Abstract

Correlations in particle production in proton-lead and lead-lead collisions at the LHC

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In high-energy heavy-ion collisions at the Large Hadron Collider (LHC), a hot and dense state of matter called the Quark-Gluon Plasma (QGP) is formed. The initial collision geometry and the subsequent expansion during the QGP stage result in the correlations of produced particles, through which the properties of the QGP can be investigated. Two analyses based on the geometrical correlations of produced particles, one in proton-lead (p–Pb) collisions and the other in lead-lead (Pb–Pb) collisions, are presented in this thesis. The data analyzed in this thesis were collected with the ALICE detector at the LHC in p–Pb collisions at a nucleon–nucleon center-of-mass energy of 5.02 TeV, and Pb–Pb collisions at a nucleon–nucleon center-of-mass energy of 2.76 TeV.

In the forward-central two-particle correlation analysis in p–Pb collisions, two-particle angular correlations between trigger particles in the forward pseudorapidity range (2.5 < |\( \eta \) | < 4.0) and associated particles in the central range (|\( \eta \) | < 1.0) are studied. The trigger particles are muon tracks reconstructed in the Forward Muon Spectrometer, and the associated particles are charged tracks reconstructed in the central barrel tracking detectors. In high-multiplicity events, the double-ridge structure, previously observed in two-particle angular correlations at midrapidity (|\( \eta \) | < 1.2), is also found in the pseudorapidity ranges studied in this analysis. The azimuthal distribution is quantified using the Fourier decomposition, and the second-order Fourier coefficients for muons in the forward pseudorapidity range in high-multiplicity events are extracted after the contributions from jet-like correlations are removed. The coefficients are measured as a function of transverse momentum (\( p_T \)) in the p-going direction and in the Pb-going direction, separately. Similar \( p_T \) dependence of the
coefficients in both directions are observed, with the Pb-going coefficients larger by $16 \pm 6\%$ independent of $p_T$. These observations further characterize the collective features in a small collision system (p–Pb). The results are compared with calculations using the AMPT model, which produces qualitatively the different $p_T$ and $\eta$ dependence of the observables.

In the analysis of the azimuthal collectivity of longitudinal structures in Pb–Pb collisions, the newly developed method is applied to investigate correlations among the longitudinal structures of produced particles in different azimuthal regions. In addition to the expansion of the QGP in the transverse direction, commonly quantified using Fourier coefficients, the initial geometry and resulting longitudinal expansion as a function of azimuthal angle enable us to better understand the full 3-dimensional evolution of heavy-ion collisions. The azimuthal angle is divided into regions in-plane and out-of-plane, and coefficients ($a_n$) of Legendre polynomials from a decomposition of the longitudinal structure at midrapidity ($|\eta| < 0.8$) on an event-by-event basis are estimated in each region for different centralities. Correlations among the longitudinal structures in different azimuthal regions are studied via the correlations among coefficients from the decomposition. The results of conditional $a_n$ measurements indicate collective features of longitudinal structure in the azimuthal direction in particular with the first and second order coefficients, which represent the forward-backward asymmetry and mid-peripheral asymmetry, respectively. The results are compared with various heavy-ion collision models, including the $N_{\text{coll}}$ PYTHIA, HIJING and AMPT models. While the AMPT model shows similar centrality dependence in the conditional $a_1$, the distinctive feature in conditional $a_2$ observed in data is absent in all models considered in this thesis.
Correlations in particle production in proton-lead and lead-lead collisions at the LHC

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To my father, Hyungseok Oh (1955–2011)
Chapter 1

Introduction

The field of relativistic heavy ion physics investigates the nature and properties of strongly interacting matter at extreme energy densities, known as the Quark-Gluon Plasma (QGP). In the QGP, quarks and gluons (partons) are thought to be freed from their confined states and interact with neighboring partons via the strong force. This state of matter can be created in relativistic heavy ion collisions in the laboratory and is equivalent to the state of the entire universe during the first few microseconds after the Big Bang \[1\]. Big Bang cosmology explains the evolution of our universe with a series of phase transitions from the state of extreme temperature and density to the current state that human life exists. Thus, understanding the state of matter in heavy ion collisions corresponds to understanding an important part of the history of our universe.

The QGP created in heavy ion collisions is not detectable via optical devices due to its size (\(10^{-15}\) to \(10^{-14}\) m) and lifetime (~ \(10^{-23}\) sec). Instead final state particles produced from collisions and reconstructed with particle detectors enable us to investigate the nature of the created medium showing specific patterns and characteristics. The aim of this thesis is to broaden our knowledge about the properties of the QGP through geometrical correlations of produced particles in proton-lead (p–Pb) and lead-lead (Pb–Pb) collisions.
Figure 1.1: Summary of measurements of $\alpha_S(Q)$ as a function of the respective energy scale $Q$ [3]. Open symbols represent Next-to-leading-order (NLO) QCD calculation, and filled symbols represent Next-next-to-leading-order (NNLO) calculations used in each analysis. The curves are from the QCD predictions.

1.1 Quantum Chromodynamics

The interaction among quarks and gluons investigated through heavy-ion collisions is generally via the strong force and described by Quantum Chromodynamics (QCD) [2]. QCD is the theory of strong interactions, and a renormalizable, i.e. calculable, gauge theory with symmetry group SU(3). Compared to Quantum Electrodynamics (QED), a quantized version of Maxwell’s electromagnetic theory, 3 color charges play the role of the electric charge and the gluon is the mediating boson in QCD while the photon is in QED. Also, QCD is a non-Abelian theory as gluons may interact among themselves (gluon self-coupling).

The characteristics of QCD can be summarized with the coupling constant of the strong force, $\alpha_s$ (Fig. 1.1) [3]. In the low energy scale regime, or equivalently at low momentum transfer or large distances in particle interactions, the coupling constant increases with lower energy scale, which can explain the fact that no isolated quarks have ever been observed.
This is called “Confinement”, meaning that quarks are bound together by the strong force with no net color charge to the outside. Once the quarks are pulled apart in high-energy scattering, the increase in the energy of the strong force field creates new quarks, which bind with the initial quarks to make up hadrons. On the other hand, in the high energy scale regime, or equivalently at high momentum transfer or short distances in particle interactions, the coupling constant becomes smaller with higher energy scale. This is called “Asymptotic freedom” \cite{4, 5}, and is able to explain the behavior of quarks such as free or weakly bound particles in this regime, observed from the experiments of deep-inelastic scattering \cite{6}.

1.2 Quark-gluon plasma and relativistic heavy-ion collisions

QCD has successfully described a wide range of phenomena, including the mass spectrum of hadrons and deep-inelastic scattering processes \cite{7, 8}. One consequence of QCD theory is that the matter in the regime of QCD should have well-defined thermodynamic properties, which is generally expressed in terms of a phase diagram in the space of thermodynamic parameters \cite{9}. However, analytic calculations of QCD to investigate the phase diagram over the entire QCD regime is not plausible due to the lack of fundamental parameters except the confinement scale $\Lambda_{\text{QCD}} \sim 1 \text{ fm}^{-1}$ in QCD. In the high energy regime, where thermodynamic properties are dominated by short-distance QCD dynamics, perturbative treatment of QCD (pQCD) based on asymptotic freedom is possible for analytic calculations. But pQCD cannot be used in the low energy regime.

The most powerful and successful tool in QCD thermodynamics, including the regime of the low energy scale, is lattice QCD \cite{10}. It is formulated on a discrete Euclidean space-time grid, and requires no parameters or field variables in the discretization. A current view of the QCD phase diagram is shown in Fig. 1.2 with axes of baryon chemical potential ($\mu_B$) and temperature ($T$) \cite{9}. This semi-quantitative sketch combines results from model calculations, empirical nuclear physics, lattice QCD calculations, and pQCD calculations in the regime.
of asymptotic freedom. Two important regions within the discussion of this thesis are the QGP, which is the intended state of matter created from heavy-ion collisions, and the region of nuclear matter in the phase diagram. Similar to the concept that liquid water changes its phase into vapor or ice depending on the temperature and pressure, nuclear matter also changes its phase depending on temperature and density. Lattice QCD estimates that the transition between hadronic matter and the QGP arises between 160 to 200 MeV. Figure 1.3 shows $\epsilon/T^4$ as a function of temperature from lattice QCD calculations, where $\epsilon$ and $T$ correspond to the energy density and temperature, respectively [11]. Since $\epsilon/T^4$ is proportional to the number of degree of freedom, the sharp increase of $\epsilon/T^4$ between 160 to 200 MeV can be interpreted as the phase transition of hadronic matter with increasing temperature. Above this temperature, the degrees of freedom are no longer from nucleons, but from quarks and gluons. Then $\epsilon/T^4$ reaches a plateau at about 90% of the non-interacting ideal gas limit (Stefan-Boltzmann limit), indicating that quarks and gluons in this phase are weakly coupled.

The attempt to create and investigate the de-confined state of quarks and gluons in the laboratory has materialized through heavy-ion collisions in multiple experiments, such as the Alternating Gradient Synchrotron (AGS) at Brookhaven National Laboratory (BNL) and the Super Proton Synchrotron (SPS) at CERN in the 1990s, the Relativistic Heavy Ion Collider (RHIC) at BNL and the Large Hadron Collider (LHC) at CERN from the
2000s to the present. Heavy-ion collisions can concentrate sufficient energy above the prediction of lattice QCD, with quarks and gluons temporarily in the de-confined state, which eventually expands and generates hadrons. With increasing energy of the collision and advances in the measurements and techniques, various aspects of QGP have been observed: from the hint of the de-confined state (SPS) to precise and systematic properties of the QGP (RHIC and LHC).

