DESTABILISING EFFECT OF LINEAR COUPLING IN THE LHC


Abstract

During operation in 2015 and 2016, some transverse instabilities were observed in the Large Hadron Collider (LHC) when either the coupling (or closest tune approach) $|\Delta^c|^{-1}$ was large, or when the tunes were moved closer together. This motivated a campaign of simulations on the effect of linear coupling on the transverse stability. Measurements made during operation and with dedicated beam time have been found to confirm the predictions. This paper will detail the results of the linear coupling studies and relate them to operation of the LHC in the future.

INTRODUCTION

In run 1 of the LHC, many transverse coherent instabilities were observed at the end of the betatron squeeze that were not fully explained [1]. A headtail instability was also irregularly observed at the HERA proton ring at the beginning of the accelerator ramp [2] where it was measured that linear coupling was an essential ingredient for the instability to occur. An explanation was proposed in the framework of "coupled Landau damping" [3], where the possibility to lose Landau damping was discussed, hence the proposed name "coupled head-tail instability" [2]. However, only a simple analytical model with an externally given tune spread was proposed and the exact mechanism for the effect of linear coupling on the tune footprint (and hence the Landau damping) from octupoles was not known at the time. It is also possible that the instabilities observed in run 1 could have been caused by the presence of linear coupling.

In 2015 instabilities were observed at injection energy in the LHC while the machine was being filled for physics operation. During this process the horizontal and vertical tunes were drifting closer together due to the Laslett tune shift [4], and once the tune separation became too small emittance blowup was occurring [5]. This confirmed that linear coupling was impacting transverse stability in the LHC, and further motivated a detailed study.

To probe this idea further and try and determine a destabilising mechanism for linear coupling, transverse stability simulations were performed using two different approaches. Firstly with PyHEADTAIL [6–8], a self-consistent time domain macroparticle tracker, as well as an estimation of the stability threshold based on the complex coherent tune shifts obtained from the frequency domain Vlasov solver DELPHI [9] and a numerical estimation of the dispersion integral using PySSD [10], taking into account the transverse amplitude detuning due to the lattice with single particle tracking using MAD-X [11]. The results of these simulations were also verified with measurements in the LHC at 6.5 TeV with a single bunch.

LINEAR COUPLING

To model linear coupling appropriately, some consistent definitions need to be introduced [12]. Firstly, to define the strength of the coupling resonance the parameter $|\Delta^c|$ is used, which is the minimum tune separation achievable in the coupled system. This is a global property of the full lattice and is easily obtained by performing a tune crossing, where a clear separation will be observed. However if the tune separation is large compared to the $|\Delta^c|$, then the effect of coupling on the transverse motion will be small.

The tune separation is defined as $Q_{sep} = |Q_x - Q_y|$, where $Q_x$ and $Q_y$ are the uncoupled horizontal and vertical fractional tunes. When coupling is present a tune shift occurs, and the new coupled tunes are defined as $Q_c$ for the peak observed in the horizontal and vertical spectra.

SIMULATIONS

The size and shape of the tune footprint is critical for determining if the unstable modes can be Landau damped [13]. If the real part of the complex tune shift lies within a projection of the tune spread on the horizontal and vertical axes, then the mode is potentially stabilised (depending on the strength of the imaginary part). However, if the real part does not lie within the tune spread, then an instability will occur. There are many effects that contribute to the tune spread (e.g. beam-beam and electron-cloud effects), but here only the effect of the Landau Octupoles (LO’s) will be considered.

To understand the effect of linear coupling on the tune spread from the LO’s, MAD-X tracking simulations were performed using the full LHC lattice for 6.5 TeV. For simplicity linear coupling was introduced by only powering one skew quadrupole, this means that there is no local variation of the coupling throughout the lattice. For each case, the horizontal and vertical tunes were rematched to the values used during collisions, $Q_x = 0.31, Q_y = 0.32$. The tune footprints calculated from this setup can be found in Fig. 1.

