Measurement of longitudinal flow correlations in Pb+Pb collisions at $\sqrt{s_{NN}} = 2.76$ and 5.02 TeV with the ATLAS detector

The ATLAS Collaboration

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Measurements of longitudinal flow correlations are presented for charged particles in the pseudorapidity range $|\eta| < 2.4$ using $7\, \mu b^{-1}$ and $22-470\, \mu b^{-1}$ of Pb+Pb collisions at $\sqrt{s_{NN}} = 2.76$ and 5.02 TeV, respectively, with the ATLAS detector at the LHC. It is found that the correlation between the harmonic flow coefficients $v_n$ measured in two separated $\eta$ intervals does not factorize into the product of single-particle $v_n$, and this breaking of factorization, or flow decorrelation, increases linearly with the $\eta$ separation between the intervals. The linear coefficients of the flow decorrelation are found to be larger at 2.76 TeV than 5.02 TeV. Higher-order moments of the correlations are also measured, and the corresponding linear coefficients for the $k^{th}$-moment of the $v_n$ are found to scale with $k$ for $n > 2$, but scale faster than $k$ for $n = 2$. The decorrelation effect is separated into contributions from the magnitude of $v_n$ changing with $\eta$ and the event-plane orientation changing with $\eta$. These two contributions are found to be comparable. The longitudinal flow correlations are also measured between $v_n$ of different order in $n$. The longitudinal fluctuations of $v_2$ and $v_3$ are found to be independent of each other, while the longitudinal fluctuations of $v_4$ and $v_5$ are found to be driven by the non-linear contribution from $v_2^2$ and $v_2 v_3$, respectively.

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1 Introduction

Heavy-ion collisions at RHIC and the LHC create hot, dense matter whose space-time evolution is well described by relativistic viscous hydrodynamics [1, 2]. Owing to strong event-by-event (EbyE) density fluctuations in the initial state, the space-time evolution of the produced matter also fluctuates event by event. These fluctuations lead to correlations of particle multiplicity in momentum space in both the transverse and longitudinal directions with respect to the collision axis. Studies of the particle correlation in the transverse plane have revealed strong harmonic modulation of the particle densities in the azimuthal angle: \( \frac{dN}{d\phi} \propto 1 + 2 \sum_{n=1}^{\infty} v_n \cos(n(\phi - \Phi_n)) \), where \( v_n \) and \( \Phi_n \) represent the magnitude and event-plane angle of the \( n^{\text{th}} \) order harmonic flow. The measurements of harmonic flow coefficients \( v_n \) and their EbyE fluctuations [3–5], as well as the correlations between \( \Phi_n \) of different order [6–9], have placed important constraints on the properties of the medium, and of transverse density fluctuations in the initial state.

The majority of previous flow studies assumed that the initial condition and space-time evolution of the matter are boost invariant in the longitudinal direction. Recent model studies of two-particle correlations as a function of pseudorapidity \( \eta \) revealed strong EbyE fluctuations of the flow magnitude and phase between two well separated pseudorapidities, i.e. \( v_n(\eta_1) \neq v_n(\eta_2) \) (forward-backward asymmetry) and \( \Phi_n(\eta_1) \neq \Phi_n(\eta_2) \) (event-plane twist) [10, 11]. The CMS Collaboration proposed an observable based on the ratio of correlations between \( \eta \) or \(-\eta\) with a common reference pseudorapidity \( \eta_{\text{ref}} \) in the forward direction, and this observable is sensitive to longitudinal correlations between \( \eta \) and \(-\eta\) [12]. The CMS results show that the longitudinal fluctuations lead to a linear decrease of the ratio with \( \eta \), and the slope of the decrease shows a strong centrality dependence for elliptic flow \( v_2 \) but very weak dependences for \( v_3 \) and \( v_4 \). Motivated by this measurement, several new observables have been proposed based on multi-particle correlations in two or more \( \eta \) intervals [13]. These observables are sensitive to the EbyE fluctuations, as a function of \( \eta \), of the initial condition as well as the non-linear mode-mixing effects. Measurement of these observables can provide new insights on the initial condition along the longitudinal direction, and will help in the development of full three-dimensional viscous hydrodynamic models.

Using these new observables, this note develops further the study of the longitudinal dynamics of collective flow in three ways. Firstly, the CMS measurement, which is effectively the first moment of the correlation between flow in separate \( \eta \) intervals, is extended to second and third moments. Secondly, a correlation between four subevents in different \( \eta \) intervals is measured to separate the contribution from fluctuation of \( v_n \) amplitude and contribution from fluctuation of \( \Phi_n \). Thirdly, measurements are also made of the correlations between harmonics of different order, e.g. between \( v_2 \) and \( v_4 \) in different \( \eta \) intervals, to investigate how the mode-mixing effects evolve with rapidity. In this way, this note presents a comprehensive measurement of flow decorrelation involving \( v_2, v_3, v_4 \) and \( v_5 \), using Pb+Pb collisions at \( \sqrt{s_{\text{NN}}}=2.76 \) and 5.02 TeV.

2 Observables

This section gives a brief summary of the observables measured in this note, and the mathematical details can be found in Ref. [13, 14]. The azimuthal anisotropy of the particle production in an event is described by harmonic flow vectors \( \mathbf{v}_n = v_n e^{i n \Phi_n} \), where \( v_n \) and \( \Phi_n \) are the magnitude and phase (or event plane), respectively. They are estimated from the observed per-particle normalized flow vector \( \mathbf{q}_n \):

\[
\mathbf{q}_n \equiv \frac{\sum_i w_i e^{i n \phi_i}}{\sum_i w_i} \equiv q_n e^{i n \Psi_n}
\]  

(1)
The sum runs over all particles in a given \( \eta \) interval of the event and \( w_i \) is the weight assigned to the \( i \)th particle. The weight accounts for detector non-uniformity and tracking inefficiency.

The longitudinal flow fluctuations are studied using the correlation between the \( k \)th-moment of the \( n \)th-order flow vectors in two different \( \eta \) intervals, averaged over events in a given centrality interval, \( r_{njk}(\eta) \):

\[
 r_{njk}(\eta) = \frac{\langle q_n^k(-\eta) q_n^k(\eta_{ref}) \rangle}{\langle q_n^k(\eta) q_n^k(\eta_{ref}) \rangle} = \frac{\left\langle v_n(-\eta) v_n(\eta_{ref}) \right\rangle^k \cos k \eta_0 (\Phi_n(-\eta) - \Phi_n(\eta_{ref}))}{\left\langle v_n(\eta) v_n(\eta_{ref}) \right\rangle^k \cos k \eta_0 (\Phi_n(\eta) - \Phi_n(\eta_{ref}))} \tag{2}
\]

where the subscript “\( n; k \)” denotes the \( k \)th moment of the flow vector of order \( n \) at \( \pm \eta \) and the \( k \)th moment of the complex conjugate of the flow vector of order \( n \) at \( \eta_{ref} \), and the \( \eta_{ref} \) is the reference pseudorapidity common to the numerator and the denominator. This correlator is sensitive to the flow decorrelation between \( \eta \) and \(-\eta \) through the reference flow vector \( q_n^k(\eta_{ref}) \). The detector effects associated with the reference \( \eta \) range are expected to be largely cancelled out in the ratio.

