of University College) which calculates the momentum and error on momentum by the Behr-Mittner method. This is carried out on either the optimum length (approximately 10 cms) of the track or on the amount of track available before a visible scatter occurs.

Each vertex with two charged tracks at it is then scanned to see whether both tracks have electron comments. If they do then a $\gamma$ ray is introduced in such a way that it appears to be going from a previously non-existent vertex to the vertex from where the electron pair emerges. The angles, momentum and errors of the $\gamma$ ray are obtained by weighting those values found on the electron tracks by the inverse of their squared errors.

Thus if $E_i$ is a squared error and $x_i$ the angle variable on an electron track,

$$\chi_{\gamma} = \frac{x_1 E_2 + x_2 E_1}{E_1 + E_2}$$

$$E_{\gamma} = \frac{2E_1 E_2}{E_1 + E_2}$$

(c) Results

The results from the geometry programme are written onto a magnetic tape, which serves as input to the hypothesis testing and kinematical fitting programmes. A comprehensive library system exists to enable magnetic tapes to be continually up-dated, the geometry programme being able to add information to tapes which have been processed by the kinematics and vice-versa. All events are stored in numerical frame number order on the tape, events of different types all being on the same tape.

6650/p
For each event there are five records written onto tape, a book-keeping list, a vertex list, a track list, a helix fit list (angles, momenta and errors) and a kinematic fit list. The geometry programme supplies the first four of these and a dummy fifth, later the kinematics programme adds to four and five.

The geometry programme has been used by the R.H.E.L. and U.C. in the T8 collaboration to study \( E^0 \) and \( E^- \) and is currently being used in the T41 collaboration.

In T8 all the possible \( E^0 \) candidates were processed through the geometry about 70 events, the most likely candidates having several different measurements made. All the \( E^- \) events, about 200 in all, have also been processed again with several measurements of some events.

At the present time the pass rate is about sixty per cent at the first attempt. The forty per cent which fail can be broken down into several categories.

1. Human mistakes at measuring
2. Computer mistakes in going from paper tape to cards
3. Human handling of cards
4. Genuine failures
5. Programme errors

Failures of type 4, requires events to be re-measured and this probably accounts for about twenty per cent.

Failures of type 1, requires either re-measurement or original output to be edited and accounts for about ten per cent.
Failures of typos 2, 3 and 5 account for the remainder, the programme faults now being relatively small - approximately three per cent.
APPENDIX

BEHR-MITTNER ELECTRON MEASUREMENTS IN THE N.I.R.N.S.
H.L.B.C. GEOMETRY PROGRAMS

Theory

In the heavy liquids used for current bubble chamber experiments, the radiation length is between ten and thirty centimeters. This leads to difficulty in measurement of electron momenta by curvature. The radiation length is of the same order as the length of track normally measured, and single bremsstrahlung collisions, in which a large fraction of the incident electron's energy is lost as a \( \gamma \)-ray, occur frequently. Thus within the measured length of a track, the electron energy may change by a large percentage, with a consequent change in curvature and perhaps a scatter.

Behr and Mittner\(^a\) in Paris analysed the process of track formation, using Heitler's approximate bremsstrahlung formula, and finding the error produced by radiation loss on three-point measurements of tracks. They give the correction to be applied to the formula for the probable momentum, an expression for the error in momentum and a method of working out the optimum measurement length.

All these formulae are dependent on the assumption that single radiations of energy greater than a certain proportion of the initial energy can be recognised by the sudden change in curvature that they cause. Tracks containing such single radiations are only to be measured over the region before the radiation. If the magnitude of the smallest single radiation

---

\(^a\) Behr's thesis - unpublished. Also paper presented by Rousset at CERN Instrumentation Conference 1962.
that can be detected by sudden change of curvature is $k_o$, then
the Behr-Mittner momentum at the beginning of the track is given
by:-

$$P = \frac{3 \cdot B \cdot l_p^2}{8 \cdot \cos \lambda} \times \left( 1 + \frac{b \cdot l_p \cdot y_o}{2 \cdot \cos \lambda} \right) \times \frac{\epsilon_{\text{coll}} \cdot l_p}{2 \cdot \cos \lambda}$$ (1)

and $y_o = - \log_e \left( \frac{k_o}{E} \right)$

The momentum error is derived from

$$\Delta \left( \frac{1}{p} \right) = \left[ \frac{b \cdot l_p \cdot y_o^2}{6 \cdot \cos \lambda} \times \left( \frac{42}{35} \right)^2 \times \frac{1}{6 \cdot l_p \cdot \cos \lambda \cdot X_o} \right]^{\frac{1}{2}}$$ (2)

Where

- $l_p$ is the length of the chord from the first to the third points of measurement, projected on the plan perpendicular to the magnetic field.
- $s$ is the sagitta at the mid-point, also projected
- $\lambda$ is the dip of the track from the plane of the chamber window
- $B$ is the field in kilogauss
- $X_o$ is the radiation length
- $b = 1/(X_o \log_e 2)$ - the Heitler radiation parameter
- $\epsilon_{\text{coll}}$ is the mean collision loss of electrons per cm in the energy region concerned.

In (1) the first term, outside the brackets, is the
momentum corresponding to a track with the given \( l_P \), \( a \), etc. in a vacuum. The second term inside the brackets is the bremsstrahlung correction, and the term \( \frac{\epsilon_{\text{coll}} l_P}{2 \cos \lambda} \) is the collision loss correction.

In (2) the first term gives the bremsstrahlung error and the second the multiple-scattering error.

Differentiating (2) shows that the error has a minimum when the two partial errors are equal, hence an optimum value of \( l_P \) is found for measurement.

\[
l_{P_{\text{opt}}} = \left( \frac{42}{35} \right) \sqrt{\frac{\log 2}{y_0}}
\]

**Constants in Current Experiments**

T8 - liquid \( \text{C}_2 \text{F}_5 \text{Cl} \) \( x_0 = 25 \text{ cms.} \) \( B \sim 17 \text{ Kg.} \)

\[
l_{P_{\text{opt}}} \approx 10 \text{ cms} \quad \left( \frac{\Delta \left( \sqrt{y_P} \right)}{y_P} \right)_{\text{opt}} = 30\%
\]

T11 - liquid \( \text{CF}_3 \text{Br} \) \( x_0 = 11.5 \text{ cms.} \) \( B \sim 17 \text{ Kg.} \)

\[
l_{P_{\text{opt}}} \approx 10 \text{ cms} \quad \left( \frac{\Delta \left( \sqrt{y_P} \right)}{y_P} \right)_{\text{opt}} = 45\%
\]

T23 - liquid \( \text{C}_3 \text{H}_6 - \text{CF}_3 \text{Br} \) mixture \( x_0 = 22.7 \text{ cms.} \) \( B \sim 13 \text{ Kg.} \)

\[
l_{P_{\text{opt}}} = 13 \text{ cms} \quad \left( \frac{\Delta \left( \sqrt{y_P} \right)}{y_P} \right)_{\text{opt}} = 37\%
\]

In all the above cases it is assumed that all single bremsstrahlung greater than 50% \( (y_0 = \log_e 2) \) can be detected by the resultant change in track curvature.

* Behr says that recent Paris measurements show \( l_{P_{\text{opt}}} \) in T11 is more like 7 cms.
DISCUSSION ON THE TALK BY J. SPARROW

Moorhead: Comparing your programme with THRESH it seems to be identical up to and including the method of corresponding points. You say that you make a linear interpolation. That is exactly what we are doing now, and what I was proposing to improve. Do you have any feeling about this?

Sparrow: Maybe the improvements will help; I don't know. We seem to be 0... at the moment. I think that the only way that this will be reflected is in the errors, and the only way that one can really find out if one's errors are good enough is to wait until events have been fitted and look at $X^2$ distributions and so on. Until one has done this really thoroughly and investigated whether measurement errors are causing trouble I don't think one can answer the question of whether something better than linear interpolation is necessary. For hydrogen chambers, I think it is.

Moorhead: I can see that it must be better for highly-curved electrons for example, unless you measure points very close together.

Sparrow: Yes. Certainly on highly-curved tracks, we would expect that the measurements were done at close enough intervals to make linear interpolation good enough.
Eaton: The point is, in this programme, one doesn't try measuring right round the 360° of an electron track anyway. One stops off at an optimum length, so that one doesn't really get involved with measuring highly-curved electron tracks.

Sparrow: An arbitrary sort of cut off is put on electron tracks, of about 7 cms, or to the first point where it visibly scatters.

Moorhead: The only point in measuring right round these electrons was to find the length. Whether this would be affected by the linear interpolation I am not sure.

Sparrow: It probably would. We aren't interested at the moment in the length of electrons. I think probably if we were, then what you say is perfectly true.
RANGE PROGRAMME

by

G. Rinaudo and A. E. Werbrouck
(Istituto Nazionale di Fisica Nucleare-Sezione di Torino)

1. INTRODUCTION

The object of the programme RANGE is to measure the range of tracks that a) turn through large angles, b) display significantly different curvature along the track, and c) undergo considerable multiple scattering.

In the usual reconstruction programme, characteristic a) would imply that no single view serves well as principal view, while b) and c) imply that it is difficult to fit an analytic curve.

Our solution is to measure the entire track in three views. This overdetermination permits the programme 1) to select the optimum principal view for each point, and 2) to reconstruct each point in the two optimum combinations, among the three available with three views. In this way we determine any gross measurement errors and obtain a precise reconstruction without utilizing an analytic curve.

The desired range is then obtained by simply adding track segments and introducing a phenomenological correction for multiple scattering and statistical measurement errors. To further reduce the effect of multiple and measurement scattering we use quadratic interpolation along tracks with at least three measured points.
If necessary the reconstructed points are also quite suitable for determining curvature.

General description

RANGE is largely a geometry programme. It can be thought to consist of three main parts: spatial reconstruction, determination of the range of a track, and a statistical determination of the probability that such a track be contained entirely within a finite chamber volume. The spatial reconstruction part is generally useful as it can be followed by any geometrical or kinematic calculation, while the latter two parts serve specifically for the tracks of the stopping particles. Recently we have successfully employed the spatial reconstruction part to measure curvature in a hydrogen bubble chamber. RANGE is written in FORTRAN for the IBM 7090. To follow the detailed programme one needs the detailed flow charts. Only a general flow chart is included here.

1) SPATIAL RECONSTRUCTION

The more interesting feature of RANGE is that a principal view is selected for the spatial reconstruction of each point. In this way it is easy to follow a track curving through any angle (even an electron spiral in hydrogen). This possibility avoids the messy procedure of reconstructing in sections a long highly curved track for which one desires the range.

