Non-leptonic three body decays at LHCb

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On behalf of the LHCb experiment

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Pion-Kaon Interaction Workshop
LHCb detector status

Spectrometer mainly dedicated to heavy flavour physics

[3 fb⁻¹ Run-I (2011/12) at 7/8 TeV]
[3.7 fb⁻¹ Run-II (2015-17) at 13 TeV]

This talk covers some recent highlights in this sector from LHCb

- Introduction to three-body decays at LHCb

- B decays to open charm, \( i.e. B \rightarrow Dhh' \) modes
  
  Dalitz plot analyses, (strange-)charm spectroscopy
  
  [PRL 113, 162001 (2014), PRD 90, 072003 (2014)]
  [PRD 92, 012012 (2015)]

- CPV measurements in charmless three-body decays involving \( K^0s \)
  
  [LHCb-PAPER-2017-033, submitted to PRL]

- Status/issues towards more DP analyses

[3 fb\(^{-1}\) Run-I (2011/12) at 7/8 TeV]
Dalitz plot analysis features

Intensity along bands indicates magnitude and the spin of the given resonance

Angular distribution of resonance related to Legendre Polynomials

Amplitude analysis can access:

- Relative phases between states
- Sensitivity to CP violating effects
- Resolve ambiguities in weak phases
- Hadron spectroscopy

Particle Data Group Collaboration
PRD 86 (2012) 010001

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Dalitz plot analysis features

Intensity along bands indicates magnitude and the spin of the given resonance

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PRD 86 (2012) 010001

Amplitude analysis can access:
- Relative phases between states
- Sensitivity to CP violating effects
- Resolve ambiguities in weak phases
- Hadron spectroscopy

Dalitz plot analysis - Isobar Model

Amplitude analysis most commonly performed in the “Isobar Model”, in which the total amplitude is approximated as coherent sum of quasi-two-body contributions:

\[ \mathcal{A}(m_{ij}^2, m_{jk}^2) = \sum_{l=1}^{N} c_l F_l(m_{ij}^2, m_{jk}^2) \]

- **CP violating**
- **Strong dynamics**
- **CP conserving**

\( c_l \): complex coefficients describing the relative magnitude and phase of the different isobars

\( F_l \): dynamical amplitudes that contain the lineshape and spin-dependence of the hadronic part

\[ F_l(L, m_{ij}^2, m_{jk}^2) = R_l(m_{ij}^2) \times X_L(|\vec{p}|r) \times X_L(|\vec{q}|r) \times T_l(L, \vec{p}, \vec{q}) \]

- Resonance mass term (e.g. Breit–Wigner)
- Barrier factors - \( p, q \): momenta of bachelor and resonance
- Angular probability distribution

Many observables can be accessed: \( \text{Re}(c_l) \) and \( \text{Im}(c_l) \) or \( |c_l| \) and \( \text{arg}(c_l) \); or derived quantities such as BF and A\(_{CP}\)
**Kπ invariant mass modelling**

- **K*(892) resonance - Relativistic Breit-Wigner**

- **Kπ S-wave contribution**

Toy simulation using Laura++ package
arXiv:1711.09854


\[
R_j(s) = \frac{\sqrt{s}}{q \cot \delta_B - i q} + e^{2i\delta_B} \frac{m_0 \Gamma_0 \frac{m_0}{q_0}}{(m_0^2 - s) - i m_0 \Gamma_0 \frac{q}{\sqrt{s} q_0}}
\]

- **EFKLLM lineshape** [PRD 79 (2009) 094005]

\[
R_j(m) = F(m) \left( \frac{c_0}{m^2} + c_1 \right)
\]

- **κ + K*0(1430) resonance** [Flatté]

NR parametrisation [PRD 92 (2015) 054010]

\[
R_j(s) = \frac{1}{1 + \frac{s}{\Lambda^2}}
\]

Single model κ + K*0(1430) + K*0(1950)
[arXiv:1701.04881]

Others (non DP), polynomial expansion
[PRD 93 (2016) 074025]
B decays to open charm, \( i.e. \, B \rightarrow Dhh' \) channels