To ensure a sizable pseudorapidity gap between the two flow vectors in the numerator and denominator of Eq. 2, \( \eta_{ref} \) is usually chosen to be at large pseudorapidity, e.g. \( \eta_{ref} > 4 \), while the pseudorapidity of \( q_n^k(-\eta) \) and \( q_n^k(\eta) \) is usually chosen to be close to mid-rapidity, \(|\eta| < 2.5\). If flow harmonics from multi-particle correlations factorize into single-particle flow harmonics, e.g. \( \langle v_n(-\eta) \rangle \approx \langle v_n(\eta) \rangle \langle v_n(\eta_{ref}) \rangle \), then it is expected that \( r_{njk}(\eta) = 1 \). Therefore a value of \( r_{njk}(\eta) \) different from one implies a factorization-breaking effect due to longitudinal flow fluctuations, and such an effect is generally referred to as “flow decorrelation”.

Based on the CMS measurement [12] and arguments in Ref. [13], the observable \( r_{njk}(\eta) \) is expected to be approximately a linear function of \( \eta \) with a negative slope, and is sensitive to both the forward-backward (FB) asymmetry in the magnitude of \( v_n \), e.g. \( v_n(-\eta) \neq v_n(\eta) \), and to the twist of the event-plane angles between \( \eta \) and \(-\eta \):

\[
r_{njk}(\eta) \approx 1 - 2kF^a_{n;k}\eta, \quad F^a_{n;k} = F^{asy}_{n;k} + F^{twi}_{n;k} \tag{3}
\]

where \( F^{asy}_{n;k} \) and \( F^{twi}_{n;k} \) represent the contribution from FB \( v_n \) asymmetry and event-plane twist, respectively. Results on \( r_{njk} \) have been obtained by the CMS collaboration for \( k = 1 \) and \( n = 2–4 \). The measured \( F^a_{n;1} \) show only a weak dependence on \( \eta_{ref} \) for \( \eta_{ref} > 3 \) at the LHC. Measuring \( r_{njk} \) for \( k > 1 \) can yield new information on how the \( v_n \) asymmetry and event-plane twist fluctuate event by event.

If \( F^{asy}_{n;k} \) and \( F^{twi}_{n;k} \) in Eq. 3 are also independent of \( k \), i.e. \( F^{asy}_{n;k} = F^{asy}_{n} \) and \( F^{twi}_{n;k} = F^{twi}_{n} \), then:

\[
r_{njk} \approx r^{k}_{n;1} \tag{4}
\]

Deviation from this relation is sensitive to detailed EbyE structure of the flow fluctuations in the longitudinal direction.

To separate the contributions of the asymmetry and twist effects, a new observable involving correlations of flow vectors in four \( \eta \) intervals is required,

\[
 R_{n,n',n'',n'}(\eta) = \frac{\langle q_n(-\eta_{ref}) q_{n'}(-\eta) q_{n''}(\eta_{ref}) q_{n'}(\eta) \rangle}{\langle q_n(-\eta_{ref}) q_n(-\eta) q_{n'}(\eta_{ref}) q_{n'}(\eta) \rangle} = \frac{\left\langle v_n(-\eta_{ref}) v_n(-\eta) v_n(\eta_{ref}) v_n(\eta) \right\rangle \cos n \left( \Phi_n(-\eta_{ref}) - \Phi_n(\eta_{ref}) + \Phi_n(-\eta) - \Phi_n(\eta) \right)}{\left\langle v_n(-\eta_{ref}) v_n(-\eta) v_n(\eta) v_n(\eta_{ref}) \right\rangle \cos n \left( \Phi_n(-\eta_{ref}) - \Phi_n(\eta_{ref}) - \Phi_n(-\eta) + \Phi_n(\eta) \right)} \tag{5}
\]
The effect of asymmetry is the same in both numerator and denominator, and so this correlator is mainly sensitive to the event-plane twist effects:

\[ R_{n,n,n} \approx 1 - 4F_{nn2}^\text{ref} \eta \]  

Therefore the two contributions can be separated by combining \( r_{n\|n;2} \) and \( R_{n,n\|n,n} \) via Eqs. 3 and 6.

Measurements of longitudinal flow fluctuations can also be extended to correlations between harmonics of different order:

\[
\begin{align*}
    r_{2,3\|2,3}(\eta) &= \frac{\langle q_2(-\eta)q_3^*(\eta_{\text{ref}})q_3(-\eta)q_3^*(\eta_{\text{ref}}) \rangle}{\langle q_3^2(\eta_{\text{ref}})q_3^2(\eta) \rangle} \\
    r_{2,2\|4}(\eta) &= \frac{\langle q_2^2(-\eta)q_4^*(\eta_{\text{ref}}) \rangle + \langle q_4^2(\eta_{\text{ref}})q_4^*(-\eta) \rangle}{\langle q_4^2(\eta_{\text{ref}})q_4^*(\eta) \rangle} \\
    r_{2,3\|5}(\eta) &= \frac{\langle q_2(-\eta)q_5^*(\eta_{\text{ref}})q_5(-\eta)q_5^*(\eta_{\text{ref}}) \rangle}{\langle q_5^2(\eta_{\text{ref}})q_5(\eta_{\text{ref}})q_5^*(\eta) \rangle} + \frac{\langle q_2(\eta)q_3(\eta)q_5^*(\eta_{\text{ref}})q_5(-\eta)q_5^*(\eta_{\text{ref}}) \rangle}{\langle q_5^2(\eta_{\text{ref}})q_5(\eta_{\text{ref}})q_5^*(\eta) \rangle}
\end{align*}
\]

If the longitudinal fluctuations for \( v_2 \) and \( v_3 \) are independent of each other, one would expect \( r_{2,3\|2,3} = r_{2,2\|3,3} \). On the other hand, \( r_{2,2\|4} \) and \( r_{2,3\|5} \) are sensitive to the \( \eta \) dependence of the correlations between \( v_2 \) and event planes of different order, for example \( \langle q_2^2(-\eta)q_6^*(\eta_{\text{ref}}) \rangle = \langle v_2^2(-\eta)v_6(\eta_{\text{ref}}) \rangle \cos 4(\Phi_2(-\eta) - \Phi_6(\eta_{\text{ref}})) \rangle \).

Correlations between different orders have been measured previously at the LHC [4, 5, 15].

It is well established that the \( v_4 \) and \( v_5 \) in Pb+Pb collisions contain a linear contribution associated with initial geometry and mode-mixing contributions from lower-order harmonics due to non-linear hydrodynamic response [4, 5, 16–18]:

\[
v_4 = v_{4L} + \beta_{2,2} v_2^2, \quad v_5 = v_{5L} + \beta_{2,3} v_2 v_3,
\]

where the linear component \( v_{4L} \) is driven by the corresponding eccentricity \( e_n \) in the initial geometry [19]. If the linear component of \( v_4 \) and \( v_5 \) is uncorrelated with lower-order harmonics, i.e. \( v_2^2 v_{4L}^* \sim 0 \) and \( v_2 v_3 v_{5L}^* \sim 0 \), one expects:

\[
r_{2,2\|4} \approx r_{2,2;2;2}, \quad r_{2,3\|5} \approx r_{2,3;2,3}.
\]

Furthermore, the \( r_{n\|n;1} \) correlators involving \( v_4 \) and \( v_5 \) can be approximated by:

\[
\begin{align*}
    r_{4\|4;1}(\eta) &\approx \langle v_{4L}(-\eta)v_{4L}^*(\eta_{\text{ref}}) \rangle + \beta_{2,2}^2 \langle v_2^2(-\eta)v_2^2(\eta_{\text{ref}}) \rangle \\
    r_{5\|5;1}(\eta) &\approx \langle v_{5L}(-\eta)v_{5L}^*(\eta_{\text{ref}}) \rangle + \beta_{2,3}^2 \langle v_2(-\eta)v_2(\eta_{\text{ref}})v_3(-\eta)v_3^*(\eta_{\text{ref}}) \rangle
\end{align*}
\]

Therefore, both the linear and non-linear components may be important for \( r_{4\|4;1} \) and \( r_{5\|5;1} \).

