PRACTICAL STABILISATION OF TRANSVERSE COLLECTIVE INSTABILITIES WITH SECOND ORDER CHROMATICITY IN THE LHC

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Abstract

The study reports on dedicated measurements made with a single nominal bunch in the LHC at 6.5 TeV. First, we show that a significant amount of second order chromaticity $Q''$ can be introduced in the machine in a well-controlled manner. Second, we demonstrate that the incoherent betatron motion spread from $Q''$ can provide beam stability through the Landau damping mechanism. This is a first step in the development of a $Q''$ knob to be potentially applied during regular physics operation in the LHC.

INTRODUCTION

A powerful stabilising mechanism against transverse collective instabilities is Landau damping. It is present when there is an incoherent spread in the betatron tunes of the particles in the beam [1, 2]. The spread is a result of non-linearities in the machine, often introduced by design through dedicated elements. In the Large Hadron Collider (LHC), magnetic octupoles are installed to produce a tune spread depending on the transverse action of the particles $\Delta Q_{x,y}(J_x, J_y)$. These Landau octupoles are extensively used for beam stabilisation during LHC operation [3, 4].

Future high energy machines will operate with beams of smaller physical transverse emittances making the Landau octupoles significantly less effective due to the reduced spread in $(J_x, J_y)$, in particular at higher energy (adiabatic damping). This may result in a loss of Landau damping of potentially performance-limiting instabilities. Alternative methods such as betatron detuning with longitudinal action $\Delta Q_{x,y}(J_z)$ are currently under development. Longitudinal action provides a much larger handle for introducing a tune spread for Landau damping due to the orders of magnitude larger spread in $J_z$ compared to $(J_x, J_y)$ of the LHC beams. After a brief explanation of the theory, the experimental procedure and the corresponding results are presented alongside numerical simulations.

Given a machine lattice with non-zero first and second order chromaticities $Q'_{x,y}$ and $Q''_{x,y}$ respectively, a particle $i$ with a relative momentum deviation of $\delta_i = \Delta p_i / p$ experiences a betatron tune change of $\Delta Q'_{x,y} = Q'_{x,y} \delta_i + Q''_{x,y} \delta_i^2 / 2$. The higher order terms $\propto O(\delta_i^3)$ are neglected here. The average betatron detuning is the relevant quantity to provide damping of the slow head-tail instabilities which develop over many synchrotron periods $T_s$ [2]

$$\langle \Delta Q'_{x,y} \rangle_{T_s} = \frac{Q'_{x,y}}{2} \frac{Q''_{x,y} p_i}{\eta R} j^i = a_{x,y}^z j^i,$$  \hspace{1cm} (1)

$Q_s$ denotes the synchrotron tune, $\eta$ is the slip factor, $R$ the machine radius, and $a_{x,y}^z \equiv Q''_{x,y} Q_s / 2\eta R$. The average detuning contains only the term with $Q''_{x,y}$. $Q''_{x,y}$ does not contribute to damping of the slow head-tail instabilities, but is instead an effective way to impact on the transverse mode coupling instability (TMCI) threshold [2].

A basic formalism for stabilisation from $Q''$ has been developed by J. Scott Berg and F. Ruggiero [5]. Their work shows that detuning with longitudinal action leads to an increase of the stable region in the complex coherent tune space $Re(\Delta Q_{coh})$ vs. $-Im(\Delta Q_{coh})$, which is equivalent to Landau damping from magnetic octupoles. Examples of stability diagrams are shown in Fig. 1 for positive and negative $Q''$ respectively, and assuming an azimuthal mode zero head-tail instability. The asymmetry of the stable regions is a result of the strictly one-sided detuning (see Eq. (1)).

![Figure 1: Stability diagrams for detuning with longitudinal amplitude from $Q'' < 0$ (red, solid), or $Q'' > 0$ (blue, dashed) respectively.](image)

EXPERIMENTAL PROCEDURE

$Q''$ is an energy dependent aberration that depends mainly on the integral of the terms $\beta'\beta''\delta k_1$, $\beta''\beta''\delta k_2$, and $D^2 k_3$ around the machine lattice [6, 7], where $k_1$, $k_2$, and $k_3$ are the quadrupolar, sextupolar, and octupolar strengths respectively, $\beta' = \partial \beta / \partial \delta$ is the derivative of the beta function and $D$ denotes the dispersion function. $\beta'$ itself depends on $k_2$, and for each arc, beam, and plane in the LHC there are two sextupole families with interleaved elements at a phase advance of about $\pi$. The main sextupoles can hence be used to enhance $\beta''\delta$ and generate $Q''$. For each arc, beam, and plane the two sextupole families are powered with opposite signs. The resulting knobs, called QPPF and QPPD, are nearly orthogonal for the two planes. This makes the independent control of $Q'$ through QPPF, and of $Q''$ through QPPD, possible. The amount of $Q''$ predicted by
MAD-X [8] as a function of the knobs is shown in Fig. 2 for the two LHC beams and both transverse planes.