Dalitz-plot analyses (\( e.g. \) spectroscopy and CKM angle measurements)

\((\text{strange-})\text{charm spectroscopy}\)

\[\text{[PRL 113, 162001 (2014), PRD 90, 072003 (2014)]}\]
\[\text{[PRD 92, 012012 (2015)]}\]
Spectroscopy of strange-charm states has been reinvigorated due to recent observations of \(D_s^{0\star}(2317)\) and \(D_s^{1}(2460)\)

**DP analysis of \(B_s^0 \to D_{s\star}\bar{D}^0 K\pi^+\)**

- \(D_{sJ}^{\star}(3040)\)
- \(D_{sJ}^{\star}(2860)\)
- \(D_s^{\star}(2700)\)
- \(D_s^{\star}(2573)\)
- \(D_s(2536)\)
- \(D_s(2460)\)
- \(D_{s0}^{\star}(2317)\)

Some discrepancies have been seen between predicted and measured values

- \(D_s^{\star}\) and \(D_{s0}^{\star}(2317)\) are too light to decay to \(D^0 K^-\)
- Neither can states with unnatural spin-parity (\(J^P = 0^-\), \(1^+\), \(2^-\), etc)
- \(D_{s2}^{\star}(2573)\), \(D_{s1}^{\star}(2700)\) and \(D_{sJ}^{\star}(2860)\) are possible
Dalitz plot analysis of $B_{s}^{0} \rightarrow D^{0}K\pi^{+}$

Analysis performed with $\sim$11K signal events and purity of 87%

Backgrounds due to Combinatorial (7.3%), $B^{0} \rightarrow D^{(*)0}\pi\pi$ (2.8%) and $\Lambda_{b}^{0} \rightarrow D^{(*)0}p\pi$ (2.3%)

LHCb PRL 113, 162001 (2014), PRD 90, 072003 (2014)

$$m^{2}(K^{-}\pi^{+}) [\text{GeV}^{2}/c^{4}]$$

$$\theta' = \frac{1}{\pi} \arccos \left( 2 \frac{m_{ij} - m_{ij}^{\text{min}}}{m_{ij}^{\text{max}} - m_{ij}^{\text{min}}} - 1 \right)$$
Efficiency (Spline) and background contributions need to be modelled for the Dalitz-plot fit

Dalitz plot analysis overview

LHCb PRL 113, 162001 (2014), PRD 90, 072003 (2014)

Combinatorial

$\bar{\Lambda}_b^0 \rightarrow \bar{D}^{(*)0} \bar{p} \pi^+$

$B^0 \rightarrow \bar{D}^{(*)0} \pi^+ \pi^-$
Dalitz plot fit results

LHCb PRL 113, 162001 (2014), PRD 90, 072003 (2014)

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Dalitz plot fit results - helicity projections

LHCb PRL 113, 162001 (2014), PRD 90, 072003 (2014)
Several spin hypotheses have been investigated for the $D_{sJ}^*(2860)$:

Two states [$D_{s1}^*(2860)^-$, $D_{s3}^*(2860)^-$] are required in the region 2.86 GeV/$c^2$ (each with a significance of 10$\sigma$)

1$^{st}$ observation of a heavy flavoured spin-3 resonance and 1$^{st}$ time a spin-3 state seen to be produced in B decay

