2.9 Building the Impedance Model of a Real Machine

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2.9.1 Introduction

While equipment heating due to beam coupling impedance is usually localized to that piece of accelerator equipment, the impedance contributions to beam instabilities add up for the whole machine. Therefore impedance models have been built for many existing machines, upgrades and projects in order to assess their stability limit.

Impedance models in various levels of complexity were developed over the years depending on the needs and knowledge at the time of implementation: for example for the CERN ISR [1], PS [2,3,4], PSB [5], SPS [6,7,8,9], LEP [10], LEP2 [11] LHC [11,12], RHIC [13] at Brookhaven National Lab, TeVatron [14] at Fermi National Lab, HERA [15] at DESY, KEKB [16], as well as many light sources: NSLS-II [17], PETRA [18] SOLEIL [19], ALBA [20], to name only a few.

Since impedance related instabilities can be a significant limitation and can drive fundamental parameters of the accelerator, impedance models are nowadays developed at the very early stage of machine design: for instance transverse instabilities required increasing the aperture of the beam screen for the Future Circular Collider project [21].

The complexity of an impedance model can range from a single number to an elaborated tool, which is able to recompute wake functions and complex impedance contributions as a function of frequency and their related thresholds with slight changes of machine configuration (e.g. energy, number of devices, beta function at the location of the devices, gaps of moving devices).

A complete impedance model should in fact compute the longitudinal, transverse driving and transverse detuning contributions (also referred to as dipolar and quadrupolar) [22] for obtaining all resistive wall, broadband and narrow band contributions over a frequency range that would span from the first potentially unstable frequency to the maximum frequency that can be excited by the various single bunch and multibunch modes. It is in particular crucial to disentangle the driving and detuning contributions as their respective impact on beam dynamics is very different: the driving impedance contributes to the growth of Headtail modes (see for instance [8]), while the detuning impedance is not expected to.

In practice, it is important to assess how the impedance model will be used in order to tailor the parameters of the computations/simulations or measurements: instability thresholds can be computed using macroparticle tracking codes or Vlasov solvers, that take impedance or wake functions as input, for single bunch or multi-bunch estimates. Several tools require fitting the impedance by one single resonator or several resonators, while other beam dynamics tools can use any type of impedance or wake as input.

Since impedance computations can be heavily time and resource consuming, it is important to identify what drives the accuracy of the impedance in the range of interest. For 3D simulations, the antagonistic requirements for (1) very short excitation source size to assess the high frequency components correctly and (2) very long wakes to take into account low frequency components and multi bunch effects accurately tend to always increase CPU needs and simulation times. For instance, single-bunch single-turn simulations with the macroparticle code HEADTAIL [23] require a wake function, which
can be computed with a very short source excitation over the length of the tracked bunch. Taking into account the multibunch/multi-turn effect requires much longer wakes, which makes it usually very difficult to keep a very short exciting source due to the very large number of mesh cells it needs. The long-range wake can therefore be assessed in a second simulation with a longer exciting source bunch, or by using a simpler model such as external circuits [24]. These two short-range and long-range wakes from these two computations can later be combined, with great care for the overlapping region and with convergence studies.

2.9.2 Assess Impedance of Individual Elements

Electromagnetic simulations are necessary to calculate the contribution to the total impedance of single accelerator components. Time domain simulations are of special interest, because the wake fields they provide can be fed directly into macroparticle simulation codes in order to predict their effects on the beam in realistic conditions.

2.9.2.1 Calculation of the Beam Couplings Impedance

The impedance can be calculated analytically or numerically. Analytical models could be used for simple geometries (resistive wall, simplified structures, pillbox). Numerical simulations are fundamental to account for all features of complex structures (e.g. kickers, collimators, cavities, beam diagnostic devices, etc.). CST Particle Studio 3D electromagnetic simulations [25] can be used to calculate wakes and/or impedances of simple accelerator structures. The software has been benchmarked with the known analytical solutions for several structures [24] (resistive wall, pillbox cavities, simplified models of kicker magnets, asymmetric chambers). Moreover the results of CST beam coupling impedance simulations were used to build the PSB, PS and SPS impedance models, which were found in very good agreement with experimental observations (beam induced heating, coherent tune shift and instability growth rate [4, 5, 8, 24]).