1.3 Nucleon-nucleus collisions

Measurements of observables in heavy-ion collisions are generally compared with the corresponding measurements in nucleon-nucleon collisions (proton-proton (p–p) at the LHC) and nucleon-nucleus collisions (proton-lead (p–Pb) at the LHC). Although the comparison with results of nucleon-nucleon collisions reveals the general properties of the QGP, effects from the structure of the initial state of the colliding nuclei can be only accessed with nucleon-nucleus collisions. In other words, one may disentangle initial-state effects, which originate from the initial structure of the nuclei, from final-states effects related to the hot and dense medium created in heavy-ion collisions through nucleon-nucleus collisions.
In addition to the usage of nucleon-nucleus collisions as a benchmark for the interpretation of the heavy-ion collisions, nuclear parton distribution functions can be studied with nucleon-nucleus collisions. Parton distribution functions (PDFs) from free protons differ from nuclear PDFs due to the shadowing and anti-shadowing of color charges, and the European Muon Collaboration (EMC) effect \cite{16} in the nucleus, and have been investigated by several collaborations, e.g. \cite{17}. With p–Pb collisions at the LHC energy, very low values in the Bjorken-\(x\) regime can be accessed and improve the knowledge of nuclear PDFs.

1.4 Outline

This thesis describes two analyses, forward-central two-particle correlation in p–Pb collisions and azimuthal collectivity of longitudinal structure in Pb–Pb collisions, commonly based on geometrical correlations of produced particles in relativistic heavy ion physics. Relevant background, previous studies and motivations to the analyses are introduced in Chapter 2. An overview of the LHC accelerator complex and the ALICE detectors are provided in Chapter 3 while Chapter 4 explains some details of the data used for the analyses. Chapter 5 and Chapter 6 describe results of forward-central two-particle correlations in p–Pb and azimuthal collectivity of longitudinal structure in Pb–Pb collisions, respectively. A summary and conclusions are provided in Chapter 7. In addition, new correction methods for finite-acceptance effects in two-particle correlation analyses are introduced in Appendix.
Chapter 2

Background and motivation

In this chapter, review of the previous measurements and theoretical framework related to the analyses in this thesis will be provided. In order to facilitate the discussion, we start with definitions of some terminologies used frequently in this thesis.

Centrality

Figure 2.1 shows a sketch of a heavy ion collision event. Not all events are head-on collisions of the two incoming nuclei, and the shape of the interaction volume depends on the impact parameter, $\vec{b}$, which is a vector quantity connecting one center of a nucleus to another in the plane perpendicular to the beam directions ($(x, y)$-plane in Fig. 2.1). Since the interacting medium created from a head-on collision (namely, central collision) is expected to have different features from that from a collision with large impact parameter (peripheral collision), events are categorized by their centrality, which is parameterized by the mag-

Figure 2.1: A sketch of a heavy-ion collision event at finite impact parameter [18].
The uncorrected multiplicity distribution of charged particles in the ALICE TPC ($|\eta| < 0.8$) \cite{19}

$nitude$ of the impact parameter, $|\vec{b}|$. However, the impact parameter is not an observable quantity in an experiment, and the centrality is often determined based on the measured particle multiplicity assuming that the multiplicity is a monotonic function of $|\vec{b}|$. Then the centrality is quantized into several centrality classes with percentile (see below) as shown in Fig. 2.2 where 0–5% class corresponds to the most central collisions and higher percentile classes correspond to more peripheral collisions.

When geometrical properties of collisions are concerned, each centrality class is characterized by the corresponding fraction percentile of the total hadronic cross section. In addition to characterizing the centrality by impact parameter or the fraction of the total hadronic cross section, centrality can also be characterized by the number of binary nucleon-nucleon collisions or the number of participating nucleons, which is defined by the number of nucleons that experience at least one inelastic collision. These quantities are calculated based on the Glauber model \cite{20}, which provides a description of the nuclear geometry, related to the impact parameter $|\vec{b}|$ of each centrality class. In practice, centrality is estimated based on signals from a certain detector or multiple detector systems. Discrepancy between the results of observables from different centrality estimators are often accounted for in the
systematic uncertainty.

**Pseudorapidity**

In experimental particle physics, the momentum of a particle, $\vec{p}$, can be decomposed into the transverse component, $p_T$, perpendicular to the beam-direction, and the longitudinal component $p_L$, along the beam direction. Also, instead of Cartesian coordinates, $(x, y, z)$ in Fig. 2.1, the kinematics of produced particles are generally expressed using the azimuthal angle, $\varphi$, in the transverse plane, and pseudorapidity, $\eta$.

Rapidity, $y$, is defined by

$$ y \equiv \frac{1}{2} \ln \left( \frac{E + p_L}{E - p_L} \right), \quad (2.1) $$

where $E$ is the energy of the particle. Rapidity contains similar information as the polar angle, $\theta$, in polar coordinates, but it has a benefit that the difference of two rapidities are conserved under Lorentz boost in the beam direction. This is particularly important, as partons from incoming nuclei carry different longitudinal momentum and each of the rest frame of parton-parton collisions is boosted differently in the beam direction. Similarly but slightly differently, pseudorapidity, $\eta$, is defined by

$$ \eta \equiv \frac{1}{2} \ln \left( \frac{|\vec{p}| + p_L}{|\vec{p}| - p_L} \right), \quad (2.2) $$

and also can be written as

$$ \eta = -\ln \left[ \tan \left( \frac{\theta}{2} \right) \right], \quad (2.3) $$

where $\theta$ is the polar angle. In other words, pseudorapidity is determined by $\theta$ only while rapidity needs the energy of the particle or equivalently the mass of the particle. In the limit that the mass of the particle is negligible compared to the momentum, rapidity is approximately the same as the pseudorapidity since $E^2 = m^2 + |\vec{p}|^2$. The value of $\eta$ is equal to 0 for a particle moving perpendicular to the beam axis, and positive (negative) if the longitudinal component of a moving particle is positive (negative). The region near
\( \eta \sim 0 \) is referred to as the midrapidity region, and positive and negative \( \eta \)-regions are called forward- and backward-rapidity regions, respectively.

### 2.1 Evolution of a heavy ion collision

The QGP created in a relativistic heavy-ion collision evolves through multiple stages. The current theoretical framework for the evolution of a heavy-ion collision [21, 22] deals with each stage discretely, and the level of our understanding in each stage varies. The current description starts from the formation of the initial state, which includes the boost of two incoming nuclei. Nuclei are highly Lorentz-contracted before the high-energy collision, and form a certain geometry of the interaction volume depending on the impact parameter and parton distributions. With the arrival of two incoming nuclei into a collision, the pre-equilibrium stage starts and lasts up to 1.5 fm/\( c \), although not precisely defined. During this non-equilibrium stage, gluon fields are thought to dominate the energy density and several models have been developed, such as the model based on a Color Glass Condensate (CGC) [23].

The pre-equilibrium stage is followed by the stage of thermally-equilibrated matter, which is the QGP. Although QGP is one of the phases in the QCD phase diagram (Fig. 1.2), the Standard Model and QCD can make predictions for only a limited number of observables regarding the QGP so far. Instead, many collective aspects and bulk properties of the QGP have been explained by effective descriptions. In the QGP, the fact that the mean-free-path of the particles is much smaller than the system size makes (viscous) hydrodynamics applicable. The QGP stage lasts up to 10 fm/\( c \) as the system continues to expand and cool down, and the system eventually becomes dissipative and hadronizes. Passing through the hadronization stage, usually simulated by a hadron cascade, the system reaches chemical freeze-out where there are no inelastic flavor-changing scatterings afterwards, i.e. the final stable hadron yields hardly change after this point. The last stage of the evolution is the kinetic freeze-out, where elastic scattering between hadrons stops, and the hadrons produced
through these stages free-stream to the detectors.

The importance of each stage varies depending on profile of the collision, as well as the characteristics of the observable in the measurement. Indeed there are various observables in relativistic heavy ion physics, and different observables may reveal different aspects in the evolution.

2.2 Collective flow

The collective expansion of the system is called flow, and it has been considered as major evidence for formation of the QGP in relativistic heavy ion collisions \[1\]. Collective flow is sensitive to properties of the early stage of the system evolution, especially providing direct information about the system properties at the QGP stage. In non-central heavy ion collisions, the initial transverse geometry of interaction region forms an anisotropic shape, namely, an almond-shaped region (Fig. 2.1). Figure 2.3 shows the time evolution of this almond-shaped volume in the transverse plane, where contours represent the energy density profile. Through the evolution, the initial almond-shaped region loses its asymmetry and changes into an almost symmetric shape. Meanwhile, the stronger expansion in the \(x\)-axis results in the change of the momentum space distribution, from approximately symmetric in the transverse plane into a preferred direction along the \(x\)-axis. In other words, the initial anisotropy in geometry creates the pressure gradients, and the pressure gradients result in the momentum anisotropy through the expansion of the created medium. The anisotropy
of the momentum space distribution is developed during the early stage of the collision, and it supports the statement that the measurement of collective flow is most related to the system properties in the QGP stage.

For quantification of the collective flow, the particle momentum distributions are correlated to the reaction plane, which is defined with the beam axis and the impact parameter ((x, z)-plane in Fig. 2.1). The azimuthal distribution of particles with respect to the reaction plane is generally decomposed with Fourier harmonics [24]:

$$E \frac{d^3N}{d^3p} = \frac{1}{2\pi} \frac{d^2N}{d p_T dy} \left( 1 + \sum_{n=1}^{\infty} 2v_n \cos(n(\varphi - \Psi_{RP})) \right), \quad (2.4)$$

where $\Psi_{RP}$ is the azimuth of the reaction plane, $p$ the momentum, $p_T$ the transverse momentum, and $v_n$ the $n$-th order coefficient of the Fourier decomposition. Although both sine and cosine terms are used in the standard Fourier decomposition, the sine terms are absent in Eq. 2.4 as they vanish due to the reflection symmetry with respect to the reaction plane. Each of the coefficients, $v_n$, parameterizes the strength of different anisotropies in the azimuthal momentum distribution, with the collective flow represented by $v_1$, $v_2$, and $v_3$ called directed flow, elliptic flow, and triangular flow, respectively. For example, the elliptical expansion shown in Fig. 2.3 results in larger population of final state hadrons in $x$-axis than $y$-axis, and $v_2$ is estimated to have a significant value.