<table>
<thead>
<tr>
<th>Spin hypothesis</th>
<th>$\Delta$NLL</th>
<th>$\sqrt{2}\Delta$NLL</th>
<th>Masses and widths</th>
</tr>
</thead>
<tbody>
<tr>
<td>1+3</td>
<td>0</td>
<td>—</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>141.0</td>
<td>16.8</td>
<td>2862, 57</td>
</tr>
<tr>
<td>0+1</td>
<td>113.2</td>
<td>15.0</td>
<td>2446, 250, 2855, 96</td>
</tr>
<tr>
<td>0+2</td>
<td>155.1</td>
<td>17.6</td>
<td>2870, 61, 2569, 17</td>
</tr>
<tr>
<td>0+3</td>
<td>105.1</td>
<td>14.5</td>
<td>2415, 188, 2860, 52</td>
</tr>
<tr>
<td>1+2</td>
<td>156.8</td>
<td>17.7</td>
<td>2866, 92</td>
</tr>
<tr>
<td>2</td>
<td>138.6</td>
<td>16.6</td>
<td>2851, 99, 3134, 174</td>
</tr>
<tr>
<td>2</td>
<td>287.9</td>
<td>24.0</td>
<td>3243, 81</td>
</tr>
<tr>
<td>2+3</td>
<td>131.2</td>
<td>16.2</td>
<td>2878, 12, 2860, 56</td>
</tr>
<tr>
<td>3</td>
<td>136.5</td>
<td>16.5</td>
<td>2860, 57</td>
</tr>
</tbody>
</table>

The presence of the state $D_{s3}^*(2860)$ has been independently confirmed in studies of $pp \rightarrow D^{*(+,0)K^0,^+X}$ (LHCb)

LHCb JHEP 02 (2016) 133
Dalitz plot fit results

LHCb PRL 113, 162001 (2014), PRD 90, 072003 (2014)
Dalitz plot fit results

LHCb PRL 113, 162001 (2014), PRD 90, 072003 (2014)

<table>
<thead>
<tr>
<th>Resonance</th>
<th>Fit fraction</th>
<th>Upper limits</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K^*(892)^0$</td>
<td>28.6 ± 0.6 ± 0.7 ± 0.9</td>
<td></td>
</tr>
<tr>
<td>$K^*(1410)^0$</td>
<td>1.7 ± 0.5 ± 0.2 ± 1.4</td>
<td></td>
</tr>
<tr>
<td>LASS nonresonant</td>
<td>13.7 ± 2.5 ± 1.5 ± 4.1</td>
<td></td>
</tr>
<tr>
<td>$K_0^*(1430)^0$</td>
<td>20.0 ± 1.6 ± 0.7 ± 3.3</td>
<td></td>
</tr>
<tr>
<td>LASS total</td>
<td>21.4 ± 1.4 ± 1.0 ± 4.7</td>
<td></td>
</tr>
<tr>
<td>$K_0^*(1430)^0$</td>
<td>3.7 ± 0.6 ± 0.4 ± 1.1</td>
<td></td>
</tr>
<tr>
<td>$K^*(1680)^0$</td>
<td>0.5 ± 0.4 ± 0.3 ± 0.8</td>
<td>&lt; 2.0 &lt; 2.4</td>
</tr>
<tr>
<td>$K_0^*(1950)^0$</td>
<td>0.3 ± 0.2 ± 0.1 ± 2.4</td>
<td>&lt; 3.7 &lt; 4.1</td>
</tr>
<tr>
<td>$K_3^*(1780)^0$</td>
<td>—</td>
<td>&lt; 0.33 &lt; 0.38</td>
</tr>
<tr>
<td>$K_4^*(2045)^0$</td>
<td>—</td>
<td>&lt; 0.21 &lt; 0.24</td>
</tr>
</tbody>
</table>
Dalitz plot analysis of $B^0 \rightarrow \bar{D}^0 K^+\pi^-$

Analysis performed with ~2.3K signal events and purity of 75%

Backgrounds due to Combinatorial (22%), $B^0 \rightarrow D^{(*)0}\pi\pi$ (2%) and $\Lambda^0_b \rightarrow D^{(*)0}\pi\pi$ (< 1%)

LHCb PRD 92, 012012 (2015)
Dalitz fit results for $B^0 \rightarrow \bar{D}^0 K^+ \pi^-$

LHCb PRD 92, 012012 (2015)
Dalitz fit results for $B^0 \rightarrow \bar{D}^0 K^+ \pi^-$

Table 8: Results for the fit fractions and their uncertainties (%). The three quoted errors are