### 3 ATLAS detector and trigger

The ATLAS detector [20] provides nearly full solid-angle coverage of the collision point with tracking detectors, calorimeters, and muon chambers, and is well suited for measurement of multi-particle correl-
measurements over a large pseudorapidity range.\footnote{ATLAS uses a right-handed coordinate system with its origin at the nominal interaction point (IP) in the centre of the detector and the $z$-axis along the beam pipe. The $x$-axis points from the IP to the centre of the LHC ring, and the $y$-axis points upward. Cylindrical coordinates $(r, \phi)$ are used in the transverse plane, $\phi$ being the azimuthal angle around the beam pipe. The pseudorapidity is defined in terms of the polar angle $\theta$ as $\eta = -\ln \tan(\theta/2)$.} The measurements were performed using the inner detector (ID), minimum-bias trigger scintillators (MBTS), the forward calorimeter (FCal), and the zero-degree calorimeters (ZDC). The ID detects charged particles within $|\eta| < 2.5$ using a combination of silicon pixel detectors, silicon microstrip detectors (SCT), and a straw-tube transition radiation tracker (TRT), all immersed in a 2 T axial magnetic field \cite{bib:atlas}. An additional pixel layer, the “Insertable B Layer” (IBL) \cite{bib:atlas, bib:ibl} installed between Run 1 and Run 2 (2013–2015), is used in the 5.02 TeV Pb+Pb measurements. The MBTS system detects charged particles over 2.1 $\leq |\eta| \leq 3.9$ using two hodoscopes of counters positioned at $z = \pm 3.6$ m. The FCal consists of three sampling layers, longitudinal in shower depth, and covers $3.2 < |\eta| < 4.9$. The ZDC, available in the Pb+Pb and $p$+Pb runs, are positioned at $\pm 140$ m from the collision point, detecting neutrons and photons with $|\eta| > 8.3$.

This analysis uses approximately $7 \mu$b$^{-1}$ and $22–470 \mu$b$^{-1}$ of Pb+Pb data at $\sqrt{s_{NN}} = 2.76$ TeV and $\sqrt{s_{NN}} = 5.02$ TeV, respectively, recorded by the ATLAS experiment at the LHC. The 2.76 TeV Pb+Pb data were collected in 2010, while the 5.02 TeV Pb+Pb data were collected in 2015.

The ATLAS trigger system \cite{bib:atlas-trigger} consists of a Level-1 (L1) trigger implemented using a combination of dedicated electronics and programmable logic, and a high-level trigger (HLT) implemented in processors. The trigger requires signals in two ZDCs or either of the two MBTS counters. The ZDC trigger thresholds on each side are set below the peak corresponding to a single neutron. A timing requirement based on signals from each side of the MBTS is imposed to remove beam backgrounds. This trigger selects $7 \mu$b$^{-1}$ and $22 \mu$b$^{-1}$ of minimum bias Pb+Pb data at $\sqrt{s_{NN}} = 2.76$ TeV and $\sqrt{s_{NN}} = 5.02$ TeV, respectively.

To increase the statistics for very central Pb+Pb collisions, a dedicated L1 trigger is used in 2015 to select events with total transverse energy ($\Sigma E_T$) in FCal above 4.54 TeV. This ultra-central trigger samples $470 \mu$b$^{-1}$ of Pb+Pb collisions at 5.02 TeV, corresponding to 0–0.1% ultra-central collisions (see Sec. 4).

### 4 Event and track selection

The offline event selection for the Pb+Pb data requires a reconstructed vertex with its $z$ position satisfying $|z_{\text{vtx}}| < 100$ mm. The selection also requires a time difference $|\Delta t| < 3$ ns between signals in the MBTS trigger counters on either side of the nominal centre of ATLAS to suppress non-collision backgrounds. A coincidence between the ZDC signals at forward and backward pseudorapidity is required to reject a variety of background processes, while maintaining high efficiency for inelastic processes. The fraction of events containing more than one inelastic interaction (pile-up) is negligible in 2.76 TeV Pb+Pb collisions, and is estimated to be less than 0.1% for 5.02 TeV Pb+Pb collisions. The pile-up contribution is studied by exploiting the correlation between the transverse energy measured in the FCal $\Sigma E_T$ or the number of neutrons $N_n$ in the ZDC and the number of tracks associated with a primary vertex $N_{\text{ch}}$. Since the distribution of $\Sigma E_T$ or $N_n$ in events with pileup is broader than that for the events without pileup, pile-up events are suppressed by a simple cut on the high tail-end of the $\Sigma E_T$ or $N_n$ distribution as a function of $N_{\text{ch}}$.

The Pb+Pb event centrality \cite{bib:centrality} is characterized using the $\Sigma E_T$ deposited in the FCal over the pseudorapidity range $3.2 < |\eta| < 4.9$ at the electromagnetic energy scale \cite{bib:em}. The FCal $\Sigma E_T$ distribution is divided into a set of variable percentile bins. A centrality interval refers to a percentile range, starting at

\[ 4 \]
0% relative to the most central collisions. Thus the 0–5% centrality interval, for example, corresponds to the most central 5% of the events. The ultra-central trigger mentioned in Sec. 3 selects events in 0–0.1% centrality with full efficiency. A Monte Carlo Glauber analysis [25, 27] is used to estimate the average number of participating nucleons, $N_{\text{part}}$, for each centrality interval. The systematic uncertainty on $N_{\text{part}}$ is less than 1% for centrality intervals in the range of 0–20% and increases to 6% for centrality intervals in the range of 70–80%. The Glauber model also provides a correspondence between the $\Sigma E_T$ distribution and sampling fraction of the total inelastic Pb+Pb cross section, allowing for the setting of centrality percentiles. For this analysis, a selection of collisions corresponding to 0–70% centrality is used to avoid possible biases from diffraction or other processes that contribute to very peripheral collisions. Following the convention of heavy ion analysis, the centrality dependence of the results in this paper is presented as a function of $N_{\text{part}}$.

Charged-particle tracks and primary vertices are reconstructed in the ID using algorithms whose implementation was improved for Run 2 of the LHC. A special reconstruction procedure, optimized for tracking in dense environments, is used for this purpose [28]. Tracks are required to have $p_T > 0.5$ GeV and $|\eta| < 2.5$. For the 2.76 TeV data, tracks are required to have at least nine hits in the silicon detectors with no missing Pixel hits and not more than one missing SCT hit, taking into account the effects of known dead modules. For the 5.02 TeV data, tracks are required to have at least two pixel hits, with the additional requirement of a hit in the first pixel layer when one is expected, at least eight SCT hits, and at most one missing hit in the SCT. In addition, for both datasets, the point of closest approach of the track is required to be within 1 mm of the primary vertex in both the transverse and longitudinal directions [29].