To conveniently select the principal view and the measured points for interpolation for each reconstructed point, it is important to maintain much symmetry in the reference systems. Diagram I (optic axes along $z$) displays several of the reference systems needed. All systems are right handed. All final output and primary geometrical constants are expressed in the chamber system $(x_{ch}, y_{ch})$, chosen to have positive event coordinates. There are

6650/p
three proper systems defined as those with an optic axis at the origin and identified by the optic axes joined in cyclic order by their respective x axes (i.e., $x_3$, points from 3 to 1). Between these there are six rotational transformations and three more from each to the chamber system as indicated by the inset table (Diagram 1). Every measured point is, after the corrections to be explained below, expressed and stored in the two proper systems whose x axes connect the optic axis of the view in which it was measured. For example, every view 1 point is expressed in systems 12 and 31, always with origin at axis 1. In this way regardless of which view is principal view, the points are expressed in a system ready for the interpolation equations. The following tables summarize the storage and utilization of measured corrected x-y points.

<table>
<thead>
<tr>
<th>View</th>
<th>Origin</th>
<th>First Proper System</th>
<th>First Storage Index</th>
<th>Second Proper System</th>
<th>Second Storage Index</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Axis 1</td>
<td>12</td>
<td>1</td>
<td>31</td>
<td>6</td>
</tr>
<tr>
<td>2</td>
<td>Axis 2</td>
<td>23</td>
<td>2</td>
<td>12</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>Axis 3</td>
<td>31</td>
<td>3</td>
<td>23</td>
<td>5</td>
</tr>
</tbody>
</table>

The principal view is simply the view opposite the x axis most nearly parallel to the track segment to which the interpolation has advanced as seen in a control view. The control view in RANGE is the view in which the projected chord is longest. One could also select the view most near the centre of the track as control view, among other choices. According to the principal view selected one has the following table of storage indices for principal and auxiliary views for the two reconstruction combinations. These storage indices serve to find the points for interpolation between the principal and auxiliary views.
<table>
<thead>
<tr>
<th>Principal view</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>First Combination</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>principal</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>auxiliary</td>
<td>4</td>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td>Second Combination</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>principal</td>
<td>6</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>auxiliary</td>
<td>3</td>
<td>1</td>
<td>2</td>
</tr>
</tbody>
</table>

In Diagram I, an illustrative example with view 1 as the principal one is given, which requires access to the points stored with indices 1, 4, 6 and 3.

The example of system 31 transferred to a fiducial mark as seen by optic axis 3 serves only as an intermediate step in the transformation of measured points from measure coordinates to those of the proper system.

To understand the logical steps of the spatial reconstruction it is also important to distinguish clearly between real, distorted, and reduced space. Real space is the actual three dimensional chamber space. Distorted space is a two dimensional projected space, considered in RANGE to be at the constant \( z \) of the distorted upper fiducial plane (DiagramII). One sees the distorted space when looking at a single projected bubble chamber frame. Reduced space is a three dimensional space parametrized by a \( z \) value in which all optical media are reduced in \( z \) dimension by their respective refractive indices \( N \). Reduced space is defined rigorously by the equation (5) below, as that space from which one passes into real space using the simplest reconstruction formulae eq. 13. The transformations from distorted to reduced to real space are illustrated in Diagrams II and III. In the former light rays are traced
from point A in real space to the film when the picture is taken and from the film to the projection table at measurement time.

Point A' is shown in reduced space after the transformation from distorted space and before that back to real space. The transformation from distorted to reduced space, i.e. from $l'_o$ to $l_o(z)$ (also called distortion correction) follows from considering Diagram II.

For the light ray from A, we have in real space

$$\sin \lambda = N_i \sin \gamma_i = N_z \sin \gamma_z = N_i \sin \gamma_i = N \sin \alpha$$

$$l_o(0) = \Sigma_i D_i \tan \gamma_i \tag{1}$$

$$l(z) = l_o(0) + z \tan \alpha$$

$$l_o(0) = \sin \lambda \cdot \frac{\Sigma_i D_i/N_i}{(1 - \sin^2 \lambda/N_i)} = \tan \lambda \frac{\Sigma_i D_i/N_i}{(1 + \sin^2 \lambda(N_i^2 - 1)/(1 - \sin^2 \lambda)N_i^2)}$$

$$\approx \tan \lambda \Sigma_i D_i N_i \left(1 - \frac{\tan^2 \lambda(N_i^2 - 1)}{2N_i^2} + \frac{3}{8} \frac{\tan^4 \lambda(N_i^2 - 1)^2}{N_i^2} \ldots \right)$$

$$= l'_o - b_0 l'_o^3 + c_0 l'_o^5 \quad b_0 > 0, \ c_0 > 0 \tag{3}$$

where $l'_o = \tan \lambda \Sigma_i D_i/N_i$. We can also write:

$$l_o(0) \geq l'_o - a_0 (l'_o)^3 \quad a_0 > 0 \tag{4}$$

which gives us

$$a_0(l'_o) = \frac{l'_o - l_o(0)}{l'_o^3} \tag{4}$$

If we know $b_0$ and $c_0$, we can find, for a projected point at a distance $l'_o$ from an optic axis the corresponding $l_o(0)$. We set $l_o(0)$ in reduced space equal to the distance from the optic axis to a real point at $z = 0$ along the ray from A. In this way our reduced space becomes completely defined by the relation

6650/0
\[ \frac{l_0(z)}{\Sigma D/N_1} = \frac{l(z)}{z/N + \Sigma D/N_1} \]  

(5)

This equation also defines \( l_0(z) \) and states that the transformation from distorted to reduced space must adjust distances from optic axes in such a way that the simplest straight line reconstruction formulae, Eq. 13, will give us the correct \( x, y \) and \( z \) in real space, if \( z \) is correct. The subscript zero refers in this development to distances measured in the upper distorted fiducial plane. In a similar manner we can define:

\[ a_2(l_0) = \frac{l_0 - l_0(z)}{l_0^3} = \frac{[l_0 - l_0(0)] + z [l_0/NH - \tan a]}{l_0^3 + z [l_0^3/NH]} \]  

(6)

\[ = \frac{PC1 + PC2 \cdot z}{PC3 + PC4 \cdot z} \text{ by eqs. 2, 4 and 5 with } H = \Sigma D/N_1 \]

However, the equation requires \( z \) which is available only after a transformation to real space. In practice we choose an intermediate \( z \) value, \( ZM \), where the beam enters the chamber and most particles interact. We thus treat at first all projected points as if they were at this depth. If the transformation to real space yields a \( z \) which differs from \( ZM \) by more than an adjustable constant \( ZST \), we use this new value of \( z \) in a second approximation to retransform to reduced and then to real space. In terms of the above definition, \( ZM \) parametrizes the first approximation reduced space.

We choose the transformation eq. 3 from distorted to reduced space. To evaluate the two constants \( b_0 \) and \( a_0 \), we must find \( a_2(l_0) \) by eq. 4 for two values of \( l_0 \). We choose RNR and RFR, near and far distances from the optic axes to projected points. (RNR > 0 because \( a_0(0) \) is indeterminate by our equations.)
Then

\[ b_0 - c_0 \cdot RNR^2 = a_0(RNR) \]
\[ b_0 - c_0 \cdot RFR^2 = a_0(RFR) \]
\[ c_0 = \frac{a_0(RNR) - a_0(RFR)}{RFR^2 - RNR^2} \]
\[ b_0 = a_0(RFR) + a_0 \cdot RFR^2 \] (7)

For the \( z \) planes at \( ZM \) and \( ZL \), we use eq. 6 twice and then eq. 7 to have \( CM, BM \), for \( ZM \) and \( CL, BL \) for \( ZL \). The constants \( a_0, b_0, CL \) and \( BL \) will serve for the inverse transformation from real to distorted space of the fiducial marks while \( CM \) and \( BM \) serve from distorted to reduced space of measured points.

For a \( z \) other than the three above, one should calculate two values of \( a_6 \) by eq. 6 and then find \( EZ \) and \( CZ \) from eq. 7. However, we find that \( CZ \) varies slowly so that we use \( EZ = a_6(RFR) + CM \cdot RFR^2 \). Thus the first and the second \( l_0(z) \) values become

\[ l_0(ZM) = l_6 - BM \cdot l_6^3 + CM \cdot l_6^5 \] (8)
\[ l_0(z) = l_6 - EZ \cdot l_6^3 + CM \cdot l_6^5 \]

and the second approximation correction to \( l_0(z) \) becomes

\[ \Delta l = l_0(z) - l_0(ZM) \approx (BM - EZ) \cdot l_6^3 \approx (BM - EZ) l_6(ZM)^3 \] (9)

Thus far, all the distortion corrections have been represented in a plane that contains the optic axis and \( l_6 \). One can write eq. 8 in vector form.
\[ l_0(z) = l_0 - a_z (l_0 \cdot l_0) l_0 \quad \text{so that} \]
\[ x_0(z) = x_0 - a_z (l_0 \cdot l_0) x_0 \quad \text{and} \]
\[ y_0(z) = y_0 - a_z (l_0 \cdot l_0) y_0 \]

The transformation from reduced space to real space is extremely simple after the above application of the consequences of eq. 5. Reduced space is always referred to a single optic axis. To pass into real space it is necessary to find corresponding points in two views by interpolation. In RANGE, there are three orders of interpolation: zeroth, or corresponding point; first, or linear; and second or quadratic. The topology, or event type, for which the detailed RANGE is written exemplifies all three orders and is the following \( K^+ \) decay in which the secondary stops and gives an electron. Point 1 is the \( K \) decay, a corresponding point; point 2 along the \( K \) track and \( n \) along the initial tangent of the electron are correlated in the three views by a linear interpolation; and quadratic interpolation is used along the secondary track. If the secondary is measured with two points only (programme condition L2P = 1) they are both treated as corresponding points. Diagram I illustrates the points in reduced space which enter the following interpolation equations.

linear: \( XAC1 = XA + (YPC1 - YA) \cdot FA1 \quad \text{where} \quad FA1 = \frac{XB - XA}{YB - YA} \) \( (11) \)

and the notation is of the form: \( XAC1 \) is \( X \) value in the auxiliary view, combination 1.

quadratic: \( XAC1 = XA + (YPC1 - YA) \left[ FA1 \cdot (YPC1 - YB) \left( \frac{F2 - FA1}{Y0 - YA} \right) \right] \)

\( (12) \)
where \( F_2 = \frac{X_C - X_B}{X_C - Y_B} \) with the condition that \( YPC_1 - Y_A \) has a different algebraic sign than \( YPC_1 - Y_B \) until \( Y_C \) is the last point. Thereafter the point designations no longer shift for the rest of the track reconstruction.

