A STOCHASTIC APPROACH IN UNCERTAINTY BUDGETING FOR ACCELERATOR PRE-ALIGNMENT – STUDY FOR THE CLIC PROJECT

I. Doytchinov, CERN, Geneva, Switzerland

Abstract

A new strategy for uncertainty budgeting estimation following the International Standard (GUM - Supplement 1) is proposed as alternative to current classical methods of error budgeting applied in the domain of accelerator components. This strategy applies stochastic modelling on the uncertainty contributing factors for providing probability density function as quantification of pre-alignment measurements uncertainty. As a case study the methodology is applied to the PACMAN pre-alignment project providing a ‘measurement specific’ uncertainty budget for accelerator pre-alignment of components according to GUM Supplement 1. With this methodology the global uncertainty budget can be determined as function of the exact conditions of each specific contributing factor (temperature and its gradients, measurement strategy, instrumentation, etc.). We believe that this methodology would provide a more accurate approach on the tight uncertainty budgeting allocated for the alignment requirements of the future particle accelerators projects. The method could be easily extrapolated/applied for uncertainty budgeting of different type metrology systems.

TARGETED PRE-ALIGNMENT MEASUREMENT UNCERTAINTY

The motivation of this work comes from the tight requirements of alignment and pre-alignment in CLIC, the Compact Linear Collider study [1]. The pre-alignment of each magnetic or electrical axis of CLIC components should be within a 17 µm to 14 µm diameter cylindrical zone rms with respect to global reference. The global reference is provided by a series of overlapping stretched wires each one of 200 meters length. These requirements lead to a target measurement uncertainty of pre-alignment of 12 µm for the magnetic axis measurement and 7 µm for the electric axis of BPM (Beam Positioning Monitor) components. The target uncertainty is lower than the pre-alignment target as it excludes 5 µm of uncertainty due to capacitive Wire Positioning Sensor (WPS) linking the measurements of the reference stretched wire [2].

In PACMAN program (Particle Accelerator Components’ Metrology and Alignment to the Nanometre scale) [3] held at CERN we are studying a method with which this targeted uncertainty could be realistically achieved. In this method we perform magnetic and electric axis metrology and geometrical survey in one place and time - the measurement environment of a CMM (Coordinate Measurement Machine) (Figure 1, a) such as

Figure 1: a) Pre-alignment metrology completed at CMM environment, b) alignment of components in tunnel using pre-alignment metrology data, c) Pre alignment metrology defined as knowledge of the magnetic axis location (X1-X2) with respect to assembly coordinate frame X0.

the Leitz PMM-C Infinity [4]. The measurements are performed at the temperature of ± 0.1 °C (class I metrology thermal environment) with low powered magnet (less than 4% of nominal current used) not thermally perturbing the measurements. In real operation, compensations for both magnetic axis and fiducials drift will be required for the working temperatures (ex. 40 °C expected in the case of CLIC tunnel, Figure 1b).

In this work we represent the target uncertainty as an ellipsoid describing the uncertainty of knowing the location of electrical or magnetic axis with respect to a physical coordinate system created by the fiducials part of each sub-assembly. The simplest possible relationship is the shortest vector between the axis and the assembly best fitted coordinate axis. The targeted uncertainty is assumed to be including the random and systematic unknown errors of the complete PACMAN pre-alignment measurements once all systematic errors have been accounted and corrected for.

In this paper we will describe the motivations of such work with respect to state of art literature. We show how GUM Supplement 1 [5] can be applied to provide uncertainty statements for the CMM measurements and environment influences.
Uncertainty of pre-alignment in state of art

Pre-alignment measurements in literature are usually divided into three parts as shown in Fig.2:

Magnetic survey: a magnetic measurement system is used to define the location of the magnetic null by best fitting a stretched wire to locate it. In PACMAN, this is achieved by the vibrating wire method [6]

Geometrical survey: a CMM with or without contact probe is used to measure the relative coordinates of all fiducials mounted on the assembly and thus create a dimensional coordinate frame and relationships between the components. In PACMAN, bridge type CMM is used for this purpose.

Link between geometrical, electric and magnetic survey: the location of the stretched wire has to be described within the coordinate system defined by the geometrical survey. In practice this is achieved by measuring fiducials kinematically linked to the stretched wire. In PACMAN this step is avoided and the link between the wire and fiducials will be made by measuring both wire and fiducials with the CMM by exchanging two different type of measurement heads: tactile for fiducials and non-contact for the wire [7].