<table>
<thead>
<tr>
<th>Resonance</th>
<th>Spin</th>
<th>Fit fraction</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K^*(892)^0$</td>
<td>1</td>
<td>$37.4 \pm 1.5 \pm 1.2 \pm 1.7$</td>
</tr>
<tr>
<td>$K^*(1410)^0$</td>
<td>1</td>
<td>$0.7 \pm 0.3 \pm 0.8 \pm 0.8$</td>
</tr>
<tr>
<td>$K_0^*(1430)^0$</td>
<td>0</td>
<td>$5.1 \pm 2.0 \pm 2.4 \pm 3.4$</td>
</tr>
<tr>
<td>LASS nonresonant</td>
<td></td>
<td>$4.8 \pm 3.8 \pm 3.8 \pm 6.7$</td>
</tr>
<tr>
<td>LASS total</td>
<td></td>
<td>$6.7 \pm 2.7 \pm 2.7 \pm 5.4$</td>
</tr>
<tr>
<td>$K_2^*(1430)^0$</td>
<td>2</td>
<td>$7.4 \pm 1.7 \pm 1.1 \pm 2.0$</td>
</tr>
</tbody>
</table>
\[ m(K\pi) \ B^0_s \rightarrow \bar{D}^0K\pi^+ \text{ vs } B^0 \rightarrow \bar{D}^0K^+\pi^- \]

**LHCb PRD 90, 072003 (2014), PRD 92, 012012 (2015)**

\[
\begin{array}{ccc}
\text{Resonance} & B^0_s \rightarrow \bar{D}^0K^-\pi^+ & B^0 \rightarrow \bar{D}^0K^+\pi^- \\
K^*(892)^0 & 4.29 \pm 0.09 \pm 0.11 \pm 0.14 \pm 0.63 & 5.13 \pm 0.20 \pm 0.15 \pm 0.24 \pm 0.60 \\
K^*(1410)^0 & 3.86 \pm 1.14 \pm 0.45 \pm 3.18 \pm 0.89 & 1.59 \pm 0.68 \pm 1.81 \pm 1.59 \pm 0.36 \\
K_0^*(1430)^0 & 2.06 \pm 0.38 \pm 0.23 \pm 0.62 \pm 0.30 & 0.71 \pm 0.27 \pm 0.33 \pm 0.47 \pm 0.08 \\
\text{LASS nonresonant} & 3.00 \pm 0.24 \pm 0.11 \pm 0.50 \pm 0.44 & 0.66 \pm 0.51 \pm 0.51 \pm 0.92 \pm 0.08 \\
\text{LASS total} & 3.21 \pm 0.21 \pm 0.15 \pm 0.71 \pm 0.47 & 0.92 \pm 0.38 \pm 0.38 \pm 0.74 \pm 0.11 \\
K_2^*(1430)^0 & 1.11 \pm 0.18 \pm 0.12 \pm 0.33 \pm 0.15 & 2.04 \pm 0.45 \pm 0.30 \pm 0.54 \pm 0.25 \\
\end{array}
\]

- Interesting different relative rates of the spin-0 and spin-2 K*’s between B^0/B^0_s
- Additional upper limits for B^0_s, *i.e.* K^*(1680), K^*_0(1950), K^*_3(1780) and K^*_4(2045)
- Systematics uncertainties on the S-wave modelling given by probing alternative lineshapes, *i.e.* EFKLLM and \( \kappa + K^*_0(1430) \)

\[
B^0_s \rightarrow \bar{D}^0K^-\pi^+ \quad B^0 \rightarrow \bar{D}^0K^+\pi^- \\
\text{Fit fractions LASS} (\%) : \quad 21.4 \pm 1.4 \pm 1.0 \pm 4.7 \quad 6.7 \pm 2.7 \pm 2.7 \pm 5.4
\]

Add/remove resonances + alternative models
Charmless three-body decays to final states containing a long-lived particle

LHCb results: $\mathcal{L} = 3 \text{ fb}^{-1} - 2011 + 2012$ dataset

Updated branching fraction of $B^0 (s) \rightarrow K^0 h^+ h^-$ decays


Dalitz plot analysis of $B^0 \rightarrow K^0 \pi^+ \pi^-$ decays

[LHCb-PAPER-2017-033, submitted to PRL]
Recent results from $B^0(s) \rightarrow K^0 h^+ h^-'$-decays

