QUALITY OF THE PSB BENDING MAGNET FIELD.

ARE STRAIGHT-COIL MEASUREMENTS SUFFICIENT?

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1. **INTRODUCTION**

During the last year many kinds of measurements have been performed on the prototype bending magnet as had been proposed\(^1\),\(^2\). First measurements were made on the original prototype\(^3\), which was later modified in view of a better radial shimming. The whole series of measurements has been collected in a file by K. Lohmann\(^4\).

For the series bending magnets a programme of measurements has been established\(^5\) in which the main emphasis is put on integrated field measurements (with long coils) and not on a three-dimensional field map (Hall probe). We explain here the reasons why these measurements integrated on \(l\) are sufficient to test the quality of the PSB bending magnet field.

2. **BEAM TRAJECTORIES**

With the long coil (2.7 m) one measures \(\int_B B \, d\ell\) on a straight trajectory from \(-l_3\) to \(+l_3\). With the intermediate coil (1.3 m) the mean value of the uniform field inside the magnet, \(B_0\), can be evaluated. Therefore the definition of the equivalent length is

\[
\ell_{equ} = \frac{\int_B B \, d\ell}{B_0} .
\]  

(1)

Corresponding to the equivalent rectangular field region (see Fig. 1a) a curved magnetic length can be defined

\[
\ell_m = k \ell_{equ} , \quad \text{with} \quad k = \frac{\pi/32}{\sin (\pi/32)}
\]  

(2)

and the curvature radius follows

\[
\rho = \frac{16}{\pi} \frac{\ell_m}{\ell_{equ}} .
\]  

(3)

In the rectangular model the integral \(\int B \, d\ell\) along the circular trajectory is

\[
\int_C B \, d\ell = B_0 \ell_m = k \int_{\ell_{equ}} B \, d\ell .
\]  

(4)
In the fringing field region from $l_1$ to $l_3$ the actual proton trajectory is different from the arc and straight line corresponding to the rectangular model. But the deflection angle after $l_3$ is exactly $\pi/32$ for both lines which are separated by a distance $\delta x$.

3. **STEP INTEGRATION ON THE THREE-DIMENSIONAL FIELD MAP**

The three-dimensional field map measured\(^1\) on the prototype magnet was used to get a consistent comparison of the trajectories.

One has successively for the field $B_z(x,z)$

$$\int B \, dl = \int B_z(0.35) \, dl = 0.94993 \, \text{Tm}$$

$$B_0 = B_z(0.35) \text{ (over 80 cm)} = 0.591322$$

$$l_{equ} = 1.60645 \, \text{m}$$

$$l_{m} = 1.60904 \, \text{m}$$

$$\rho = 8.19474 \, \text{m}$$

$$B_0 l_m = 0.951458 \, \text{Tm}.$$  

The integration along the actual trajectory was performed on a circle with radius $\rho$ starting with $x_0 = z_0$ for $0 \leq \lambda \leq l_1$, and by Runge-Kutta integration (2 cm steps) for $l_1 \leq \lambda \leq l_3$, where the particle trajectory $x(\lambda)$ satisfies the equation\(^2\)

$$\frac{d^2 x}{d\lambda^2} = -\frac{B(x,\lambda)}{B_0 \rho} \left[ 1 + \left( \frac{dx}{d\lambda} \right)^2 \right]^{\frac{3}{2}}. \tag{5}$$

Results are as follows:

a) $$\int_{\text{traj}} B \, dl = 0.951530 \, \text{Tm},$$

which is different from $B_0 l_m$ by less than $10^{-4}$. We then propose to consider Eqs. (1), (2) and (3) as the final definition of those quantities.

\(^1\) This is just a consistent example of values and they do not supersede those in the parameter list.
In the future \( p \) will be regarded as fixed to the value given in the parameter list\(^7\) and \( \int B \, dl \) will be adjusted consistently for each magnet.

b) The distance between geometrical and actual trajectories is \( \delta x = 0.11 \) mm. Such a small value is of no significance and we shall in the future always use the geometrical trajectory made of arcs and straight lines.

c) Parallel trajectories were also computed with \( x_0 = \pm 4 \) cm around the central orbit. Their respective distances at \( s_3 \) only differ by 0.04 mm at maximum, so that any beam envelope can be referred to a unique central orbit defined under (b).

4. NON-LINEAR FIELDS

The test on field quality is described in an earlier paper\(^8\). \textit{A priori} it was not clear whether the effective non-linearities of the field were well represented by their integrals \( \int B \, dl \) (over 1.3 m and 2.7 m, respectively). An attempt was made here to see whether a different non-linear behaviour of the beam would appear when the field is integrated along curved trajectories (parallel to the central orbit).