The first part of Diagram III illustrates the geometrical situation after interpolation (First combination).

\[
\frac{-XAC_1}{H} = \frac{AB}{H+z/N}, \quad \frac{XPC_1}{H} = \frac{BC}{H+z/N} = \frac{x_0(z)}{H}
\]

The minus sign is necessary because the optic axis corresponding to the view in which the points were measured remains the origin in both proper systems.

\[
z = NH \left( \frac{AC}{XPC_1 - XAC_1} - 1 \right)
\]

\[
x = XPC_1 \frac{AC}{XPC_1 - XAC_1}
\]

\[
y = YPC_1 \frac{AC}{XPC_1 - XAC_1}
\]

For the second combination an additional set of \( x \ y \ z \) values results. A simple average is used to combine the \( x \) and \( y \) values since the factor \( \frac{AC}{XPC_1 - XAC_1} \) is not sensitive to measurement errors. To combine the \( z \) values a weighted average is used based on the following considerations. One can rewrite the expression \( (XPC_1 - XAC_1) \) as \( AC - \Delta x \) so that \( z \) becomes

\[
z = \frac{NH}{AC} \frac{\Delta x}{1 - \Delta x/AC}
\]

The denominator is slowly varying since \( \Delta x \ll AC \), so that \( \sigma_z = \frac{NH}{AC} \sigma_{\Delta x} \) where \( \sigma_{\Delta x} = \sigma_{\text{prin}}^2 + \sigma_{\text{aux}}^2 \).

6650/p
The $x$ error in the principal view is taken as a constant, 

$$\sigma_{\text{prin}}^2 = \sigma_x^2 = \sigma_y^2.$$ 

Errors in $x$ and $y$ are set equal because the original $x$, $y$ measurement errors are quite scrambled after the reconstruction due to the combinations of systems in which the points are stored and interpolated. The auxiliary $x$ value is approximately on the line

$$x_{\text{aux}} = FA_1 \cdot y + B$$

where $B$ is a dummy intercept, $FA_1 = \frac{XB -XA}{YB - YA}$ in the programme for first combination, $FB_1$ the same for second, while $XB$, $XA$, $YB$ and $YA$ are illustrated in Diagram I. In this way $\sigma_{\text{aux}}^2 = \sigma_x^2 \left(1 + FA_1^2\right)$ and $\sigma_{\Delta x}^2 = \sigma_x^2 \left(2 + FA_1^2\right)$. The product $NH$ is a constant but the optic axis separation varies with the combination so that the final weight for a $z$ value becomes

$$W = \frac{AC^2}{2 + FA_1^2}$$

and finally

$$z = \frac{W_1 z_1 + W_2 z_2}{W_1 + W_2}$$

In terms of our notation, the standard error in the combined $z$ becomes

$$\sigma_z = \frac{NH\sigma_x}{\sqrt{W_1 + W_2}}$$

The equations for the second approximation correction as applied to the auxiliary views also keep account of the slope of the interpolated line, eq. 15. In the principal view in reduced space, eq. 9 becomes by eq. 10.
\[ x_0(z) = x_0(ZM) \left[ 1 + (BM - EZ) \frac{y_0^2(ZM)}{\text{prin}} \right] \]

\[ y_0(z) = y_0(ZM) \left[ 1 + (BM - EZ) \frac{y_0^2(ZM)}{\text{prin}} \right] = y_0(ZM) + \Delta y_0 \text{prin} \tag{18} \]

and in the auxiliary view

\[ x_0(z) = x_0(ZM) \left[ 1 + (BM - EZ) \frac{y_0^2(ZM)}{\text{aux}} \right] + \Delta y_0 \text{prin} \cdot FA1 \tag{19} \]

The subscript to the bracket indicates the view in which \( l_0 \) is calculated. The second part of Diagram III illustrates the effects of the second approximation distortion correction in an exaggerated manner. The curves \( l_0(z) \) and \( l_0(ZM) \) for each view are the results of transforming the curve \( l_0 \) from distorted space to reduced spaces parametrized by \( z \) and \( ZM \). The second approximation corrections are indicated by the points on the \( l_0(z) \) and \( l_0(ZM) \) curves connected by dashed segments.

After the values of \( x, y \) and \( z \) in the two combinations are found and transferred to the same system (chamber system) they are compared. Differences exceeding adjustable limits result in the printing of an error code. For an excessive error in \( z \), the quantities \( \overline{w}_1, \overline{w}_2 \) and \( \sigma_x/\sigma_z = NH/\sqrt{\overline{w}_1 + \overline{w}_2} \) are calculated and printed. These quantities are considered before requesting a remeasure. The weight for a single combination may be low due to significantly different projections, resulting from high dip or high curvature at the end of the track. A number of error codes exceeding an adjustable limit causes the programme to pass to the next event.

For the fiducial marks, the transformation situation is reversed as their positions are known in real space and must be found in distorted space in order to serve as reference for measured projected points. This transformation is effected once at the beginning of each run.
\[ l'_6 = l'_6(0) + a_6(l'_6) l'_6^3 \]  

where \( l'_6(0) \) is the true space distance from optic axis to fiducial mark. We calculate:

\[ l'_7 = l'_6(0) + a_6(l'_6(0)) l'_6^2 \]

and then

\[ l'_6 = l'_6(0) + a_0(l'_7) l'_7^2 \]

where \( a_6(1) = b_0 - c_6 \cdot l'^2 \).

These two iterations give sufficient precision. Thereafter the separations, positions, and orientations of the fiducial pairs to which can be referred measured events are calculated and stored as secondary constants. The latter name is in contrast to primary constants supplied by the programmer. Fiducial marks in the lower real fiducial plane are first projected upward in reduced space by eq. 5 and then transformed to distorted space using BL and CL in the determination of \( a'_{2L}(1) = BL - CL \cdot l'^2 \) and in two iterations as above. In this way all fiducial marks are in the same distorted space (that of the upper distorted fiducial plane) independent of their physical position. Thus the processing of events is the same regardless of the plane of the fiducial marks of reference.

2. **DETERMINATION OF RANGE**

The first approximation to the range of the stopping track in the bubble chamber medium is the sum of the segments (TOL in programme). Due to measurement errors and multiple scattering, the end points of the segments deviate from a smooth trajectory. We apply to TOL a phenomenological correction based on the following considerations. The length of each segment will be overestimated by

6650/p
a quantity $\delta l$. To evaluate $\delta l$ let us assume that the track is contained in the $yz$ plane (setting $x$ and $y$ errors equal gives us rotational symmetry about the $z$ axis) and that its dip angle is $i$ (assumed constant along the track). We then rotate about $x$ to a system with $\eta$ axis along the track.

![Diagram](image)

Here $2'$ is the actual point, 2 the measured one. Thus

$$\rho_2 = \delta y \sin i + \delta z \cos i, \quad \xi_2 = \delta x$$

When $i \neq \pi/2$, the error distribution in the constant $\eta$ plane is elliptical because of the larger errors in $z$. Then

$$1 + \delta l = \sqrt{(\eta_2 - \eta_1)^2 + (\xi_2 - \xi_1)^2 + (\rho_2 - \rho_1)^2} \quad (21)$$

on the average, $\eta_2 - \eta_1 = 1$,

$$\overline{(\xi_2 - \xi_1)^2} = \overline{\xi_2^2} + \overline{\xi_1^2} - 2\overline{\xi_1 \xi_2} \approx 2\overline{x^2}$$

$$\overline{(\rho_2 - \rho_1)^2} = \overline{\rho_2^2} + \overline{\rho_1^2} - 2\overline{\rho_1 \rho_2} \approx 2\overline{\rho^2} \approx 2(\delta y \sin i + \delta z \cos i)^2 =$$

$$\approx 2(\delta y^2 \sin^2 i + \delta z^2 \cos^2 i)$$

If there is no correlation between points 1 and 2, $\overline{\xi_1 \xi_2} = 0$. This is not exact since points 1 and 2 are correlated slightly through the quadratic interpolation.
Then \[ l + \delta l \approx l \sqrt{1 + \frac{2 \delta x^2}{l^2} + \frac{2(\delta y^2 \sin^2 i + \delta z^2 \cos^2 i)}{l^2}} \]

For purposes of this correction we can reduce the above relation (eq. 17) between \( \delta z \) and \( \delta x \) to

\[ \delta z^2 = k^2 \delta x^2 \]

Thus expanding and keeping the first term, we find

\[ \delta l \approx \frac{\delta x^2}{l} (1 + \sin^2 i + k^2 \cos^2 i) = \frac{\delta x^2 (k^2 + 1)}{l} (1 - \frac{(k^2 - 1)}{k^2 + 1}) \sin^2 i \]  

(22)

with the dependence on dip controlled by the term \((k^2 - 1)/(k^2 + 1)\) which varies little since \(k^2 >> 1\). For the Lagarrigue chamber we take \(k = 3.5\). We adjust experimentally \(\delta x^2 (k^2 + 1)\) until the range is stationary for a track measured several times each time with a different number of points. Instead of the true dip of the track, we use the dip of the chord in the length correction. The expansion assumes \(l >> \delta l\). Thus a segment shorter than an adjustable constant SML (SML = 1 cm now) is not added (unless it is the last segment) and the length to the next reconstructed point is calculated. If the segment was the last segment, the correction becomes \(\approx \frac{1}{\text{SML}}\).

3. **MONTE-CARLO CORRECTION**

Probability of containing the reconstructed track in the finite chamber volume. This probability is necessary to reproduce a decay spectrum as its inverse is the weight of the individual observed event.

Given the range (RANGE), the chord (CHD) and the initial point \((X_0, Y_0, z_0)\), the track is reoriented ME times and the programme

6650/p
calculates how many times NE the entire track is contained in the fiducial volume. This reorientation is effected systematically by fixed intervals instead of by random number selection. There are two difficulties due to the presence of the magnetic field:

1 - the track may have the end points inside the chamber, but have some intermediate part outside

2 - the ratio RANGE/CHD is not constant, but depends on the dip of the track.

To carry out the Monte-Carlo calculation we approximate the track to a helix analytically from the CHD and RANGE. The actual track deviates in orthogonal projection from a circle as shown below. By plotting measured points we find this deviation not serious for this scope. For a given range and mass the change of direction in the (x,y) plane is independent of dip, if the dip does not change significantly along the track.

\[
\text{RANGE} = \int ds = \int \frac{\sqrt{dx^2 + dy^2}}{\sqrt{dx^2 + dy^2}} \frac{ds}{\sqrt{dx^2 + dy^2}}
\]

where:
- RHOA = projected analytic radius of curvature
- SINC = sine of polar angle = cosine of dip angle = x,y projection of track
- DEL = analytic rotation angle of the track in the plane
- RHO = analytic radius of curvature of a track with the same range and zero dip angle.

First of all, DEL of the track is calculated from the CHD and the
RANGE of the track:

\[
\frac{\text{CHD SINC}}{2} = \sin \frac{\delta}{2} \text{ RHOA}
\]

\[
\text{CHD SINC} = \frac{\sin \frac{\delta}{2}}{\frac{\delta}{2}} \text{ RANGE SINF}
\]

SINC = x,y projection of the chord

The ratio is called TRF (transcendental function) and is treated as a constant of the motion

\[
\text{TRF} = \frac{\text{CHD SINC}}{\text{RANGE SINF}} = \frac{\sin \frac{\delta}{2}}{\frac{\delta}{2}}
\]

From this function, which is slowly varying in the \( \delta \), the value of \( \delta \) is deduced through a table. From the analytic radius of curvature RHO is calculated. The Monte-Carlo calculation is then done in the following way: the value of COSP is increased by a constant quantity PINT; from the new value of COSP the \( z \) value of the end point of the new track is calculated and checked whether it is comprised in the \( z \) limits (VZU and VZL).

