NNLO corrections for LHC processes

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To fully profit from the remarkable achievements of the experimental program at the LHC, very precise theoretical predictions for signal and background processes are required. In this contribution, I will review some of the recent progress in fully exclusive next-to-next-to-leading-order (NNLO) QCD computations. As an example of the phenomenological relevance of these results, I will present LHC predictions for \( t \)-channel single-top production and Higgs boson production in association with one hard jet.

1 NNLO computations: status and prospects

The impressive achievements of the LHC program at CERN, culminated with the discovery of the Higgs boson\(^1,2\), allow us to scrutinize the Standard Model (SM) to a level of accuracy never seen before. Although interesting \textit{per se}, the ultimate goal of such scrutiny is to find deviations from expected predictions which point to new physics beyond the standard model. So far, no significant deviation has been observed, indicating that new physics, if present, may be hiding in subtle effects. Disentangling such small deviations requires very good theoretical control on standard model predictions.

In general, computing perturbative corrections to a given process requires two steps. First, all relevant tree and loop amplitudes must be known. Second, a consistent framework to combine real and virtual corrections into finite predictions for physical observables is needed. In order for theoretical predictions to describe the complicate experimental environments in a reliable way, the framework must be able to cope with arbitrary (infra-red safe) fiducial cuts on the final state and be valid for any (infra-red safe) differential distribution. In other words, the framework should allow for fully exclusive predictions. This requirement proved to be very challenging to deal with and as a result for a a long time fully exclusive NNLO predictions were only available for processes with a trivial color flow, like Higgs\(^3,4\) or Drell-Yan production\(^5,6\). In the recent years however, thanks to a big effort in the theory community, significant progress has been made\(^7\) and it is now possible, at least in principle, to deal with processes of arbitrary complexity. In practice, predictions for genuine \( 2 \rightarrow 2 \) processes are now doable, and indeed in the recent past NNLO computations for top-quark\(^8\), dijet\(^9\), single-top\(^10\) and Higgs plus Jet\(^11\) started to appear.

To go beyond \( 2 \rightarrow 2 \) processes, several issues must be addressed. First, two loop amplitudes must be computed. So far the state-of-the art is \( 2 \rightarrow 2 \) amplitudes with up to one internal/external mass scale (like e.g. amplitudes for \( pp \rightarrow tt \) production\(^12\)) or \( 2 \rightarrow 2 \) amplitudes with up to two \textit{external} mass scales (like e.g. amplitudes for \( pp \rightarrow WZ \) production\(^13\)). Going beyond that will require substantial improvements of our current technology. Second, to compute NNLO correction to \( pp \rightarrow X \) one needs one-loop amplitudes for \( pp \rightarrow X \) + parton, and they have to be stable enough in the unresolved regime where the extra parton is very soft / collinear. In the recent past, we witnessed a lot of progress in automatic one-loop computations\(^14\). It is an interesting question whether the results obtained from these automatic tools are stable and
efficient enough to cope with the demands of NNLO computations. Third, it is not clear if existing implementations of subtraction frameworks will *in practice* work with high multiplicity final states. Although all major conceptual issues have been now more or less solved, yet all existing computations require significant amount of computing resources to deal with \( 2 \rightarrow 2 \) kinematics. It is hence far from obvious that the same frameworks will successfully handle genuine \( 2 \rightarrow 3 \) and more complicated processes.

Fortunately, a lot of processes for which NNLO accuracy is desirable fall in the \( 2 \rightarrow 2 \) category (see e.g. the Les Houches wish-list\(^\text{15}\)). In the following, I will briefly illustrate the recent progress in NNLO computations and its phenomenological implications by discussing two of such processes, \( t \)-channel single top production and Higgs boson production in association with one hard jet.

### 2 NNLO predictions: phenomenological examples

#### \( t \)-channel single-top

The relevance of the top physics program at the LHC has already been discussed extensively in these proceedings. The main difference w.r.t. the Tevatron program is that at the LHC the signal yield for tops as the LHC is relatively large, thus allowing for very precise studies of this particle and its properties. At hadron colliders, tops can be produced both through strong interactions, which lead to \( t\bar{t} \) pair production, and through weak interactions, which lead to single-top production. The cross-section for \( t\bar{t} \) is larger, yet the yield for single-top is still sizable at the LHC. Moreover, measuring the single-top cross-section gives direct access to the \( V'tb \) CKM matrix element.

For single-top production, most of the cross-section comes from the so-called \( t \)-channel topology. NLO corrections to the total \( t \)-channel cross-section are known to be small, at the percent level. Naively, this would suggest per mill level NNLO corrections, well beyond the accuracy goal of actual measurements. Unfortunately, there are many indications that the smallness of NLO corrections to the total cross-section is accidental. Moreover, NLO corrections to more differential quantities can be as large as 20%. In order to achieve a percent level perturbative control, higher order predictions are thus needed.

