A review of the recent experimental results in the field of rare kaon decays is given. The prospects for future progress is discussed.

1 Motivations

The relevance of rare kaon decays is at least three-fold:

- To search for explicit violations of the Standard Model. The posterchild of this class is the search for Lepton Flavour Violation.

- To study of CP-Violation and quark-mixing. This is done by studying Flavour Changing Neutral Current (FCNC) reactions that uniquely probe the $s \rightarrow d$ transition and are affected by small theoretical uncertainty.

- To study the structure of the hadrons and the characteristics of the strong interaction at low energy.

2 Lepton Flavour Violation

The puzzling replication of lepton and quark generations has stimulated significant theoretical interest and Lepton Flavour Violation (LFV) is foreseen by many extension of the Standard Theory. A popular example is the interaction mediated by generation-changing gauge bosons. Theories based on Technicolor, Compositeness and Super-symmetry also predict LFV. Based on this theoretical interest, a round of experimental searches has taken place during the past decade. So far no LFV was reported in kaon decays and the best limits are reported in Table 1.
Table 1: Most recent upper limits on Lepton Flavour Violating processes.

<table>
<thead>
<tr>
<th>Decay</th>
<th>Upper limit (90 % CL)</th>
<th>Experiment</th>
<th>Limit on $M_X$ (TeV/$c^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$BR(K^0_L \rightarrow \mu\nu)$</td>
<td>$&lt; 4.7 \times 10^{-12}$</td>
<td>AGS-E871$^2$</td>
<td>$&gt; 150$</td>
</tr>
<tr>
<td>$BR(K^+ \rightarrow \pi^+\mu^+\nu)$</td>
<td>$&lt; 1.2 \times 10^{-11}$</td>
<td>AGS-E865$^3$</td>
<td>$&gt; 80$</td>
</tr>
<tr>
<td>$BR(K_L^0 \rightarrow \pi^0\mu^+\nu)$</td>
<td>$&lt; 3.3 \times 10^{-10}$</td>
<td>KTeV$^4$</td>
<td>$&gt; 37$</td>
</tr>
</tbody>
</table>

The lower limits on the mass of the hypothetical generation-changing bosons are derived assuming that the interaction takes place at tree level. For instance, for the reaction $K^+ \rightarrow \pi^+\mu^+\nu$ the following relation holds:

$$
\frac{BR(K^+ \rightarrow \pi^+\mu^+\nu)}{BR(K^+ \rightarrow \pi^0\mu^+\nu)} = 16 \frac{1}{\sin^2\theta_c} \left( \frac{g_X}{g_W} \right)^4 \left( \frac{M_W}{M_X} \right)^4
$$

From Eq. 1, and assuming that the coupling constant $g_X$ is equal to the electro-weak coupling, the lower limit $M_X > 80$TeV/$c^2$ is obtained. The discovery of neutrino oscillations have given new motivation to the study of LFV. Progress in the kaon sector is unlikely to continue since the experiments start to be background limited. Further progress is expected in the study of $\mu \rightarrow e\gamma$ and muon-electron conversion.

3 Rare decays and the Standard Model

3.1 CKM matrix and CP-Violation

The mixing of quarks is described by the Cabibbo–Kobayashi–Maskawa matrix. CP-Violation is automatically incorporated in the phenomenology if three or more quark generations exist because an irreducible phase makes some of the couplings complex. Two crucial consequences of this description are the prediction of direct CP-Violation and the existence of CP-Violation in the $B$ meson system. Both predictions were experimentally verified during the last decade, by the measurement of direct CP-Violation in two-pion decays of the neutral kaons and by the measurement of a large CP-asymmetry in the $B^0 \rightarrow (\bar{B}^0) \rightarrow J/\psi K_S$ decays. All results are consistent with the CKM paradigm. To further test the Standard Model one needs to look for inconsistencies measuring independent observables affected by small theoretical uncertainties and different sensitivity to new physics. The class of decays theoretically most clean is represented by the $K \rightarrow \pi\nu\bar{\nu}$ transitions. Unfortunately these decays are very challenging experimentally because they lack a clean signature.