If it is not, the trial has failed and we go to the next trial. If it is comprised the new parameters of the track are calculated, that is the chord, the projected radius of curvature, the projected chord, etc. The azimuthal angle of the CHD is then obtained increasing by a constant the azimuthal angle of the original chord. Instead of the azimuthal angle of the track, we can use the angle of the chord (SINT and COST) since DEL is constant. We then calculate the \( x \) and \( y \) coordinates of the end point XF, YF and we check if they fall within the limits (VXU, VX L; VYU, VYL). Since the fiducial volume of the chamber has a slope in the \( x,z \) plane (present figure and Diagram IV), the \( x \) exit depends on the \( z \) value of the track. If the point is outside the fiducial volume, the track is rejected, if not, we must check that no point falls outside. First of all we

\[
\text{BC2} = AC2 \cdot X - Z
\]

\[
\text{BC1} = AC1 \cdot X - Z
\]
we check if the point A, at the crossing of the tangents at the initial and the end points of the track is inside. If it is inside, the entire track is inside, so we accept the track and the number of successes is increased by one. If A is outside the fiducial volume, the track might still be inside. We therefore calculate if a circle with radius = the projected curvature radius (RHOA) intercepts the x and y limits of the chamber. AH and BI are the x and y coordinates of the centre of the circle at C.

\[
\frac{(x - AH)^2}{(y - BI)^2} = RHOA^2
\]

\[
x = AH \sqrt{RHOA^2 - (VYU - BI)^2}
\]

If \(RHOA < |VYU - BI|\) there is no solution and we accept the trial, otherwise we calculate the x value of the intersection point. If it is comprised between XD and XF the trial is unsuccessful, otherwise it is accepted.

The calculation is more complicated for the x limits, since they are a function of the z value of the point. The situation is shown in Diagram IV. The straight line which now represents the fiducial limit is obtained first projecting the chord on the sloping wall (XDB and XFD are the coordinates of the projected D and F points) and then projecting the resultant intersection into the x,y plane

\[
x = EF \cdot y + GI \quad \text{where} \quad EF = \frac{XF_B - XDB}{YF - YD} \quad GI = XFB - EF \cdot YF
\]

The intersection is then done as for the y limits; only, both the solutions must now be considered, since there is the possibility that only one of the two falls between YF and YD: in this case the
track does not intersect the wall. Not all the tracks follow this entire calculation. First of all, we check if a sphere of radius = RANGE is entirely comprised in the chamber volume. If it is, the weight is = 1 and no further calculation is needed. The tracks are divided in very short (RANGE < TAU), short (RANGE < RANS), medium and long. For the very short tracks we assume the chord to be equal to the range: therefore, we do not calculate $\delta$, $RHO$ etc., and we simply check that the end point of the reoriented track is inside the volume. For the very long tracks (ILM = 3, RANGE > RANL) we do not check the point A on the tangents, it being quite unlikely that it falls inside the chamber.

The number of trials is different for the different kind of tracks (20 for the short ones, 80 for the medium, 1600 for the long). This is done because the long tracks have generally very large weights. Besides, the interval of $\cos \varphi$ and $\theta$, by which the angles are increased, must be sufficiently small so that the original track can be considered a good example of random track within the volume; otherwise the other tracks obtained from the original one with the above described method can no longer be considered randomly oriented.

A card is punched with the necessary parameters to repeat the above calculation for another volume if one finds the first choice not the wisest, or if one wants to check that the spectrum is invariant to the confining volume.

4. ACKNOWLEDGEMENTS

One of us (A.W.) wishes to thank Drs. Wilson Powell and Howard White for their hospitality during the summer of 1962, and for the opportunity to study in Berkeley the FOG system from which certain aspects of this reconstruction are derived.

6650/p
DIAGRAM I
X-Y DIAGRAM OF CHAMBER COORDINATE SYSTEM, PROPER SYSTEM OF EACH OPTIC AXIS, NINE ROTATIONAL TRANSFORMATIONS, DASHED ORTHOGONAL PROJECTION OF TRACK WITH CORRESPONDING PROJECTIONS INTO FIDUCIAL PLANE AS SOLID CURVES

NOTE: X AXES JOIN OPTIC AXES IN CYCLIC ORDER
VIEW 1 IS PRINCIPAL VIEW
FIRST COMBINATION 1 AND 2 FOR ILLUSTRATIVE EXAMPLE
SECOND " 1 AND 3 " EXAMPLE

<table>
<thead>
<tr>
<th>ROTATION</th>
<th>FROM</th>
<th>TO</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>12</td>
<td>31</td>
</tr>
<tr>
<td>2</td>
<td>23</td>
<td>12</td>
</tr>
<tr>
<td>3</td>
<td>31</td>
<td>23</td>
</tr>
<tr>
<td>4</td>
<td>12</td>
<td>23</td>
</tr>
<tr>
<td>5</td>
<td>23</td>
<td>31</td>
</tr>
<tr>
<td>6</td>
<td>31</td>
<td>12</td>
</tr>
<tr>
<td>7</td>
<td>12</td>
<td>CHAMBER</td>
</tr>
<tr>
<td>8</td>
<td>23</td>
<td>&quot;</td>
</tr>
<tr>
<td>9</td>
<td>31</td>
<td>&quot;</td>
</tr>
</tbody>
</table>

AXIS 2

AXIS 3
FIDUCIAL MARKS IN DISTORTED SPACE OF OPTIC AXIS 3
Diagram II

OPTIC AXIS

FILM PLANE

LIGHT PATH FOR PROJECTING PICTURE

UPPER FIDUCIAL PLANE

UPPER REAL FIDUCIAL PLANE

DISTORTED PLANE

LOWER REAL FIDUCIAL PLANE

$D_1 / N_1$

$D_2 / N_2$

$Z / N$

$e_0$

$e_0(0)$

$e_0(z)$

$e(z)$

$e(z)$

$e(z)$

$\lambda_1$

$\gamma_1$

$\gamma_2$

$Z$

$ZM$

$ZL$
THE PROGRAM CONSISTS OF A MAIN PROGRAM, CONTAINING INPUT - OUTPUT AND OVERALL LOGICAL CONTROL, AND OF SEVERAL SUBROUTINES.

**MAIN PROGRAM**

1. **READ PRIMARY CONSTANTS**
2. **READ, PREPARE, STORE INPUT DATA**

**SUB-PROGRAMS**

1. **FLOW CHART II - SECON**
   - DERIVE SECONDARY CONSTANTS
2. **FLOW CHART III**
3. **FLOW CHART IV - COMBIN**
   - CORRECT FOR MAGNIFICATION
   - TRANSFORM TO OPTIC AXIS Centered SYSTEMS
   - CORRECT FOR OPTICAL DISTORTION
   - ROTATE TO SECOND PROPER SYSTEM
   - SELECT CONTROL VIEW
4. **FLOW CHART V AND VI - INTER**
   - SELECT PRINCIPAL VIEW
   - INTERPOLATE POINTS FOR RECONSTRUCTION IN TWO COMBINATIONS
5. **FLOW CHART VII - RECON**
   - RECONSTRUCT POINTS IN TWO COMBINATIONS
   - COMPARE AND COMBINE THE TWO RESULTS
6. **FLOW CHART VIII - ACCUM**
   - CALCULATE CHORD
   - EVALUATE LENGTH CORRECTION FACTOR
   - ADD AND CORRECT SEGMENT LENGTHS
   - FIND DIP AND AZIMUTH ANGLES
7. **FLOW CHART IX, X, XI - MONCAR**
   - STATISTICAL EVALUATION OF THE PROBABILITY OF CONTAINING THE TRACK IN GIVEN GEOMETRY
8. **PRINT AND PUNCH FINAL OUTPUT**
9. **MEMORY DUMP AND STOP**

**LAST EVENT PROCESSED?**

**YES**
A NEW GEOMETRY PROGRAMME

by

M. Heybrechts

(I.I.Sc.N. Laboratoire des Hautes Energies, Bruxelles)

The programme, called STAN, can be divided into two distinct parts:

1) reconstruction of points in space from the measurements of non-corresponding points;

2) estimation of parameters characterizing the track, as curvature, dip and azimuthal angle.

Concerning the first point, I want only to mention that the programme works with three views and calculates near corresponding points by second order interpolation. However, it makes no correction for the refraction of light rays and its input is not adapted to the output of the RHAP. It is better, therefore, to neglect the first part of the programme and to transform the second part into a sub-routine which can be introduced into the THRESH instead of the helix-fitting sub-routine (first and second order approximation).

We give, therefore, only a description of the second part of the programme, which calculates estimates for the parameters characterizing the tracks from the statistical analysis of simple quantities, as angles between chords, without any curve fitting.
1. **Estimation of Curvature**

We start with coordinates of points in space, reconstructed by the first part of THRESH, and, as we are interested in the estimation of the curvature, we project these points on a plane Oxz perpendicular to the mean direction of the magnetic field (i.e., a plane parallel to the front glass of the chamber).

The simplest quantities depending on magnetic curvature which we can form with the coordinates $x_i, y_i$ of the projected points are the angles between successive chords, given by the expression:

$$\omega_i = \frac{(y_{i+1} - y_i)(x_{i+1} - x_i) - (y_i - y_{i-1})(x_{i+1} - x_{i-1})}{(x_{i+1} - x_i)(y_{i+1} - y_i) + (y_i - y_{i-1})(x_{i+1} - x_{i-1})}$$

It is easy to see that the average value of $\omega_i$ is $\mathbb{E} [\omega_i] = \Theta, m_i$, where $\Theta = \frac{1}{p}$ is the magnetic deflection per unit length and

$$m_i = \frac{i}{2} \left( \sqrt{(x_{i+1} - x_i)^2 + (y_{i+1} - y_i)^2} + \sqrt{(x_{i+1} - x_{i-1})^2 + (y_{i+1} - y_{i-1})^2} \right)$$

**Maximum likelihood estimator**

If we suppose that the multiple scattering angle and the measurement errors have both gaussian distributions, it is possible to show, by using the Maximum Likelihood method, that the best estimator for $\Theta$ is a weighted average of the form

$$\Theta^* = \sum_i \frac{\omega_i}{m_i} p_i$$

The weights $p_i$ have been calculated for equidistant points and take

* M. Huybrechts, Bull. Acad. Roy. Belg. (Cl. des Sciences) 1961 - 7 pp 739-749
into account the correlation between successive angles \( \omega_i \). They depend on the number of measurements and on a parameter \( a^2 \), which is a function of the ratio between the multiple scattering and the measurement errors.