However, the impact parameter or the reaction plane may not always represent the transverse geometry of a collision, since the distribution of nucleons is not a continuous and even function, but quantized with nucleons on an event-by-event basis. The positions of the participating nucleons in a collision fluctuate event-to-event, and the geometry of the participant plane is represented in Fig. 2.4. Instead of the impact parameter, the azimuthal angle of the $n$-th order axis of symmetry of the participating nucleon distributions corresponds to the $n$-th order participant plane. In other words, $\Psi_{RP}$ in Eq. 2.4 is replaced by the $n$-th order participant plane, $\Psi_{n,PP}$, and $\Psi_{n,PP}$ is determined by the geometrical distribution of the participating nucleons.
The reaction plane and participant planes are not directly observable as they are defined from the initial geometry of the collision or initial distribution of participating nucleons. Experimentally, the final state hadrons, not the initial profile of the collision, are measured in detectors. As a consequence, the $n$-th order event plane, $\Psi_{n,EP}$, is introduced as an approximation of the reaction plane and participant planes, and defined by the azimuthal angle of the $n$-th order axis of symmetry of the distribution of produced hadrons. More details on how to measure the event plane are discussed in Chap. 6.

2.2.1 Measurement of $v_n$

The $v_n$ in Eq. 2.4 are quantifying parameters of different anisotropy in the azimuthal momentum distribution, and characterize the anisotropic expansion of the system. They conceptually have a clear definition, but how to measure them in an experiment are a distinct problem. Measurement of the $v_n$ are based on particles reconstructed in the detectors, but the momentum distribution of particles does not solely originate from the collective flow. Contributions other than collective flow to the momentum distributions are usually referred to as non-flow, and contain various physics mechanisms, such as jets, resonance decay, the
Hanbury-Brown and Twiss (HBT) effect, and momentum conservation \cite{25, 26}. Also, the \( v_n \) for a given impact parameter or a given centrality fluctuate due to the random nature of the initial geometry of participant nucleons in the interaction volume. The effects from fluctuations are generally referred to as flow fluctuations.

Over the past two decades, measurement methods of \( v_n \) have advanced significantly to account for effects from non-flow and flow fluctuations. The development of new methods not only considers the ability to suppress the contributions from non-flow or flow fluctuations, but also computing power needed for the calculations. Methods of the \( v_n \) measurement include the event plane method \cite{27}, pair-wise correlation method (two-particle correlation method) \cite{28, 29}, scalar product method \cite{30}, multi-particle cumulant method \cite{31, 32}, and Lee-Yang Zeros method \cite{33, 34}. In practice, measurements relying on a certain method borrow the underlying procedures of the corresponding method, but various steps are usually added for further characterization of the observables.

Different methods have different sensitivity to non-flow and flow fluctuations. More details on each method and comparison of the sensitivity among these methods are summarized in \cite{22}. Figure 2.5 shows the results of \( v_2 \) measurements with various methods as a function of centrality in heavy ion collisions. The discrepancy among points from different methods reflects the different sensitivity of each method to the effects of non-flow and flow fluctuations. Meanwhile, the general increase of positive \( v_2 \) values from the most central to the mid-central classes indicates the elliptic expansion of the system in non-central collisions as expected in Fig. 2.3.

Measurements of \( v_n \) have been made for various conditions: collision energy, centrality, momentum, particle species, rapidity, etc. Of particular interest are the results shown in Fig. 2.6 \( v_2 \) for the 20–30% centrality class as a function of collision energy. The \( v_2 \) increases continuously above 2 GeV, but most dramatically from negative to positive value near \( \sqrt{s_{NN}} = 4 \) GeV. This transition may indicate the onset of a nuclear phase change \cite{35}. Also, positive \( v_2 \) at high energy justifies the hydrodynamic treatment of the system.

In (viscous) hydrodynamics, the system is often characterized by the ratio of shear viscos-
Figure 2.5: The $v_2$ of charged particles in the $p_T$ range $0.2 < p_T < 5.0$ GeV/c as a function of centrality in Pb–Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV measured by the ALICE Collaboration using various methods. Data points from the STAR Collaboration are for Au–Au collisions at $\sqrt{s_{NN}} = 200$ GeV, in the $p_T$ range $0.15 < p_T < 2.0$ GeV/c [19].

Figure 2.6: The $v_2$ for the 20–30% centrality class at 2.76 TeV for Pb–Pb collisions (ALICE) compared with results at lower energies taken at similar centralities [19].
Figure 2.7: The transverse momentum dependence of $v_2$ measured by the STAR collaboration and the corresponding calculations based on viscous hydrodynamics [18].

...ity to entropy density, $\eta/s$. While $\eta/s$ can be investigated using (viscous) hydrodynamics and $v_2$ measurements [36], calculations based on the anti-de Sitter/conformal field theory (AdS/CFT) correspondence sets a lower bound of $\eta/s > 1/4\pi$ [37]. Figure 2.7 shows an example of the $v_2$ calculation based on viscous hydrodynamics, compared to the experimental measurement. The fact that the extracted $\eta/s$ is close to the lower limit indicates that the QGP is nearly a perfect fluid.

With the help of particle identification capability in detectors, $v_2$ for various species can be separately measured and exhibit distinguishing features. One important result from identified particle $v_2$ measurements is shown in Fig. 2.8. While $v_2$ as a function of $p_T$ or the transverse kinetic energy, $KE_T = m_T - m$, for different species do not agree with each other, they lie on the same curve once they are scaled by the number of constituent quarks ($n_q$), which differs for mesons versus baryons. This scaling property indicates that the degrees of freedom in the collective flow are quarks instead of hadrons.
2.2.2 Two-particle correlations

The two-particle correlation (or pair-wise correlation) method is one of the most widely used methods in relativistic heavy-ion physics for various reasons. It is an effective method to investigate not only the collective flow [29], but also jet-like correlations [39] depending on its usage. In the case of collective flow studies, $v_n$ is extracted based on the fit of the two-particle azimuthal distribution:

$$\frac{dN_{\text{pair}}}{d\Delta \varphi} \propto \left( 1 + \sum_{n=1}^{\infty} 2 v_n^2 \cos(n \Delta \varphi) \right), \quad (2.5)$$

where all pairs of particles in a given momentum region are included in the left side of the equation and $\Delta \varphi$ is the difference between azimuthal angles of two particles in a pair. To suppress the non-flow effects in the extraction of $v_n$, various techniques are further added, such as using only large $\Delta \eta$ pairs. Figure 2.9 shows the correlation function, $C(\Delta \varphi)$, which is equivalent to the left side of Eq. 2.5 from 2.76 TeV Pb-Pb collisions with 0–2% centrality and its decomposition with the first to the fifth order Fourier harmonics [40].

Generally, the two-particle correlation study begins with generating the two-particle correlation function (or equivalently, the per-trigger-particle associated yield [41]) in $(\Delta \varphi, \Delta \eta)$-space. The differences in $\varphi$ and $\eta$ between trigger and associated particle provides input...
to this function, namely, $\Delta \varphi$ and $\Delta \eta$. Figure 2.10 provides examples of the correlation functions in different collisional systems. The regions near $\Delta \varphi \sim 0$ and near $\Delta \varphi \sim \pi$ are called near side and away side, respectively. If two particles of a pair have the same or similar (opposite) azimuthal angles, this pair contributes to the near side (away side) of the correlation function. Particle correlations originating from resonance decays, jet fragmentation, and Bose-Einstein correlations are generally localized over a small range in $(\varphi, \eta)$, or opposite $\varphi$ angle (e.g. back-to-back jets, called di-jets), and they are categorized as short-range correlations (SRC). On the other hand, correlations that influence wide $\eta$ regions, such as anisotropic flow, are referred to as long-range correlations (LRC). Each region of correlation function has different levels of contributions from different correlation mechanisms. For example, a low-multiplicity p–Pb collision event or equivalently a p–p collision event is not expected to form a QGP or exhibit collective flow, and the dominant correlation mechanisms are SRC. The correlation function from low multiplicity p–Pb collisions (Fig. 2.10 top) agrees with this expectation, where a sharp peak near $(\Delta \varphi, \Delta \eta) \sim (0,0)$ region is explained by jet-fragmentation and the ridge structure in the away side $(\Delta \varphi \sim \pi)$ extended over $\Delta \eta$ is explained by back-to-back di-jet signals. Meanwhile, the correlation function from Pb–Pb collisions (Fig. 2.10 bottom) contains distinguishable structure on
Figure 2.10: Two-particle correlation functions from p–Pb collisions (top) [42] and Pb–Pb collisions (bottom) [40]. Detailed conditions, such as collisional energy, centrality, and $p_T$ ranges for trigger and associated particles ($p_T^t$ and $p_T^a$) are included in plots.
the near side ($\Delta \varphi \sim 0$) at large $|\Delta \eta|$. Contributions from SRC also exist in the correlation function, but the ridge structure in the near side over the extended $\Delta \eta$ cannot be attributed to SRC.

Considering the dependence on the correlation mechanism in each region in $(\Delta \varphi, \Delta \eta)$-space, two-particle correlation functions can be used to investigate specific physics mechanisms. Jet-like correlation studies deal with regions near $(\Delta \varphi, \Delta \eta) \sim (0, 0)$ and the away side after some treatment, while collective flow studies consider large $|\Delta \eta|$ regions where SRC contributions significantly decrease. For example, the extraction of $v_n$ in Fig. 2.9 uses only the $|\Delta \eta| > 1.0$ regions of the correlation function.