It can be seen that as the coupling is increased, the detuning coefficients from the LO’s are changed. When the $|\Delta^c| \approx Q_{sep}/2$, the angle of the spread becomes less than 90 degrees. This occurs because linear coupling can be modelled as a rotation of the coordinates of motion from horizontal and vertical to a plane where there is no coupled motion. This decreases the effect of normal octupoles and increases the effect of skew octupoles. The abrupt reduction in the spread can potentially cause a loss of Landau damping.

Before continuing, an important assumption must be made regarding the specific case of the LHC. Here, only the case where the horizontal and vertical planes are approximately symmetric (with regards to the impedance, the chromaticity
Figure 1: Tune footprints from MAD-X tracking simulations for different values of $|C^-|$. For each case the tunes were rematched to $Q_u = 0.31, Q_v = 0.32$. The black point gives a typical coherent unstable tune (whose value depends on impedance, damper and chromaticity) that needs to be lie within the projection of the footprint in the horizontal and vertical axes in order to allow Landau damping. The octupole currents are constant for each case.

and the transverse feedback) will be considered. In this specific case, the rotation to the reference frame that coupling causes does not change the dynamics or the characteristics of the instability [14]. This allows an uncoupled stability theory to be applied to the coupled case. It is clear that in the case where there is a strong asymmetry in the two planes, then the process shown here is not valid and a more complete stability theory will need to be defined as the instability growth rates will be shared between the two planes [15].

We now describe the setup for the time and frequency domain simulations.

For PyHEADTAIL a single skew quadrupole was introduced to provide coupling and its strength was calibrated against the measured $|C^-|$. To accurately recreate the LHC tune feedback, an optimisation routine was performed for each step of $|C^-|$ to ensure that the tunes remain at $Q_u = 0.31, Q_v = 0.32$, which is only possible when the closest tune approach is smaller than $Q_{sep}$. This mimics the tune feedback which would always strive to keep the measured tunes at these values, regardless of whether coupling is present or not. For each coupling step, a scan of the octupole current was performed for $N_{turns} = 1 \times 10^6, N_b = 1 \times 10^4 \text{p/b}, Q_{x,y} = 1$ and $\xi_{x,y} = 2 \mu m$ and no transverse feedback to determine the threshold current.

For PySSD, the footprint is obtained from MAD-X single particle tracking simulations for a range of LO currents. A list of the unstable mode real and imaginary frequencies is computed from DELPHI for the specific optics and impedance model that is being considered, here it is the LHC for flat top optics at 6.5 TeV. A stability diagram model is then applied and a coherent stability factor is calculated. This factor is the largest relative distance in the complex plane among all the coherent modes’ tune shifts with respect to the stability limit given by Landau damping, values greater than 1 mean all the modes were not stabilised, values less than 1 mean they were stabilised. The path where the factor is 1 gives the threshold. In this method, the uncoupled dispersion integral is used to calculate the stability diagram which assumes there are no resonances present in the beam motion but this is used with the coupled tune footprint.

Figure 2 compares the results of both sets of simulations. The y-axis is normalised to the uncoupled threshold, meaning the figure represents the relative increase in stability threshold for constant coupled tunes of $Q_u = 0.31, Q_v = 0.32$. The clear behaviour is that as the $|C^-|$ increases, the required octupole current for stabilisation increases by up to a factor of 2 for $|C^-| = 0.009$. For the final point at $|C^-| = 0.01$ there is a discrepancy between PyHEADTAIL and PySSD, a possible explanation could be that the theoretical assumption that single particle motion is not affected by resonances when implementing the dispersion integral breaks down. This is under further study.

Figure 2: PySSD (contours) and PyHEADTAIL (black line) results showing the relative stabilising octupole required as a function of coupling.

MEASUREMENTS

Measurements were made in the LHC with a single bunch with $N_b = 1.2 \times 10^{11} \text{p/b}, \xi_x = 1.5 \mu m, \xi_y = 1.76 \mu m$ and $4 \tau_d = 1.2 \text{ns}$ [16]. The chromaticity was set to $Q' = 9/9$, with the transverse feedback operating with a damping time of $\tau_d \approx 100 \text{ turns}$. For a bunch with these parameters at an energy of 6.5 TeV in the LHC, the instability threshold had been verified with simulations and measurements as $J_{oct,th,x} \approx 100 \text{A}$ and $J_{oct,th,y} \approx 90 \text{A}$ (based on the different emittances). The intention of the measurements was to introduce large coupling at the LO’s, and create an instability with high current in the LO’s by reducing the tune separation.

