Status of mixing and CP-violation measurements at LHCb

A. Bertolin on behalf of the LHCb collaboration

Outlook:
- short LHCb and CPV introduction
- sin 2β, sin 2βs LHCb results review
- LHCb γ results:
  - a time-integrated γ measurement (used in the latest LHCb γ combination)
  - the latest LHCb γ combination results
- an updated time-dependent γ measurement
- matter antimatter differences in beauty baryon decays
- summary and future prospects
optimized for beauty and charm physics at $2 < \eta < 5$:

- momentum resolution $(\sigma(p)/p \approx 0.5 - 0.8 \%, p < 100 \text{ GeV/c})$
- impact parameter resolution $(\sigma(IP) \approx 20 \mu\text{m at high } p_T)$
- primary and secondary vertices reconstruction
- decay time resolution $(\sigma(t) \approx 50 \text{ fs})$
- ‘global’ PID: $e / \mu / \pi / K$
  $(K \text{ id } \approx 95 \% \pi \text{ mis-id } \approx 5 \%, p < 100 \text{ GeV/c})$
- $\gamma$ and $\pi^0$ reconstruction
CP violation

CP violation is one requirement for explaining the baryon asymmetry we observe today. A process must have been in place that took us from the equal amounts of matter - anti-matter produced in the Big Bang to the matter dominated Universe we are leaving in.

Charged current weak interactions between quarks are described by a matrix: 3 x 3, unitary (\( \leftrightarrow 3 \) angles and 1 phase), the **CKM matrix**

\[
\begin{pmatrix}
1 - \lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\
-\lambda & 1 - \lambda^2/2 & A\lambda^2 \\
A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1
\end{pmatrix}
\]

CKM expansion in \( \lambda \approx 0.22 \) up to \( \mathcal{O}(\lambda^3) \)

\( \rho - i\eta \): gives the **CKM phase**, only source of CP violation in the SM quark sector.

Multiple measurements allow to over constrain the few ‘free’ parameters of the SM and hence allow to look for new physics effects distorting their values ... due for example to new particles / mediators being exchanged in loops ...

Why B mesons / hadrons? Related unitary triangles are less squeezed hence expect larger sensitivity to any CP violation effect.
LHCb sin 2β measurement, 3 ifb

\[ \beta \equiv \arg[-(V_{cd}V_{cb}^*)/(V_{td}V_{tb}^*)] \]

\( B^0 \rightarrow J/\psi K_0^* \) "golden mode" for this measurement

\[ A(t) = \frac{\Gamma(B^0(t) \rightarrow J/\psi K_0^*) - \Gamma(B^0(t) \rightarrow J/\psi K_S^0)}{\Gamma(B^0(t) \rightarrow J/\psi K_0^*) + \Gamma(B^0(t) \rightarrow J/\psi K_S^0)} = \frac{S \sin(\Delta m t) - C \cos(\Delta m t)}{\cosh(\frac{\Delta \Gamma}{2} t) + A \Delta \Gamma \sinh(\frac{\Delta \Gamma}{2} t)} \]

\[ S = \sin 2\beta \]

\[ S = 0.731 \pm 0.035 \text{ (stat)} \pm 0.020 \text{ (syst)} \quad (\delta \beta \approx 1.16^\circ) \]

BaBar [PRD 79 (2009) 072009] = 0.69 ± 0.03 (stat) ± 0.01 (syst)

Belle [PRL 108 (2012) 171802] = 0.67 ± 0.02 (stat) ± 0.01 (syst)

overall result as good as BaBar and Belle
LHCb $\sin 2\beta_s$ measurements

$$\beta_s = \arg[-(V_{ts} V_{tb}^*)/(V_{cs} V_{cb}^*)]$$

interference between the amplitudes of decays of $B^0_s$ mesons to $c\bar{c} X(s)$ CP eigenstates directly or via mixing

$\phi_s \approx -2 \beta_s$ (SM+ ignoring subleading penguin contributions)

LHCb analyses, all updated to 3 ifb, give the most precise results:

$B^0_s \to J/\psi \pi^+ \pi^-$ [PLB 736 (2014) 186]

$B^0_s \to D_s^+ D_s^-$ [PRL 113 (2014) 211801]

$B^0_s \to J/\psi K^+ K^-$, $m(K^+ K^-)$ in the $\phi$ region [PRL 114 (2015) 041801]