3.2 $K \rightarrow \pi\nu\bar{\nu}$: Theory in Standard Model

Details about the calculation of the $K \rightarrow \pi\nu\bar{\nu}$ decays can be found in the original literature and in very good reviews. For the purpose of this article it is important to recall only the main properties which make these decays such an exciting opportunity. In general, calculations involving meson transitions are marred by the uncertainties due to the hadronic matrix elements which cannot be calculated reliably, but for $K \rightarrow \pi\nu\bar{\nu}$ the hadronic matrix elements are measured from the isospin-related semileptonic decays. Isospin breaking effects are known to be small.

The other advantage of studying these decays is that the $\nu\bar{\nu}$ pair in the final state suppresses long distance contributions to the total rate. In the notation of, the Branching Ratios can be written as:

$$
B(K^+ \rightarrow \pi^+\nu\bar{\nu}) = \kappa_+ \left[ \left( \frac{\lambda \lambda_t}{\lambda^5} \right)^2 + \left( \frac{\lambda \lambda_t}{\lambda^5} P_0(X) + \frac{\lambda \lambda_t}{\lambda^5} X(z_1) \right)^2 \right],
$$

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where:

\[
B(K_{L}^{0} \rightarrow \pi^{0} \nu \bar{\nu}) = \kappa_{0} \left( \frac{2 \lambda_{t}^{2}}{\lambda_{s}^{2}} X(x_{t}) \right)^{2},
\]

(3)

\[
\kappa_{+} = \frac{r_{K^{+}}}{2 \pi^{2} \sin^{2} \theta_{W}} = 4.11 \times 10^{-11},
\]

(4)

\[
k_{L} = \frac{r_{K^{+}}}{r_{K_{L}^{0}}} \frac{\tau(K_{L}^{0})}{\tau(K^{+})} = 1.80 \times 10^{-10},
\]

(5)

and

\[
\lambda_{t} = V_{ts}^{*} V_{td}.
\]

(6)

Recent numerical estimates\(^{16}\) give:

\[
B(K^{+} \rightarrow \pi^{+} \nu \bar{\nu}) = (8.0 \pm 1.1) \times 10^{-11}
\]

(7)

\[
B(K_{0}^{0} \rightarrow \pi^{0} \nu \bar{\nu}) = (3.0 \pm 0.6) \times 10^{-11}
\]

(8)

Where the errors are completely dominated by the uncertainty on the CKM elements and not by the theory errors. The \(K_{L}^{0} \rightarrow \pi^{0} \nu \bar{\nu}\) decay is particularly clean because it is CP-Violating and completely dominated by the top quark contribution. The theoretical error for the charged mode is larger (about 8%) because of the charm quark contribution, but will be reduced to less than 5\% by NNLO calculations\(^{13}\).

3.3 A Clean Test of the Standard Model

Possibly the cleanest test in the flavour sector of the Standard Model can be constructed if both the charged and neutral \(K \rightarrow \pi \nu \bar{\nu}\) modes are precisely measured. The phase \(\beta\) derives from \(Z_{0}\) \(\Delta F = 1\) diagrams whereas in the CP-asymmetry \(A(J/\Psi K_{S})\) it originates from the box diagrams which are \(\Delta F = 2\). Any non-minimal contribution to \(Z_{0}\) diagrams would be signalled by a violation of the relation:

\[
\sin(2\beta)_{J/\Psi} = \sin(2\beta)_{\pi \nu \bar{\nu}}
\]

(9)

A deviation from the predicted rates of the Standard Model would be a clear indication of new physics. Rare kaon decays form a complementary programme to the high energy frontier: when new physics appears at the Tevatron/LHC, the rare decays may help to understand the nature of it.

4 Experimental Status

4.1 \(K^{0} \rightarrow \pi^{0} \nu \bar{\nu}\)

The best published limit was derived by the KTeV collaboration\(^{17}\):

\[
BR(K^{0} \rightarrow \pi^{0} \nu \bar{\nu}) < 5.9 \times 10^{-7} \quad 90\% \text{ CL}
\]

(10)

This result was obtained studying the \(\pi^{0} \rightarrow \gamma e^{+}e^{-}\) decay in order to be able to reconstruct the kaon decay vertex and to impose a constraint to the \(\pi^{0}\) transverse momentum. The search region is shown in Figure 1. The experimental bound is still quite far from the model independent upper limit derived by Grossman and Nir\(^{8}\):

\[
BR(K_{L}^{0} \rightarrow \pi^{0} \nu \bar{\nu}) < 4.4 \times BR(K^{+} \rightarrow \pi^{+} \nu \bar{\nu}) = 1.1 \times 10^{-8}.
\]

(11)
Figure 1: Data-Monte Carlo comparison of the KTeV search for $K_L^0 \rightarrow \pi^0 \nu \bar{\nu}$.