More precisely, \( a^2 \) is defined by the relation \( a^2 = \frac{3 \delta^2}{\sigma^2 \ell_0^2} \), where \( \delta \) is the r.m.s. measurement error of the ordinate of a point reconstructed in space, \( \sigma \) is the r.m.s. of the projected multiple scattering angle (angle between tangents to the projected track) per unit length and \( \ell_0 \) is the length of the part of the track considered for curvature estimation.

If we write \( \sigma = \frac{C}{p_{xy}^\beta} \), \( p_{xy} \) being the projected momentum on the Oxy plane, we obtain

\[
a^2 = \frac{3p_{xy}^2 \beta^2 \delta^2}{\sigma^2 \ell_0^3}
\]

**Determination of the weights \( p_i \)**

The exact values of the Maximum Likelihood weights are sometimes difficult to obtain, but it is possible to use approximate values:

a) We can take \( p_i = \frac{1}{n} \), \( n \) being the number of angles. In this way we do not use all the information available, but the programme becomes very simple. Moreover, it seems that this approximation is satisfactory when the multiple scattering is very important compared to the measurement errors; the loss of information would then be of the same order of magnitude as with curve fitting by the method of least squares.

6650/p
b) The weights \( p_i \) become nearly independent from \( \alpha^2 \) for \( \alpha^2 > 0.1 \) (at least if \( n \gtrsim 10 \)). It is thus possible in many cases to use the values calculated for \( \alpha^2 >> 1 \).

By introducing in the programme the average value of the magnetic field and one or more mass hypotheses, it is easy to calculate the corresponding values of \( \alpha^2 \) to verify that the approximation is justified.

c) If a better estimator is wanted, an iterative procedure can be used, a first estimate of \( \Theta \) giving a value for \( \alpha^2 \) from which the weights \( p_i \) are then deduced by interpolation between values of a table. It is even possible to calculate the weights for each track taking into account the real distances between the points.

2. TANGENT AT THE APEX

In the present version of the programme, the points in space are projected orthogonally on a cylinder of radius \( \rho = \frac{1}{\Theta} \), perpendicular to the Oxy plane and having the apex and the last measured point of the track on its surface.

From the distances of the points of the track to the cylinder and the cylindrical coordinates of their projections on the cylinder, estimates are deduced for the azimuthal angle \( \varphi \) and the dip angle \( \alpha \).

As for the curvature, the estimators for \( \varphi \) and \( \alpha \) are weighted averages of simple functions of the coordinates of the points, the weights in this case being calculated also by the
Maximum Likelihood method. They depend on the number of measurements and on the parameter $a^2$. As in §1, approximate values for the weights can be used.

The use of weighted averages for the determination of the tangent plays more or less the same role as the choice of an optimal length. When kinks are detected by the programme (see §5) the last point before the first kink is taken as last point of the track.

The cylinder on which the points are projected is only introduced to simplify the calculations. When the track differs very much from a helix, it is possible to use another method. To estimate $\varphi$, for instance, we consider then the angles of the projected chords with the Ox axis,

$$\varphi_i = \text{arc tg} \left( \frac{y_i - y_{i-1}}{x_i - x_{i-1}} \right)$$

and from the expression for the mean value of $\varphi_i$, $E[\varphi_i] = \varphi + f_i(\rho)$ we deduce an estimate for $\varphi$, the curvature radius $\rho$ being estimated as explained in §1.

An estimate for the dip angle $\alpha$ can be calculated in the same way, from the angles of the chords in space with the Ox plane.

3. **RANGE**

The range in space and the projected range are estimated from the distances between successive points. These quantities are always calculated, even when kinks are detected.
4. **ERROR MATRIX**

The error matrix, giving the variances and correlations of the estimators of the parameters $\rho$, $\varphi$ and $\alpha$, has been calculated theoretically for equidistant points and Maximum Likelihood weights.

The comparison of internal errors calculated by the programme with the theoretical variances allows the rejection of badly measured tracks.

5. **DETECTION OF KINKS**

As the programme calculates angles between the successive chords, it should be possible to introduce a test for kinks, based on the detection of large deviations of these angles from their mean value.

However, as kinks can influence considerably this mean value as calculated in §1, it is better to calculate first the median of the angles between chords and to use this to eliminate the large angles, before calculating the weighted average giving the estimate of $\rho$.

It is of interest to remark that this procedure makes it possible to distinguish between real kinks and measurement errors as the latter give two successive large angles with opposite signs.

6. **ENERGY LOSS**

If a mass hypothesis is introduced into the programme it is possible to calculate corrections to the weights, for curvature

$6650/\rho$
and angles estimation, in order to take into account the energy loss suffered by the particle along its track. It should also be possible, if the number of measurements is large enough, to detect energy loss by comparison of the curvatures of the first and second part of the tracks.

7. **CONCLUSION**

The programme STAN, especially written for heavy liquid chambers, permits the analysis of tracks differing greatly from a helix. Therefore, it does not need to measure the tracks in several pieces.

It is unavoidable that the presence of multiple scattering reduces the precision of the estimation of curvature and other parameters, but the programme tends to use all the information available, by the statistical analysis of simple quantities, based on the Maximum Likelihood method.

For hydrogen bubble chambers also, this last point can be of importance because such an analysis can increase the precision of estimation and gives estimators with known distribution, which allows theoretical calculation of the error matrix.
DISCUSSION ON THE TALK OF M. HUYBRECHTS

Werbrouck: You don't use the length of the individual segments, but do you assume that the segments are of equal length?

Huybrechts: There is a correction made to take into account the real distance between points.

Goldschmidt-Clermont: Is it right that in the limit where the measuring errors and multiple scattering would go to zero, then the weights would be equal?

Huybrechts: Yes, this is so.

Goldschmidt-Clermont: If the magnetic field would be zero, then you have an estimation of the $p^\mu$ of the track from the multiple scattering?

Huybrechts: No, I take the angles with their signs and not only their absolute values. The information given by multiple scattering is not used.

6650/p
Glasser: How different are the weights for a typical track?

Huybrechts: When $a^2 = 0$, which signifies that there is only multiple scattering, and $n = 10$, the weight function is more or less as in a curve below.

When $a^2 \gg 0.1$, one obtains curve b.

Bingham: These are weights for the momentum estimates; for the angular estimates there must be much larger weights for the first points.

Huybrechts: There is a maximum in the weighting function; this plays the same role as the optimum length.

Werbrouck: Has this problem been tried with tracks in heavy liquid?

Huybrechts: No, only in hydrogen.
MISEURE PAR COURBURE DE L'ENERGIE DES ELECTRONS

DANS LES CHAMBRES A LIQUIDES LOURDS

par

L. Behr et P. Mittner
(Ecole Polytechnique - Paris)

INTRODUCTION

Si les chambres à liquides lourds permettent une reconnaissance aiséee des électrons et la matérialisation de nombreux photons, la mesure des événements ainsi révélés pose un problème délicat: en effet, puisque des pertes d'énergie radiatives importantes ont fait spiraliser les trajectoires des électrons, une mesure par courbure dans un champ magnétique suppose l'apport de certaines corrections et l'estimation de l'incertitude sur la détermination du moment, une fois la correction effectuée.

La méthode que nous exposons ici est utilisée au Laboratoire de l'Ecole Polytechnique depuis plus d'un an et a donné des résultats très satisfaisants.

1. CALCULS ET NOTATIONS

Nos résultats ont été atteints de deux manières:

- une méthode de Monte-Carlo qui consistait à reconstituer, par tirage au sort des pertes d'énergie radiatives, des familles de trajectoires électroniques, sur lesquelles étaient calculés tous les paramètres utiles.
- un calcul analytique qui a donné des formules faciles à introduire dans un programme de reconstitution géométrique.

Ces deux façons de procéder sont étroitement liées: la construction par Monte-Carlo d'un nombre limité de trajectoires a montré que les approximations faites dans le calcul étaient justifiées; dès lors on peut penser que, dans certaines limites que nous préciserons, les expressions analytiques de la correction et de l'incertitude sont correctes.

Position simplifiée du problème

Nous supposons l'impulsion initiale perpendiculaire au champ magnétique, et que la trajectoire est suffisamment peu déformée par le scattering multiple pour rester plane. En outre nous négligeons les pertes d'énergie par ionisation.

Copiant en cela les calculs effectuées pour le scattering multiple nous avons raisonné sur la flèche \( f \) de la trajectoire.

En fait, il est intéressant de faire intervenir \( \frac{f}{f_0} \) où \( f_0 \) est la flèche sur la même longueur \( L \) de trajectoire du cercle que décrivait un électron ne perdant pas d'énergie. Le
calcul de \( \frac{f}{f_0} \) n'étant pas facile nous avons trouvé l'approximation

\[
\frac{f}{f_0} \approx \frac{\Theta}{\Theta_0},
\]

\( \Theta \) et \( \Theta_0 \) étant les angles de rotation des deux trajectoires précédentes. Le Monte-Carlo a montré que cette approximation est justifiée si les angles de rotation ne sont pas trop grands, et si les variations de courbure ne sont pas trop importantes. \( \frac{\Theta}{\Theta_0} \) peut s'exprimer facilement:

\[
\frac{\Theta}{\Theta_0} = \frac{\int_0^L \frac{d\ell}{R(\ell)}}{\frac{L}{R_0}} \text{ avec } \quad R(\ell) = p(\ell)/3B
\]

\( R \text{ en cm, } p \text{ en MeV/c, } B \text{ en Tesla} \)

soit

\[
\frac{\Theta}{\Theta_0} = \frac{1}{L} \int_0^L \frac{R_0}{p(\ell)} \, d\ell
\]

donc indépendant de \( B \).

Un résultat classique de la théorie du bremstrahlung est que \( \frac{p(\ell)}{p_0} \) est une fonction aléatoire de \( \ell \), indépendante de \( p_0 \) en première approximation.

Donc notre paramètre est indépendant de \( B \) et du moment initial. Il s'écrit avec les notations de Heitler

\[
\frac{f}{f_0} \approx \frac{\Theta}{\Theta_0} = \frac{1}{L} \int_0^L y(\ell) \, d\ell
\]

\( y(\ell) \) étant une variable aléatoire, dont on connaît la répartition.

\[
\bar{y}(y(\ell)) = e^{-y(\ell)} y(\ell) \frac{\ell}{x_0 \log 2} - 1 \quad dy(\ell)/\Gamma \left( \frac{x_0}{x_0 \log 2} \right)
\]

où \( \Gamma \) est la fonction Eulerienne de première espèce.
2. **NECESSITE D'UNE COUPURE**

Si nous essayons de calculer la valeur moyenne de \( \frac{f}{f_0} \), nous trouvons qu'elle n'a pas de valeur finie. Évidemment \( \frac{f}{f_0} \) se comporte différemment puisque les flèches gardent des valeurs finies. Cependant le Monte-Carlo donne pour des longueurs raisonnables de mesure des incertitudes relatives sur les flèches de l'ordre de plusieurs unités (200 à 300 %). Nous avons alors introduit une coupure qui limite les pertes d'énergie admissibles le long d'une trajectoire. Nous ne mesurons un électron sur une longueur \( L \) (encore indéterminée) qu'à la condition que pour toute longueur \( \ell < L \)

\[
p(\ell + d\ell) > p(\ell) \cdot e^{-\gamma_0}
\]

Cette coupure présente les avantages suivants:

- réduire d'une façon appréciable les incertitudes de mesure
- faire que l'approximation \( \frac{f}{f_0} \approx \frac{\Theta}{\Theta_0} \) est valable puisque les variations de courbure ne sont pas trop importantes
- permettre de donner des expressions analytiques de la correction et de l'incertitude.