$B^0_s \to \psi(2S) \phi$ [PLB 762 (2016) 253-262]

$B^0_s \to J/\psi K^+ K^-$, $m(K^+ K^-) > m(\phi)$ [https://arxiv.org/abs/1704.08217]

- time dependent
- flavor tagging
LHCb γ combination

\[ \gamma \equiv \arg\left[ -V_{ud}V_{ub}^*/V_{cd}V_{cb}^* \right] \]

- can be measured using only tree-level processes
- assuming new physics is not present in tree-level decays, negligible theoretical uncertainty
- disagreement between direct measurements and the value inferred from global CKM fits would spot new physics beyond the SM
- can be determined by exploiting the interference between favored \( b \rightarrow c \ W (V_{cb}) \) and suppressed \( b \rightarrow u \ W (V_{ub}) \) transition amplitudes

<table>
<thead>
<tr>
<th>( B ) decay</th>
<th>( D ) decay</th>
<th>Method</th>
<th>Ref.</th>
<th>Status since last combination [28]</th>
</tr>
</thead>
<tbody>
<tr>
<td>( B^+ \rightarrow D h^+ )</td>
<td>( D \rightarrow h^+h^- )</td>
<td>GLW/ADS</td>
<td>[44]</td>
<td>Updated to 3 fb(^{-1})</td>
</tr>
<tr>
<td>( B^+ \rightarrow D h^+ )</td>
<td>( D \rightarrow h^+\pi^-\pi^+\pi^- )</td>
<td>GLW/ADS</td>
<td>[44]</td>
<td>Updated to 3 fb(^{-1})</td>
</tr>
<tr>
<td>( B^+ \rightarrow D h^+ )</td>
<td>( D \rightarrow h^+h^-\pi^0 )</td>
<td>GLW/ADS</td>
<td>[45]</td>
<td>New</td>
</tr>
<tr>
<td>( B^+ \rightarrow D K^+ )</td>
<td>( D \rightarrow K^0_s h^+h^- )</td>
<td>GGSZ</td>
<td>[46]</td>
<td>As before</td>
</tr>
<tr>
<td>( B^+ \rightarrow D K^+ )</td>
<td>( D \rightarrow K^0_s K^-\pi^+ )</td>
<td>GLS</td>
<td>[47]</td>
<td>As before</td>
</tr>
<tr>
<td>( B^+ \rightarrow D h^+\pi^-\pi^+ )</td>
<td>( D \rightarrow h^+h^- )</td>
<td>GLW/ADS</td>
<td>[48]</td>
<td>New</td>
</tr>
<tr>
<td>( B^0 \rightarrow D K^{*0} )</td>
<td>( D \rightarrow K^+\pi^- )</td>
<td>ADS</td>
<td>[49]</td>
<td>As before</td>
</tr>
<tr>
<td>( B^0 \rightarrow D K^+\pi^- )</td>
<td>( D \rightarrow h^+h^- )</td>
<td>GLW-Dalitz</td>
<td>[50]</td>
<td>New</td>
</tr>
<tr>
<td>( B^0 \rightarrow D K^{*0} )</td>
<td>( D \rightarrow K^0_s \pi^-\pi^- )</td>
<td>GGSZ</td>
<td>[51]</td>
<td>New</td>
</tr>
<tr>
<td>( B^0_s \rightarrow D^+_s K^- )</td>
<td>( D^+_s \rightarrow h^+h^-\pi^+ )</td>
<td>TD</td>
<td>[52]</td>
<td>As before</td>
</tr>
</tbody>
</table>

LHCb inputs for the \( \gamma \) combination:
- \( D \ K \): excludes
  - \( B^+ \rightarrow D \ \pi^+ \)
  - \( B^+ \rightarrow D \ \pi^+ \pi^- \pi^+ \)
- \( D \ h \): full list
from the two upper plots:

\[ A^f_{h} = \frac{\Gamma(B^- \to [f]_D h^-) - \Gamma(B^+ \to [\bar{f}]_D h^+)}{\Gamma(B^- \to [f]_D h^-) + \Gamma(B^+ \to [\bar{f}]_D h^+)} \]

\[ A^K_{\pi\pi} = \chi^{\pi\pi} \]

the significance of CP violation in \( B^- \to [\pi^- K^+]_D K^- \) is 8.0 \( \sigma \)
γ from $B^\pm \rightarrow [h h' \pi^0] K / \pi$, 3 ifb, time integrated