The efficiency, $\epsilon(p_T, \eta)$, of the track reconstruction and track selection cuts is evaluated using Pb+Pb Monte Carlo events produced with the HIJING event generator [30]. The generated particles in each event are rotated in azimuthal angle according to the procedure described in Ref. [31] to produce harmonic flow consistent with previous ATLAS measurements [8, 32]. The response of the detector is simulated using GEANT4 [33, 34] and the resulting events are reconstructed with the same algorithms applied to the data. For the 5.02 TeV Pb+Pb data, the absolute efficiency increases with $p_T$ by 5% between 0.5 GeV and 0.8 GeV, and varies only weakly for $p_T > 0.8$ GeV. The efficiency varies more strongly with $\eta$ and event multiplicity. For $p_T > 0.8$ GeV, it ranges from 75% at $\eta = 0$ to 50% for $|\eta| > 2$ in peripheral collisions, while it ranges from 71% at $\eta = 0$ to about 40% for $|\eta| > 2$ in central collisions. The detailed description of tracking efficiency in 5.02 TeV data can be found in Refs. [35]. The tracking efficiency for the 2.76 TeV data has similar dependence on $p_T$ and $\eta$ and is taken from Ref. [36]. However the correlation analysis in this note is not particularly sensitive to the efficiency.

The rate of falsely reconstructed tracks (“fakes”) is also estimated and found to be significant only in central collisions and low $p_T$, where it ranges from 2% at $|\eta| < 1$ to 8% at largest $\eta$. The fake rate drops rapidly for higher $p_T$ and towards more peripheral collisions. The fake rate has been accounted for in the tracking efficiency correction following the procedure in Ref. [35].

5 Data analysis

Measurement of the longitudinal flow dynamics requires the calculation of the flow vector $q_n$ via Eq. 1 in the ID and the FCal. The flow vector from the FCal serves as the reference $q_n(\eta_{\text{ref}})$, while the ID provides the flow vector as a function of pseudorapidity $q_n(\eta)$. 

6
In order to account for detector inefficiencies and non-uniformity, particle weight for the \(i\)-th-particle in the ID for the flow vector (Eq. 1) is defined as [36]

\[
w_i^{\text{ID}}(\eta, \phi, p_T) = d_{\text{ID}}(\eta, \phi) / \epsilon(\eta, p_T).
\] (14)

The determination of track efficiency \(\epsilon(\eta, p_T)\) is described in Section 4. The additional weight factor \(d_{\text{ID}}(\eta, \phi)\) corrects for variation of tracking efficiency or non-uniformity of detector acceptance as a function of \(\eta\). For a given \(\eta\) interval of 0.1, the distribution in azimuthal bins, \(N(\phi, \eta)\), is built up from all reconstructed charged particles with \(0.5 < p_T < 3\) GeV. The weight factor is then obtained as \(d_{\text{ID}}(\eta, \phi) \equiv \langle N(\eta) \rangle / N(\phi, \eta)\), where \(\langle N(\eta) \rangle\) is the average of the \(N(\phi, \eta)\). This “flattening” procedure removes most \(\phi\)-dependent non-uniformity from track reconstruction, which is important for any azimuthal correlation analysis. Similarly, the weight in the FCAL for the flow vector (Eq. 1) is defined as:

\[
w_i^{\text{FCAL}}(\eta, \phi) = (E_T)_i d_{\text{FCAL}}(\eta, \phi)
\] (15)

where \((E_T)_i\) is the transverse energy measured at the \(i\)-th tower in the FCAL at \(\eta\) and \(\phi\). The azimuthal weight \(d_{\text{FCAL}}(\eta, \phi)\) is calculated for each \(\eta\) interval of approximately 0.1, in a similar way to what is done for the ID. It ensures that the \(E_T\)-weighted distribution, averaged over all events in a given centrality interval, is uniform in \(\phi\). The flow vectors \(q_n(\eta)\) and \(q_n(\eta_{\text{ref}})\) are further corrected by an event-averaged offset, \(q_n = q_n - \langle q_n \rangle_{\text{evts}}\) [4].

The flow vectors obtained after these reweighing and offset procedures are used in the correlation analysis. The correlation quantities used in \(r_{n,\eta,\phi}\) are calculated as:

\[
\langle q_n^k(\eta) q_n^{*k}(\eta_{\text{ref}}) \rangle = \langle q_n^k(\eta) q_n^{*k}(\eta_{\text{ref}}) \rangle_s - \langle q_n^k(\eta) q_n^{*k}(\eta_{\text{ref}}) \rangle_b,
\] (16)

where subscripts “\(s\)” and “\(b\)” represent the correlator constructed from the same event (“signal”) and from the mixed-event (“background”), respectively. The mixed-event quantity is constructed by combining \(q_n^k(\eta)\) from each event with \(q_n^{*k}(\eta_{\text{ref}})\) obtained in other events with similar centrality (within 1%) and similar \(\Delta z_{\text{vtx}}\) (\(|\Delta z_{\text{vtx}}| < 5\) mm). The \(\langle q_n^k(\eta) q_n^{*k}(\eta_{\text{ref}}) \rangle_b\), which is typically more than two orders of magnitude smaller than the corresponding signal term, is subtracted to account for any residual detector non-uniformity effects that correlate between different \(\eta\) ranges. This mixed-event procedure is used in previous correlation analyses, for example Refs. [4, 5].

For correlators involving flow vectors in two different \(\eta\) ranges, mixed events are constructed from two different events. For example, the correlation for \(r_{2,3/5}\) is calculated as:

\[
\langle q_2(\eta) q_3(\eta) q_5^*(\eta_{\text{ref}}) \rangle = \langle q_2(\eta) q_3(\eta) q_5^*(\eta_{\text{ref}}) \rangle_s - \langle q_2(\eta) q_3(\eta) q_5^*(\eta_{\text{ref}}) \rangle_b.
\] (17)

The mixed-event correlator is constructed by combining \(q_2(\eta) q_3(\eta)\) from one event with \(q_5^*(\eta_{\text{ref}})\) obtained in another event with similar centrality (within 1%) and similar \(\Delta z_{\text{vtx}}\) (\(|\Delta z_{\text{vtx}}| < 5\) mm). On the other hand, for correlators involving more than two different \(\eta\) ranges, mixed events are constructed from more than two different events, one for each unique \(\eta\) range. One such example is \(R_{n,n_{\eta},n_{\phi}}\), for which each mixed event is constructed from four different events with similar centrality and \(\Delta z_{\text{vtx}}\).

Due to the symmetry of the Pb+Pb collision system, most correlators can be symmetrized to improve statistical precision. For example, instead of Eq. 2, the actual measured observable is:

\[
r_{n,\eta,\phi}(\eta) = \frac{\langle q_n^k(-\eta) q_n^{*k}(\eta_{\text{ref}}) + q_n^k(\eta) q_n^{*k}(-\eta_{\text{ref}}) \rangle}{\langle q_n^k(\eta) q_n^{*k}(\eta_{\text{ref}}) + q_n^k(-\eta) q_n^{*k}(-\eta_{\text{ref}}) \rangle}
\] (18)
The symmetrization procedure also allows further cancellation of possible differences between $\eta$ and $-\eta$ in the tracking efficiency or detector acceptance.