- BF measurements of quasi-two-body states
  - $[B^0\rightarrow K^{*\pm}h^\mp] - New J. Phys. 16 (2014) 123001$
  - $[B^0_s \rightarrow K^{*0}K^0_S] - JHEP 01 (2016) 012$
- Long term: time-dependent amplitude analysis for CKM-phases measurement
- Short term: time-integrated analysis
  - [sensitive to CP in flavour-specific contributions]

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Dalitz plot analysis of \(B^0 \rightarrow K^0\pi^+\pi^-\)

Analysis performed with \(~3.2K\) signal events and purity of 85-95%

Backgrounds due to Combinatorial (3-13%) and cross-feed (2-3%)

Signal region: \(\pm3\sigma\) around nominal mass is considered for the Dalitz plot fit

Quasi-flavour-specific final state
Dalitz plot analysis of $B^0 \rightarrow K^0\pi^+\pi^-$

<table>
<thead>
<tr>
<th>Resonance</th>
<th>Parameters</th>
<th>Lineshape</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K^*(892)^-$</td>
<td>$m_0 = 891.66 \pm 0.26$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\Gamma_0 = 50.8 \pm 0.9$</td>
<td>RBW</td>
</tr>
<tr>
<td>$(K\pi)_0^-$</td>
<td>$\Re(\lambda_0) = 0.204 \pm 0.103$</td>
<td>EFKLLM</td>
</tr>
<tr>
<td></td>
<td>$\Im(\lambda_0) = 0$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\Re(\lambda_1) = 1$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\Im(\lambda_1) = 0$</td>
<td></td>
</tr>
<tr>
<td>$K_2^*(1430)^-$</td>
<td>$m_0 = 1425.6 \pm 1.5$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\Gamma_0 = 98.5 \pm 2.7$</td>
<td>RBW</td>
</tr>
<tr>
<td>$K^*(1680)^-$</td>
<td>$m_0 = 1717 \pm 27$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\Gamma_0 = 332 \pm 110$</td>
<td>Flatté</td>
</tr>
<tr>
<td>$f_0(500)$</td>
<td>$m_0 = 513 \pm 32$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\Gamma_0 = 335 \pm 67$</td>
<td>RBW</td>
</tr>
<tr>
<td>$\rho(770)^0$</td>
<td>$m_0 = 775.26 \pm 0.25$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\Gamma_0 = 149.8 \pm 0.8$</td>
<td>GS</td>
</tr>
<tr>
<td>$f_0(980)$</td>
<td>$m_0 = 965 \pm 10$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$g_\pi = 0.165 \pm 0.025$ GeV</td>
<td>Flatté</td>
</tr>
<tr>
<td></td>
<td>$g_K = 0.695 \pm 0.119$ GeV</td>
<td></td>
</tr>
<tr>
<td>$f_0(1500)$</td>
<td>$m_0 = 1505 \pm 6$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\Gamma_0 = 109 \pm 7$</td>
<td>RBW</td>
</tr>
<tr>
<td>$\chi_{c0}$</td>
<td>$m_0 = 3414.75 \pm 0.31$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\Gamma_0 = 10.5 \pm 0.6$</td>
<td>RBW</td>
</tr>
<tr>
<td>Nonresonant (NR)</td>
<td>Phase space</td>
<td></td>
</tr>
</tbody>
</table>

EFKLLM: $R_j(m) = F(m) \left( \frac{\lambda_0}{m^2} + \lambda_1 \right)$

Asymmetry observable derived from usual isobar parameters

$$A_{\text{raw}} = \frac{|\bar{c}_j|^2 - |c_j|^2}{|\bar{c}_j|^2 + |c_j|^2}$$

The CP asymmetry is related to the raw asymmetry by $A_{CP} = A_{\text{raw}} - A_{\Delta}$