LHCb can cope with this $\pi^0$ ‘challenge’ additional inputs for the combined measurement
LHCb $\gamma$ combination: D K (D h) combination results, freq. approach

- 71 (89) observables, 32 (38) parameters for D K (D h)
- from the $\chi^2$ value at the minimum and the n.d.f., goodness of fit is $p = (72.9)$ % for D h (D K)

\[ \mathcal{A}_{\text{sup}} / \mathcal{A}_{\text{fav}} = r_B^X e^{i(\delta_B^X \pm \gamma)} \]

uncertainty shrinks due to the large fitted value of $r_B^{D\pi}$
even if $B^+$ is dominant should continue to use both $B^+ B^0$ and $B^0_s$ modes to further access the robustness of the measurement

$LHCb$ $\gamma$ combination: result by $B$ decay mode (for D K) and final outcome

final outcome: quote for $\gamma$ the value obtained from the D K combination

\[ \gamma = (72.2^{+6.8}_{-7.3})^\circ \]

65.33 [+0.96 -2.54]

CKMfitter, ICHEP16
\( \gamma \) from \( B_s \rightarrow D_s K \), 3 ifb, time dependent

(with will be included in the next D K LHCb \( \gamma \) combination)

\[
\frac{d\Gamma_{B_s \rightarrow f}(t)}{dt} \propto e^{-\Gamma_{s}t} \left[ \cosh \left( \frac{\Delta \Gamma_{s}s}{2} \right) + A_{f}^{\Delta \Gamma} \sinh \left( \frac{\Delta \Gamma_{s}s}{2} \right) + C_{f} \cos(\Delta m_{s}t) - S_{f} \sin(\Delta m_{s}t) \right]
\]

decay time distributions

\[
\frac{d\Gamma_{B_s \rightarrow f}(t)}{dt} \propto e^{-\Gamma_{s}t} \left[ \cosh \left( \frac{\Delta \Gamma_{s}s}{2} \right) + A_{f}^{\Delta \Gamma} \sinh \left( \frac{\Delta \Gamma_{s}s}{2} \right) - C_{f} \cos(\Delta m_{s}t) + S_{f} \sin(\Delta m_{s}t) \right]
\]

- blue line: fit
- dashed red: signal
- dark pink: combinatorial
- violet: \( B_s \rightarrow D_s \pi \)
- yellow: \( \Lambda _{b} \rightarrow \Lambda _{c} K \)
$\gamma$ from $B_s \rightarrow D_s K$, 3 ifb, time dependent (cont.)

red curve: time acceptance obtained from $B_s \rightarrow D_s \pi$ DATA and corrected for the $B_s \rightarrow D_s K$ to $B_s \rightarrow D_s \pi$ MC time acceptance ratio

black curve: fit to the decay time distribution

folded asymmetry plots for $D_s^+ K^-$ and $D_s^- K^+$

red curve: fit result

CP violation: different phase for $\tau = 0$ ps

strictly speaking probing $\gamma - 2 \beta_s$, using in addition $\phi_s = -2 \beta_s$ and $\phi_s$ from $B_s \rightarrow J/\psi \ K K / \pi \pi$

$$\gamma = (127^{+17}_{-22})^\circ$$

$$\delta = (358^{+15}_{-16})^\circ \ \ \ \ r_{D_s K} = 0.37^{+0.10}_{-0.09}$$
Matter antimatter differences in beauty baryon decays, 3 ifb

First observation of this decay mode

LHCb

**Full fit**
- $\Lambda_b^0 \rightarrow p\pi^-\pi^+\pi^-$
- Part-rec. bkg.
- Comb. bkg.
- $B^0 \rightarrow K^+\pi^-\pi^-\pi^+
- \Lambda_b^0 \rightarrow pK^-\pi^+\pi^-

Events (9 MeV/c^2)

yield: 6646 ± 105

First observation of this decay mode

LHCb

**Full fit**
- $\Lambda_b^0 \rightarrow p\pi^-K^+K^-$
- Part-rec. bkg.
- Comb. bkg.
- $B^0 \rightarrow K^-K^+K^+\pi$
- $B_s^0 \rightarrow K^-K^+\pi^-\pi^+$
- $\Lambda_b^0 \rightarrow pK^-K^-$
- $\Lambda_b^0 \rightarrow pK^-\pi^+\pi^-