Table 1 gives a summary of the set of correlators measured in this note. The analysis is performed in intervals of centrality and the results are presented as a function of $\eta$ for $|\eta| < 2.4$. The main results are obtained using 5.02 TeV Pb+Pb data. The 2.76 TeV Pb+Pb data are limited, and therefore are used only to obtain $r_{n,j;1}$ and $R_{n,n,k,n}$ to compare with results obtained with the 5.02 TeV data and study the dependence on collision energy.

<table>
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</tr>
<tr>
<td>$R_{n,n,k,n}$ for $n = 2,3$</td>
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<tr>
<td>$r_{2,2}[4]$, $r_{2,3}[5]$, $r_{2,3}[2,3]$</td>
<td>5.02 TeV</td>
</tr>
</tbody>
</table>

Figures 1 and 2 show the sensitivity of $r_{2,2}[4]$ and $r_{3,3}[1]$, respectively, to the choice of reference pseudorapidity $\eta_{\text{ref}}$ in the FCal. A smaller $\eta_{\text{ref}}$ value implies a smaller pseudorapidity gap between $\eta$ and $\eta_{\text{ref}}$. The values of $r_{n,j;1}$ generally decrease with decreasing $\eta_{\text{ref}}$, possibly reflecting the contributions from non-flow associated with the away-side jet. However such contributions should be reduced in the most central collisions, due to large charged-particle multiplicity and jet-quenching effects. Therefore the decrease of $r_{n,j;1}$ in the most central collisions could also be due to the $\eta_{\text{ref}}$ dependence of $F_2^{*}$, as defined in Eq. 3. In this analysis, the reference flow vector is calculated from 4.0 $< |\eta_{\text{ref}}| < 4.9$, which should reduce adequately the effect of dijets and allow good statistical precision. For this choice of $\eta_{\text{ref}}$ range, the $r_{2,2}[4]$ and $r_{3,3}[1]$ show a linear decrease as a function of $\eta$ in most centrality intervals, indicating a significant breakdown of the factorization of two-particle flow harmonics into single-particle flow harmonics.

Figures 3 and 4 show $r_{2,2}[4]$ and $r_{3,3}[1]$ calculated for several $p_T$ ranges of the charged particles in the ID. If the longitudinal flow asymmetry and twist reflect global properties of the event, the values of $r_{n,j;1}$ should not depend strongly on $p_T$. Indeed little dependence is observed, except for $r_{2,2}[4]$ in the most central collisions and very peripheral collisions. The behaviour in central collisions may be related to the factorization breaking of the $n_2$ as a function of $p_T$ [8,12]. The behaviour in peripheral collisions is due to increasing relative contributions from jets and dijets at higher $p_T$ and for peripheral collisions. Unless specified otherwise, the measurements presented are performed using charged particles with $0.5 < p_T < 3$ GeV.

6 Systematic uncertainties

Since all observables are found to follow an approximately linear decrease with $\eta$, i.e. $D(\eta) \approx 1 - c\eta$ for a given observable $D(\eta)$ where $c$ is a constant, the systematic uncertainty is presented as the relative uncertainty for $1 - D(\eta)$ at $\eta = 1.2$, the mid-point of the $\eta$ range. The systematic uncertainties in this analysis arise from the event-mixing, track selection, and reconstruction efficiency. Most of the systematic uncertainties enter the analysis through the particle weights in Eqs. 14 and 15. In general, the uncertainties
Figure 1: The $r_{22,1}(\eta)$ measured for several $\eta_{	ext{jet}}$ ranges. Each panel shows the results for one centrality range. The error bars are statistical only.

Figure 2: The $r_{33,1}(\eta)$ measured for several $\eta_{	ext{jet}}$ ranges. Each panel shows the results for one centrality range. The error bars are statistical only.
Figure 3: The $r_{2\eta\eta}(\eta)$ measured in five $p_T$ ranges. Each panel shows the results for one centrality range. The error bars are statistical only.

Figure 4: The $r_{3\eta\eta}(\eta)$ measured in five $p_T$ ranges. Each panel shows the results for one centrality range. The error bars are statistical only.
for $r_{n;k}$ increase with $n$ and $k$, the uncertainties for $R_{n,n;n,n}$ increase with $n$, and all uncertainties are larger in the most central and more peripheral collisions. For $r_{2,3;2,3}$, $r_{2,2;4}$ and $r_{2,3;5}$, the uncertainties are significantly larger than for the other correlators. Each source is discussed separately below.

The effect of detector azimuthal non-uniformity has been accounted for by the weight factor $d(\phi, \eta)$ in Eqs. 14 and 15. The effect of reweighting is studied by setting the weight to unity and repeating the analysis. The results are consistent with the default (weighted) results within statistical uncertainties. Possible residual detector effects for each observable are further removed by subtracting those obtained from mixed events as described in Sec. 5. Uncertainties due to the event-mixing procedure are evaluated by varying the criteria for matching events in centrality and $z_{\text{mix}}$. The resulting uncertainty is in general found to be smaller than the statistical uncertainties. The event-mixing uncertainty for $r_{2;2,k}$ and $r_{3;3,k}$ is $< 1\%$ for $k = 1$ and increases to about 0.4–8\% for $k = 2$ and 0.6–10\% for $k = 3$, while the uncertainty for $r_{4;4,1}$ and $r_{5;5,1}$ is in the range of 1.5–3\% and 5–13\%, respectively. The uncertainty for $R_{n,n;n,n}$ is 1.5–6\% for $n = 2$ and 3–14\% for $n = 3$. The uncertainties for $r_{2,3;2,3}$, $r_{2,2;4}$ and $r_{2,3;5}$ are typically larger, they are in the range of 1–4\%, 1.5–16\% and 3–15\%, respectively.

The systematic uncertainty associated with the track quality selections is evaluated by tightening or loosening the requirements on $|d_0|$ and $|z_0 \sin \theta|$ used to select good tracks. In each case, the tracking efficiency is re-evaluated and the analysis is repeated. The difference is observed to be largest in the most central collisions where the flow signal is smallest and the influence of falsely reconstructed tracks is higher. The difference is observed to be less than 0.2–12\% for $r_{2;2,k}$ and $r_{3;3,k}$, 1.1–2\% for $r_{4;4,1}$, 3–6\% for $r_{5;5,1}$, 0.5–13\% for $R_{n,n;n,n}$, and 1–14\% for $r_{2,3;2,3}$, $r_{2,2;4}$ and $r_{2,3;5}$, respectively.

From previous measurements [8], it is well known that the $v_n$ signal has a strong dependence on $p_T$ but relatively weak dependence on $\eta$. Therefore a $p_T$-dependent uncertainty in the track reconstruction efficiency $\epsilon(\eta, p_T)$ could affect the measured longitudinal flow correlation, through the particle weights. The uncertainty in the track reconstruction efficiency is due to differences in the detector condition and material uncertainty between data and simulations. The uncertainty in the efficiency varies between 1\% and 4\%, depending on $\eta$ and $p_T$ [35]. The systematic uncertainty for each observable in Table 1 is evaluated by repeating the analysis with the tracking efficiency varied up and down by its corresponding uncertainty. For $r_{n;n;k}$ the uncertainties are in the range of 0.1–2\%, depending on $n$ and $k$. For $R_{n,n;n,n}$ the uncertainties are in the range of 0.1–1\%. For $r_{2,3;2,3}$, $r_{2,2;4}$ and $r_{2,3;5}$, the uncertainties are in the range of 0.1–2\%.