On peut donc calculer (et le résultat est assez bien confirmé par le Monte-Carlo en calculant \( \frac{P_\alpha}{P_\alpha} \) au lieu de \( \frac{P}{P_0} \), où \( P_0 \) est le moment initial, et \( P \) le moment correspondant au cercle adapté par les points d'abscisses curvilignes \( 0, \frac{L}{2} \) et \( L \))

\[
< \frac{f}{f_0} > = \frac{A_4(y_0) b L}{A_4(y_0) b L - 1}
\]

\[
\sigma^2 (\text{radiation}) = < \left( \frac{f}{f_0} \right)^2 > - < \left( \frac{f}{f_0} \right)^2 > \approx \frac{A_5(y_0) - 2A_4(y_0)}{3} b L
\]

(approximation linéaire)
ou: \( b = (X_0 \log 2)^{-1} \)

\( A_4 \) et \( A_5 \) sont deux fonctions de \( y_0 \):

\[
A_4 (y_0) = \log \gamma + \log y_0 - E_1 (-y_0) \\
A_5 (y_0) = \frac{E_1}{y_0} (y_0) - E_1 (-y_0)
\]

\((E_1 \text{ et } \frac{E_1}{y_0} \text{ sont les fonctions exponentielles intégrales.})\)

Si \( y_0 \) n'est pas trop grand \( (y_0 < 1) \), de même que \( bL \) on peut donner les approximations:

\[
< \frac{f}{f_0} > \simeq 1 + \frac{y_0 bL}{2} \\
\sigma^2 \text{ (radiation)} \simeq \frac{1}{6} y_0^2 bL
\]

En introduisant maintenant le scattering multiple on a:

\[
< \frac{f}{f_0} > \simeq 1 + \frac{y_0 bL}{2} \quad \text{(inachangée)}
\]

\[
\sigma^2 \text{ (total)} = \frac{K}{LX_0} + \frac{1}{6} y_0^2 bL
\]

ou \( K \) est une constante dépendant du champ magnétique.

L'incertitude de mesure est minimum pour une longueur que nous appellerons la longueur optimum. (Voir figure 1 et figure 2. Sur chaque courbe est indiquée la valeur de \( e^{-y_0} \), soit le pourcentage maximum admissible de perte d'énergie.)

On voit que si la coupure n'est pas trop large, le minimum est assez plat pour que la longueur de mesure ne soit pas critique.

Le graphique n° 3 donne en fonction de la longueur de
trajectoire et de la coupure le nombre d'électrons qui satisfont à cette coupure. Il est évident que si la longueur optimum est \( L_0 \), la fraction d'électrons qui ne satisfait pas à la coupure sur cette longueur n'est pas à rejeter définitivement : on peut soit mesurer sur une longueur plus courte en conservant la même valeur de la coupure, dans la limite où l'incertitude due au scattering multiple ne devient pas trop importante, soit choisir une valeur plus élevée de la coupure.

Ajoutons que le Monte-Carlo nous a permis de constater que la distribution des flèches déformées par scattering et bremstrahlung est quasi-gaussienne au voisinage de la longueur optimum (Figure n° 4).

**Choix de la coupure**

Les valeurs de la correction et de l'incertitude étant assez sensibles à la coupure il est avantageux de la choisir la plus faible possible. On est en fait limité par le scattering multiple qui fixe une valeur minimum aux pertes d'énergie pouvant être appréciées le long d'une trajectoire : plus faible sera la longueur de radiation, plus élevée sera la valeur minimum de la coupure. (Notons cependant que si la disposition géométrique le permet, la matérialisation des photons émis permettra, dans le cas de très faibles longueurs de radiation, d'éviter le choix de coupures très élevées.)

Jusqu'ici nos tentatives de détermination théorique de la coupure minimum dans un milieu déterminé n'ont pas donné de résultats satisfaisants. Par contre la coupure que nous avions estimée raisonnable pour \( X_0 = 22 \) cm, et qui était de 50% en perte relative d'énergie, nous a permis d'obtenir une bonne mesure de la masse du \( \pi^0 \) (nous avons trouvé un résultat voisin


6650/p
de 137 MeV, avec des corrections de 15% en moyenne, c'est-à-dire que le résultat sans corrections eût été de 120 MeV environ. En outre la largeur obtenue expérimentalement est on assez bon accord avec les erreurs calculées.

3. **PROCEDE PRATIQUE DE MESURE**

   Donc, une fois la valeur de la coupure fixée, il faudra pour chaque électron faire l'opération suivante:

   - vérifier qu'avant la longueur optimum de mesure il satisfait à la coupure et dans ce cas le mesurer sur la longueur optimum.

   - Sinon le mesurer sur la plus grande longueur satisfaisant à la coupure.

   Le résultat de la mesure de courbure en 3 points est à multiplier par le facteur de correction correspondant à la longueur utilisée.

   Tous les résultats précédents négligeaient les pertes d'énergie par ionisation: en réalité, le Monte-Carlo montre qu'il suffit d'effectuer la correction due au bremsstrahlung et à apporter ensuite la correction de perte d'énergie par ionisation.

   Dans le cas de traces dipées, on peut voir que la longueur optimum projetée reste inchangée; le facteur correctif est celui qui correspond à la longueur réelle, et les incertitudes sont multipliées par $(\cos \phi)^{-1/2}$.
DISCUSSION ON THE TALK OF L. BEHR

Rousset:
Je voudrais parler au nom des gens qui travaillent dans le fréon dont la longueur de radiation est de 11 cm dans une grande chambre: ils voient les γ de bremstrahlung un peu plus loin, donc peut-être pourra-t-on donner une coupure qui soit au moins égale à 50%, du moins nous l'espérons.

Werbrouck:
What is the lowest momentum measured with this programme?

Behr:
Finallement, on l'a appliqué à tous les électrons (évidemment c'est faux puisque la longueur de radiation devient plus grande aux basses énergies). J'ai oublié de dire que l'introduction des portes d'énergie par ionisation ne change rien à la valeur de l'erreur. En faisant séparément la correction \( \frac{f}{f_0} \) de bremstrahlung et celle de porte par ionisation, on obtient le bon résultat.

Nikolić:
Est-ce que vous avez considéré l'influence de bremstrahlung sur les erreurs sur les angles?

Behr:
Effectivement, j'ai fait ce calcul. Une fois que l'on a fait la correction moyenne sur le moment de l'électron, il n'y a plus de correction à apporter sur l'angle. Si on cherche la longueur optimum de mesure des angles, on peut voir qu'elle n'est très différente de celle des autres traces: pour un \( \pi \) de \( p\beta c = 150 \text{ MeV} \) on a la courbe en trait plein, pour un electron de \( p\beta c = 150 \text{ MeV} \) la courbe pointillée.
En outre, l'erreur n'est pratiquement pas modifiée.

Rouset:

N'est-ce pas seulement valable pour des longueurs de radiation grandes, de l'ordre de 20 cm?

Behr:

Ce qu'on peut dire, c'est que pour 20 cm l'effet est faible; je pense que cela ne doit pas changer grand-chose pour 10 cm, par exemple.

(Englishman ?);

Two questions about the electron which you reject. First, have you considered measuring after a scattering which would happen near the origin of the pair? Second, have you any thought about getting the energy from range, if there are too many scatters?

Behr:

Si le scattering est trop près de l'origine, on rejette l'électron. Quant à la mesure par parcours, un article de Wilson², qui avait fait les calculs par Monte-Carlo, donne approximativement des incertitudes de l'ordre de 50% au mieux.
Je pense préférable de mesurer par la courbure; nous avons ainsi 35%, pour une longueur de radiation d'environ 20 cm.

---

² Robert R. Wilson, Phys. Rev. 84, 100 (1951)
MEASUREMENT OF THE TOTAL TRACK LENGTH OF ELECTRON SHOWERS

USING A NEW FIRST APPROXIMATION IN THRESH

by

H. Burmeister

INTRODUCTION

In many elementary particles' reactions (e.g. decay of the hypothetic intermediate boson), it is important to determine the momentum of the \( \pi^0 \) involved. For the normal decay mode into two \( \gamma \)'s, these must be converted within the visible fiducial region of the bubble chamber. Therefore, one has to choose a liquid with a short radiation length. (The CERN HLB uses Freon 13 b, CF\(_3\)Br, with \( x_0 = 11 \) cm). On the other hand, the normal method of momentum determination from the curvature becomes more inaccurate with smaller radiation length. Even with the bremsstrahlung correction of Behr and Mittner, one has an error of at least 45% in Freon 13 b. Another possibility is to find the \( \gamma \)-energy from the total track length of the electron shower produced. This method is to be described in another paper. Therefore, I give here only a rough idea of it and then discuss the details of the measuring procedure itself.

THE ENERGY TRACK LENGTH RELATION

In the application of the well-known formula

\[
E_0 = cT
\]

\( (E_0 = \text{primary energy}, \ T = \text{total track length}) \)
the following difficulties exist:

1) The linearity of this formula is not quite correct
2) One does not observe low energy electrons below a certain cut-off energy $E_0$.
3) In a finite chamber, part of the shower escapes from the visible region (mainly photons).

Points 2) and 3) give rise to an increase of the factor $c$ and a fluctuation of the track length. The best way to overcome these difficulties is to measure showers with known energy and to find an empirical relation

$$T = T(E_0)$$

This relation depends obviously on the size of the chamber and where the shower is produced.

We have made some measurements with electrons of 600 MeV/c entering the CERN HLBC and have found a projected length

$$T_p = 152 \text{ cm} \pm 13\%.$$  

If one cuts off the showers at 3, 4, 5, 6 radiation lengths, one obtains the relations between the length $\delta$ available for the shower development, and the accuracy and the track length, respectively (Fig. 1 and 2). The curve in Fig. 1 shows that for 600 MeV/c this method has a better accuracy than that of Behr-Mittner, as long as at least three radiation lengths are available for the development of the shower. One can expect that for lower electron energies the accuracy becomes still better. To find the energy dependence of the curves, we are going to investigate
π°'s from the reaction

\[ K^- \rightarrow \pi^0 + \pi^- \]

and also electron showers calculated with a Monte-Carlo programme.

**THE MEASURING PROCEDURE**

As it is not possible to fit a smooth curve to the electron tracks, one has to split them up into pieces. This could be done by looking for real corresponding points, but this needs much scanning work. Therefore, we have decided to measure each track in a straight-forward manner and then find points in space by the method of near corresponding points of the normal THRESH.