Events (9 MeV/c^2)

yield: 1030 ± 56

CKM expansion in $\lambda \approx 0.22$ up to $\mathcal{O}(\lambda^3)$
Matter antimatter differences in beauty baryon decays (cont.)

\[ C_{\tilde{T}} = \vec{p}_{p} \cdot (\vec{p}_{h_{1}}^{-} \times \vec{p}_{h_{2}}^{+}) \]
\[ \overline{C}_{\tilde{T}} = \vec{p}_{p} \cdot (\vec{p}_{h_{1}}^{+} \times \vec{p}_{h_{2}}^{-}) \]

\[ A_{\tilde{T}}(C_{\tilde{T}}) = \frac{N(C_{\tilde{T}} > 0) - N(C_{\tilde{T}} < 0)}{N(C_{\tilde{T}} > 0) + N(C_{\tilde{T}} < 0)} \]
\[ \overline{A}_{\tilde{T}}(\overline{C}_{\tilde{T}}) = \frac{\overline{N}(\overline{C}_{\tilde{T}} > 0) - \overline{N}(\overline{C}_{\tilde{T}} < 0)}{\overline{N}(\overline{C}_{\tilde{T}} > 0) + \overline{N}(\overline{C}_{\tilde{T}} < 0)} \]

\[ a^{\tilde{T}_{\text{odd}}} = \frac{1}{2} (A_{\tilde{T}} + \overline{A}_{\tilde{T}}) \quad a^{\tilde{T}_{\text{CP}}} = \frac{1}{2} (A_{\tilde{T}} - \overline{A}_{\tilde{T}}) \]

searches for localized P or CP violation
1. \( p \pi K K \): no significant P or CP violation (not shown)
2. \( p \pi \pi \pi \):
   - no significant P violation (white boxes)
   - CP violation at the 3.3 \( \sigma \) level (black dots)

- first evidence of CP violation in the baryon sector
- indicates an asymmetry between baryonic matter and antimatter
## Summary and future prospects

current LHCb sensitivities and expected ones in 2018 and beyond

<table>
<thead>
<tr>
<th>Observable</th>
<th>LHCb Run I</th>
<th>LHCb Run II</th>
<th>Upgrade (50 fb⁻¹)</th>
<th>Theory uncertainty</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \gamma(B \to D^{(<em>)} K^{(</em>)}) )</td>
<td>( \sim 7° )</td>
<td>4°</td>
<td>0.9°</td>
<td>negligible</td>
</tr>
<tr>
<td>( \gamma(B_s^0 \to D_s K) )</td>
<td>( \sim 20° )</td>
<td>11°</td>
<td>2°</td>
<td>negligible</td>
</tr>
<tr>
<td>( \beta(B^0 \to J/\psi K_S) )</td>
<td>1.16°</td>
<td>0.6°</td>
<td>0.2°</td>
<td>negligible</td>
</tr>
<tr>
<td>( 2\beta_s(B_s^0 \to J/\psi \phi) )</td>
<td>0.049</td>
<td>0.025</td>
<td>0.008</td>
<td>( \sim 0.003 )</td>
</tr>
</tbody>
</table>

- **2011-2012**
  - 3 ifb
  - 3.5 / 4.0 TeV

- **2015-2018**
  - + 5 ifb
  - 6.5 TeV
  - (factor 2 increase of \( |\sigma| \) for beauty)

\[ \gamma[\text{CKMfitter, ICHEP16}] = 65.33 \pm [0.96 -2.54] \]

- expect to be close when integrating up to 2018 data
- at the same level with the upgrade

+ start to see CPV also for baryons

+ if you add also Belle 2 in this game I really feel we have exciting times ahead of us
Backup
LHCb $\gamma$ combination: auxiliary inputs

