Abstract

Collision scheme with a large crossing angle is being very popular in design of future colliders in combination with the crab waist scheme. We discuss that a strong wake field with correlation between turns is induced by the beam-beam interaction. Recently strong-strong beam-beam simulations have shown a strong coherent instability in head-tail mode in collision with a large crossing angle. The wake field explains the mechanism of the coherent head-tail instability. Study of this instability is essential for collider designs based on a large crossing angle and crab waist scheme.

INTRODUCTION

A coherent head-tail instability has been seen in collision with a large crossing angle in strong-strong beam-beam simulation. We try to explain this beam-beam instability using wake field induced by the beam-beam interaction. When a positron bunch with a dipole moment $\mu^+(z')$ collide with an electron bunch, parts of the electron bunch ($z_-$) experiences a momentum kick as a function of $z_-$ and $\rho^+(z')$. In collision with a large crossing angle, the kick depends on $z$ and $z'$. We present two kinds of wake field in the beam-beam collision.

One is single beam approach, and second is two beams approach. In the single beam approach, a beam interacting with another beam regarded with a particle cloud. A wake field is obtained by the similar way with the electron cloud. The beam particle cloud interacts with the beam in many turns. The wake field contains a turn-by-turn correlation.

Figure 1: Sketch for evaluation of wake field induced by beam-beam interaction in single beam approach.

In second approach, a wake field describes correlation between two beams. The wake field is evaluated by calculating kick of positron/electron beam induced by a delta function like dipole moment of electron/positron beam. Figures 1 and 2 shows schematic views of the wake field models.

Figure 2: Sketch for evaluation of wake field induced by beam-beam interaction in two beam approach.

WAKE FIELD AND HEAD-TAIL INSTABILITY IN BEAM-BEAM COLLISION WITH A LARGE CROSSING ANGLE

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about in 5 turns. For \( t = 0 \), \( \Delta p_x(0, 0) = -5.93 \times 10^{-6} \) and \( \Delta p_x(z < 0, 0) \sim \pm 7 \times 10^{-9} \) correspond to the tune shift and short range wake in a bunch, respectively. The short range wake is 2 order smaller than that for \( t \geq 1 \). The momentum kick has a peak near \( z = 0 \) and oscillate turn by turn. Picture (b) depicts the peak momentum kick as function of turn. The frequency and quality factor are estimated to be \( \nu = 0.61 \) and \( Q = 5.7 \). The frequency is reasonable with considering the horizontal tune (\( \nu_x = 0.54 \)), the synchrotron tune (\( \nu_x = 0.018 \)) and beam-beam tune shift (\( \xi_x = 0.024 \)). 

Linearity and translational invariance of the wake field is checked as shown in Figure 4. Wake field for the displacement 1, 2 and 3\( \sigma_x \) is plotted in Picture (a) Linearity for the displacement is satisfied well, though it is not perfect. Translational invariance, which guarantees the function form \( W(z - z') \), is also satisfied well: that is, the wake field shift for changing \( z' = 0, \pm 2.4, \pm 4.8 \) mm.

Figure 3: (a) Momentum kick of micro-bunches at \((z, t)\) for displacement (\( \Delta x = \sigma_x = 10^{-5} \) m) of a micro-bunch at \( z' = 0, t = 0 \), where \( n_{mb} = 100 \). Wake field is given by \( W_x(z, t) [m^{-1}] = -10^6 \Delta p_x(z, t) \). (b) peak momentum kick as function of turn. \((t)\).

Figure 4: (a) Wake field for the displacement 1, 2 and 3\( \sigma_x \). (b) Wake field \( W(z - z') \) for \( z' = 0, \pm 2.4, \pm 4.8 \) mm.

Figure 5: (a) evolution of \( \langle x^2 \rangle \) for various \( \beta_x^* \) after 1000 turn (\( \beta_x^* = 0.5 \) m), and (b) Particle distribution in \( z - \delta p/p - x \) phase space.