A great summary and comparison of stated measurement uncertainties of different state of the art methods can be seen in [8]. There, the authors investigate the quoted size of both geometrical survey and magnetic measurement uncertainties. What they find is that in general the uncertainties reported are used in a ‘lose’ sense as the best case scenario achieved. A rigorous assessment of the absolute accuracy involving systematic comparisons with reference standard often is not available. There is currently no international standard for magnetic axis or pre-alignment measurements. Thus it is hard to compare uncertainties statements of different laboratories on magnetic measurement. For the uncertainties of the geometrical survey, authors often use the Maximum Permissible Error (MPEE) quoted by the manufacturer. This qualification parameter, however, does not include the real uncertainty of their specific measurement (task specific uncertainty explained in next section). Such lack of rigorous assessment and reference/traceability to general standard leads to understatements of the uncertainty. Currently the lowest uncertainties stated are in the range of 10 µm with requirements far bigger than that – 20 µm or more. This leaves a comfortable safety factor for the current cases ensuring that even if uncertainties are understated, there is a sufficient margin. For the CLIC study such assumptions cannot be taken. The really tight targets of near 7 µm at 1σ of pre-alignment uncertainties has to be achieved over meter sized assemblies (up to 2 m girders) on industrial scale (close to 20000 modules). This is beyond the current state of art and leaves no allowance for fuzzy margin of the stated uncertainties. A methodology that can provide such accuracy and traceability in uncertainty statements is shown in the next sections.

TASK SPECIFIC UNCERTAINTY OF PRE-ALIGNMENT

The “task specific uncertainty” as defined in the field of coordinate measurement is the measurement uncertainty that results, computed according to the ISO Guide to the Expression of Uncertainty in Measurement (GUM), when a specific feature/measurand is measured using a specific inspection plan, by a specific system, according to a specific calibration and environment conditions. This is the most full and accurate statement of the uncertainty and in order to be correct, a traceability to international standards should be ensured. Such statement can be produced in several methods: expert judgement, sensitivity analysis by substitution via calibrated objects, or computer simulation. [9]

The GUM, Supplement 1 [5] was selected as the most practical way of propagating the uncertainties of the complex measurements. This includes the application of the Monte Carlo method for the propagation of the uncertainties of measurement. By this standard the metrologist has to represent the measurement by a mathematical model. All input parameters uncertainties including the model accuracy have to be described by probability density functions. Once estimated, they can be propagated through the measurement model via random (or more advanced such as Latin Hyper Cube) sampling from their probability distributions. The result would be the measurand described by a probability density function defining the standard uncertainty associated with this virtual measurement. (Fig.3).

To be correct such statement of uncertainty should be expressed by the close as possible the real measurement mathematical model.
The error between the model and the real system has to be described as an additional input uncertainty with a probability density function. Such model needs to be validated by comparing its stated uncertainty to traceable reference standards. This method is already successfully validated and applied in the field of CMM metrology.

**Leitz-Infinity CMM Task specific uncertainty for PACMAN fiducial measurements.**

In parallel to our first CMM measurements, we provided a study of the expected task specific uncertainty of measuring the stretched wire with respect to the coordinate system created by the fiducials of the smallest CLIC magnet assembly. For this study we used PUNDITCMM™ (the only commercial virtual CMM software currently available) kindly provided to us by METROSAGE™.

We made a comparison for the expecting uncertainty of knowing a thin cylindrical object (representing wire) and a sphere respect to dimensional reference coordinate frame created by the centre of the farthest 4 fiducials symmetrically distributed around the assembly. The uncertainty was evaluated in 3 ways. First the MPEE equation given by the manufacturer was used to calculate the uncertainty for the given distances.

\[ U_{mpee} = 0.3 \mu m + \frac{L}{1000} \text{, where } L \text{ is the length in mm. (1)} \]

Second we used PUNDITCMM™ with calibration data for 10 cm long multi-tip-stylus. Then we performed the same simulation but with the calibration data of 5 cm long stylus. The comparison can be seen in Fig.4.

In orange one can see the expected uncertainty accordance to the manufacturer MPEE. In dark and light blue the real task specific uncertainty evaluated using PUNDITCMM™ for the two different types of touch probes modelled.