**Fig. 3.**

Camera 1

\[ \Psi_{12} \]

\[ \Psi_{23} \]

\[ \psi-\gamma \]

\[ \text{segment} \]

\[ x-axis \]

6650/p
 Afterwards the distances between the space points are added together. Up to now we have not added any correction as Rinaudo and Werbrouck have done. It is not too necessary, for we have a higher accuracy of measurement.

We use the same first order interpolation, as is done in THRESH, but we do not choose the views in the same way. In addition we choose the pivotal and antipivotal view for each track segment, because of the strongly curved nature of the tracks. We start with the first segment in all views, i.e. the connection between the two first points and determine the difference of the angle between this segment and the x-axis and the angle between the projected connection of the second point with the considered camera (Fig. 3). Then we look for the largest of these differences according to the equation

\[ |\sin (\psi - \gamma)| = \text{maximum} \]

and call the corresponding view a pivotal camera. Next one takes the second point (the first is supposed to be the apex) and finds near corresponding points in the other views, but uses for calculation of the space point only that view, in which the projected connection between the camera and the pivotal camera makes the larger angle with the track segment

\[ |\sin (\phi_{ij} - \psi)| = \text{maximum} \]

The programme then forgets the first point of the pivotal view, and in the others all the points up to whose which are found as near corresponding points. After this, the whole procedure can be started again, etc., until the last point is found.

Obviously the method described can be used for measuring the track length of all other particles but electrons. In addition, the calculated points can be used as good first approximation in THRESH.

6650/6
Hennossy: May I remind you of a naïve method of measuring $\pi^0$ energies. When one has a star and there are two $\gamma$'s pointing, then one measures the electron curvatures with a template, and punches the sum of momenta of both electrons on a card containing already the directions of the $\gamma$, and then puts it into the kinematics. You calculate in such a way the mass of $\pi^0$; and we got for 150 pairs of $\gamma$'s such a curve.

It may be interesting for simple cases.
(We had a rad. length of about 50.)

Burmeister: For shorter radiation length it becomes worse.

Czyżewski: I would like to ask you the following: what is the ratio of the energy release into the chamber to all the energy of the electron used for calibration?

Burmeister: About two-thirds.

Roussel: How many radiation lengths have you in the thickness of the beam window?

Burmeister: About one.
MEASUREMENT OF $\gamma$-RAY ENERGY IN A Xe BUBBLE CHAMBER

by

O. Czyżewski, J.A. Danysz, Z. Strugalski

presented by

O. Czyżewski

This paper presents a simple and quick method of $\gamma$-ray energy measurements in the energy interval from 20 to 1000 MeV. This work was performed in the Joint Institute for Nuclear Research in Dubna, U.S.S.R., on the JINR Xenon bubble chamber photographs obtained in a 9 GeV/c $\pi^-$ experiment.

The method of measurement is based on the fact that the sum of electron ranges produced in a shower is roughly proportional to the energy of its primary photon(1):

$$E = K \cdot \Sigma R + 2 n m_e c^2$$

where $K$ - proportionality coefficient

$\Sigma R$ - the sum of electron ranges

$n$ - number of $e^+ e^-$ pairs.

Because of the strong effects of multiple scattering it is very difficult and sometimes impossible to measure the track length by the ordinary method. We measured the length of track projection on scanning tables by means of a curvimeter. The sum of electron ranges is given by:

$$\Sigma R = k \Sigma r / \cos \lambda$$

where $\Sigma r$ - sum of lengths measured on the projection table

$k$ - magnification coefficient

$\lambda$ - dip angle of the original photon

6650/p
To find the proportionality coefficient $K$ the sample of 272 positrons from $\mu^+ \rightarrow e^+ + \nu + \bar{\nu}$ decay was used. The comparison of the mean positron range $<R>$ with expected average energy $<E>$ gave $K = \frac{<E>}{<R>} = (0.59 \pm 0.02) \text{ MeV cm}^{-1}$. The experimentally obtained energy spectrum is presented in Fig. 1. The continuous curves are calculated from the theoretical spectrum with the assumption of different errors. The best agreement was obtained for the error value 25\%.

In many cases the shower development occurs only partly inside the chamber. The distance between the photon conversion point and the plane limiting the visibility region, measured in the direction of photon flight, will further on be called "development length" $d$. 100 showers were chosen fulfilling the following criteria:

1. The photograph not overloaded by the background tracks.
2. The shower develops totally inside the chamber.
3. The dip angle of the shower axis is less than 25°.

Some showers of more than 600 MeV energy have not fulfilled the condition (2). The total energy of these showers was determined assuming the exponential decrease of energy fraction escaping from the chamber.

Each shower was artificially cut on different development lengths $d$, every 2 cm, and for every $d$-value $\Sigma R$ was determined. The thus obtained curves averaged in the energy intervals are presented in Fig. 2.

By means of interpolation between these curves we obtained the family of curves $\Sigma R = f(d)$, presented in Fig. 3. To determine the photon energy one has to measure $\Sigma R$ and $d$. Using these two values one can find on Fig. 3 the point on the curve corresponding to photon energy.
The error in the measurement of summary length of electron tracks varies from 5 to 50%; it should be estimated in every case, since it depends on the shower energy, visibility conditions and background.

The upper limit of errors due to the fluctuations of radiation and ionisation losses is approximately equal to the single pair error (≈ 20%) divided by the square root of the secondary pair number.

The third source of errors are the fluctuations in the shower development. The dependence of this last error on $d$ for different photon energies is presented in Fig. 4.
References

(1) Z.S. Strugalski, JINR report R 796/61, Dubna 1961

L.P. Konovalova et al., JINR report R 700, Dubna 1961
Fig 1. Positron energy spectrum from $\mu^+ \rightarrow e^+ + \nu + \bar{\nu}$.

Fig 2. Experimental dependence $\Sigma R$ on $d$. 

ADM/248
Fig. 3. Diagram for determination of the photon energy $E$ in function of $\Sigma R$ and $d$. The curves of energy above 1 GeV are only approximate.
Fig. 4. Errors due to the shower fluctuations versus d.

\[ \delta = \frac{\Delta E}{vN} \]
DISCUSSION ON THE TALK OF O. CZYŻEWSKI

Burmeister: From where did you know the energy for such a shower of 500 MeV?

Czyżewski: We assume that there is a proportionality between the total length of the track and the energy. For energies of electrons between 0 and 50 MeV, we have calibrated this relation. We assume that this relation is linear.

Worbruck: Do you have any analytic correction for the average depth of the shower?

Czyżewski: It is included in the coefficient; it is calculated individually in each case.
Goldschmidt-Clément: I would like to make a preliminary list of subjects that may be usefully discussed, particularly those that are common to several of the papers presented today:

- the choice of the principal and auxiliary view
- the determination and use of the optimum length for calculating angles
- the validity and usefulness, especially for heavy liquids, of the helix fit which was found extremely useful for hydrogen, but which becomes perhaps less valid as the radiation length decreases
- the non-convergence of the computing methods for some types of tracks
- the detection of kinks, possibly by programmes instead of by physicists
- the question of compatibility of programmes and formats.

The following remark may perhaps launch the discussion. The optimum length $t_0$ is the
distance along the track at which the best
determination of angle can be made. The
error due to multiple scattering is propor-
tional to \( t^\frac{3}{2} \), while the measuring error
is proportional to \( t^{-1} \). Combining these
errors quadratically, gives a function that
decreases rapidly to a minimum (which defines
the optimum length \( t_o \)) and then increases
slowly. The corresponding weighting function
is thus expected to start from a very small
value, to reach a maximum at the optimum
length, and then to decrease slowly.

I noticed that some of the programmes mentioned
today first find the optimum length, and then
make a fit that uses the measurements up to
the optimum length only. This neglects valu-
able information, as the points immediately
after the optimum length have a fairly high
weight.

Hennessy:

Once the optimum length is selected the whole
question of intermediate points becomes
irrelevant, because

a) the intermediate points are used to cal-
culate the optimum length, and

b) the angles are given unambiguously using
the apex point, the optimum point and the
radius of curvature - at least it is so in
the Paris programme.
Sparrow: To calculate the optimum length for the angle we use the momentum found from the whole track length. As we always deal with angles at the beginning of a track, I feel that one should calculate these from the first section and not from the whole length, thus making the weights far from the origin zero. If one used angles at the middle of the track, the weights of the later points would be significant.

Goldschmidt-Clermont: My remark was applicable to the determination of angles at the beginning of the track. If you use only the first part of the track you will probably get angles which are fairly good, but you could do still better by using all points with an appropriate weighting.

Böck: The problem of optimum length reduces to that of weighting anyway. If you have good weights the whole calculation of optimum length is not necessary, and if the weights go to zero at the end of the track, obviously this is the same thing.

Huybrechts: When I calculated the weights I got a maximum near the optimum point, but it was a little strange to me to find negative weights at the beginning and the end of the track. I think it can be explained by the correlation between measurements. I did not pay so much attention to the weights, however, but concentrated on the errors obtained.
Moorhead: I should like to point out that FOG does not do a refit after having found the optimum length. Once the optimum point is found a circle with the found radius is drawn through this point and the starting point, and the angles are found from this.

Hennessy: So do we.

Nikolić: We have heard that many of the geometry programmes use the mass assignment. I would like to ask what consequences this would have for the kinematical calculations if one uses GRIND and if GRIND can be modified to accept mass dependent geometry output.

Böck: From the point of view of GRIND there is no difficulty to come in with fitted results at the vertex which hence will not be corrected for mass dependent momentum loss etc. This is a simple modification. In fact, I was going to propose the same, i.e. for heavy liquid programmes to have the mass hypothesis in the geometry part. Usually there are not so many different mass assignments in heavy liquid chamber experiments.

Sparrow: For each charged track, we use either the actual mass (if given) or the three masses $\pi$, $K$, $p$ to calculate parameters and errors. In this latter case we then will have three sets of results. At a later stage the hypothesis programme will select which of these to use in a given hypothesis.
Honnessy: What is the order of magnitude of difference between the angles, etc. for the different mass hypotheses?

Sparrow: Between 0.5 and 1 degree.

Honnessy: That is also what we find.

Goldschmidt-Clermont: Does anybody want to make comments on the choice of the principal view?

Werbrouck: The selection of the control view is still an open question. We choose the largest projected chord as control view. In this view the principal view becomes that opposite the x-axis most parallel to the segment which has as end point the next point to be reconstructed, or the point which will enter in the next interpolation. We have considered but not tried using as control view the one whose optical axis is nearest to the centre of the track. Burmeister's idea of using the view in which the segment is most nearly perpendicular to the line joining its centre to the optical is also very interesting and may be better.

Honnessy: There are two principles in choosing the views, namely

a) let the physicist do it

b) let "George" do it, "George" being the machine.
If you let the physicist do it you are more flexible, you may try the various possibilities and see the difference. We have found no great difference unless the track is nearly parallel to one of the lines joining the cameras.