2.3 Motivation: Double-ridge structure in p–Pb collisions

Shortly after the p–Pb collision period at the LHC in early 2013, LHC collaborations independently reported the observation of extended near-side structure in the $\Delta \eta$-direction in high-multiplicity p–Pb events [41,42]. Figure 2.11 shows the associated yield per trigger particle in $(\Delta \varphi, \Delta \eta)$ in the highest multiplicity class (0–20%) and the lowest multiplicity class (60–100%) [42]. Similar to the correlation function in Pb–Pb in Fig. 2.10, the near-side ridge structure in large $|\Delta \eta|$ is observed in the high multiplicity classes. To isolate the contribution from LRC in high multiplicity classes, the associated yield in a lowest multiplicity class is subtracted from that in the high multiplicity classes. This is done since the associated yield in the lowest multiplicity class is dominated by the SRC, and the level of the SRC is assumed to be invariant in all multiplicity classes. The subtraction results in producing symmetric double-ridge structures in the near and away sides (Fig. 2.12). The $v_2$ can be properly extracted from this yield (Fig. 2.13), and found to have comparable values to those measured in heavy-ion collisions. The double-ridge structure is considered as the possible evidence for the LRC or collectivity in p–Pb collisions. With similar observations in high-multiplicity p–p collisions [44] and deuteron-gold collisions [45], these results triggered intense debate about the origin of the collectivity in small systems. Experimentally,
Figure 2.11: The associated yields per trigger particle in $(\Delta \phi, \Delta \eta)$ for pairs of charged particles in p–Pb collisions at $\sqrt{s_{NN}} = 5.02$ TeV. The $p_T$ ranges and multiplicity classes are shown in each plot [42].

Figure 2.12: The associated yield per trigger particle in $(\Delta \phi, \Delta \eta)$ for pairs of charged particles in p–Pb collisions at $\sqrt{s_{NN}} = 5.02$ TeV in 0–20% multiplicity class after subtracting the yield [43].
the existence of LRC in small systems is further characterized by correlation measurements with the multi-particle cumulants methods \cite{46,48} or measurements with identified particles \cite{49,51}. The current experimental knowledge about the small system is summarized in \cite{52}. Meanwhile, several theoretical models have addressed the LRC structures in small systems, including hydrodynamic models \cite{53,58}, models based on initial-state effects such as Color Glass Condensate (CGC) Effective Field Theory (EFT) \cite{59,64}, and models based on final-state parton-parton induced interactions \cite{65,68}.

Additional insight into the production mechanism of the long-range correlation structures, namely the double-ridge structure in p–Pb collisions, can be achieved by studying the $\eta$-dependence of the ridges, as different models have different predictions for it. The CMS Collaboration reported preliminary results on the $\eta$-dependence (Fig. 2.14), but the $\eta$-acceptance of this measurement is limited to the trigger particle $\eta$ range $2.0 < |\eta| < 2.4$, and associated particle $\eta$ range $|\eta| < 2.0$ \cite{43}. Also, the STAR Collaboration extended the $\eta$-coverage in the $v_2$ measurement from two-particle correlations in d–Au collisions using the Forward Time Projection Chamber (FTPC), and reported $v_2$ at $2.0 < |\eta| < 4.5$ in both d-going and Au-going directions \cite{69}. Meanwhile, predictions from 3+1 dimensional viscous hydrodynamics and a multi-phase transport model (AMPT) were made for $v_2$ for the $\eta$ range, $2.5 < |\eta| < 4.0$, in p-going and Pb-going direction in 5.02 TeV p–Pb collisions \cite{70}.
Figure 2.14: Self-normalized anisotropy parameters, $v_2(\eta_{\text{lab}})/v_2(\eta_{\text{lab}}=0)$ (left panel) and $v_3(\eta_{\text{lab}})/v_3(\eta_{\text{lab}}=0)$ as a function of $\eta_{\text{lab}}$ in p–Pb collisions at $\sqrt{s_{\text{NN}}}=5.02$ TeV. Data points (curves) are results with (without) low-multiplicity data subtraction [43].

In ALICE, the Forward Muon Spectrometer (FMS) covers the $\eta$ region $-4.0 < \eta < -2.5$ and $v_2$ can be measured in this region in 5.02 TeV p–Pb collisions extending the previous measurement beyond mid-rapidity $|\eta| < 1.2$. The corresponding results have been published in [71], and details will be described in Sec. 4 and 5.

2.4 Measurements in the longitudinal-direction in heavy-ion collisions

The extensive measurements of $v_n$ in heavy-ion collisions have revealed various characteristics of medium expansion in the transverse direction. Meanwhile, less work has been done both experimentally and theoretically in the longitudinal direction, which is defined by the direction spanned by pseudorapidity. As the initial geometry and the subsequent medium expansion in the transverse direction are investigated with many interesting observables and models, the initial geometry in the longitudinal direction, namely density fluctuations in pseudorapidity [72], and the consequent response of the medium through the expansion may suggest different aspects of a heavy-ion collision.

A few previous works in the longitudinal direction include the pseudorapidity density distribution [73,75] and forward-backward (FB) multiplicity correlation [76,77]. Figure 2.15
Figure 2.15: The $dN_{ch}/d\eta$ per centrality bin from 2.76 TeV Pb–Pb collisions with predictions from models [75].

shows the pseudorapidity density distribution of charged particles measured by the ALICE Collaboration, and corresponding predictions from three different models [75]. While this measurement utilizes the average pseudorapidity distribution from given centrality classes, the FB multiplicity correlation analysis is based on the simultaneous measurement of multiplicities in two $\eta$-windows on an event-by-event basis. The correlation strength, $b$, is defined by

$$b = \frac{\langle N_f N_b \rangle - \langle N_f \rangle \langle N_b \rangle}{\langle N_f^2 \rangle - \langle N_f \rangle^2}, \quad (2.6)$$

where $N_f$ and $N_b$ are the multiplicity in the forward and backward pseudorapidity windows, respectively, and the bracket corresponds to the average over a given centrality class. A significant correlation strength at central to mid-central heavy ion collisions is observed, and it adds additional complexity to theoretical models.

In principle, the underlying concepts of these two measurements in the longitudinal direction can be applied in the transverse direction in the same manner. The pseudorapidity density distribution becomes the azimuthal density distribution in the transverse direction, but it yields a flat distribution due to the random nature in the azimuthal angle of the impact
parameter. If the azimuthal density distribution is evaluated with respect to the azimuthal angle of the impact parameter, equivalently $\Psi_{RP}$, it is similar to measuring the average $v_n$ for a given centrality. If the method of FB correlation measurement is used in the transverse direction, multiplicities in two $\varphi$ windows are simultaneously measured in each event. If $\varphi$ windows can be determined with respect to the $\Psi_{RP}$, the corresponding correlation strength may reveal similar characteristics of the expansion of the system as the $v_n$ measurement does.

As the above examples demonstrate, several approaches can exist to investigate the same underlying physics mechanism. However, different approaches have different sensitivity or effectiveness, as $v_n$ describes the collective flow more intuitively than the correlation strength between two $\varphi$ windows.

More recently, measurements in the longitudinal direction earned more field-wide attention in various conferences and workshops [78,79], along with the development of the new methodologies. One interesting measurement is discussed in [80]. Zero Degree Calorimeters (ZDC) in ALICE detectors estimate the neutron spectators, neutrons in incoming nuclei not participating in a collision, in both beam directions, and the discrepancy in the ZDC signals in the two directions indicates the asymmetry in the number of participating nucleons in a collision. The discrepancy in the ZDC signals in two directions is quantified in each event, and $dN/d\eta$ in events with the large discrepancy is observed to be distinguished from that with little discrepancy. In other words, the asymmetry in the number of participating nucleons, or equivalently asymmetry in the initial longitudinal geometry, is observed to influence the pseudorapidity distribution of the final state particles.

Of particular importance is the newly-developed approach to the longitudinal structure via decomposition with the orthogonal polynomials [81]. As the azimuthal distribution is decomposed with the Fourier harmonics and each of the corresponding coefficients parameterizes a specific transverse pattern, so is the longitudinal distribution decomposed with the orthogonal polynomials, such as Legendre polynomials. Although the initial proposal from [81] suggested to use the Chebyshev polynomials, any orthogonal polynomials can be used. Coefficients from the decomposition are the quantifying parameters of the longitudinal structure, and proposed to be estimated by the two-particle pseudorapidity correlation [81].
The coefficients from the longitudinal decomposition, $a_n$, via two-particle longitudinal correlations have been measured in various collision systems (p–p, p–Pb, and Pb–Pb) by the ATLAS Collaboration [83] and some theoretical event generators [84]. The two-particle longitudinal correlation function utilizes the $\eta$ of the two particles in each pair, and extracts $\langle a_n a_m \rangle$ from the correlation function $C(\eta_1, \eta_2)$. Further details of the pseudorapidity two-particle correlation function and related observables are summarized in Appendix C. Figure 2.16 shows the $\langle a_n^2 \rangle$ and $\langle a_n a_{n+2} \rangle$ from the two-particle longitudinal correlation function in various collision systems [83]. The longitudinal particle distribution is further characterized by separating the contributions from SRC and LRC as done in the transverse distribution analysis.
2.5 Motivation: Azimuthal dependence of the longitudinal structure in heavy-ion collisions

Correlation measurements through $v_n$ and $a_n$, coefficients of decompositions in the transverse and the longitudinal direction, respectively, quantify the transverse and longitudinal structures of produced particles independently. However, both approaches can be considered to limit the structures of produced particles into a certain direction, or use the projection of the 3-dimensional structure into a certain plane. If the 3-dimensional expansion of the created medium can be indeed studied with the 3-dimensional approach on an event-by-event basis, our understanding of the medium will be raised to a different level.

The $\eta$-dependent $v_n$ measurements in Pb–Pb collisions [85] are an example of the 3-dimensional approach in the sense that they consider both the transverse and the longitudinal directions. However, $v_n$ at each $\eta$ are averaged over all events in a given centrality bin, which loses information on the event-by-event $\eta$-dependence of $v_n$. The $v_n$ are parameters quantifying the transverse structure, and an event-by-event expansion in the 3-dimensional space is distinguished from what was studied in $v_n(\eta)$. A more advanced forward-backward correlation study in p–p collisions, which utilizes the multiplicity correlations in two $(\eta, \phi)$ windows, is closer to the definition of the 3-dimensional approach [86]. However, as the longitudinal correlation study is improved and generalized from the forward-central multiplicity correlation in two $\eta$ windows [76, 77] to measurements via longitudinal decomposition [81, 83], further generalization is possible. Instead of either transverse or longitudinal decomposition, the 3-dimensional decomposition with spherical harmonics is one possible scenario [87].