Powering the main sextupoles does not only introduce \( Q'' \), but also creates non-negligible detuning coefficients \( \alpha_{mm} = \partial Q_m / \partial(2J_n) \), \( m, n \in \{x, y\} \), which give rise to detuning with transverse amplitude and hence Landau damping in the same manner as magnetic octupoles. For the studies shown here, the latter is an undesired side effect and needs to be disentangled from the stabilisation through \( Q'' \). This is achieved by means of tracking simulations.

Experiment

The aim of the study is to demonstrate stabilisation of a single nominal bunch at flat top by means of Landau damping predominantly from detuning with longitudinal rather than transverse amplitude. The LHC impedance-driven head-tail instabilities are characterised by \( \text{Re}(\Delta Q_{coh}) < 0 \) in both planes. From theory (Fig. 1) and tracking simulations, the most effective way to stabilise them is by using \( Q''_{x,y} \ll 0 \).

To measure the amount of \( Q'' \) present in the machine, an rf modulation is applied to vary the momentum deviation \( \delta(t) \) of the beam. This translates into time-varying betatron tunes \( Q_{x,y}(t) \), obtained from the base-band tune metre (BBQ). Figure 3 (left) shows an example measurement of BBQ spectral data, where several synchrotron side bands can be seen. One side band is selected consistently throughout the modulation period as illustrated by the overlaid red markers in Fig. 3, and binned with respect to \( \delta \). \( Q' \) and \( Q'' \) are extracted from a weighted second order polynomial fit of \( Q \) vs. \( \delta \) (see Fig. 3, right).

Four bunches (two in each beam) were accelerated to 6.5 TeV. For QPPF = QPPD = 0 and with the Landau octupoles turned off, the amount of \( Q''_{x,y} \) at flat top is approximately zero, both in measurements and MAD-X simulations (Table 1, upper half) [9]. To introduce the desired amount of \( Q'' \) several iterations were made by varying the sextupole knobs, re-measuring \( Q'' \), and comparing the results to MAD-X simulations. At that stage, the octupoles were powered with \( I_{oct} = 320 \) A to ensure beam stability. The aim was to set QPPF and QPPD in such a way as to end up with \( Q''_{x,y} \approx -40'000 \) in both beams once the current in the Landau octupoles would be reduced to zero. The additional contribution to \( Q'' \) from the octupoles in dispersive regions was taken into account using MAD-X. The final values of \( Q''_{x,y} \) are summarised in Table 1 (lower half) and compared to MAD-X calculations showing a good agreement and the success of the procedure. Once the desired sextupole settings were reached, the currents in the Landau octupoles were reduced in steps of 40 A down to 0 A. At 0 A, a horizontal instability occurred in beam 1 for one of the two bunches while all the other bunches remained stable. The observed head-tail instability was an azimuthal mode \( m = -1 \) with a rise time of \( \tau \approx 28.5 \) s [9]. The top right plot in Fig. 4 displays the overlapping traces acquired from the Head-Tail monitor [10] showing a pattern with three nodes.

The study was continued with another fill with lower values of QPPF and QPPD. This time, at zero Landau octupole current all the bunches remained stable. They could be made deliberately unstable by a strong reduction of \( Q'' \) [9].

Table 1: \( Q'' \) MAD-X simulations vs. measurements for the two LHC beams (B) without and with powering of the main sextupoles (MS) for zero Landau octupole current.

<table>
<thead>
<tr>
<th>B</th>
<th>MS</th>
<th>QPPF</th>
<th>QPPD</th>
<th>( Q''_x ) [10^3]</th>
<th>( Q''_y ) [10^3]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>-0.4 ± 2.1</td>
<td>-0.3 ± 2.1</td>
</tr>
<tr>
<td>2</td>
<td>0.0</td>
<td>0.0</td>
<td>0.1</td>
<td>-0.3 ± 2.1</td>
<td>-0.1 ± 2.2</td>
</tr>
<tr>
<td>1</td>
<td>1.5</td>
<td>1.9</td>
<td>-37.5</td>
<td>-43.9 ± 5.6</td>
<td>-38.6 ± 3.6</td>
</tr>
<tr>
<td>2</td>
<td>1.5</td>
<td>2.6</td>
<td>-36.9</td>
<td>-31.3 ± 4.9</td>
<td>-36.8 ± 3.6</td>
</tr>
</tbody>
</table>

ANALYSIS AND INTERPRETATION

In 2016, the single-bunch instability threshold was measured to be at a current of \( I_{oct}^d = -I_{oct}^d = 80^{+35}_{-20} \) A in the focusing and defocusing Landau octupoles respectively [3]. This is for flat top optics at 6.5 TeV, nominal bunch parameters, and \( 11 \leq Q' \leq 14 \). Using the LHC impedance model, PyHEADTAIL [11] simulations predict a Landau octupole threshold of 105 ± 5 A. The contribution from the Landau octupoles to the \( Q'' \) are included in the simulation (\( Q''_x \approx 5'000, Q''_y \approx -1'400 \) at \( I_{oct} = 105 \) A). Both experimental and simulation data show that the most unstable mode is a weak head-tail instability with azimuthal and radial mode numbers (0, 2) respectively. The results are fully consistent which demonstrates the reliability of the model for the machine configuration used during the \( Q'' \) experiments.

