Hollow longitudinal phase space distributions have a flat profile and hence reduce the impact of transverse space charge. Dipolar parametric excitation with the phase loop feedback systems provides such hollow distributions under reproducible conditions. We present a procedure to create hollow bunches during the acceleration ramp of CERN’s PS Booster machine with minimal changes to the operational cycle. The improvements during the injection plateau of the downstream Proton Synchrotron are assessed in comparison to standard parabolic bunches.

Abstract

Particles in a transversely Gaussian bunch are detrained due to direct space charge (SC), the incoherent tune spread expression reads

\[ \Delta Q_u(z) = \frac{r_p \lambda(z)}{2 \pi \beta^2 \gamma^3} \left\{
\begin{array}{l}
\beta_x(s) \\
\sigma_x (s)
\end{array}
\right\} \left( \sigma_x (s) + \sigma_y(s) \right) \]

with \( u = x, y \) for the horizontal resp. vertical plane (and the classical particle radius \( r_p \), velocity \( \beta \) and the Lorentz factor \( \gamma \)). The line charge density \( \lambda(z) \) can vary along the longitudinal position \( z \) within the bunch. The betatron functions \( \beta_x(s) \) depend on the path length \( s \) around the accelerator ring, they shape the transverse beam sizes \( \sigma_x(s) \). In presence of dispersion \( D_x(s) \), the distribution of the relative momentum \( \delta \) contributes to \( \sigma_x \). For a Gaussian distributed \( \delta \) one obtains the well-known beam size expression

\[ \sigma_x(s) = \sqrt{\frac{\beta_x(s) \epsilon_x}{\beta_x(s) \epsilon_x + (D_x(s) \delta_{\text{rms}})^2}} \]

where \( \epsilon_x \) is the normalised beam emittance.

The Idea: Hollow Bunches

Hollow distributions in longitudinal phase space \( (z, \delta) \) reduce \( \Delta Q_u \) compared to Gaussian or parabolic bunches as in Fig. 1. They project to:

- intrinsically flat bunch profiles (reduced \( \lambda_{\text{max}} \)).
- broader momentum profiles (larger \( \delta_{\text{rms}} \) and \( \sigma_x \)).

To create hollow bunches, we excite a longitudinal dipolar parametric resonance by phase modulation via the phase loop feedback system. We modulate the RF reference phase \( \phi_Q \) around the synchro phase \( \phi_s \):

\[ \phi_Q(t) = \phi_s + \phi_{\text{drive}} \sin(\omega_{\text{drive}} t) \]

To excite the beam, the driving frequency \( \omega_{\text{drive}} \) needs to satisfy the resonance condition

\[ m \omega_{\text{drive}} \approx n \omega_s \]

with \( \omega_s \approx \cos(\phi_s) \) the angular synchrotron frequency. The integer numbers \( m \) and \( n \) characterise the \( m:n \) parametric resonance. The actual \( \omega_{\text{drive}} \) decreases with larger synchrotron amplitudes due to the bucket non-linearities towards the separator. Below transition, longitudinal SC additionally reduces \( \omega_s \).

PyHEADTAIL Simulations for PSB

![PyHEADTAIL Simulations for PSB](image)

Figure 2: Longitudinal phase space \( (z, \delta) \) during the simulation. The bunch is excited to larger synchrotron amplitudes via a dipolar parametric resonance. The non-linear bucket leads to a frequency spread over the particles and they spread azimuthally around the bucket center.

Implementation in PSB

![Implementation in PSB](image)

Figure 3: phase modulation excites the dipolar resonance.

![Implementation in PSB](image)

Figure 4: Longitudinal phase space \( (z, \delta) \) reconstructed via tomography at different stages in the PSB (measurements).

Horizontal Emittance

\[ \delta \] is not Gaussian distributed for hollow bunches, Eq. (2) is not valid. Instead, we convolve the measured \( \delta \) distribution with a guessed Gaussian betatron profile. The horizontal emittance \( \epsilon_x \) can then be found by least-squares when varying the betatron \( \sigma_x \). Applying this procedure to both beams reveals that Eq. (2) underestimates \( \epsilon_x \) by 24.8% to 34.8%.

Results for PS Injection Plateau

The decreased maximum line density from hollow bunches is expected to mitigate transverse space charge impact during the 1.2 s PS injection plateau.

1. Bunch Length Scan

Over many shots, we vary the bunch length for both beam types by adiabatically ramping the total RF voltage during the initial 15 ms to values between the initial 25 kV and 80 kV. Hollow bunches consistently achieve peak line densities depressed by a factor 0.9. A theoretically ideal rectangular profile of \( \sigma_x \) length would yield a \( \sqrt{3 \pi / 4} \approx 0.63 \) depression factor compared to a perfect Gaussian. Both extrema are plotted in Fig. 8a for comparison. We want to compare the impact of space charge for both beam types for fixed \( B_{L1} \) and \( N \) and \( \epsilon_y \).

To unify this set in one quantity, we choose to evaluate \( \Delta Q_u^{\text{max}} \) assuming a 6D Gaussian distributed beam in Eq. (1). Hence we apply (2) and use the Gaussian peak line density \( \lambda_{\text{max}} = N/(\sqrt{2 \pi} \sigma_x) \) with \( \sigma_x = B_{L1}/A \). Hollow bunches provide lower vertical emittances for the same unified reference tune shift \( \Delta Q_u^{\text{max}} \). The real tune shift of the hollow bunches is a factor 0.88 lower than their reduced \( \lambda_{\text{max}} \) and the larger \( \sigma_x \). In contrast, the parabolic bunches are rather well represented by the Gaussian approach (factor 0.97 lower real tune shift).

2. Intensity Scan

Keeping the maximum RF voltage 80 kV, we scan the intensity by varying the injected turns in the PSB. Figure 8c exhibits the emittance blow-up \( \epsilon_x \) \( \epsilon_y \) across the injection plateau versus the brightness, which is again lower for the hollow bunches.

![Results for PS Injection Plateau](image)

Figure 5: Wire scanning comprising betatron and dispersive part.

(a) Gaussian distribution

(b) hollow distribution

(a) Normalised peak line charge density vs. total bunch length.

(b) Vertical emittances (end of injection plateau) vs. reference space charge tune shift.

(c) Vertical emittance blow-up vs. brightness (at full RF voltage \( V_{\text{RF}} = 80 \) kV).