<table>
<thead>
<tr>
<th>Decay</th>
<th>Parameters</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D^0-\bar{D}^0$-mixing</td>
<td>$x_D, y_D$</td>
<td>HFAG</td>
</tr>
<tr>
<td>$D \rightarrow K^+\pi^-$</td>
<td>$r_D^{K\pi}, \delta_D^{K\pi}$</td>
<td>HFAG</td>
</tr>
<tr>
<td>$D \rightarrow h^+h^-$</td>
<td>$A_{KK}^{dir}, A_{\pi\pi}^{dir}$</td>
<td>HFAG</td>
</tr>
<tr>
<td>$D \rightarrow K^+\pi^+\pi^+\pi^-$</td>
<td>$\delta_D^{K^3\pi}, \kappa_D^{K^3\pi}, r_D^{K^3\pi}$</td>
<td>CLEO+LHCb</td>
</tr>
<tr>
<td>$D \rightarrow \pi^+\pi^-\pi^+\pi^-$</td>
<td>$F_{\pi\pi\pi\pi}$</td>
<td>CLEO</td>
</tr>
<tr>
<td>$D \rightarrow K^\pm\pi^0$</td>
<td>$\delta_D^{K^2\pi}, \kappa_D^{K^2\pi}, r_D^{K^2\pi}$</td>
<td>CLEO+LHCb</td>
</tr>
<tr>
<td>$D \rightarrow h^+h^-\pi^0$</td>
<td>$F_{\pi\pi\pi^0}, F_{KK\pi^0}$</td>
<td>CLEO</td>
</tr>
<tr>
<td>$D \rightarrow K_S^0K^-\pi^+$</td>
<td>$\delta_D^{K_S\pi}, \kappa_D^{K_S\pi}, r_D^{K_S\pi}$</td>
<td>CLEO</td>
</tr>
<tr>
<td>$D \rightarrow K_S^0K^-\pi^+$</td>
<td>$\kappa_D^{K_S\pi}$</td>
<td>LHCb</td>
</tr>
<tr>
<td>$B^0 \rightarrow DK^{*0}$</td>
<td>$\kappa_B^{DK^{*0}}, \bar{R}_B^{DK^{*0}}, \Delta\delta_B^{DK^{*0}}$</td>
<td>LHCb</td>
</tr>
<tr>
<td>$B_s^0 \rightarrow D_s^{+}K^\pm$</td>
<td>$\phi_s$</td>
<td>LHCb</td>
</tr>
</tbody>
</table>

- taken from HFAG or other experiments
- more and more often taken from LHCb itself
LHCb $\gamma$ combination: D $h$ vs D $K$ combination results (Bayesian analysis)

$r_B^{D\pi}$ “small” so no shrinkage
Table 1: **Definition of binning scheme A for the decay mode** $A_b^0 \rightarrow p\pi^-\pi^+\pi^-$. Binning scheme A is defined to exploit interference patterns arising from the resonant structure of the decay. Bins 1-4 focus on the region dominated by the $\Delta(1232)^{++} \rightarrow p\pi^+$ resonance. The other eight bins are defined to study regions where $p\pi^-$ resonances are present (5-8) on either side of the $\rho(770)^0 \rightarrow \pi^+\pi^-$ resonances (5-12). Further splitting for $|\Phi|$ lower or greater than $\pi/2$ is done to reduce potential dilution of asymmetries, as suggested in Ref. [19]. Masses are in units of GeV/c^2.