Figure 3: Left: BBQ spectral data acquired during beam momentum modulation overlaid with the selected tune peaks (red). Right: Quadratic fit (blue line) to data (red).

Figure 2: \( Q'' \) values predicted by MAD-X in both beams and planes as a function of QPPF and QPPD.

Figure 3: Left: BBQ spectral data acquired during beam momentum modulation overlaid with the selected tune peaks (red). Right: Quadratic fit (blue line) to data (red).
At the experimental working point of QPPF = 1.5 and QPPD = 1.9 (beam 1), the detuning coefficients \( \alpha_{mn} \) generated by the sextupoles are comparable to LHC Landau octupole currents of \( |I_{oct}| \approx 50 \, \text{A} (\alpha_{xx}) \), \( \approx 20 \, \text{A} (\alpha_{yy}) \) and \( \approx 30 \, \text{A} (\alpha_{xy}) \) respectively. They are all well below the above mentioned single-bunch octupole stability threshold which indicates that an important contribution to beam stability must have been provided by \( Q'' \). To understand the horizontal instability observed in one of the bunches, simulation studies are required. Optics results from MAD-X and PTC [12], among them the dependencies of \( Q''_{xy} \) and \( \alpha_{mn} \) on the QPPF and QPPD knobs, are fed into PyHEADTAIL such that both the stabilising effects from detuning with transverse and longitudinal amplitude are modelled. In all the studies, the Landau octupoles are switched off.

In a first set of simulations, the tune spread from \( Q'' \) is deliberately excluded to assess whether the detuning from only the coefficients \( \alpha_{mn} \) introduced by QPPF, QPPD is enough to provide stability at the working point. The results are summarised in Fig. 5 (top) for the two planes of beam 1. The colour code shows the emittance growth over the simulation period (1.8 \( \times 10^6 \) turns) where blue means stable and white means unstable. The dots represent the azimuthal mode number of the instability. Most of the area is unstable, in particular the working point (orange). This demonstrates that the \( \alpha_{mn} \) from sextupoles indeed do not provide sufficient Landau damping. The instability is a mode \( m = 0 \) with two nodes in the head-tail pattern, consistent with earlier LHC experiments. Figure 4 (left) shows a comparison between the head-tail mode patterns acquired during the experiments (top) and in simulations (bottom).

The second set of simulations includes also the effects from \( Q'' \). Two main observations can be made. First, large regions of stability are created, separated by an unstable band in both planes showing a different head-tail mode \( m = -1 \) (red). The reason for that is that \( Q'' \) changes the effective impedance, similarly to \( Q' \). This effect is described by the Vlasov formalism [2] and is currently under more detailed study. It affects the complex coherent tune shift and can also change the most unstable mode. The results shown in Fig. 5 are hence a combination of a change in effective impedance, and Landau damping, both introduced by \( Q'' \). The stable region between the two unstable bands \( m = 0 \) and \( m = -1 \) arises from sufficient Landau damping of both modes. The further increase of QPPF (QPPD), however, leads to a change of the effective impedance, such that Landau damping is lost for the \( m = -1 \) mode. For larger amounts of \( Q'' \), however, all the instabilities are suppressed. The second observation is that the working point, although essentially stable, lies very close to the unstable band of \( m = -1 \). Indeed, the experimental data clearly revealed the observed horizontal instability to be of mode \( m = -1 \). The LHC Head-Tail monitor signal is in excellent agreement with the predictions from simulations. Both of them feature a three-node coherent oscillation pattern along the bunch (Fig. 4, right). Thus, experiment and simulation agree both in the azimuthal as well as the radial mode number of the excited instability.

CONCLUSIONS

Stabilisation of nominal bunches from \( Q'' \) was studied experimentally in the LHC. It was demonstrated that \( Q'' \) can be introduced in a well-controlled manner. Beam dynamics simulations clarify that detuning from transverse amplitude alone cannot explain the observations made in the machine and that \( Q'' \) contributes to beam stability. Simulations, experiments, and ongoing analytical studies show that \( Q'' \) introduces both, Landau damping, and a change of the effective impedance. However, further studies are needed to assess the practicality of stabilisation with \( Q'' \).

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REFERENCES