The technique of representing the B.M. field errors by three thin non-linear lenses (one positioned in the centre, two at the edges) has been described in detail\(^8\). For convenience, let us just repeat the essential points: the short-coil measurement results

\[
\begin{align*}
\int_{-\ell_1}^{\ell_1} B(x,z) \, dl - 2B_0 \ell_1
\end{align*}
\]

are concentrated in the non-linear central lens. The field deviations measured with the long coil

\[
\begin{align*}
\int_{+\ell_3}^{-\ell_3} B(x,z) \, dl - B_0 \ell_3
\end{align*}
\]

must be equal to the sum of the field errors in all three thin lenses together. Therefore the strength of either non-linear lens on the edge is equal to \( \frac{1}{4}[ (7) - (6) ] \), namely
One may see from Fig. 1b that expression (8) represents the field error integral on a straight line between \( l_1 \) and \( l_3 \), whereas the particle follows a very different trajectory (dotted line). The straight line between \( l_1 \) and \( l_3 \) is considered as sort of "average" of the real trajectory.

For the three thin lenses approach as given by expressions (6), (7) and (8), we do not need the three-dimensional field map, but only long- and short-coil measurements. In order to judge whether this approach is sufficient for determining the field quality, i.e. the effect of the B.M. field errors on beam parameters, one has to compare the field error map given by expression (8) to the deflections the particles experience on the real trajectories between \( l_1 \) and \( l_3 \). This study is based on a three-dimensional fringing field map measured by K. Lohmann in June 1969. (This field map is only considered as a representative example, suitable for a consistent exercise.)

For \( l_1 \leq l \leq l_3 \), \( B(x, z, l) \) is given for the coordinate values

\[
\begin{align*}
x &= -70, -60, -50, -30, 0, 30, 50, 60, 70 \text{ mm.} \\
z &= 15, 35, 55 \text{ mm}
\end{align*}
\]

and for \( l \) in steps of 1 cm, for gap 3 at transfer energy. From the field map one may calculate the following two error maps:

a) **Non-linear field integrated on straight lines between \( l_1 \) and \( l_3 \)**

Compute

\[
\frac{1}{2} \left[ \int_{l_3}^{l_1} B(x, z, l) \, dl - \int_{l_3}^{l_1} B(x, z) \, dl - 2B_0(l_2 - l_1) \right].
\]

In order that the Hamiltonian in transverse phase space be conserved, expressions (6) and (7) are approximated by a multipole development in \( x \) and \( z \) (satisfying Laplace's equation). Building the difference between corresponding coefficients yields the multipole development for expression (8).
straight line in Fig. 1b

\[ \int_{\ell_1}^{\ell_3} B(-20,z,\ell) \, d\ell . \]

The straight line field error map is

\[ \delta_1(x,z) = \frac{\int_{\ell_1}^{\ell_3} B(x,z,\ell) \, d\ell - \int_{\ell_1}^{\ell_3} B(-20,z,\ell) \, d\ell}{\int_{\ell_1}^{\ell_3} B(-20,z,\ell) \, d\ell} , \]

which corresponds to the error field map derived from expression (8).

b) Non-linear field integrated on actual trajectories between \( \ell_1 \) and \( \ell_3 \)

When integrating Eq. (5) between \( \ell_1 \) and \( \ell_3 \) for various initial values \( x(\ell = 0) = x_0 \) (assuming a constant field \( B_0 \) for \( 0 \leq \ell \leq \ell_1 \)) we get the bending angle of the particle between \( \ell_1 \) and \( \ell_3 \) to be

\[ \Delta \theta(x_0,z) = \arctg \left( \frac{d x(\ell)}{d \ell} \bigg|_{\ell = \ell_1} \right) - \arctg \left( \frac{d x}{d \ell} \bigg|_{\ell = \ell_3} \right) , \]

which is of course proportional to

\[ \int_{\ell_1}^{\ell_3} B[\mathbf{x}(\ell),z,\ell] \, d\ell \]

with initial value \( x_0 \).

Taking as reference the bending angle on the central orbit defined by \( x_0 = +20 \) mm, \( \Delta \theta(20,z) \), we get the trajectory field error map to be

\[ \delta_2(x,z) = \frac{\Delta \theta(x_0 = x + 40,z) - \Delta \theta(20,z)}{\Delta \theta(20,z)} . \]

Now, if the straight-coil measurements were an exact representation of the deflection errors experienced by the protons on their trajectories, \( \delta_1(x,z) \) had to be equal to \( \delta_2(x,z) \). This turned out to be not true, so we have to continue our study in simulating the beam behaviour when it is influenced by \( \delta_1(x,z) \) or \( \delta_2(x,z) \).
5. ARE STRAIGHT-COIL MEASUREMENTS SUFFICIENT?

With the three-dimensional fringing field map under consideration and \( \lambda_1 = 0.67 \text{ m}, \lambda_3 = 1.47 \text{ m} \), it turns out that field error maps \( \delta_1(x, z) \) and \( \delta_2(x, z) \) differ slightly. To get a more precise answer, we simulate the behaviour of the proton beam for the field error maps \( \delta_1(x, z) \) and \( \delta_2(x, z) \), respectively.