CST Studio Suite is a commercial 3-D electromagnetic Computer Aided Design (CAD) software. In particular, the Wakefield solver of Particle Studio solves Maxwell's equations in Time Domain (TD), using a particle bunch as excitation of the structure under study. Standard outputs of the code are the wake potentials produced by the exciting Gaussian bunch (called “source”) as a function of the time delay $\tau$ with respect to the passage of the source (i.e. the integrated electromagnetic force felt by a witness charge that goes through the structure at a time $\tau$ behind the source) and its equivalent in Frequency Domain (FD), the Fourier transform normalized to the bunch spectrum, i.e. the beam coupling impedance. Besides, since the code allows defining separately the transverse position of the exciting bunch and that of the computation point, we can also separately simulate the driving and the detuning terms of the transverse wake potentials. The main interest of simulating accelerator structures in time domain lies in the fact that the output of the simulation in terms of wake function may be directly used in particle tracking simulations to study the impact of these elements on the beam stability. In particular, wake functions in form of tables can be fed as an input into the HEADTAIL code, which is typically used for studying collective effects in beam dynamics. Since the wake functions are needed, the source bunch used in CST simulations should be short enough to be consistent with the length of the bunch slices simulated in HEADTAIL. This obviously limits the analysis to a maximum frequency defined by the bunch slicing, but it can be
applied when no higher frequency mechanisms are expected in the given beam dynamics problem.

Resistive wall, Indirect Space Charge (ISC), magnetic kickers, collimators, stripline kickers, RF cavities, beam pipe transitions and beam diagnostics elements like wire scanner and Beam Position Monitors (BPMs), constitute a non-exhaustive list of classical impedance sources. Some of these elements could be not relevant for the overall impedance budget but could suffer of beam induced heating and then limit the beam intensity at which these elements can be used (e.g. wire scanners).

Resistive wall: in general, the resistive wall impedance describes the coupling between the beam and an external chamber with finite conductivity. Analytical derivations are possible when dealing with chambers with simple geometries. Many of the existing theories are based on the field matching technique [12, 26, 27, 28, 29]. Alternative models for resistive wall calculations are based on the Transmission Line (TL) theory [24, 30, 31, 32]. For more complicated geometries (asymmetries, small beam pipe ceramic breaks or thin inserts [33, 34], holes [35], etc.), a theoretical estimation without involving numerical electromagnetic simulations becomes more involved. An example of interest in this sense is the LHC beam-screen where CST 3D simulations were used for the impedance estimation [24].

Indirect space charge: usually it is computed analytically. For complex vacuum chambers it can be computed numerically using form factors which could be obtained performing non-relativistic CST simulations [25] or using the BeamImpedance2D code [36].

Kicker magnets: they are the most important impedance source in the CERN SPS. In a very simple approximation a SPS ferrite loaded kicker can be modelled as two parallel plates of ferrite. For this simple geometrical model all the impedance terms (longitudinal, driving and detuning horizontal and vertical impedances) can be calculated analytically [37, 38, 39]. CST 3D simulations were found to be in very good agreement with the analytical results [24]. The excellent agreement between analytical model and numerical simulations can be read as an important benchmark for the simulation code in the correct solution of electromagnetic problems involving dispersive materials such a ferrite. In the framework of an improvement of the SPS kicker impedance model a step by step simulation study has been performed starting from the simplest model and introducing one by one the new features that make the model gradually closer to reality. This approach allows for a good understanding of the different contributions brought to the kicker impedance by the different aspects. First, the ferrite is assumed to be C-shaped and the whole finite length device is inserted in the vacuum tank and equipped with an inner conductor [40]. In order to further approach a more realistic model other aspects have to be included: the cell longitudinal structure, also called segmentation, which determines a significant increase of the beam coupling impedance for the SPS injection kickers (due to the short cell length) and the serigraphy for the SPS extraction kickers. More details about the SPS kicker impedance model can be found in Ref. [24].

Collimators: in the case of the LHC [12], the collimation system represents the largest source of impedance in the machine due to the collimator jaws proximity to the beam. Detailed calculations could proceed with CAD models and codes like CST [25], HFSS [41], GdfidL [42] or ACE3P [43]. A first approximation is considering the jaws of the collimator as two parallel plates for which codes like [12] can provide accurate impedance calculation. One should note that, for very small collimator gaps, the nonlinear wakefield introduced by the beam distribution may play an
important role [44].

Stripline kickers: analytical models could be used for a first estimation [45, 46, 47]. For the final calculation a 3D EM simulation should be performed in order to include all the relevant features of the kicker.

RF cavities: a model could be developed based on the RF parameters of the cavity. However, a good estimation of the beam coupling impedance of the RF cavities would require measuring the actual cavities or simulate the 3D geometry of the tuned cavity.

Transitions between different vacuum chambers: the beam coupling impedance of these elements can be accurately calculated by using CST Particle Studio. Analytical models are available for simple geometries of the vacuum chambers [24, 48, 49].

Beam diagnostic elements: for complex devices a simplified calculation is strongly advised in order to have a good understanding of the beam coupling impedance contribution.