Another way to pursue the 3-dimensional structure is based on the correlation among $\eta$ structures in separate $\phi$ bins. This is fundamentally distinguished from $\eta$-dependent $v_n$ measurements in [85], not only by using the longitudinal decomposition, but more importantly by using the event-by-event correlations in two orthogonal directions. Since the event plane can be determined in each event, the $\phi$ angle can be divided into several bins with respect to the event plane. The $\eta$ structure in each $\phi$ bin is decomposed with orthogonal
polynomials, and the correlation among extracted coefficients at each $\varphi$ bin is equivalent to the correlation among $\eta$ structures in different $\varphi$ bins. This underlying idea is developed further, and the results will be presented in Chap. 6.
Chapter 3

The LHC Complex and the ALICE Experiment

3.1 The Large Hadron Collider

The Large Hadron Collider (LHC) is by far the largest human-made particle collider, located beneath the France-Switzerland border near Geneva, Switzerland. It is designed to address some of the fundamental concepts in physics, including the Higgs mechanism, supersymmetry, dark matter, Quark-Gluon Plasma and others. It can be considered as a successor of the Large Electron-Positron Collider (LEP) as it inherited the 27km tunnel of the LEP. It essentially consists of two separate rings with separate magnet systems, and accelerates two counter-rotating hadrons. This design with two separate rings makes the LHC capable of accelerating complex species of hadrons and operating various beam configurations up to the energy of a 7 TeV proton beam. The LHC includes four main intersection regions, two high luminosity experiments (ATLAS and CMS), one B-meson experiment (LHCb), and one dedicated heavy-ion experiment (ALICE) as shown on Fig. 3.1.

The initial discussion on the possibility of a proton-proton collider using the LEP tunnel
Figure 3.1: Overall view of the LHC with its four main detectors [96].

appeared in 1983, and the official approval of the LHC Project by CERN (European Organization for Nuclear Research) Council was made in 1994. After the design stage of the experiments and machines, the LHC was constructed between 1998 and 2008. Through the first operational run from late 2009 to early 2013, experiments of particle collisions with several beam energies and configurations were conducted. This produced fruitful physics results such as the discovery of Higgs boson [97,98]. Some essential systems and components of the LHC will be introduced below.

**Beam preparation**

Figure 3.2 shows a schematic view of the LHC including the four main detectors and beam preparation systems [89]. Before particles are injected into the main LHC rings, they are accelerated by a series of systems increasing their energy up to 450 GeV [88]. At the first stage, LINear ACcelerator (LINAC) 2 and 3 generate proton and ion beams, respectively. The proton beam is passed to the Proton Synchrotron Booster (PSB), and the ion beam
Figure 3.2: Schematic view of the LHC with its four main detectors and beam preparation systems [89].
to the Low Energy Ion Ring (LEIR), and both are consequently directed into the Proton Synchrotron (PS). Finally, both beams are further accelerated in the Super Proton Synchrotron (SPS) and injected into the main LHC rings. The energy of the beams is increased at each stage, and beams are steered by transfer lines in between systems.

The magnets system

As in other circular accelerators, magnets play an essential role in the operation of the LHC. Magnetic force,

\[ \vec{F} = q\vec{v} \times \vec{B}, \quad (3.1) \]

steers the charged particle, and superconducting magnets of the LHC generate desired beam properties through magnetic fields. There are three types of the LHC magnets, these are dipole magnets, quadrupole magnets, and correction elements. Dipole magnets bend the trajectory of charged hadrons to keep them in the beam channel, and quadrupole magnets focus hadrons into tight beams. Correction elements provide smaller dipolar correction to the beam, as well as prevent them from nonlinear resonances and beam instabilities.

These magnets are installed throughout the LHC and approximate 10,000 in number. Another schematic view of the LHC in Fig. 3.3 shows the more detailed layout of the main LHC rings [88]. The LHC consists of eight arcs (ARC), straight sections (Long Straight Section, LSS) and dispersion suppressor cells (DSL, DSR) in a 27 km circumference. As shown in Fig. 3.3, sections are labelled with numbers from 1 to 8, and detectors are located at different sections (ATLAS at point 1, ALICE at point 2, CMS at point 5, LHCb at point 8). Also, beam injections into the LHC rings take place near Interaction Point (IP) 2 for ring 1 and IP8 for ring 2. Each arc consists of 23 LHC cells, and the schematic layout of 106.9 m cell is depicted in Fig. 3.4. Six dipole magnets in each cell keep the beam in the semi-circular orbit, and the maximum field strength of 8.3 Tesla of the dipole magnet is capable of a maximum energy per proton beam of 7 TeV. Each straight section is about 528 m long and provides room for detectors and utilities insertions, such as the Radio Frequency system. Dispersion suppressor cells (DSL, DSR) are located either side of the arcs for the
Figure 3.3: Schematic layout of the LHC main rings with sections [88].

Figure 3.4: Schematic layout of one LHC cell. (MQ: Lattice Quadrupole, MO: Landau Octupole, MQT: Tuning Quadrupole, MQS: Skew Quadrupole, MCB: Combined Lattice Sextupole or skew sextupole(MSS) and Orbit Corrector (MCB), BPM: Beam position monitor, MBA: Dipole Magnet Type A, MBB: Dipole magnet Type B, MCS: Local Sextupole corrector, MCDO: Local combined decapole and octupole corrector) [99].
reduction of the horizontal dispersion with quadrupoles.

**Radio frequency system**

The direction of particles can be controlled by the magnets system, but magnetic field cannot increase the energy of particles. After the beam is injected into the LHC rings, it is further accelerated in the Radio Frequency (RF) system, located at point 4 [88][89]. The RF system consists of 16 RF cavities; a single RF cavity is essentially a metallic chamber that has an oscillating electromagnetic field in it. Due to the two-ring design of the LHC, RF systems are designed to work on two rings independently and capable of controlling beams of different species. They provide 16 MV circumferential voltage at high energy, and the electromagnetic field in RF systems oscillates at 400 MHz. Protons and ions are accelerated or decelerated, depending on its arrival to the RF systems, until they reach the target energy, and the particle beam is sorted into discrete bunches that are spaced by 25 ns apart. During this energy-boosting process, the magnetic field of the LHC also increases.
to keep particles in the orbit. Particles reach the top energy in 15 minutes, and they pass the RF system approximately 1 million times through the accelerating process.

**Other systems and components**

Besides the above beam preparation, magnets, and RF systems, the LHC contains many more systems and components based on cutting-edge technology. Power supplies for each of the systems are as important as the system itself for the operation. Machine protection and interlock systems are carefully designed considering the destructive power of the high energy beam. The vacuum system provides the required condition of the beam pipe, as the beam pipe has to be at ultrahigh vacuum to avoid the scattering of beam particles from collisions with gas molecules in the pipe (beam-gas interaction). The cryogenic system cools down components of the LHC, especially for operations of magnets, RF cavities, and beam pipes. More details of the above systems, and additionally the monitoring system and beam dump system can be found in [88].

### 3.2 ALICE

ALICE (A Large Ion Collider Experiment) is a dedicated heavy-ion detector located at IP2 of the LHC, and is designed to study the nature and properties of the quark-gluon plasma [15,95,101]. The ALICE Collaboration consists of a thousand scientists and engineers from 143 member institutes over 41 countries. Compared to ATLAS and CMS, ALICE utilizes lower beam luminosity. The number of events per second in particle accelerators is generally given by

\[
N_{\text{event}} = \mathcal{L}\sigma_{\text{event}},
\]

where \(\sigma_{\text{event}}\) is the cross section of the event and \(\mathcal{L}\) is the luminosity of the machine. Luminosity in ALICE is determined from visible cross sections measured in van der Meer scans [102,104], and aimed to be \(\mathcal{L}(\text{ALICE}) = 10^{30}\text{ cm}^{-2}\text{s}^{-1}\) for p–p collisions and \(10^{27}\text{ cm}^{-2}\text{s}^{-1}\) for Pb–Pb collisions, while ATLAS and CMS have \(\mathcal{L}(\text{ATLAS, CMS}) = 10^{34}\text{ cm}^{-2}\text{s}^{-1}\) for
p–p collisions.

Figure 3.6 shows a schematic layout of the ALICE detectors. The Inner Tracking System (ITS) determines the primary and secondary vertices, and provides information on the particle tracking along with the Time Projection Chamber (TPC), which surrounds the ITS. On the outside of the TPC, the Transition Radiation Detector (TRD) is used for electron identification and the Time-Of-Flight (TOF) detector provides pion, kaon and proton identification. The PHOton Spectrometer (PHOS), the Electro-Magnetic CALorimeter (EMCAL), and the High-Momentum Particle Identification Detector (HMPID) are three single-arm detectors in the central region. In addition, the Forward Muon Spectrometer (FMS), Zero Degree Calorimeter (ZDC), Photon Multiplicity Detector (PMD), Forward Multiplicity Detector (FMD), and VZERO detector located in the forward regions provide various information on the collision, such as trigger information, particle tracking, and particle identification. The performance of the ALICE detector during the run between 2009 and 2013 is summarized in [106]. In the following subsections, selected detectors, especially used in analyses in this thesis, will be described further.
3.2.1 Inner Tracking System

Semiconductor detectors are based on crystalline semiconductor materials, usually silicon or germanium [107]. The functioning of semiconductor detectors relies on the formation of a semiconductor junction, better known as a diode. When a pn-junction is created, for example by the juxtaposition of a p-type semiconductor with an n-type material for simple descriptions of the functioning, there is an initial diffusion of holes into the n-region and electrons into the p-region. Then diffused electrons fill up the holes in the p-region while diffused holes catch the electrons on the n-region, and this makes the electric field gradient across the junction. The formation of this electric field gradient continues until the diffusion process of holes and electrons stops due to the electric field itself, and the potential difference in the end due to the electric field is known as the contact potential. The depletion zone corresponds to the region across the pn-junction, where there is a certain electric potential gradient, and this zone can be used for particle detection purposes. When a charged particle traverses the diode and ionizing radiation enters this zone, electron-hole pairs are liberated and swept out by the electric field. With external electric contacts on either side of the pn-junction, an electric current signal proportional to the ionization can be detected.