A key distinction needs to be made between local and global coupling in the LHC. It has been shown in prior simulations that the local coupling at the position of the spread generating element (i.e. LO’s) is what determines the modification of the tune spread rather than specifically the $|C^-|$. Available in the LHC control room are knobs to vary

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the real and imaginary components for the $f_{1001}$ resonance driving term at the location of the tune measurement pickups. In the case where the coupling is well corrected, the local variation in coupling is large. However when the knobs are varied to increase the global coupling, it has the effect of increasing the local coupling everywhere in the ring, which reduces the impact of local variations. In this particular regime, the $|C^-|$ becomes an adequate parameter to describe the level of coupling at the LO’s.

First, the $|C^-|$ was measured by performing a crossing of the horizontal and vertical tunes. The minimum tune separation in this case was measured to be $|C^-| = 0.0015$. The coupling knobs were varied to increase the minimum tune separation, and the tunes were then crossed again to provide the new measurement, which was $|C^-| = 0.0106$. These measurements can be found in Fig. 3. The bunch did not go unstable during the second tune crossing because it occurred very rapidly, typically the instabilities have rise times on the order of approximately 20s, whereas the crossing was completed in less than 6s.

The tunes were then slowly moved closer together with a constant current in the LO’s of $J_{oct} = 283$A. An instability developed in horizontal when $Q_{sep} = 0.019$. This corresponds to an increase in the relative instability threshold of 2.8 for these machine settings. Typically, the emittance measurements can have a 10% error bar attached to it, which is the dominant source of error out of all the bunch and machine parameters. This error can be propagated through to give a measured increase of $2.8 ± 0.3$.

An acquisition with the LHC headtail monitor rendered a signal with 2 distinct nodes along the bunch indicating a radial mode 2. A tune analysis on the beam oscillations gave an azimuthal mode number 0. Indeed this mode has been predicted and is in excellent agreement with simulations and the underlying impedance model [17]. This observation is the origin of the term "coupled head-tail instability" proposed in the study at HERA [2], which means it is purely a loss of Landau damping, rather than a new instability mechanism.

The PySSD analysis was re-done with the same conditions as found in the measurement. The results can be found in Fig. 4 and show that a relative increase in the threshold of 2.9 is expected. This is compared to the measured increase of 2.8. There are a variety of factors that could contribute to the small discrepancy between the two values. Firstly, and perhaps most importantly, is that the DELPHI mode frequencies are calculated from the LHC impedance model, within which there is still a discrepancy between simulations and measurements. The second is that by moving the tunes, a shift in the $|C^-|$ occurs which is not considered in the calculation. This is expected to be quite small, (on the 1-2%) but is a highly non-linear effect.

This measurement shows that if the $|C^-|$ begins to approach the tune separation, Landau damping can be lost even in the presence of a large spread from LO’s.

**IMPACT ON OPERATION**

Due to this study, applications were developed that fulfilled the role of maintaining a large tune separation and a low coupling at injection energy in the LHC. With these applications in place, no instabilities relating to linear coupling have been observed at injection. Some instabilities were occasionally observed at the end of the betatron squeeze, but these were quickly identified and dedicated optics measurements were re-conducted at the part of the cycle where high coupling was present. The instabilities were then suppressed after correction of the coupling.

**CONCLUSION**

Linear coupling can cause transverse coherent instabilities by modifying the tune spread created by the Landau Octupoles, reducing their effectiveness. This was verified with both simulations and measurements in the LHC. Linear coupling is now a vital part of the LHC stability model and must be carefully controlled in all aspects of the machine cycle.
REFERENCES


[8] https://github.com/PyCOMPLETE/PyHEADTAIL