4.2 $K^+ \rightarrow \pi^+ \nu \bar{\nu}$

The first observation of the decay $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ was made by the BNL experiment E787\textsuperscript{19}. The study was extended by the subsequent experiment E949 which collected data in 2002. From these observations, a measurement of the Branching Ratio is extracted\textsuperscript{20}:

$$BR(K^+ \rightarrow \pi^+ \nu \bar{\nu}) = 1.47^{+1.30}_{-0.89} \times 10^{-10},$$

(12)

in agreement with the Standard Model given the large experimental errors. These measurements were performed by employing decays at rest and represent the culmination of a very long research programme at BNL. In Figure 2 the data from E787 and E949 are presented.

The signal area contains three candidates. The events clustering at a kinetic energy of about 108 MeV are due to $K^+ \rightarrow \pi^+ \pi^0$ decays. The small dots represent the signal as simulated by Monte Carlo. The experimental technique employed is based on the complete control of event kinematics, and the combined measurement of the charged pion range, momentum and kinetic energy. This measurement already constraints $\rho$ and $\eta$ as shown, for instance, in Figure 3 provided by the UTfit Collaboration\textsuperscript{21}. The precision of the theoretical prediction sets the scale for the next round of experiments. To make a decisive test of the Standard Model a measurement of about 100 signal events and high signal to background ratio has to be envisaged. The initiatives foreseen to further measure this decay will be described in a later section of the article.

4.3 $K_L^0 \rightarrow \pi^0 l^+ l^-$

With respect to the $\pi \nu \bar{\nu}$ modes, the extraction of the CKM parameters from $K_L^0 \rightarrow \pi^0 l^+ l^-$ is complicated by the presence of long distance effects\textsuperscript{22}. CP conserving effects are small and the CP-Violation introduced by mixing was recently measured\textsuperscript{23,24} from the study of the corresponding $K_S$ decays modes. NA48/1 has measured seven and six signal events for the decays $K_{S}^{0} \rightarrow \pi^{0} e^{+} e^{-}$ and $K_{S}^{0} \rightarrow \pi^{0} \mu^{+} \mu^{-}$ with an expected background of about 0.15 and 0.22 events respectively. The data are shown in Figures 4 and 5 and the resulting Branching Ratios were found to be:

$$BR(K_{S}^{0} \rightarrow \pi^{0} e^{+} e^{-}) = (5.8^{+2.8}_{-2.3} \pm 0.8) \times 10^{-9},$$

(13)
Figure 2: Range versus Energy scatter plot by the E787 and E949 on $K^+ \rightarrow \pi^+\nu\bar{\nu}$.

Figure 3: The constraint imposed on the $\rho$ and $\eta$ plane by the measurements of E787 and E949. From UTfit Collaboration.
Figure 4: The seven candidates measured by NA48/1 for the decay $K^0_L \to \pi^0 e^+ e^-$. 

and

$$BR(K^0_S \to \pi^0 \mu^+ \mu^-) = (2.9^{+1.4}_{-1.2} \pm 0.2) \times 10^{-9},$$  \hspace{1cm} (14)

Thanks to these $K^0_S$ measurements, the $K^0_L$ rates can be predicted in the Standard Model. The direct and indirect CP-Violation components interfere but the question whether the interference is constructive or destructive cannot be answered by the experiment. Two theoretical analyses prefer constructive interference\textsuperscript{22,25}. Assuming constructive interference, the Standard Model predictions\textsuperscript{26} for the electronic and muonic modes are:

$$BR(K^0_L \to \pi^0 e^+ e^-) = 3.7^{+1.1}_{-0.9} \times 10^{-11},$$  \hspace{1cm} (15)

and

$$BR(K^0_L \to \pi^0 \mu^+ \mu^-) = 1.5 \pm 0.3 \times 10^{-11}.$$  \hspace{1cm} (16)