If you let the machine do it, it will always select the same views, and the only possibility to prevent it from doing so, is to measure two views only. I think it would be worthwhile to try this on some of the "let George do it" programmes.

Werbrouck: There is an essential difference: if the physicist does it he chooses one principal view for the whole track, whereas in our programme and in that of Burmeister there is no single principal view for a track that turns through, say, 140°. If the physicist wants to choose the principal view instead of our programmes and compare the results he also has to choose where to shift from one view to another and measure the track in several segments and then join together. This is a lot more work.

Hennessy: But I should like to see the difference when only measuring two views for tracks which curve through, say, less than 90°. Have you tried... that?
Werbrouck: Measuring a few thousand events, we want a simple measuring system, as we don't yet have an automatic logic. In our system the measurer must only select a suitable fiducial pair and the rest is determined. Therefore we have not tried it.

Hennessy: What do you do if for some reason one of the views is useless?

Werbrouck: In that case we don't use the film.

Goldschmidt-Clormont: In measuring electrons in hydrogen we have changed from always using the same pair of views to automatic change of the views. Could Miss Cnops tell us about her experience with this?

Cnops: For very curved tracks we found it necessary to change at least one view. In fact we keep the principal view for the whole track and change only the secondary view. I think that changing both views would be even better for heavy liquid.

Burmeister: I think THRESH could be a little improved if one first selected the anti-pivotal view as that where \( \sum (F_i^2 + F_j^2) \) = min and then find the pivotal view as that where the connection between the cameras is most perpendicular to the track. But the method which chooses the pivotal view as that in which the connection between the camera and the track is most perpendicular to the track, may be a little bit better.
Nikolić: From our experience with collaborations we know that very often films or partial results have to be sent from one laboratory to another in order to be further analysed. Therefore, I think some sort of standardisation is necessary. This does not mean that other laboratories should change their programmes, but some common points should be kept, perhaps in the form of converting sub-routines, by which it is possible to jump from one system to the other. Could we use the occasion to discuss these points?

Sparrow: It is difficult to separate our geometry from our kinematics, thus making a common point between them, but it is possible to put the different measurements in a form acceptable to our geometry programme.

Werbrouck: May we have the experts' opinion on the possibility of using the input to GRIND as a standard?

Böck: As I said before there is no difficulty to get fitted results in to GRIND. If corrections due to ionization loss, bremsstrahlung, etc. are done before, one can simply throw away the first part of GRIND. That's all.

Goldschmidt-Clermont: I think the question was if one could agree on a special format as input for GRIND which the various geometry programmes could produce.
Böck: At present GRIND reads absolute binary with a non-standard input routine. If people are prepared to make their output in this form they can go into GRIND without any change. Otherwise this input routine has to be changed. At present the format contains three blocks of information concerning the event, the points and the tracks, one could add one more containing mass dependent information on tracks if one wants. I am not sure that the actual form on the tape can be standardized, it depends too much on the geometry programme.

Rousset: Our present difficulties are probably due to the fact that our programmes have developed from existing programmes written for hydrogen. If one should start all over again and make up a new system for heavy liquid chambers, I think it is natural to keep a clear distinction between a) what we call the geometry part, which includes calculation of the geometrical parameters of the track with errors, including the mass dependent ones, and b) what we call the kinematics with fitting at decay and production vertices etc.

In heavy liquids we have many special mass dependent problems, such as scatters and optimum lengths etc., which I think are not suitable for GRIND, but should be kept in the geometry part.
Goldschmidt-Clermont: Could we discuss now the question of non-convergence for certain tracks? Should we assume that they have to be remeasured, or should we try to improve our methods?

Hennessy: In Paris about 10% of the tracks do not converge the first time. They do converge, however, when they are remeasured.

Rousset: This is certainly important, but I think it is as important to discuss all the special problems of heavy liquids which should be treated at the end of the geometry part of the system.

Goldschmidt-Clermont: May we have some comments on the two different ways of treating the geometrical optics? There are programmes which attempt to solve the problem exactly and there are others which treat it by polynomial approximations like FOG and Werbrouck's programme. Can we have some comments on this?

Moorhead: It might be better to adopt more from FOG and Werbrouck. It would mean more programming work for us, but reconstruction of corresponding points might be slightly better.

Goldschmidt-Clermont: Could you also comment on the difference in the way the interpolation is made?

Moorhead: I think Werbrouck's method of taking two auxiliary views and weighting them is very desirable.
Wilkinson: Is there any geometry programme which does not assume that the optical axes are perpendicular to the front glass?

Glasser: The Berkeley programme PANG does not assume perpendicularity, in fact they have a tilted window.

Werbrouck: That the fiducial points are not in a single plane and normal to the optical axes is no problem for us. In our programme all fiducial marks are projected into one plane for reconstruction independent of where they are located in real space.

Goldschmidt-Clermont: Can anybody comment on the possibilities of looking for kinks with the programme. It is used in FOG, I think.

Moorhead: It was used in an early version of FOG, but not very successfully.

Glasser: Why should the programme try? It would only be possible for the programme to compete with the human eyes' ability in this regard if one had a density of digitizations comparable to the HPD, and if one considers that problem we would go far afield.

Hennessy: Our programme detects kinks in the sense that if a measured point is far away from the fitted curve it throws it away and flags it. Then you look at the picture and find the kink.
Fett:

THRESH has a similar feature except that it does not tell that a point is thrown away. This is very dangerous as it is impossible to see the difference between a kink and a bad measurement. If you really have a kink the track sometimes converges after having thrown away a point, but the parameters given are wrong since the points after the kink are used.

I should like to add that an important advantage of Huybrechts' programme is that it studies angles between successive chords, rather than deviations from a curve. This allows us to study kinks and even energy loss in a much more systematic way.

Bingham:

I would like to ask experts on programmes using the sum of distances between approximate corresponding points to get the range of a track, what precision is possible on approximate corresponding points, e.g. how much worse is it than for real corresponding points? How does the precision obtained this way compare with that of even rather poor helix or parabola fit? (Clearly many short segments with large errors would give worse precision than a helix approximately following a track between the kinks.)

Werbrouck:

Due to the precision in our measuring machine we must apply a phenomenological correction to find the range. This correction is described in our report. It is determined by requiring that the length be invariant to the number of points.
measured and how close they may be. If two points are too close, that segment is not used as the length correction diverges for zero length tracks.

Burmeister: I don't think it is necessary to correct length if a sufficiently high order interpolation is made. Therefore, I think we should do more effort on this interpolation.

Bingham: But do you know the difference in accuracy between near corresponding points and real corresponding points?

Burmeister: I do not know, since we have not yet measured real electron showers.

Goldschmidt-Clermont: Before closing the meeting, it is a pleasure to express our thanks to the chairman who conducted the meetings, to the speakers who presented us with most interesting papers, to the contributors who enlightened us on several useful points in the discussion, and to all others who listened with constant interest to a wealth of information.
<table>
<thead>
<tr>
<th>Name</th>
<th>Institution</th>
</tr>
</thead>
<tbody>
<tr>
<td>M. Aderholz</td>
<td>CERN</td>
</tr>
<tr>
<td>M. Basin</td>
<td>Ecole Polytechnique</td>
</tr>
<tr>
<td>L. Behr</td>
<td>Ecole Polytechnique</td>
</tr>
<tr>
<td>W. Bonsch</td>
<td>CERN</td>
</tr>
<tr>
<td>H. H. Bingham</td>
<td>CERN</td>
</tr>
<tr>
<td>V. Bisi</td>
<td>Università di Torino</td>
</tr>
<tr>
<td>R. Böck</td>
<td>CERN</td>
</tr>
<tr>
<td>F. Bruyant</td>
<td>CERN</td>
</tr>
<tr>
<td>H. Burmeister</td>
<td>CERN</td>
</tr>
<tr>
<td>E. Castelli</td>
<td>Università di Trieste</td>
</tr>
<tr>
<td>A. M. Chops</td>
<td>CERN</td>
</tr>
<tr>
<td>S. Coletti</td>
<td>Università di Milano</td>
</tr>
<tr>
<td>G. A. Cooklin</td>
<td>University College, London</td>
</tr>
<tr>
<td>W. A. Cooper</td>
<td>CERN</td>
</tr>
<tr>
<td>M. di Corato</td>
<td>Università di Milano</td>
</tr>
<tr>
<td>B. Crung</td>
<td>University of Bergen</td>
</tr>
<tr>
<td>D. C. Cundy</td>
<td>CERN</td>
</tr>
<tr>
<td>O. Czyzowski</td>
<td>CERN</td>
</tr>
<tr>
<td>J. Daup</td>
<td>CERN</td>
</tr>
<tr>
<td>E. Deher</td>
<td>L.P.C.H.E. - Saclay</td>
</tr>
<tr>
<td>M. G. Eaton</td>
<td>University College, London</td>
</tr>
<tr>
<td>M. Ferrero</td>
<td>Università di Torino</td>
</tr>
<tr>
<td>E. Fett</td>
<td>CERN</td>
</tr>
<tr>
<td>W. Fiekingor</td>
<td>L.P.C.H.E. - Saclay</td>
</tr>
<tr>
<td>A. Fridmann</td>
<td>CERN</td>
</tr>
<tr>
<td>R. G. Glassor</td>
<td>CERN</td>
</tr>
<tr>
<td>Y. Goldschmidt-Clermont</td>
<td>CERN</td>
</tr>
<tr>
<td>P. Guerdoni</td>
<td>Università di Roma</td>
</tr>
<tr>
<td>A. Haatuft</td>
<td>University of Bergen</td>
</tr>
<tr>
<td>J. Hennessy</td>
<td>Ecole Polytechnique</td>
</tr>
</tbody>
</table>
J. M. Howie  CERN
M. Haybrochts  I.M.E.B. - Brussels
P. Jacquot  Ecole Polytechnique
W. Koch  CERN
Ph. Krajbich  Ecole Polytechnique
R. Lestienne  Ecole Polytechnique
Ch. Letortre  CERN
G. R. Macleod  CERN
J. V. Major  CERN
D. J. Miller  University College, London
W. G. Moorhead  CERN
D. Morellet  Ecole Polytechnique
F. Muller  CERN
G. Myatt  CERN
M. Nikolić  CERN
C. Ounnès  Laboratoire Joliot-Curie
M. Paty  CERN
G. Pichon  Laboratoire Joliot-Curie
E. Renne  CERN
R. Rousset  Ecole Polytechnique
K. Schultze  CERN
V. Simak  CERN
J. Six  Ecole Polytechnique
O. Skjøgestad  CERN
K. Soop  CERN
J. Sparrow  N.I.R.N.S. R.H.E.L.
G. Tomasini  Università di Genova
A. Werbrouck  Università di Torino
C. A. Wilkinson  University of Oxford

V. Cooper  Conference Secretary

6650/p