NNLO corrections to \( t \)-channel single top, for stable tops, were computed in\(^\text{10}\) using the five flavor number scheme in the factorized approximation. Using the MSTW2008 parton set, the NNLO total cross section for \( m_t = 173.2 \) GeV is \( \sigma_{\text{NNLO}} = 54.2^{+0.5}_{-0.2} \) pb, to be compared with the NLO prediction \( \sigma_{\text{NLO}} = 55.1^{+0.5}_{-0.9} \) pb and the LO one \( \sigma_{\text{LO}} = 53.8^{+3.6}_{-4.3} \) pb. Errors refer to scale variations of a factor of two around \( \mu = m_t \). As anticipated, NNLO corrections are as large as the NLO ones. All these predictions are in excellent agreement with the current experimental measurement presented in these proceedings \( \sigma_t = 53.85 \pm 9\% \). At NNLO, the residual perturbative uncertainty due to scale variation is greatly diminished compared to previous orders, thus making it basically irrelevant in the full budget of theoretical error, which is dominated by uncertainty on parameters like parton distribution functions or the \( b \)-quark mass. A thorough estimate of such uncertainties is currently under way. As we briefly mentioned before, NLO corrections can be significantly larger for more exclusive quantities. It is thus interesting to investigate whether NNLO corrections are enough to stabilize the perturbative expansion in these cases or not. An example is shown in Fig. 1, where the top \( p_\perp \) cumulative cross-section is shown. Despite NLO corrections at high \( p_\perp \) are very large and outside the LO uncertainty band, NNLO corrections are very stable throughout the full spectrum.

So far, NNLO predictions are only known for stable top. However, NNLO corrections to top decay are known\(^\text{16}\). Combining these results with the one just described will allow for a complete \( pp \rightarrow t + X \rightarrow Wb + X \) NNLO computation, in the narrow width approximation. The result of such combination would be very interesting, as it will allow for a proper study of single-top production at the LHC in the actual measured fiducial region.
Figure 1 – Top $p_T$ cumulative cross-section at LO (red), NLO (green) and NNLO (blue). Colored bands represent scale variation uncertainty, in the range $\mu_r = \mu_f \in [m_H/2, 2m_H]$. See text for details.

2.1 Higgs boson production in association with one jet

Studying the properties of the recently found Higgs boson is clearly one of the main goals of the LHC experimental program. From a theoretical point of view, Higgs production at the LHC is quite hard to properly describe because the perturbative series has a poor convergence. For example, the NLO corrections for inclusive Higgs production in gluon fusion are as large as the LO rate. To obtain reliable predictions, computation of enough terms in the perturbative expansion is thus mandatory. For fully inclusive Higgs production, corrections up to $N^3\text{LO}$ were recently computed\(^\text{17}\). Unfortunately, in most cases the knowledge of the total cross-section alone is not enough and more differential information is required. Predictions in these situations are much more involved, and as a consequence the theoretical control is much worse. A particularly interesting class of processes in this family are Higgs boson produced in association with one or more hard jets. On one hand a proper modeling of such processes is very important for experimental analysis in channels like the $H \rightarrow WW$ or $H \rightarrow \tau\tau$ where a jet-bin categorization is fundamental for increasing the sensitivity. Furthermore, Higgs boson in association with one hard jet gives direct access to the Higgs transverse momentum spectrum.

In this contribution I will present results of a recently completed computation of Higgs boson plus jet which includes all the ingredients relevant for reliable phenomenology at the LHC. More specifically, NNLO corrections are computed for the $gg$ and $qg$ channels, which are expected to account for about 99% of the total cross-section. The computation is carried out in the Higgs effective field theory where the top quark is integrated out and a point-like interaction $ggH$ is considered. Such an approximation is reliable at the percent level up to transverse momenta of the order of $p_T \sim 150$ GeV\(^\text{18}\). Results are obtained using the NNPDF21LO, NNPDF23NLO and NNPDF23NNLO parton sets and values of $\alpha_s$. We use the central scale $\mu_r = \mu_f = m_H = 125$ GeV. Jets are reconstructed using the anti-$k_T$ algorithm with $p_T > 30$ GeV and $R = 0.5$. We find the total cross-section to be $\sigma_{\text{LO}} = 3.9^{+1.7}_{-1.1}$ pb, $\sigma_{\text{NLO}} = 5.6^{+1.3}_{-1.1}$ pb and $\sigma_{\text{NNLO}} = 6.7^{+0.5}_{-0.6}$ pb at LO, NLO and NNLO respectively where the upper(lower) results are for the scale choices $\mu_r = \mu_f = m_H/2$ (2$m_H$). NNLO corrections are sizable, around 20% on top of NLO, but smaller than the NLO ones indicating a convergence of the perturbative series. The residual uncertainty due to scale variation is significantly reduced to the $\sim 10\%$ level.

In Fig.2 the cumulative one-jet cross-section (left pane) and the Higgs $p_T$ spectrum in the 1-jet bin (right pane) are shown. These plots show a clear improvement of scale uncertainties also for more differential observables. The convergence of the perturbative series is reasonable:
while the LO and the NLO bands only partially overlap, the NLO/NNLO overlap is substantial. Interestingly enough, the NNLO to NLO ratio is not constant: the bulk of the corrections lies in the low $p_{T}$ region, while the NLO and NNLO curves tend to converge to the same value at high $p_{T}$.

### 3 Conclusion

The LHC program requires very accurate theoretical modeling of complex experimental environments. Such predictions are mandatory to scrutinize the structure of the Standard Model and hopefully find deviations pointing to new physics. An essential ingredient is refined higher order predictions for fully exclusive reactions. Thanks to a big effort in the theoretical community, first NNLO predictions for genuine $2 \rightarrow 2$ colorful processes recently started to appear. In this contribution, I illustrated the recent progress by showing accurate phenomenological predictions for $t$-channel single top and Higgs boson production in association with one jet at the LHC. These are only examples chosen among a rapidly growing number of processes computed to NNLO accuracy, which are pushing farther the boundaries for phenomenological studies at hadron colliders.

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### References