$A_{\Delta} = A_P(B^0) + A_D(\pi)$

$(-0.35 \pm 0.81)\% \quad 0.25\%$

Fit fractions:

$$F_i = \frac{\int \int_{\text{DP}} |c_i F_i(s_+, s_-)|^2 ds_+ ds_-}{\int \int_{\text{DP}} \left| \sum_j c_j F_j(s_+, s_-) \right|^2 ds_+ ds_-}$$
Dalitz plot fit results

\[ \mathcal{A}_{CP}(K^*(892)^-\pi^+) = -0.308 \pm 0.060 \pm 0.011 \pm 0.012 \]
\[ \mathcal{A}_{CP}((K\pi)_0^-\pi^+) = -0.032 \pm 0.047 \pm 0.016 \pm 0.027 \]
\[ \mathcal{A}_{CP}(K_2^*(1430)^-\pi^+) = -0.29 \pm 0.22 \pm 0.09 \pm 0.03 \]
\[ \mathcal{A}_{CP}(K^*(1680)^-\pi^+) = -0.07 \pm 0.13 \pm 0.02 \pm 0.03 \]
\[ \mathcal{A}_{CP}(f_0(980)K_S^0) = 0.28 \pm 0.27 \pm 0.05 \pm 0.14 \]

First observation of CP violation in \(B^0 \rightarrow K^*(892)\pi\) with ~ 6 sigma

World average
\[ \mathcal{A}(K^*(892)\pi) = -0.23 \pm 0.06 \]
Dalitz plot fit results

Fit fractions:

\[
\begin{align*}
\mathcal{F}(K^*(892)^-\pi^+) & = 9.43 \pm 0.40 \pm 0.33 \pm 0.34 \% \\
\mathcal{F}((K\pi)\pi^+) & = 32.7 \pm 1.4 \pm 1.5 \pm 1.1 \% \\
\mathcal{F}(K^*_2(1430)^-\pi^+) & = 2.45 \pm 0.10 \pm 0.08 \pm 0.14 \pm 0.12 \% \\
\mathcal{F}(K^*(1680)^-\pi^+) & = 7.34 \pm 0.30 \pm 0.31 \pm 0.06 \% \\
\mathcal{F}(f_0(980)K^0_s) & = 18.6 \pm 0.8 \pm 0.7 \pm 1.2 \% \\
\mathcal{F}(\rho(770)^0K^0_s) & = 3.8 \pm \frac{1.1}{1.6} \pm 0.7 \pm 0.4 \% \\
\mathcal{F}(f_0(500)K^0_s) & = 0.32 \pm \frac{0.40}{0.08} \pm 0.19 \pm 0.23 \% \\
\mathcal{F}(f_0(1500)K^0_s) & = 2.60 \pm 0.54 \pm 1.28 \pm 0.60 \% \\
\mathcal{F}(\chi_c^0K^0_s) & = 2.23 \pm \frac{0.40}{0.32} \pm 0.22 \pm 0.13 \% \\
\mathcal{F}(K^0_s\pi^+\pi^-)^{NR} & = 24.3 \pm 1.3 \pm 3.7 \pm 4.5 \%
\end{align*}
\]

- The resonance state K*(1680) has been included in the model (not used in previous analyses)


- Alternative LASS modelling for the S-wave has been examined:

\[ -2\Delta \ln \mathcal{L} = 85 \]

No systematic is assigned to the choice of the model, but only uncertainties associated to fixed parameters
Status/issues foreseen for CPV measurements in charmless three-body decays

LHCb $\mathcal{L} = 3\text{ fb}^{-1}$ – 2011 + 2012 dataset

Prospects for Dalitz-plot analyses
Towards [more] charmless DP analyses

Efforts on understanding the large asymmetries seen in $B^+ \rightarrow h^+ h^+ h^+$ decays


$A_{CP}^{reg}(B^\pm \rightarrow \pi^\pm \pi^+ \pi^-) = 0.584 \pm 0.082 \pm 0.027 \pm 0.007$

$A_{CP}^{reg}(B^\pm \rightarrow \pi^\pm K^+ K^-) = -0.648 \pm 0.070 \pm 0.013 \pm 0.007$
Towards [more] charmless DP analyses