Simulation for beam instability is performed using the wake field. Particles (~ 10k) are generated with Gaussian distribution for the design emittance and beta in the 6 dimensional phase space. The kick induced by the wake field is calculated turn by turn using Eq.(1), where the beam dipole moments \( \rho_x(z', n') \) are recorded for the past several turns. After the kick (effective collision), coordinate of particles are multiplied by revolution matrix. Figure 5 (a) shows evolution of \( \langle x^2 \rangle \) for various \( \beta_x^* \). Exponential growth in \( \langle x^2 \rangle \) and \( \langle z^2 \rangle \) is seen. Note that this wake field model is linear for betatron amplitude. Actually since the beam-beam force is nonlinear and is saturates at several \( \sigma_x \).

Figure 5(b) shows particle distribution in \( z - \delta p/p - x \) phase space. Complex head-tail motion is seen clearly. The amplitude is huge, since linear wake model is used.

\( \Delta p_x(\Delta x, 0) = -\int_0^t W_x(z, \Delta x) \rho_x(z, \Delta x) dz'. \) \( (3) \)

We consider that a part of positron bunch \( \rho_0(z, \Delta x) \delta(z, \Delta x) \) deviates \( \Delta x \).

\( \Delta p_x^*(\Delta x) = -W_x(z, \Delta x) \rho_0(z, \Delta x) \Delta x. \) \( (4) \)
Effect of the deviation in the momentum kick is given by the beam-beam force,
\[ \Delta p_x^{(-)} = \frac{N_x \rho_0 (z_+) r_e}{\gamma} (F(x_+ - x_+ - \Delta x) - F(x_- - x_+)). \]  
(5)

For a transverse Gaussian beam, \( F(x, y) \) is represented by complex error function as follows,
\[ F(x, y) = F_y + i F_x = \frac{2\sqrt{\pi}}{\Sigma} \left[ w \left( \frac{x + iy}{\Sigma} \right) \right] \]
\[- \exp \left( -\frac{x^2}{2\sigma_x^2} - \frac{y^2}{2\sigma_y^2} \right) \]
\[ w \left( \sigma_x y / \sigma_y + i \sigma_x y / \sigma_y \right) \]
\[ \exp \left( -\frac{y^2}{2\sigma_y^2} \right) \]
where \( \Sigma = \sqrt{2(\sigma_x^2 - \sigma_y^2)} \). The beam sizes are convoluted ones of two beams \( \sigma_x^{(x)} = \sqrt{\sigma_x^{(x)+} \sigma_x^{(x)+}}, \)
\( \sigma_x^{(x)+} \approx \sigma_x^{(x)} \theta_c \)
for collision with the half crossing angle \( \theta_c \).
\[ F_x (\langle x_+ - x_+ \rangle \theta_c - \Delta x, 0) - F_x (\langle x_+ - x_+ \rangle \theta_c, 0) \]
\[ = -\frac{\partial F_x (x, 0)}{\partial x} \bigg|_{x = (z_+ - z_+)} \Delta x \]  
(7)

The wake force is expressed by derivative of the beam-beam force,
\[ W_x (z_+ - z_+) = \frac{N_x r_e}{\gamma} \frac{\partial F_x (x, 0)}{\partial x} \bigg|_{x = (z_+ - z_+)} \theta_c \]
(8)

For \( z_+ = z_- \), \( W(z) \) is the minimum value,
\[ W_x (0) = \frac{N_x r_e}{\gamma} \frac{2}{\sigma_x (\sigma_x + \sigma_y)} \]
(9)

\[ W(z) = 0 \] at \( z = \pm 1.3 \theta_c / \sigma_x \), and \( W \) is the maximum \( \approx 0.28 |W_x (0)| \) at \( z = \pm 2.2 \sigma_x / \theta_c \). Figure 6 shows the wake field. The wake field is also calculated by a numerical method. The wake linearly depends on \( \Delta x \) around \( \Delta x \leq 3 \sigma_{x,+} \).

Particle tracking simulation using the wake in Fig.6 was carried out. Figure 7 shows evolution of the horizontal bunch size and \( \langle xz \rangle \) correlation. The growth of the beam size is very fast (\( \sim 20 \) turns) and the head-tail phase of two bunches was the same. This behavior is consistent with a strong simulation.

Figure 8 shows distribution of electron/positron bunches after 230 revolutions. The distributions of two bunches are mostly identical.

CONCLUSION

Wake fields induced by beam-beam collision with a large crossing angle were evaluated. A head-tail instability is caused by the wake fields. The instability explains the strong simulation results.

ACKNOWLEDGEMENT

The authors thank fruitful discussions with Dr. D. Shatilov.

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