![Figure 3: CLIC T1 magnet assembly within Leitz Infinity CMM as part of PACMAN pre-alignment measurements](image)

![Figure 4: Uncertainty of thin cylinder (representing wire) and sphere location in assembly coordinate frame](image)

**Environment related uncertainty**

One of the biggest error sources in metrology is the thermally induced error. It is not only important to know what the length of an object is but as well at what temperature (both of object and comparator) the measurement was done. Currently 20°C is accepted as the universal reference for the temperature at which international standards of length are defined. Thus all traceable standards measurements and calibration are defined at their lowest uncertainty at this temperature. All measurements done at different temperature would inherently have a thermally induced error proportional to the differential expansion between the measuring device and the measurand [11]. In the case of CLIC pre-alignment the measurement will be performed as close as possible to 20°C at CMM class 1 environment (+0.1°C). However, this measurement will be used when the measurand is operational in the tunnel at air temperatures close to 40°C with magnet temperatures close to 45°C. This creates a significant thermal gradient of 25°C (from CMM measurement temperature) which can cause deformations in the order of 100s of µm over the meter sized assemblies. The so called Total Thermal Uncertainty will be a function of the relative drift between the magnetic axis and the
fiducials due to the thermal effects. For this study we have accepted to evaluate them independently with respect to a common reference, a granite mounting base thus:

The Total Thermal Uncertainty or (TTU) = Total Fiducial Thermal Error (TFTE) + Total Magnetic Axis Thermal Error (TMATE). In next two sub sections examples how GUM Supplement 1 standard can be applied for the uncertainty evaluation via modelling for the (TFTE) and empirical testing and uncertainty propagation for (TMATE).

**Total Fiducial Thermal Error**

Can be defined as: $\text{TFTE} = (\text{NDE} + \text{UNDE})$ (2) Where,

- TFTE = total fiducial thermal error
- NDE = nominal differential expansion and
- UNDE = uncertainty of NDE. Where

\[ \text{NDE} = [\text{NE}]_{\text{cmn scale}} - [\text{NE}]_{\text{assembly}}. \]

$[\text{NE}]_{\text{cmn scale}}$ is the nominal expansion and thermal deformation of the CMM scales as influenced by the temperature. This is at the CMM environment temperature. $[\text{NE}]_{\text{assembly}}$ is the nominal expansion and 3D deformation of the measurand/magnet assembly at the pre-alignment conditions (45 °C mean magnet temperature). In PACMAN the measurement is performed with magnet powered at very low current (4 A) without water cooling and thus close to the +/- 0.1 °C CMM assembly temperature, thus: the $\text{NDE} \approx [\text{NE}]_{\text{assembly}}$

If compensation for this thermal error is applied for the NDE then depending on the method used, the TFTE is equal to the Uncertainty of this compensation method.

\[ \text{TFTE} = \text{UNDE}_{\text{assembly compensation}} \] (3)

The $[\text{NE}]_{\text{assembly}}$ was calculated in two ways. First by the simplest linear scaling law that is widely used in metrology for simple corrections:

\[ NE_x or y or z = [\alpha \times L \times \Delta T]_{x or y or z} \] (4)

Where $\alpha$ = average coefficient of thermal expansion CTE. $L$ = Length of measurand metrology loop in the coordinate direction measured. $\Delta T$ = Temperature deviation from measurement temperature.

As application of GUM Supplement 1 we have attempted to mix stochastic modelling and Finite Element Modelling (Fig6).

A deterministic FEM model of CLIC T1 magnet Assembly [1] is used as the mathematical description of the behaviour of the measurand. All input to the FEM model parameters such as initial, boundary conditions, control parameters such as the CTE, heat convection coefficients are described not by mean values but by distributions (probability density functions) quantifying their uncertainty. Thermal measurement data from real heat cycle experiment are used as input boundary condition for the simulation.

The uncertainty due to four different scenarios was studied and it can be seen in Fig.7. One can see the directional uncertainty in X and Y with respect to the global reference frame (the granite base of the assembly).

At the most left is the TFTE experienced by the assembly at 25 °C gradient if no correction for this was applied (as estimated by the FEM deterministic model). The biggest error component is in Y and is over 100 μm.

Second, left to right in Fig.8 is the uncertainty of applying the linear compensation law (LCL) for compensation. The uncertainty in Y is significantly reduced. However surprisingly the uncertainty in Z is actually increased. This is due to the non-linear manner in which the assembly deforms in reality due to the internal gradients in the structure.

Figure 7: Comparison between TTU and different thermal error compensation methods

Figure 6: Propagation of Probability Density Function (PDF) through FEM model via Monte-Carlo like sampling to produce PDF
In this study, the Stochastic FEM method was applied to predict the deformation and the associated uncertainty with this deformation. As input standard uncertainties met in state of art literature for the various parameters were used. Uncertainty of the CTE coefficient of both steel and aluminium were taken as Gaussian distributions with 1σ close to 10% of the value of their CTE coefficient as an example of 1 type of parameter.