Applying the methods discussed in Ref. 8, both \( \delta_1(x, z) \) and \( \delta_2(x, z) \) are approximated by multipole coefficients up to order 8. The results of this polynomial approximation are shown in Figs. 2a and b. The coordinate "EPS" represents the fringing field error expressed in terms of total bending power, i.e. for the straight-coil approach (Fig. 2a)

\[
\text{EPS1}(x, z) = \delta_1(x, z) \frac{\Delta \theta(20, z)}{\pi/32} \times 1000
\]  

(12a)

and for the trajectory approach (Fig. 2b)

\[
\text{EPS2}(x, z) = \delta_2(x, z) \frac{\Delta \theta(20, z)}{\pi/32} \times 1000
\]  

(12b)

The field errors evaluated from the three-dimensional field map with Eqs. (12a) and (12b) are indicated with -, 0, + for \( z = 15, 35, 55 \text{ mm} \), respectively. These values are the input data for the multipole approximation (up to the 20-pole), the results of which are also plotted in Figs. 2a and b. Closed orbit distortions (for simulating the real machine) are produced by six dipoles per period, powered at random.

Assuming that the field error maps \( \delta_1(x, z) \) and \( \delta_2(x, z) \) do not change significantly between injection and transfer, emittance increase and Q-shifts were calculated for \( \varepsilon_H = 130, \varepsilon_V = 40 \) (injection), \( \varepsilon_H = 28, \varepsilon_V = 8.5 \) (transfer) \( \pi \text{ mrad mm} \).

Results of the computer simulation for either field error map are summarized in Table 1. For \( Q_H = 4.70, Q_V = 4.809 \), the following average and r.m.s. values for emittance increase \( \Delta \varepsilon_H, \Delta \varepsilon_V \) and Q-shifts \( \Delta Q_H, \Delta Q_V \) were obtained after simulating 100 revolutions. (Closed orbit distortions \( |x_{\max}| = 11.2 \text{ mm}, |z_{\max}| = 7.7 \text{ mm} \).)
<table>
<thead>
<tr>
<th>Momentum</th>
<th>Non-linear field map taken</th>
<th>Emittance increase</th>
<th>Q-shift</th>
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<tbody>
<tr>
<td></td>
<td></td>
<td>$\Delta \varepsilon_H \ [\pi , \text{mrad mm}]$</td>
<td>$\Delta \varepsilon_V$</td>
</tr>
<tr>
<td>Injection</td>
<td>$\delta_1$ (straight)</td>
<td>23.79 4.23</td>
<td>19.39 1.83</td>
</tr>
<tr>
<td>$\varepsilon_H = 130$</td>
<td>$\varepsilon_V = 40$</td>
<td>$\delta_2$ (trajectory)</td>
<td>22.69 4.74</td>
</tr>
<tr>
<td>Transfer</td>
<td>$\delta_1$ (straight)</td>
<td>5.44 0.94</td>
<td>6.54 2.04</td>
</tr>
<tr>
<td>$\varepsilon_H = 28$</td>
<td>$\varepsilon_V = 8.5$</td>
<td>$\delta_2$ (trajectory)</td>
<td>4.56 1.10</td>
</tr>
</tbody>
</table>
Discussion of results:

a) Comparing $\Delta z_{\text{H}}$ and $\Delta z_{\text{V}}$, at either injection and transfer energy obtained with $\delta_1$ or $\delta_2$, we find that corresponding mean values differ by less than their r.m.s. values, that is to say that the differences cannot be regarded as significant.

b) The corresponding Q-shifts, however, differ by more than their r.m.s. values, but are less significant.

6. CONCLUSIONS

The numerical studies presented in this paper were based on three-dimensional fringing field maps measured on the Booster B.M. prototype by means of a Hall probe. We emphasize again that only the accuracy of the straight-coil measurement representation of the errors in bending power is under discussion, but not the quality of the prototype bending magnet field itself.

Two questions had to be answered by this study:

a) Is this circle-straight line approach a good representation of the real trajectory?

b) Are straight-coil measurements sufficient for describing the deflection errors experienced by the particles?

Answers:

a) Runge-Kutta integration of the trajectory Eq. (5) proved that the distance between idealized and real trajectory when leaving the B.M. fringing field region was $\delta x = 0.11 \text{ mm}$, a value of no significance.

b) We see from Table 1 that the computer simulation of the proton beam when passing through the non-linear lenses $\delta_1(x,z)$ and $\delta_2(x,z)$, respectively, did not result in significantly different values for the emittance increase. Hence, the use of long-coil and short-coil measurements for testing the field quality in the series bending magnet is adequate.
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5) C. Bovet, M. Giesch and K.D. Lohmann, Memorandum of 31.8.70.

6) R. Keyser, Particle trajectories in the CPS magnetic field, October 1963.

7) C. Bovet and K.H. Reich, SI/Note DL/70-9 Rev.

8) C. Bovet and K. Schindl, SI/Int. DL/70-4.
Fig. 2  a) Non-linear field integrated on straight lines [for definition, see Eqs. (10) and (12a)].

b) Non-linear field integrated on trajectories [for definition, see Eqs. (11) and (12b)].