- DP analysis is clearly required, in particular to understand the origin of the strong phase difference
- Such analyses are currently ongoing at LHCb!
  
  \[ i.e. (1) B^\pm \rightarrow \pi^\pm \pi^\mp \pi^\pm, (2) B^\pm \rightarrow \pi^\pm K^+ K^- \ldots ]

- Unprecedented statistics (e.g. 5-200 K events for \( B^\pm \rightarrow \pi^\pm K^+ K^- \), \( K^\pm \pi^+ \pi^- \)): simplified theoretical descriptions are no longer sufficient

  - How to model non-resonant contributions, final state interaction, re-scattering effects? Connect two (or all) different final states?
  
  - How to obtain an accurate description of the S-wave?
    
    \[ e.g. \ for \ B^\pm \rightarrow \pi^\pm K^+ K^- \ already \ limited, \ NR+ K^*_0(1430) \ or \ k + K^*_0(1430)? \]
Towards [more] charmless DP analyses

Many other channels that have never been explored in the B-factories are gradually being investigated

- Untagged time-integrated analysis of $B^0_s \rightarrow K^0_S K \pi$ decays
  - Only flavour averaged fit fractions
  - Modelling of S-wave of crucial relevance, \textit{i.e.} LASS/EFKLLM

- Amplitude analysis of $\Lambda_b \rightarrow K^0_S p \pi^-$
  - Large $A_{CP}(pK^*) \sim 20\%$ predicted

[PRD 91 (2015) 11, 116007]
General conclusions

- Enormous wealth of physics to be found in three-body hadronic decays of b-hadrons (e.g. Spectroscopy, CP violation)

- Note that LHCb has also a large programme of Dalitz plot analyses in D mesons [e.g. LASS vs GLASS in $D^0 \rightarrow K^0_s K\pi$; PRD 93, 052018 (2016)]

- Of course, not only DP analyses, e.g. $B^0 \rightarrow [K\pi] l^+l^-$, $B^0_s \rightarrow [K\pi][K\pi]$

- Some very interesting and intriguing results obtained recently in B to open-charm and charmless decays

  - However, many open questions on how to properly model the $K\pi$ system. Unprecedented statistics will require a better understanding

  - Larger datasets from the LHCb Run-II+ upgrade will provide in the future the possibility to fully explore the potential of the field
Detector acceptance modelling

Since each detector has its own acceptance, such effects need to be properly accommodated when performing a Dalitz plot fit

- Two general approaches: 2-D polynomial (or similar) function or a 2-D histogram

\[ L = \prod_{l=1}^{N_{\text{evts}}} \frac{\epsilon(m_{ij}, m_{jk})}{\int_{D_P} \epsilon(m_{ij}, m_{jk})} \frac{\sum_{m=1}^{N_R} c_m F_m(m_{ij}, m_{jk})^2}{dD_P} \]

LHCb general strategy has been based on Spline these distributions in the squared DP
Detector acceptance modelling

Preferential co-ordinate transformation to improve sensitivity modelling in the DP borders
Background contributions

Contributions from different backgrounds can be presented and need to be modelled

\[
\mathcal{L} = \prod_{l=1}^{N_{\text{events}}} \left[ \frac{\epsilon(m_{ij}^2, m_{jk}^2) \left| \sum_{m=1}^{N_R} c_m F_m(m_{ij}^2, m_{jk}^2) \right|^2}{\int_{DP} \epsilon(m_{ij}^2, m_{jk}^2) \left| \sum_{m=1}^{N_R} c_m F_m(m_{ij}^2, m_{jk}^2) \right|^2 dDP} + \frac{\sum_{m=1}^{N_B} |k_m|^2 |B_m(m_{ij}^2, m_{jk}^2)|^2}{\int_{DP} \sum_{m=1}^{N_B} |k_m|^2 |B_m(m_{ij}^2, m_{jk}^2)|^2 dDP} \right]
\]

Signal (Coherent)  
Background (Incoherent)
How LHCb reconstruct $K^0$ mesons?