The Inner Tracking System (ITS) consists of six layers of silicon detectors [95, 108, 109], directly surrounding the beam pipe in the ALICE detector as shown in Fig. 3.7.
innermost layers use Silicon Pixel Detectors (SPD), while the two intermediate layers use Silicon Drift Detectors (SDD) and the two outermost layers contain Silicon Strip Detectors (SSD). These six layers are spaced between 3.9 cm and 43.0 cm in radii, and cover the full azimuth and the pseudorapidity range $|\eta| < 0.9$ for all vertices in nominal interaction regions. The main task of the ITS is to provide the precise vertex of a collision and the secondary vertices from the decay of hyperons and heavy-flavor mesons, as well as information on position and momentum of tracks. Especially, very low momentum particles ($< 200$ MeV/$c$) can be reconstructed in the ITS, while these particles do not reach the TPC. The ITS alignment to achieve the high precision of vertex and track reconstruction is extensively discussed in [95], and some features of the SPD, SDD, and SSD are described in the following (with more details in [95]).

**Silicon Pixel Detector**

The two innermost layers with the SPD play an essential role in the determination of the vertex position as well as the secondary vertices from the decay of hyperons and heavy-flavor mesons. Figure 3.7 shows the overall layout of the SPD and beam pipe (inner circle). The 10 radial devices correspond to carbon-fibre support sectors, where six staves are attached
to each sector: two on the inner layer and four on the outer layer. Two SPD modules are mounted together into a half-stave, and two half-staves form a stave. Each of the SPD modules consists of a two-dimensional sensor matrix of $256 \times 160$ reverse-biased silicon detector diodes bump-bonded to readout chips. The resolution values along $r\phi$- and $z$-direction are 12 and 100 $\mu$m, respectively. These SPD layers operate in a region where the track density is the highest of all ALICE detectors, 50 tracks/cm$^2$, and the design of the SPD considers this condition, including its high radiation levels.

**Silicon Drift Detector**

The SDD provides information on tracking and $dE/dx$. The schematic layout of the basic module of SDD is shown on Fig. 3.9. It consists of the central high voltage cathode strip and two drift regions, where electrons move away from the cathode strip under the electric field. While the SPD uses rectangular semiconductor diodes in the fundamental cell, strips of $p^+$-type silicon (+ for heavily doped material) are attached to the bulk of $n$-type silicon on both sides in the SDD. Six of the SDD modules are mounted on a single ladder of the inner SDD layer, and eight modules on a single ladder of the outer SDD layer. As shown on Fig. 3.9, the inner SDD layer consists of 14 ladders and the outer SDD layer consists of 22 ladders with some overlap regions for full angular coverage.

**Silicon Strip Detector**

The SSD is crucial for the matching of the track reconstruction with the TPC and ITS. The fundamental sensor of the SSD is made of $n$-type bulk, where $p^+$-type and $n^+$-type silicon strips are implanted on each side. When a charged particle traverses the sensitive volume and creates electron-hole pairs, electrons and holes are collected in semiconductor strips and transferred into electric signals. Similarly to the SDD ladder and its modules, the SSD modules are mounted on ladders. The inner SSD layer contains 34 ladders and the outer layer is equipped with 38 ladders. Each SSD module has one sensor, which consists of 768 double-sided strip detector connected to the front-end electronics. There are also overlap regions between ladders for full azimuthal coverage.
Figure 3.9: Scheme of a SDD module (top) and scheme of the SDD layers (bottom).
3.2.2 Time Projection Chamber

Time Projection Chamber (TPC) is the most sophisticated tracking and ionization detector capable of providing information on the trajectory and energy loss, $\text{d}E/\text{d}x$, of particles \cite{107}. It has been played an essential role among modern particle detectors, and in ALICE. Conceptually, the TPC combines ideas from the drift chamber and Multiwire Proportional Chamber (MWPC). It consists of a large gas-filled cylinder with a uniform electric field along the beam axis and a parallel magnetic field, and the MWPC endcaps consisting of sector arrays of proportional anode wires and rectangular cathode pads.

The 3-dimensional layout of the ALICE TPC field cage, which is the backbone structure of the TPC, is shown in Fig. 3.10 \cite{95, 111, 112}. The design of the ALICE TPC follows the conventional structure of a TPC, but is innovative in many aspects to work for nuclear collisions at high luminosities and energies at the LHC. The functioning of the TPC starts from the high voltage electrode, located at the center of the 5 m drift volume. It creates a uniform electric field along the beam axis in the directions of both endcaps. Particles generated from collisions at the interaction point first pass through the ITS, and then traverse the TPC cylinder volume producing free electrons by ionization of the gas in the chamber. Free electrons are drifted into both endcaps by the electric field, and detected
by the anode wires as in a MWPC. The position of the ionization of the electrons can be reconstructed by the combination of the position of the firing anode wire, the signal induced on the cathode pad, and the drift time of the ionization electrons in the cylinder volume. Each charged particle moving in the TPC generates multiple ionization, and the trajectory of each particle can be reconstructed combining these space points.

The dimensions of the ALICE TPC are 85 cm inner radius, 250 cm outer radius, and 500 cm of length along the beam axis, and the drift time is about 90 µs. It covers the full azimuth and pseudorapidity range |\eta| < 0.9 for tracks. It is surrounded by the large L3 magnet that provides a \( B = 0.5 \) T field, and is capable of detecting trajectories of particles with \( p_T \) from 0.1 GeV/c to 100 GeV/c with good momentum resolution. The cylinder volume is filled with a Ne – CO\(_2\) – N\(_2\) mixture at atmospheric pressure. The two endplates of the cylinder are 32.5 m\(^2\) in area and divided into 18 sectors holding two readout chambers each.

The ALICE TPC is designed to operate in conditions up to 200 Hz collisional rate of central Pb-Pb collisions with very high multiplicities, which has never been required in past. Careful optimization of the TPC design considering these extreme conditions resulted in the current ALICE TPC, and more details including TPC parameters, electronics, gating grids, laser calibration, and temperature control system are summarized in [112]. Figure 3.11 depicts trajectories of particles from a central Pb-Pb collision, reconstructed with the ITS (tracks in white) and the TPC (tracks in red, color coding according to \( p_T \)).

### 3.2.3 VZERO Detector

The VZERO detector in ALICE is a scintillation detector, composed of two arrays, VZERO-A and VZERO-C [95,114,115]. In general, a scintillation detector is based on the physics property that certain materials emit a flash of light, known as a scintillation process, when struck by a charged particle (or a photon) or radiation. These scintillations are coupled to an amplifying device, usually a photomultiplier, and converted into electrical pulses in order to be analyzed. One of the distinguishing features of a scintillation detector compared to other types of detectors is its fast time response. The fact that its response and recovery
times are shorter than other detectors allows determination of the timing information from different events with great precision, and makes the scintillation detector ideal for use as a triggering source.

The VZERO detector in ALICE provides triggers for the experiment and separates the nominal beam-beam interactions from background beam-gas interactions. It also participates in the measurement of beam luminosity, and centrality based on charged particle multiplicity and azimuthal distribution, such as an event-plane.

The two VZERO arrays are located 329 cm from the nominal vertex on the side of the muon spectrometer (VZERO-C) and 86 to 88 cm on the opposite side (VZERO-A) (Fig 3.12). Each array consists of four rings in the radial direction, and is divided into eight sections in the azimuthal direction. (Fig 3.13). Pseudorapidity coverage and the radius of each ring is summarized in Tab. 3.1. The VZERO-A and VZERO-C have different designs for the elementary counter (Fig. 3.14) considering their different constraints in the installation, such as their position with respect to other detectors and space availability. The scintillation material of the counter is BC404 plastic scintillator from Bicron with a thickness of 2.5 cm for VZERO-A and 2.0 cm for VZERO-C. BCF9929a Wave-Length Shifting (WLS) fibers from

\footnote{Bicron, Saint Gobain Crystals, 104 route de Larchant BP 521, 77794 Nemours Cedex, France.}
Figure 3.12: Position of the two VZERO arrays with respect to other ALICE detectors [114].

Figure 3.13: Sketches of VZERO-A and VZERO-C arrays showing their segmentation [114].

Table 3.1: Pseudorapidity ($\eta$), angular acceptance (degree), radius (cm) and z (cm) position of VZERO-A and VZERO-C [114].
Bicron are attached to the scintillator differently for VZERO-A and VZERO-C as shown in Fig. 3.14. Fibers are embedded in both faces of the scintillator with 1 cm space in between for VZERO-A, and glued along the two radial edges for VZERO-C. Through the fibers, the light is transferred to the R5946070 photomultiplier (PMT) from Hamamatsu\textsuperscript{2} and then output of the PMT is linked to the Front-End Electronics (FEE). A single elementary counter occupies each of the 32 sectors of the VZERO-A (Fig. 3.13), and 48 counters are distributed in VZERO-C, one counter for a sector of two inner rings and two counters for a sector of two outer rings.

### 3.2.4 Forward Muon Spectrometer

The Forward Muon Spectrometer (FMS) is designed to study open heavy flavor production and quarkonia production ($J/\psi$, $\psi'$, $\Upsilon(1S)$, $\Upsilon(2S)$, and $\Upsilon(3S)$) via the $\mu^+\mu^-$ decay channel\textsuperscript{95,116,117}. In addition, the muon reconstructed in the FMS can be used to investigate the azimuthal correlations as in the analysis described in Chap. 5. It covers the pseudorapidity range $-4.0 < \eta < -2.5$, and consists of absorbers, a muon magnet, a tracking system, a muon filter, and a trigger system (Fig. 3.15).