| Phase space bin | $m(p\pi^+)$ | $m(p\pi^-_{\text{slow}})$ | $m(\pi^+\pi^-_{\text{slow}})$, $m(\pi^+\pi^-_{\text{fast}})$ | $|\Phi|$ |
|-----------------|--------------|---------------------------|-------------------------------------------------|---------|
| 1               | (1.07, 1.23) | (0, 1.23)                 |                                                | (0, $\frac{\pi}{2}$) |
| 2               | (1.07, 1.23) | (1.07, 2.00)              | $m(\pi^+\pi^-_{\text{slow}}) < 0.78$ or $m(\pi^+\pi^-_{\text{fast}}) < 0.78$ | (0, $\frac{\pi}{2}$) |
| 3               | (1.23, 1.35) | (0, 1.23)                 |                                                | (0, $\frac{\pi}{2}$) |
| 4               | (1.23, 1.35) | (1.23, 2.00)              |                                                | (0, $\frac{\pi}{2}$) |
| 5               | (1.35, 5.34) | (1.35, 5.34)              | $m(\pi^+\pi^-_{\text{slow}}) < 0.78$ or $m(\pi^+\pi^-_{\text{fast}}) < 0.78$ | (0, $\frac{\pi}{2}$) |
| 6               | (1.35, 5.34) | (1.07, 2.00)              | $m(\pi^+\pi^-_{\text{slow}}) < 0.78$ or $m(\pi^+\pi^-_{\text{fast}}) < 0.78$ | (0, $\frac{\pi}{2}$) |
| 7               | (1.35, 5.34) | (1.07, 2.00)              | $m(\pi^+\pi^-_{\text{slow}}) > 0.78$ and $m(\pi^+\pi^-_{\text{fast}}) > 0.78$ | (0, $\frac{\pi}{2}$) |
| 8               | (1.35, 5.34) | (1.07, 2.00)              | $m(\pi^+\pi^-_{\text{slow}}) > 0.78$ and $m(\pi^+\pi^-_{\text{fast}}) > 0.78$ | (0, $\frac{\pi}{2}$) |
| 9               | (1.35, 5.34) | (2.00, 4.00)              | $m(\pi^+\pi^-_{\text{slow}}) < 0.78$ or $m(\pi^+\pi^-_{\text{fast}}) < 0.78$ | (0, $\frac{\pi}{2}$) |
| 10              | (1.35, 5.34) | (2.00, 4.00)              | $m(\pi^+\pi^-_{\text{slow}}) < 0.78$ or $m(\pi^+\pi^-_{\text{fast}}) < 0.78$ | (0, $\frac{\pi}{2}$) |
| 11              | (1.35, 5.34) | (2.00, 4.00)              | $m(\pi^+\pi^-_{\text{slow}}) > 0.78$ and $m(\pi^+\pi^-_{\text{fast}}) > 0.78$ | (0, $\frac{\pi}{2}$) |
| 12              | (1.35, 5.34) | (2.00, 4.00)              | $m(\pi^+\pi^-_{\text{slow}}) > 0.78$ and $m(\pi^+\pi^-_{\text{fast}}) > 0.78$ | (0, $\frac{\pi}{2}$) |
CPV in decays (A)
Need CP invariant (strong) phase $\delta$ and CPV (weak) phase $\phi$.

$A(P \rightarrow f) = a_1 e^{i\delta_1} e^{i\phi_1} + a_2 e^{i\delta_2} e^{i\phi_2}$

$A(\bar{P} \rightarrow \bar{f}) = a_1 e^{i\delta_1} e^{-i\phi_1} + a_2 e^{i\delta_2} e^{-i\phi_2}$

$\rightarrow |A|^2 \propto \sin(\delta_1 - \delta_2) \sin(\phi_1 - \phi_2)$

e.g.: $B^+ \rightarrow J/\psi K^+$ vs $B^- \rightarrow J/\psi K^-$. 

CPV in mixing (B)
Mass eigenstates vs flavour eigenstates:
$|P_{L,H}\rangle = p |P^0\rangle \pm q |\bar{P}^0\rangle$

$\rightarrow$ CPV if $|q/p| \neq 1$

e.g.: Lepton charge asymmetry in $B_s^0 \rightarrow D^+_s \mu^+\nu\mu X$ decays.

CPV in interference between decay and mixing (C)
Neutral meson decaying into non-flavour specific states.

$$\frac{A(\bar{P} \rightarrow f) - A(P \rightarrow f)}{A(\bar{P} \rightarrow f) + A(P \rightarrow f)} = \frac{C_f \cos(\Delta m t) - S_f \sin(\Delta m t)}{\cosh\left(\frac{\Delta m t}{2}\right) + D_f \sinh\left(\frac{\Delta m t}{2}\right)}$$

$S_f$ and $D_f$ coefficients: interference between mixing and decay. $C_f$ coefficient: direct CPV.
**Efficiency and mistag:**

\[
\epsilon_{tag} = \frac{N_{tag}}{N_{tag} + N_{untag}}, \quad \omega = \frac{N_{wrong}}{N_{right} + N_{wrong}}
\]

Tagging power: \( \epsilon_{eff} = \epsilon_{tag} D^2 = \epsilon_{tag} \langle (1 - 2\omega)^2 \rangle \)

Statistical uncertainty: \( \sigma_{stat} \propto 1/\sqrt{\epsilon_{eff} N} \)

Mistag: dilution of time-dependent asymmetries.

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