2.9.2.2 Bench Measurements of the Beam Coupling Impedance: the Wire Method

Ideally measurements of beam coupling impedances of a device should be done by exciting the device with the beam itself [50]. However, in most cases this solution is not possible and one must resort to bench measurement techniques in which the beam is simulated by a current pulse flowing through a wire stretched along the beam axis. For beam coupling impedance evaluations, the Wire Method (WM) is a common and appreciated choice.

This technique was proposed in the first half of the 70’s, based on intuitive considerations. By means of WM, Faltens et al. [51] measured the wall contributions to the beam coupling impedance. M. Sands and J. Rees (1974) [52] measured the energy loss of a stored beam to a cavity due to the higher mode excitation. Moreover, at BNL and at CERN, the method was employed to measure the longitudinal and transverse beam coupling impedance [53, 54] of a kicker in the frequency domain. The method of Sands and Rees requires a complex numerical manipulation to obtain the beam coupling impedance from the measured quantities, because of the presence of multiple reflections in the measuring devices [52]. An improved method of measurement that does not need this manipulation was proposed by V.G. Vaccaro [55].

Since many years it has become customary to use the coaxial wire method [52, 55] to measure the beam coupling impedance of accelerator devices (e.g. [56, 57, 58, 59, 60, 61, 62, 63]). Nevertheless, the results obtained from wire measurements might not entirely represent the solution of our initial problem, because the presence of the stretched wire perturbs the electromagnetic boundary conditions. The most evident consequence of the presence of another conductive medium in the center of the device under study is the fact that it artificially allows TEM propagation through the device, with zero cut-off frequency. The presence of a TEM mode among the solutions of the electromagnetic problem will have the undesired effect to cause additional losses during the measurement. Theoretical studies about the validity limits of the Sands and Rees method in relation to the presence of the central wire that simulates the beam can be found in Ref. [64], where, by means of a general theoretical approach, the effect of the central conductor with small but finite radius has been studied. As results for an example of application (pill-box cavity with a radius of 15 cm using a wire with a diameter of 1.12 mm) the longitudinal beam coupling impedance of the DUT, calculated with this approach, is very similar to the impedance obtained with the Sands and Rees formula (Fig. 2 of Ref. [64]). Details about the measurement setup used in [64] can be found in [65]. However, the mode analyzed in this study is above the cut off
frequency of the beam chamber. For modes below this frequency, due to the TEM propagation introduced by the wire, the WM is found to provide inaccurate results, as investigated in Ref. [66] comparing the impedance with and without a wire for a Copper pillbox, by means of the Mode Matching Technique.

2.9.2.3 Summing the Different Impedance Sources to Build the Impedance Model of a Real Machine

The impedance model of the machine can be obtained summing the contributions of all the impedance sources analysed. In the transverse plane the sum has to be weighted by the respective beta functions as expressed in the following formula (see e.g. [12]):

\[ Z_\perp(f) = \sum_{i=1}^{N} \frac{\beta_i}{\langle \beta_\perp \rangle} Z_\perp^i(f) \]

where \( \beta_i \) is the beta function of the \( i^{th} \) element (since the element extends over a certain length the beta function should be averaged over the element length), \( \langle \beta_\perp \rangle \) is the average beta function of the ring and \( Z_\perp^i(f) \) is the transverse impedance of the \( i^{th} \) element.

As an example, Fig. 1 shows the full SPS impedance model, which includes kicker magnets, wall, BPMs, RF cavities and broadband impedance from step transitions for the horizontal and vertical driving and detuning impedances [8].

![Figure 9: Horizontal (left) and vertical (right) SPS impedance model.](image)

2.9.3 Compute Beam Observables and Compare with Measurements

When an impedance model is produced for an accelerator machine, it allows to perform beam stability predictions and to calculate the impact of upgraded (or removed) equipment in the machine. A series of benchmark can be performed in order to ensure the good agreement between impedance model and beam observables. In the following we will introduce some of the most common procedures used to validate an impedance model.

2.9.3.1 Single Bunch Tune Shift / Growth Rate versus Intensity

Measuring the complex betatron coherent frequency shift with intensity gives information on the total transverse impedance according to Sacherer’s theory [67]. Given a
beam with Gaussian longitudinal distribution we can calculate the total machine tune shift with intensity as

$$\Delta Q_\perp = -\frac{qZ\bar{I}T_0}{8\pi^2\alpha E_0 \sigma_t} \sum_k \beta_{\perp k} Z_{\perp k}^{\text{eff}},$$  