The front absorber, made of carbon and concrete, is located in the solenoid magnet from 90 cm to 503 cm with respect to the interaction point. It protects other ALICE detectors from secondary particles produced within the absorbing material, and reduces the initial flux of primary hadrons from nominal collisions into the tracking chambers. Meanwhile, the muon filter located right after the last tracking chamber is designed to protect trigger

\textsuperscript{2}Hamamatsu Photonics, 360 Foothill Rd Bridgewater, NJ 08807, U.S.A.
chambers and made of iron. To determine muon momenta in the tracking system, a large dipole with a nominal field of 0.7 T in the perpendicular direction to the beam is installed. With the help of the magnetic field from the dipole magnet, the tracking system has a spatial resolution of 100 µm and a momentum resolution of about 1%. The tracking system consists of 5 stations, while each station is made of two chamber planes. Each chamber plane is essentially a MWPC as the endcap of the TPC, consisting of 5 mm gas gap drift MWPC with segmented cathode plane. Two stations are placed before, one inside, and two after the dipole magnet.

Since the FMS system is limited to operate up to 1 kHz event rate, proper triggering of the system is important to take advantage of the full luminosity of the LHC beam. There exist 6 trigger signals that are delivered to the ALICE Central Trigger Processor (CTP) from the FMS, each requiring a single low- (high-)\(p_T\) muon, low- (high-)\(p_T\) like-sign pairs, and low- (high-)\(p_T\) unlike-sign pairs. The trigger system (red in Fig. 3.15) consists of four Resistive Plate Chamber (RPC) planes [118] with trigger electrodes in two stations behind the muon filter. With the help of the trigger electronics, the RPC has a time resolution of 1-2 ns for the identification of an event, and the trigger system has space resolution better than 1 cm.
Chapter 4

Experimental details

The two analyses described in this thesis are based on different collision systems, proton-lead (p–Pb) and lead-lead (Pb–Pb) collisions. Both collisional systems were measured during the LHC Run 1 (2009–2013), and shared some event and track selection criteria. In this chapter, details on event and track selections used in this thesis are summarized.

4.1 Event selection

As described in Sec. 3.1, the bunch crossing of two beams at the LHC occurs every 25 ns at top rate. Compared to the two high-luminosity experiments (ATLAS and CMS), ALICE has a lower luminosity capability as it is mainly designed for heavy-ion collisions. During the LHC Run 1, the luminosity at ALICE was adjusted to its optimal level by varying parameters, such as the number of interacting bunches, the value of the amplitude function at the interaction point, crossing angles, and the separation of colliding beams in the transverse plane [106]. The luminosity reached $2.8 \times 10^{25} \text{ cm}^{-2}\text{s}^{-1}$ in the Pb–Pb run in 2010, $4.6 \times 10^{26} \text{ cm}^{-2}\text{s}^{-1}$ in the Pb–Pb run in 2011 and $5 \times 10^{27} \text{ cm}^{-2}\text{s}^{-1}$ in the p–Pb run in 2013.

With the working luminosity, collision events were recorded after the selection by the ALICE Central Trigger Processor (CTP) [95, 119]. The CTP is able to select events with
various different features at rates that can be suitably reconstructed in accordance with the capability of ALICE detectors. The trigger decision is based on signals in multiple detectors and information about the LHC bunch filling scheme, and consists of three different levels, Level 0 (L0), Level 1 (L1), and Level 2 (L2). The L0 decision is made from signals in fast detectors, such as SPD, VZERO, T0, EMCal, PHOS, and MTR, at 0.9 µs after the collision. The events accepted at L0 are passed to L1, where the decision is made from the TRD, EMCal and ZDC at 6.5 µs after the L0 decision. The final L2 decision is made approximately 100 µs after the collision, providing a specific time window between event reconstructions to reject the events from multiple interactions in the same bunch crossing (pile-up events). The event selection during the experimental run is referred to as the on-line selection, and generally followed by the off-line selection afterwards. Conditions for on-line and off-line event selections vary by the collisional system, period, and the characteristics of the analysis. Two relevant conditions to the analyses in this thesis are further described in the following.

4.1.1 p–Pb collisions in 2013

5.02 TeV p–Pb data used in this thesis were collected in early 2013, which was the first nucleon-nucleus collision period at the LHC following the pilot run in September 2012. The collisions between protons with a beam energy of 4 TeV and fully stripped $^{208}\text{Pb}$ ions with a beam energy of 1.58 TeV per nucleon resulted in a center-of-mass energy of $\sqrt{s_{NN}} = 5.02$ TeV and the shift of the nucleon-nucleon center-of-mass reference system by a rapidity of 0.465 in the direction of the proton beam. Two beam configurations were used during the period: the proton beam direction in the negative $\eta$ (denoted with p–Pb), and in the positive $\eta$ (Pb–p) direction in the ALICE coordinate system. Having two configurations is particularly important considering the asymmetric $\eta$ coverage of the Forward Muon Spectrometer (FMS) in ALICE, which is $-4.0 < \eta < -2.5$. With the p–Pb configuration, final state particles in the direction of the proton beam are reconstructed in the FMS, while particles in the direction of the lead beam are reconstructed in the FMS in the Pb–p configuration.
The period of the p–Pb beam configuration was further divided into a dedicated minimum-bias (MB) block and a rare-trigger block, while the period of Pb–p beam configuration utilized rare-triggers only. The two triggering modes had different interaction rates, 10 kHz for the MB trigger and 200 kHz for the rare-triggers. The on-line event selection combines the MB trigger and two muon triggers out of various rare triggers. The MB selection in the p–Pb period required signals in VZERO-A and VZERO-C simultaneously and covered 99.2% of the non-single-diffraction (NSF) cross section. Two muon triggers required at least one muon above a $p_T$ threshold in addition to the simultaneous VZERO-A and VZERO-C signals, with low $p_T$ (0.5 GeV/$c$) and high $p_T$ (4.2 GeV/$c$) thresholds. The integrated luminosities from the high $p_T$ muon trigger and the low $p_T$ muon trigger were 5.0 nb$^{-1}$ and 0.28 nb$^{-1}$, respectively, for the p–Pb beam configuration, and 5.8 nb$^{-1}$ and 0.26 nb$^{-1}$ for the Pb–p beam configuration.

The off-line event selection was subsequently applied to further clear out the event sample. The analysis on p–Pb data requires at least one muon track in each event, and MB events without a muon track were rejected although only 5% of the MB events had one or more muon tracks. Also, the off-line selection required the primary-vertex position within 7 cm from the nominal interaction point in the beam direction. The tracking efficiency in the central-barrel system (TPC + ITS) depends on the primary-vertex position, which is reconstructed by the SPD clusters, and limiting the vertex position in a certain range ensures consistency of the tracking efficiency throughout events. Pile-up events were effectively removed by rejecting events with more than one reconstructed vertex.

Selected events were then classified into multiplicity classes, similar to the centrality classes in heavy ion collisions. The classification of events was based on multiplicity in the VZERO detectors, but only partial rings of the VZERO detectors were used for consistent results in p–Pb and Pb–p configurations. VZERO-A and VZERO-C cover the $\eta$ range, 2.8 < $\eta$ < 5.1 and −3.7 < $\eta$ < −1.7, respectively. If these full ranges are converted into the center-of-mass reference system, they cover different ranges in the two beam configurations. Therefore only ring 2 and 3 of VZERO-A (2.8 < $\eta$ < 3.9) and ring 0 and 1 of VZERO-C (−3.7 < $\eta$ < −2.7) were used for multiplicity classification providing symmetric $\eta$ ranges for p–Pb and Pb–p.
configurations. The multiplicity class based on the partial usage of VZERO detectors is called V0S in the following discussion.

4.1.2 Pb–Pb collisions in 2011

Although 2.76 TeV Pb–Pb collisions were collected in 2010 and 2011, only data from the 2011 Pb–Pb run were used in the analysis due to the larger statistics in central classes. During the 2011 Pb–Pb run, the interaction rate reached 4 kHz, and ALICE ran with the MB trigger, rare triggers, and centrality triggers concurrently. The on-line selection for the analysis combined the MB trigger and two centrality triggers. The MB trigger required two hits in the outer layer of the SPD and signals in VZERO detectors, as well as signals in both ZDCs. The ZDC signal requirement in the MB trigger suppressed events from only electromagnetic interactions between the lead ions. Two centrality triggers, namely, central and semi-central triggers were based on the above MB trigger conditions with the additional VZERO requirement. The central and semi-central triggers selected events of 0–10% and 0–50% central classes in Pb–Pb collisions, respectively, based on the VZERO multiplicity, and required signals in both VZERO-A and VZERO-C sides.

In addition, the off-line selection limited the primary-vertex position within 8 cm from the nominal interaction point. In general, the overall data consist of smaller chunks of data, and each chunk is assigned a specific run number. It is observed that different run numbers occasionally have low tracking efficiency regions, e.g. run 170387, 170268, 169498, and 168105 in Fig. 4.1 and these runs were not used in the analysis. After the on-line and off-line selection, approximately $10^7$ events with centrality 0–50% were selected for the analysis.

Events were then classified into centrality classes based on the multiplicity in VZERO detectors, and 0–5%, 5–10%, 10–20%, 20–30%, 30–40%, 40–50% centrality classes were used for the analysis. The event distribution as a function of centrality is shown in Fig. 4.2. Since the MB and two centrality triggers are used for the analysis, especially the central trigger for 0–10% centrality, the number of events in each centrality bin varies. The effects...
Figure 4.1: \((\eta, \varphi)\) distributions of the charged tracks from some run numbers in the 2011 Pb–Pb collision period.