Progress on the $K^0_L$ modes was made by the KTeV experiment at Fermilab. The combined 1997 and 1999 data set led to the publication of the following upper limit\textsuperscript{27}:

$$BR(K^0_L \to \pi^0 e^+ e^-) < 2.8 \times 10^{-10} \text{ 90\% CL},$$  \hspace{1cm} (17)

which is about an order of magnitude above the Standard Model prediction. In the 1999 data sample KTeV found one candidate, tantalisingly placed at the middle of the signal region, as shown in Figure 6. Unfortunately, this is consistent with the expected background which was calculated to be 0.99±0.35 events. As far as the muonic mode is concerned, KTeV has published the best upper limit\textsuperscript{28}:

$$BR(K^0_L \to \pi^0 \mu^+ \mu^-) < 3.8 \times 10^{-10} \text{ 90\% CL},$$  \hspace{1cm} (18)

based on the observation, consistent with the expected background, of two events in the signal region.

The prediction of the Standard Model rates are currently limited by the precision achieved on the $K^0_S$ modes which hopefully can be improved by KLOE at the Frascati $\phi$ factory. The $K^0_L$ searches are background limited and progress will therefore not be linear with the kaon exposure. A 100-fold increased in kaon flux, coupled with clever experimental technique such as the study of the time-dependent interference\textsuperscript{29}, is needed to be able to test the Standard Model predictions.
Figure 5: The six candidates measured by NA48/1 for the decay $K^0_L \rightarrow \pi^0 \mu^+ \mu^-$. 

Figure 6: KTeV search for $K^0_L \rightarrow \pi^0 e^+ e^-$. 

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5 Prospects

5.1 $K^{+}_L \rightarrow \pi^0 \nu \bar{\nu}$

A large window of opportunity exists. As described above, the current upper limit is four order of magnitude above the Standard Model prediction. A first round of dedicated experiments is taking place and significant improvements are to be expected soon. In particular the experiment E391a at KEK has recently collected data. The experiment is a pilot project to assess the feasibility of studying this reaction at the future J-PARC facility employing the technique of photon vetoes and pencil neutral beam. A sketch of the experiment is shown in Figure 7.

Figure 7: Layout of experiment E391a. The experimental technique exploits hermetic photon vetoes and a pencil beam.

A different approach is proposed by the BNL KOPIO experiment. KOPIO plans to measure as much information as possible from the initial and the final state including the momentum of the neutral kaon by means of time of flight and the direction of the two photons by means of a pre-radiator placed in front of the electromagnetic calorimeter.

5.2 $K^{+} \rightarrow \pi^+ \nu \bar{\nu}$

The situation for $K^{+}$ is quite different with respect to $K^{+}_L \rightarrow \pi^0 \nu \bar{\nu}$ because it is already known that the rate of the decay agrees within errors to the magnitude expected from the Standard Model. There is a window of opportunity to accumulate more data by E949 at BNL. Eventually, in order to make significant progress, the next experiments have to be capable of collecting on the order of 100 signal events to extract a precise measurement of the CKM parameter $|V_{td}|$.

Plans to pursue the stopped kaon technique, which is well established, are contained in a Letter of Intent submitted to KEK. The extrapolation to O(100) of a stopped kaon technique is not straightforward because of the very small acceptance associated with this type of experiment. A different approach is to exploit the high acceptance and high sensitivity which can be reached from in flight decays. This technique was proposed, together with an RF-separated kaon beam, by the CKM experiment at Fermilab. The experiment was approved by Fermilab but not ratified by the HEPAP-P5 committee for cost reasons. If the attempt of separating the kaons from the other beam species is abandoned, there are advantages to propose the experiment at higher secondary beam momentum. A new proposal, CERN-P326, plans to employ a 75
GeV/c charged beam to collect $4.8 \times 10^{12}$ $K^+$ decays in about two years of data taking starting in 2009. The challenge of this experiment is to be able to perform tracking of the beam particles at a rate approaching one GHz in the beam spectrometer. The main advantage to performing the experiment at high energy is the large acceptance and the large amount of electro-magnetic energy deposited in the photon counters which allows one to veto with high confidence the possible backgrounds originating from the $K^+ \rightarrow \pi^+\pi^0$ reaction. The experimental layout of P326 is shown in Figure 8.

Acknowledgments

I wish to thank the organizers for a very interesting programme and all my experimental and theoretical colleagues for keeping rare kaon decays a very special field. Many thanks to Jennifer Jackson for editing the manuscript.

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