(2)

where $\Delta Q_\perp$ is the complex machine transverse betatron tune, $q$ the elementary charge, $\bar{I} = qZN_b f_0$ the beam current (with $N_b$ number of particles, $Z$ the charge number, and $f_0 = 1/T_0$ the revolution frequency), $\beta$ relativistic beta factor, $E_0 = \gamma A m_u c^2$ is the energy of a traveling ion with $m_u$ nucleon rest mass, $A$ number of mass, $\gamma$ the relativistic gamma and $c$ speed of light, $\sigma_t$ is the rms bunch length, $z_{\perp k}^{\text{eff}}$ the effective transverse impedance of the $k^{th}$ element weighted by the corresponding betatron function $\beta_{\perp k}$. The effective impedance is defined as

$$Z_{\text{eff}} = \frac{\int_{-\infty}^{\infty} Z(\omega) \|S(\omega)\|^2 d\omega}{\int_{-\infty}^{\infty} \|S(\omega)\|^2 d\omega},$$  

(3)

where $\omega$ is the angular frequency, $\|S(\omega)\|^2$ the beam power spectrum. For a Gaussian beam distribution we have

$$\|S(\omega)\|^2 = e^{-\omega^2 \sigma_t^2}.$$  

(4)

From Eq. (2), we infer that a measurement of the tune frequency shift versus intensity can give information on the imaginary part of the total transverse effective impedance of the machine, while the measurement of the corresponding growth rate would give information on the real part. In practice, this is usually done injecting in the machine few bunches of different intensity and measuring their tune frequency shift versus intensity. If the machine duty cycle is high, the measurement could be done injecting one bunch per cycle with different intensity. The growth rate can be measured moving the machine to negative (positive) chromaticity if operating above (below) the transition energy. The agreement or disagreement with the impedance model predicted tune shift can point to a lack of knowledge of the machine impedance in which case one would have to refine it including those elements not yet included in the model or performing impedance localization measurements. In Fig. 2 we show the progressive refinement of the SPS impedance model accounting for kickers and, progressively, wall, BPMs, RF cavities and flanges impedance [24].
Figure 2: Measured tune shift in the CERN SPS compared with impedance model simulations accounting for kickers and, progressively, wall, BPMs, RF cavities and flanges impedance.

For particular equipment, where the impedance presents Higher Order Modes (HOMs) with considerably high shunt impedance, the tune shift estimation may not be enough to get the full picture of the equipment impact on the machine performance. In this case the full Sacherer’s theory should be applied, especially studying the effect of the HOMs on the most unstable couple bunch growth rate. As an example, the HL-LHC crab cavities are planned to be installed close to the LHC interaction points IP1 and IP5 [68] where large $\beta$ functions are expected. The HOMs introduced by the cavities are therefore magnified by the $\beta$ function according to Eq. (1) and the impact on the corresponding coupled bunch most unstable mode has been addressed [69].

### 2.9.3.2 Impedance Localization Techniques

An extension of the tune shift method for measuring the reactive part of transverse localized impedances was proposed the first time in 1995 at CERN [70]: measuring the impedance-induced betatron phase advance shift with intensity, the LEP RF sections were found to be important impedance contributors. A similar method, based on the impedance-induced orbit shift with intensity, was proposed in 1999 in the Novosibirsk VEPP-4M electron-positron storage ring [71] and in 2001 in the Argonne APS synchrotron accelerator [72]. Later in 2002, the same method was tried in the Grenoble ESRF [73]. The CERN research on the impedance localization method using phase advance shift with intensity was continued in 2004 in the SPS [74, 75] and in BNL RHIC [76]. The method has been recently successfully employed in the CERN PS [77] as shown in Fig. 3 and in Alba [78] and extended to the use of AC dipoles in order to achieve a higher signal to noise ratio and sufficient measurement resolution to detect impedance sources [79].
Figure 3: Impedance-induced phase advance beating (top) and corresponding reconstruction with detected impedance sources along the PS ring (bottom).

2.9.3.3 Single Bunch Octupole Threshold

In machines like the LHC, the beam stability at high energy is ensured by the combined stabilizing effect of transverse damper and Landau octupoles [80]. The threshold for stability as a function of the damper gain and/or octupole current can give information of the accuracy of the impedance model. This approach assumes the impedance to be the only source of instability, which is not always the case, especially when operating with train of bunches where electron cloud effects are not negligible, or two beams, where beam-beam effects play a role. As long as single bunches are accelerated, the octupole threshold can be measured and correlated with the machine impedance. Figure 4 shows the overall picture of the LHC octupole threshold as a function of chromaticity [81] measured and simulated with DELPHI [82]: a good agreement is obtained for positive chromaticities while discrepancies are present for negative and close to zero chromaticities where, probably, a refined damper model is required and currently being investigated.
Figure 3: LHC octupole current threshold $J_{oct}$ as a function of $Q'$ for different damper gains $d$ measured at flat top (FT) and end of the squeeze (EOS).

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