The (2σ) of the probability density function of the stochastic modelling of the resulted deformation is taken as its uncertainty. The results are in the range of 24 – 16 µm. Even without calibrating with measurements but using state of the art knowledge, this method gives better results than the LCL.

Finally, a theoretical study was performed to determine the limit of this method by selecting the lowest possible input uncertainties of the control parameters. Those are the limit uncertainties at which the input parameters of the FEM model can be measured/calibrated. This can reduce the uncertainty of the thermal error compensation with a further order of magnitude to reach close to 2.5 µm (2σ).

This study showed that if sufficient engineering rigour is spent in the evaluation of the input parameter uncertainties, stochastic modelling can be calibrated/validated to provide correction of the thermal errors with impressively low uncertainty for those dimensions/thermal gradients. More than that the uncertainty statement will be accurate and in accordance with GUM Supplement 1 international metrology standard. The major method benefit is that it can relatively quickly provide estimation of the expected thermal error compensation uncertainty for large variety of magnet and assembly designs.

**Magnetic Axis Thermal Error**

In this section we used Monte Carlo method to propagate the uncertainty of empirical compensation formula made for the magnetic axis drift.

Thermal deformation in magnet and its assembly change the location of the magnet poles geometry. This leads to a global change of the magnetic axis location with respect to the common base. We performed a study to measure the magnetic axis drift as function of temperature. We measured the drift only due to the magnet behaviour with disassembled nano-positioning system and external frame. (Fig8). For this experiment the magnet was placed directly on a granite support. The magnet was powered with nominal power (125 A). Water chiller was used to control the coils cooling water temperature. The magnet was stabilised over eight different steady states temperatures from 10 to 35 ºC. Once magnet was thermally stabilised at each of the test temperatures, the stretched wire system was used to best fit the wire to the current magnetic axis. A separate pair of capacitive Wire Positioning Sensors (WPS) were used to determine the relative and absolute location of the wire at each of the test temperatures. From the data we did least square fit for X and Y thermal drift (Fig11).

We wrote Monte Carlo script in MATLAB that propagates the evaluated measurement uncertainties trought the least square equation including the equations defining the least square adjustments a, and b (Fig9) below.

\[
Y_{\text{drift}} = a \cdot X + b
\]

\[
a = \frac{(n \sum xy) - (\sum x \sum y)}{(n \sum x^2) - (\sum x)^2}
\]

\[
b = \frac{(\sum y \sum x^2) - (\sum x \sum xy)}{(n \sum x^2) - (\sum x)^2}
\]

![Figure 8: Measurement of magnet axis drift due to temperature setup](image)

![Figure 9: Propagation of probability density functions of measurement parameters through least square equations to evaluate uncertainty of line of best fit](image)

In Fig10 the orange lines represent the least square line equations that can be used for the compensation of magnet axis drift. The black lines represent the evaluated uncertainty, by the MATLAB program, of the current model at 2σ.

A separate simulation was performed this time with the theoretical best thermal and drift measurements propagated. In Fig.10 the dashed line representing the theoretical best uncertainty of the empirically evaluated compensation. In Fig.11 on next page the comparison between the two can be observed.

The current study shows first the magnetic axis drift manly in vertical direction and close to 2 µm/ Cº. This is significant especially for the large thermal gradients to be experienced by CLIC pre-alignment. Furthermore, once nano-positioning system is installed the magnetic axis drift...
is expected to increase even further due to the additional components thermal deformation in Z and Y. Another important conclusion from this study is that although empirical modelling can provide really low uncertainty, it would be only valid if it is applied for the exact configuration of magnet/assembly. For any different magnet or assembly configuration, the lengthy test procedure has to be repeated. This is an inherent limitation of the empirical error compensation method that hopefully could be answered in future by the calibrated Stochastic FEM.

**CONCLUSIONS AND FUTURE WORK**

The studies performed proved that appropriate error compensation and uncertainty evaluation can be critical for highly demanding alignment applications. For the case of CLIC the biggest and most decisive error contributor remains the thermally induced deformation errors and uncertainties. The application of Monte Carlo like techniques provides promising initial results. Currently further studies are performed in order to verify/validate the methodology (Stochastic FEM) as appropriate for accurate and traceable uncertainty statements production.

If this proven by the planned experiments, it can provide tool with which uncertainty statements for various accelerator assembly’s designs and operational conditions can be evaluated, at well-defined operational conditions. Thus the methodology could be in future used as powerful design and decision making tool.
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REFERENCES