The systematic uncertainties from the different sources described above are added in quadrature to give the total systematic uncertainty for each observable. They are listed in Tables 2–4.
Table 2: Systematic uncertainties in percent for $1 - r_{2;2}$ and $1 - r_{3;3}$ at $\eta = 1.2$ in selected centrality intervals.

<table>
<thead>
<tr>
<th></th>
<th>$r_{2;2}$</th>
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<th></th>
<th>$r_{3;3}$</th>
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<tr>
<td></td>
<td>0-5%</td>
<td>20-30%</td>
<td>40-50%</td>
<td>0-5%</td>
<td>20-30%</td>
<td>40-50%</td>
</tr>
<tr>
<td>event mixing (%)</td>
<td>0.8 0.2 0.3</td>
<td>2.2 0.4 0.6</td>
<td>6.0 0.6 2.1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>track resolution (%)</td>
<td>0.4 0.3 0.2</td>
<td>1.5 0.4 0.9</td>
<td>9.4 1.0 2.4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>reco. efficiency (%)</td>
<td>0.3 0.1 0.1</td>
<td>0.4 0.1 0.1</td>
<td>0.9 0.1 0.1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>total (%)</td>
<td>1.1 0.4 0.4</td>
<td>3.1 0.8 1.3</td>
<td>14 1.5 3.7</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 3: Systematic uncertainties in percent for $1 - R_{2;2,2}$, $1 - R_{3;3,3}$, $1 - r_{4;4}$, and $1 - r_{5;5}$ at $\eta = 1.2$ in selected centrality intervals.

<table>
<thead>
<tr>
<th></th>
<th>$R_{2;2,2}$</th>
<th></th>
<th></th>
<th>$R_{3;3,3}$</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>0-5%</td>
<td>20-30%</td>
<td>40-50%</td>
<td>0-5%</td>
<td>20-30%</td>
<td>40-50%</td>
</tr>
<tr>
<td>event mixing (%)</td>
<td>6.1 1.5 1.5</td>
<td>4.6 2.9 14</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>track resolution (%)</td>
<td>3.5 0.4 0.7</td>
<td>2.0 3.2 13</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>reco. efficiency (%)</td>
<td>0.2 0.1 0.1</td>
<td>0.1 0.2 0.5</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>total (%)</td>
<td>8.4 1.7 2.0</td>
<td>6.1 5.5 25</td>
<td></td>
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<td></td>
</tr>
</tbody>
</table>

Table 4: Systematic uncertainties in percent for $1 - r_{2;2}$, $1 - r_{3;3}$, $1 - r_{4;4}$, and $1 - r_{5;5}$ at $\eta = 1.2$ in selected centrality intervals.

<table>
<thead>
<tr>
<th></th>
<th>$r_{2;2}$</th>
<th></th>
<th></th>
<th>$r_{3;3}$</th>
<th></th>
<th></th>
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</thead>
<tbody>
<tr>
<td></td>
<td>0-5%</td>
<td>20-30%</td>
<td>40-50%</td>
<td>0-5%</td>
<td>20-30%</td>
<td>40-50%</td>
</tr>
<tr>
<td>event mixing (%)</td>
<td>4.1 1.7 3.2</td>
<td>16 1.5 2.4</td>
<td>15 3.4 7.8</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>track resolution (%)</td>
<td>1.4 0.5 2.0</td>
<td>12 1.6 1.5</td>
<td>14 2.0 7.4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>reco. efficiency (%)</td>
<td>0.1 0.0 0.1</td>
<td>1.6 0.1 0.1</td>
<td>1.2 0.1 0.1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>total (%)</td>
<td>4.4 1.8 3.8</td>
<td>25 2.7 3.2</td>
<td>26 4.7 13</td>
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</table>

7 Results

The presentation of the results is structured as follows. Section 7.1 presents the results for $r_{n;1}$ and $R_{n,n;1,n}$ and the comparison between the two collision energies. Section 7.2 shows the results on $r_{n;1}$ for $k > 1$. The scaling relation Eq. 4 is tested and the contributions from $v_n$ FB asymmetry and event-plane twist are separated. Results on the mixed-harmonic correlators, Eqs. 7–9, are presented in Sec. 7.3 and
checked for consistency with the hydrodynamic picture. Unless specified otherwise, the measurements are performed using charged particles in $0.5 < p_T < 3$ GeV, and the reference flow vector is calculated in $4.0 < |\eta_{ref}| < 4.9$. Most results are shown for the $\sqrt{s_{NN}} = 5.02$ TeV Pb+Pb dataset, which has the better statistical precision. The results for $\sqrt{s_{NN}} = 2.76$ TeV Pb+Pb dataset are shown only for $r_{n,n,1}$ and $R_{n,n,n,n}$.

7.1 $r_{n,n,1}$ and $R_{n,n,n,n}$ at two collision energies

Figure 5 shows $r_{2;1}$ in various centrality intervals at the two collision energies. The correlator shows a linear decrease with $\eta$, except in the most central collisions. The decreasing trend is weakest around the 20-30% centrality range, and is more pronounced in both more central and more peripheral collisions. This centrality dependence is related to the centrality dependence of the $v_2$ associated with the average elliptic geometry. The decreasing trend at $\sqrt{s_{NN}} = 2.76$ TeV is slightly stronger than that at $\sqrt{s_{NN}} = 5.02$ TeV, which is expected as the collision system becomes less boost-invariant at lower collision energy.

Figures 6 and 7 show the results for $r_{3;1}$ and $r_{4;1}$ respectively at the two collision energies. A linear decrease as a function of $\eta$ is observed for both correlators, and the rate of the decrease is approximately independent of centrality. The lack of centrality dependence could be due to the fact that $v_3$ and $v_4$ are driven mainly by fluctuations in the initial state. The rate of the decrease is also observed to be slightly stronger at lower collision energy.

The decreasing trend of $r_{n,n,1}$ for $n = 2–4$ in Figs. 5–7 indicates significant breakdown of the factorization of two-particle flow harmonics into single-particle flow harmonics. However, the size of the factorization breakdown depends on harmonic order $n$, collision centrality and collision energy.
Figure 6: The \( r_{3|3;1}(\eta) \) compared between the two collision energies. Each panel shows results from one centrality interval. The error bars and shaded boxes are statistical and systematic uncertainties, respectively.

Figure 7: The \( r_{4|4;1}(\eta) \) compared between the two collision energies. Each panel shows results from one centrality interval. The error bars and shaded boxes are statistical and systematic uncertainties, respectively.

Figures 8 and 9 show \( R_{2,2;2,2} \) and \( R_{3,3;3,3} \) in several centrality intervals. Both observables follow a linear decrease with \( \eta \) and the decreasing trends are stronger at lower collision energy.

The measured \( r_{n|n;1} \) and \( R_{n,n|n,n} \) are parameterized with linear functions,

\[
r_{n|n;1} = 1 - 2F_{r,n;1} \eta, \quad R_{n,n|n,n} = 1 - 2F_{R,n} \eta, \tag{19}
\]

For \( n = 2 \), the \( r_{n|n;1} \) and \( R_{n,n|n,n} \) show significant non-linear behavior in the 0-20% centrality range. However, the slope parameter from the fit in Eq. 19 for a given observable \( D(\eta) \) is always found to be consistent with a linear-regressing coefficient \( F \approx \frac{\sum(1-D(\eta)) \eta}{2 \sum \eta^2} \). Therefore \( F_{n;1} (F_{R,n}) \) always characterizes the
average $\eta$-weighted deviation of $r_{n|n;1}(\eta)$ ($R_{n,n|n,n;1}(\eta)$) from one, whether or not the observable is linear in $\eta$.

The extracted slope parameters $F_{r,n;1}$ and $F_{R,n}$ are plotted as a function of centrality in terms of $N_{\text{part}}$, in Figs. 10 and 11, respectively. The data show that the extracted values of $F_{r,2;1}$ and $F_{R,2}$ first decrease and then increase as a function of increasing $N_{\text{part}}$. The larger values in central and peripheral collisions is related to the fact that $v_2$ is more dominated by the initial geometry fluctuations. The slopes for higher-order harmonics are significantly larger. As a function of $N_{\text{part}}$, a slight decrease is observed for $F_{r,3}$ and $F_{R,3}$ for $N_{\text{part}} > 200$, as well as an increase of $F_{r,4;1}$ for $N_{\text{part}} < 100$. The values of $F_{r,n;1}$ and $F_{R,n}$ are larger at lower collision energy. An increase of $F_{r,n;1}$ with lowering $\sqrt{s_{NN}}$ has been predicted in hydrodynamic
model calculations [37]. This energy dependence is quantified in Fig. 12 via the ratio of $F_{r_{2;1}}^r$ and $F_{r_{2}}^R$ between the two energies. The results of a fit to a constant in the range of 30 < $N_{\text{part}}$ < 400 are given in Table 5. Compared to $\sqrt{s_{\text{NN}}} = 5.02$ TeV, the values of $F_{r_{2;1}}^r$ and $F_{r_{2}}^R$ at $\sqrt{s_{\text{NN}}} = 2.76$ TeV are about 10% higher, and the values of $F_{r_{3;1}}^r$ and $F_{r_{4;1}}^r$ are about 16% higher.

Figure 10: Centrality dependence of $F_{r_{2;1}}^r$ (left panel), $F_{r_{3;1}}^r$ (middle panel) and $F_{r_{4;1}}^r$ (right panel) for Pb+Pb at 2.76 TeV (circles) and 5.02 TeV (squares). The error bars and shaded boxes are statistical and systematic uncertainties, respectively. The widths of the centrality intervals are not fixed but have been optimized to reduce the uncertainty.

Figure 11: Centrality dependence of $F_{r_{2}}^R$ (left panel), $F_{r_{3}}^R$ (middle panel) and $F_{r_{4}}^R$ (right panel) for Pb+Pb at 2.76 TeV (circles) and 5.02 TeV (squares). The error bars and shaded boxes are statistical and systematic uncertainties, respectively. The widths of the centrality intervals are not fixed but have been optimized to reduce the uncertainty.

Table 5: Result and total uncertainties of the average of the ratio of $F_{n;1}^r$ and $F_{n}^R$ between the two $\sqrt{s_{\text{NN}}}$ in the range of 30 < $N_{\text{part}}$ < 400.

<table>
<thead>
<tr>
<th></th>
<th>$n = 2$</th>
<th>$n = 3$</th>
<th>$n = 4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F_{r_{n;1}}^r (2.76 \text{ TeV})/F_{r_{n;1}}^r (5.02 \text{ TeV})$</td>
<td>$1.100 \pm 0.010$</td>
<td>$1.153 \pm 0.011$</td>
<td>$1.17 \pm 0.04$</td>
</tr>
<tr>
<td>$F_{r_{n}}^R (2.76 \text{ TeV})/F_{r_{n}}^R (5.02 \text{ TeV})$</td>
<td>$1.106 \pm 0.027$</td>
<td>$1.19 \pm 0.08$</td>
<td>--</td>
</tr>
</tbody>
</table>
Figure 12: Centrality dependence of ratio of $F_{n;1}$ (left panel) and $F_{n}^{R}$ (right panel) between 2.76 TeV and 5.02 TeV. The lines are a linear fit in the range of $30 < N_{\text{part}} < 400$. The error bars and shaded boxes are statistical and systematic uncertainties, respectively.

7.2 Higher-order moments

The longitudinal correlations of higher-order moments of harmonic flow carry information about the EbyE flow fluctuations in pseudorapidity. In the simple model described in Ref. [13], the decrease of $r_{n;k}$ is expected to scale with $k$ as given by Eq. 4.

Figure 13 compares the results of $r_{2;2;k}$ for $k = 1–3$ (solid symbols) with $r_{2;1;k}$ for $k = 2–3$ (open symbols). The data follow the scaling relation, Eq. 4, in the most central collisions (0–5% centrality) where $v_2$ is driven by the geometry fluctuations. In other centrality intervals, where the average geometry is more important for $v_2$, the $r_{2;2;k}$ ($k = 2$ and 3) data show stronger decreases with $\eta$ than $r_{2;1;k}$.

A similar study is performed for third-order harmonics and the results are shown in Fig. 14. The data follow approximately the scaling relation Eq. 4 in all centrality intervals.

To quantify the difference between $r_{n;k}$ and $r_{n;1;k}$, the $r_{n;k}(\eta)$ distributions are parameterized with a linear function:

$$r_{n;k} = 1 - 2k F_{n;k} \eta,$$

and the values of $F_{n;k}$ for $k = 1, 2$ and 3 are compared with each other as a function of centrality in Fig. 15.

For second-order harmonics, data clearly show that $F_{2;3} > F_{2;2} > F_{2;1}$ except in the most central and the most peripheral collisions, where they approach each other. On the other hand, the slightly opposite trend for the third-order harmonics, $F_{3;3} > F_{3;2} > F_{3;1}$, is observed in mid-central collisions ($150 < N_{\text{part}} < 350$).

Following the discussion in Sec. 2, the measured $F_{n;2}^{F}$ and $F_{n}^{R}$ values can be used to estimate the separate contributions from FB asymmetry and event-plane twist, $F_{n;2}^{\text{asy}}$ and $F_{n;2}^{\text{twi}}$, respectively, via the relation:

$$F_{n;2}^{\text{asym}} = F_{n}^{R} / 2, \quad F_{n;2}^{\text{twi}} = F_{n;2}^{F} - F_{n}^{R} / 2,$$

The results are shown in Fig. 16. The contributions from the two components are similar to each other for $n = 2$, for which the harmonic flow arises primarily from the average collision shape, as well as for $n = 3$, for which the harmonic flow is driven mainly by fluctuations in the initial geometry.
Figure 13: The $r_{2j}^{k}$ for $k = 1–3$ compared with $r_{2j}^{k}$ for $k = 2–3$ in various centrality intervals for Pb+Pb collisions at 5.02 TeV. The error bars and shaded boxes are statistical and systematic uncertainties, respectively. The data points for $k = 2$ or 3 in some centrality intervals are rebinned to reduce the uncertainty.

Figure 14: The $r_{3j}^{k}$ for $k = 1–3$ compared with $r_{3j}^{k}$ for $k = 2–3$ in various centrality intervals for Pb+Pb collisions at 5.02 TeV. The error bars and shaded boxes are statistical and systematic uncertainties, respectively. The data points for $k = 2$ or 3 in some centrality intervals are rebinned to reduce the uncertainty.
Figure 15: The values of $F_{n;k}$ for $k = 1,2$ and 3 for $n = 2$ (left panel) and $n=3$ (right panel), respectively. The error bars and shaded boxes are statistical and systematic uncertainties, respectively. The widths of the centrality intervals are not fixed but have been optimized to reduce the uncertainty.

Figure 16: The event-plane twist component $F_{n;2}$ and FB asymmetry component $F_{n;2}$ as a function of $N_{\text{part}}$ for $n = 2$ and 3 for Pb+Pb collisions at 5.02 TeV. The error bars and shaded boxes are statistical and systematic uncertainties, respectively.

7.3 Mixed-harmonics correlation

Figure 17 compares the $r_{2,3;2,3}$ with the product of $r_{2;2,1}$ and $r_{3;3,1}$. The data show that they are consistent with each other, suggesting the longitudinal fluctuations of $v_2$ and $v_3$ are uncorrelated. Figure 18 compares $r_{2;2,2}$ with the mixed-harmonic correlator $r_{2,2;4}$, as well as $r_{4;4,1}$. As discussed in Sec. 2 in the context of the relation in Eq. 10, if $v_4$ is dominated by the non-linear contribution associated with $v_2^2$, then $r_{2,2;4}$ would be expected to be similar to $r_{2;2,2}$. This is indeed confirmed in Fig. 18, which supports relation Eq. 10. Figure 18 also shows that the $\eta$ dependence for $r_{4;4,1}$ is stronger than for $r_{2;2,2}$ in all centrality intervals, suggesting that the decorrelation effects are stronger for the linear component of $v_4$ than the non-linear component.
Figure 17: $r_{2,3,3}$ (circles) and $r_{2,1,3;1}$ (squares) as a function of $\eta$ for several centrality intervals. The error bars and shaded boxes are statistical and systematic uncertainties, respectively. The $r_{2,3,3}$ data in the 50–60% centrality interval are rebinned to reduce the uncertainty.

Figure 18: Comparison of $r_{2,2,2}$, $r_{2,2,4}$ and $r_{4,4,1}$ for several centrality intervals. The dashed-lines indicate a linear fit. The error bars and shaded boxes are statistical and systematic uncertainties, respectively. The data points in some centrality intervals are rebinned to reduce the uncertainty.
A similar study of the influence of the linear and non-linear effects for $v_5$ has also been performed, and results are shown in Fig. 19. The three observables, $r_{2,3|2,3}$, $r_{2,3|5}$, and $r_{5|5;1}$ show similar values in all centrality intervals, albeit with large statistical uncertainties.

Figure 19: Comparison of $r_{2,3|2,3}$, $r_{2,3|5}$ and $r_{5|5;1}$ for several centrality intervals. The dashed-lines indicate a linear fit. The error bars and shaded boxes are statistical and systematic uncertainties, respectively. The $r_{5|5;1}$ data in the 30–40% centrality interval are rebinned to reduce the uncertainty.

8 Summary

Measurements of longitudinal flow correlations are presented for charged particles in the pseudorapidity range $|\eta| < 2.4$ using $7 \mu b^{-1}$ and $22 \mu b^{-1}$ of Pb+Pb data at $\sqrt{s_{NN}} = 2.76$ and 5.02 TeV, respectively, as well as $470 \mu b^{-1}$ of Pb+Pb data at 5.02 TeV collected via an ultra-central trigger. The factorization of two-particle azimuthal correlations into single-particle flow harmonics $v_n$ is found to be broken as a function of $\eta$, and the amount of factorization breakdown increases approximately linearly as a function of the $\eta$ separation between the two particles. The slope of this dependence is nearly independent of centrality and $p_T$ for $n > 2$. But for $n = 2$, the effect is smallest in mid-central collisions and increases toward more central or more peripheral collisions, and in central collisions the effect also depends strongly on the $p_T$. Furthermore, the effects are found to be slightly larger at 2.76 TeV than 5.02 TeV for all harmonics.

The higher moments of the $\eta$-dependent flow correlations are also measured and the corresponding linear coefficient of the $\eta$ dependence is extracted. The coefficient for the $k^{th}$-moment scales with $k$ for $n > 2$, but scales faster than $k$ for $n = 2$. The factorization breakdown has been separated into contributions from forward-backward asymmetry of the flow magnitude and event-plane twist, which are found to be comparable to each other.

The longitudinal flow correlations are also measured between harmonic flow of different order. The correlation of $v_2v_3$ between two $\eta$ ranges is found to factorize into the product of the correlation for $v_2$
and correlation for $v_3$, suggesting that the longitudinal fluctuations of $v_2$ and $v_3$ are independent of each other. The correlations between $v_4$ and $v_2^2$ suggest that the longitudinal fluctuations of $v_4$ are driven by the non-linear contribution from $v_2$, i.e. $v_4 \propto v_2^2$. Similarly, the correlations between $v_5$ and $v_2 v_3$ suggest that the longitudinal fluctuations of $v_5$ are driven by the non-linear contribution from $v_2 v_3$, i.e. $v_5 \propto v_2 v_3$. The results presented in this note provide new insights on the fluctuations and correlations of harmonic flow in the longitudinal direction, which can be used to improve full three-dimensional viscous hydrodynamic models.

References

Appendix

Figure 20: The $r_{[4,1]}(\eta)$ measured for several $\eta_{jet}$ ranges for Pb+Pb collisions at 5.02 TeV. Each panel represents one centrality range. The error bars are statistical only. The data points in some centrality intervals are rebinned to reduce the uncertainty.

Figure 21: The $r_{[4,1]}(\eta)$ measured in five $p_T$ ranges for Pb+Pb collisions at 5.02 TeV. Each panel shows the results for one centrality range. The error bars are statistical only. The data points in some centrality intervals are rebinned to reduce the uncertainty.
Figure 22: The $r_{|n;1}$($\eta$) (left panel) and $R_{n|n,n}$($\eta$) (right panel) in ultra-central Pb+Pb collisions at 5.02 TeV. The error bars and shaded boxes are statistical and systematic uncertainties, respectively.

Figure 23: The values of $r_{2|2;1}$ (top row) and $r_{3|3;1}$ (bottom row) compared between ATLAS and CMS [12] for Pb+Pb collisions at 2.76 TeV.
Figure 24: The slope parameter of $F_{n\perp}$ as a function of $N_{\text{part}}$ for various flow harmonics for Pb+Pb collisions at 2.76 TeV (left panel) and 5.02 TeV (right panel). The error bars and shaded boxes are statistical and systematic uncertainties, respectively. The widths of the centrality intervals are not fixed but have been optimized to reduce the uncertainty.

Figure 25: The slope parameter of $F_{Rn}$ as a function of $N_{\text{part}}$ for various flow harmonics for Pb+Pb collisions at 2.76 TeV (left panel) and 5.02 TeV (right panel). The widths of the centrality intervals are not fixed but have been optimized to reduce the uncertainty.