Figure 4.2: Centrality distribution of events in 2.76 TeV Pb–Pb collisions based on VZERO multiplicity with MB, Central, and Semi-central triggers.
4.2 Track selection

Analyses in this thesis utilize tracks in the central barrel (TPC + ITS), tracklets from SPD layers, and muon tracks from the FMS. The overall flow of the reconstruction for tracks is shown schematically in Fig. 4.3 and described in detail in [95, 106]. The reconstruction of tracks starts from the clusterization in each detector, where the electronic signals such as positions, signal amplitudes, timing information, and errors, are combined into clusters. Then the preliminary interaction vertex is determined based on clusters in the SPD, and consequently track finding and fitting is performed with clusters in the TPC and ITS with the Kalman filter technique [120, 121]. The space points reconstructed in the TPC are used as the seeds in the technique, and resultant tracks are further matched to clusters in the ITS and fitted. However, the reconstruction efficiency in the TPC is very low for low $p_T$ particles (lower than 0.2 GeV/c for pions and 0.4 GeV/c for protons), and these tracks are not used in the matching. Therefore, a standalone ITS reconstruction is performed and resultant ITS tracks are propagated into the TPC in the form of the seed in the Kalman filter, and propagated inward again for the final determination of the vertex position.

For muon tracks, the track reconstruction follows similar procedures as above, but uses clusters in 5 tracking chambers in the FMS and the vertex position from the SPD. Also, multiple scattering and energy loss of muons in the front muon absorber are accounted for.
in the muon track reconstruction \cite{122}.

4.2.1 p–Pb collisions in 2013

The analysis for the p–Pb collisions uses the central barrel tracks, tracklets in the SPD, and muon tracks together.

Central barrel track

In order to achieve uniform tracking efficiency, charged tracks were reconstructed using ITS and TPC clusters, namely with hybrid track selection. This selection has been used in various ALICE analyses \cite{42,49} and the quality of the track with the selection has been studied extensively \cite{123,124}. Tracks were required to have at least 70 space points in the TPC, the ratio of the number of recorded points to the findable points larger than 80\%, and a distance of closest approach (DCA) to the reconstructed vertex smaller than 2.4 cm in the transverse direction and 3.2 cm in the longitudinal direction. In the process of matching TPC tracks with ITS clusters, two approaches were followed: tracks reconstructed with at least one hit in the SPD, and tracks without associated SPD hit but the reconstructed vertex position in the fit. Finally, tracks in the fiducial region $|\eta| < 1$ and $0.5 < p_T < 4.0$ GeV/c were selected for the analysis.

Although the correlation observables in the analysis are insensitive to the single-particle efficiency \cite{123}, the efficiency and purity of the particles after the selection procedures were studied from a Monte Carlo (MC) simulation using the DPMJET event generator \cite{125} and GEANT3 \cite{126} for particle transport through the detectors. The efficiency of the reconstruction is about 82\% at $0.5 < p_T < 1.0$ GeV/c, and decreases to 79\% at 4 GeV/c. The level of contamination from interactions in the detector material or weak decays remained near 1 to 2\% throughout the considered $p_T$ range.

Tracklet

Tracklets are short track segments reconstructed based on the clusters in two layers of SPD and the primary vertex position, not relying on any TPC clusters. The main reason that
tracklets are used in the analysis is the limited availability of tracks during the 2013 p–Pb run period. As mentioned in Sec. 4.1.1, 2013 p–Pb run period consisted of two alternating beam directions, p–Pb and Pb–p. The TPC and SDD detectors have significantly longer deadtime compared to the FMS, SPD, and VZERO, and therefore, the number of events with TPC clusters was particularly low in Pb–p collisions. For the comparison of the observables in p–Pb and Pb–p collisions, tracklets are good candidates to replace tracks although momentum and spatial resolution are not as good as tracks.

For the tracklet selection, the difference in azimuth between hits in two SPD layers, $\Delta \varphi_h$, was used, requiring $\Delta \varphi_h < 5$ mrad for tracklet candidates. This $\Delta \varphi_h$-cut reduced the contamination from secondary tracks from weak decays and from detector material to below 2.5%. At the same time, it biased the $p_T$ of the selected particles to a higher value, as the high $p_T$ particles generally have smaller curvature in the magnetic field. The mean $p_T$ of the selected tracklets were estimated to be 0.75 GeV/$c$ from the MC simulation with DPMJET and GEANT3.

In addition, the use of tracklets influenced the selection of events, in particular, for the primary vertex position. The acceptances of two SPD layers strongly depend on the primary vertex position in the beam direction ($z_{vtx}$), and the event cut of $z_{vtx} < 7$ cm was applied to achieve the uniform $\eta$-coverage throughout the analysis. The corresponding $\eta$ range of tracklets is $|\eta| < 1$.

**Muon track**

Muon tracks are reconstructed based on the primary vertex position and the clusters in the Forward Muon Spectrometer (FMS), which covers the $\eta$ range, $-4.0 < \eta < -2.5$ in the ALICE system. Various selection criteria were applied for the muon track selection, similarly to the central-barrel tracks. Tracks were required to exit the front absorber at a radial distance from the beam axis, $R_{abs}$, between 17.6 cm and 89.5 cm, to avoid regions with large material density. Tracking information from the five tracking chambers were matched to the track segment in the trigger chambers, which ensured a muon purity higher than 99%. Without this matching, only 84% of reconstructed particles were muons, with
The FMS is initially designed to measure light ($\omega$ and $\phi$) and heavy ($J/\psi$ and $\Upsilon$ families) mesons through their $\mu^+\mu^-$ decay channel. The combination of the absorber and filter wall is optimized to detect muons with $p_T > 4\,\text{GeV}/c$, but particles with lower $p_T$ have never been utilized in any of the previous ALICE analyses \cite{106}. Indeed, the tracking efficiency in $p_T$ lower than $4\,\text{GeV}/c$ becomes significantly low ($\sim 1\%$). However, the observables used in this analysis do not depend on the tracking efficiency of muon tracks, and this justifies the usage of tracks with $p_T$ lower than $4\,\text{GeV}/c$ with additional considerations.

While the central barrel tracks are mostly hadrons such as pions, kaons, and protons, muons reconstructed in the FMS are decayed from their parent particles. The composition of parent

**Figure 4.4:** ($\varphi, \eta$) distribution of selected muon tracks for different $p_T$ selections. Low efficiency due to the effects from the dipole magnet on low $p_T$ particles is visible near $\varphi = 1/2$ and $\varphi = 3/2$ \cite{124}.
particles are distinguished from the composition of central barrel tracks, as showed in the left panel of Fig. 4.5 which is generated from the MC simulation with DPMJET event generator and GEANT3 for the detector response. Heavy-flavor particles are observed to take the dominant contribution in parent particle composition at $p_T$ larger than 2.0 GeV/c, and similar results are obtained from the MC simulation with the AMPT event generator instead of the DPMJET event generator. Also, the relative reconstruction efficiencies for muons from pion and kaon decays to those for muons from heavy flavor decays are shown in the right panel of Fig. 4.5. These relative efficiencies are aimed to facilitate future comparison with theoretical models, and will be explained further in Chap. 5.

4.2.2 Pb–Pb collisions in 2011

The analysis of Pb–Pb collisions uses central-barrel tracks only. The hybrid track selection explained in the above is also used with Pb–Pb data with an additional requirement that the maximum fraction of shared clusters in the TPC is lower than 40%. The $p_T$ and $\eta$ ranges of tracks are $0.2 < p_T < 5.0$ GeV/c and $-0.8 < \eta < 0.8$, respectively.

The efficiency and contamination of tracks with Pb–Pb data are further studied using the HIJING event generator and GEANT3. Mathematically, the contamination, $\lambda$, is
defined as

\[ \lambda = \frac{N_{\text{rec}}^{\text{secondaries}}}{N_{\text{rec}}^{\text{primaries}} + N_{\text{rec}}^{\text{secondaries}}} , \quad (4.1) \]

where \( N_{\text{rec}}^{\text{secondaries}} \) is the number of reconstructed secondary particles, which originate from weak decays or material interactions, and \( N_{\text{rec}}^{\text{primaries}} \) is the number of reconstructed primary particles. Then the raw efficiency, \( \epsilon' \), is

\[ \epsilon' = \frac{N_{\text{rec}}^{\text{primaries}}}{N_{\text{gen}}^{\text{primaries}}} , \quad (4.2) \]

where \( N_{\text{gen}}^{\text{primaries}} \) is the number of generated primary particles. Finally, considering secondary particles, the true efficiency, \( \epsilon \), is defined by

\[ \epsilon = \frac{\epsilon'}{1 - \lambda} . \quad (4.3) \]

These \( \epsilon \) and \( \lambda \) for the hybrid tracks are calculated in each bin of \( p_T, \eta, \) and \( \varphi \) and shown in Fig. 4.6. The efficiency is used in particular for the correction procedures of the observables, and its usage will be explained further in Chap. 6.

Meanwhile, different track selection schemes are also tested: ITS-standalone and TPC-standalone track selections, which reconstruct tracks based only on information in the ITS and TPC, respectively. Tracks in the TPC-standalone selection were required to have at least 50 space points, and a DCA to the reconstructed vertex smaller than 2.4 cm in the transverse direction and 3.2 cm in the longitudinal direction. Corresponding efficiency and contamination values are shown in Fig. 4.7 for ITS-standalone selection and Fig. 4.8 for TPC-standalone selection. ITS-standalone and TPC-standalone selections are observed to have significantly larger \( \lambda \) values throughout all ranges, compared to the results with the hybrid track selection. This also results in \( \epsilon \) at certain \( p_T \) bin larger than 1 in ITS-standalone and TPC-standalone selections, which can be explained by the contribution from contamination. Therefore, only tracks with hybrid track selections are considered in the analysis.

Summaries of event and track selection for proton-proton and lead-lead collisions are pro-
Figure 4.6: Tracking efficiency ($\epsilon$) for hybrid track selection as a function of $p_T$ (left), $\eta$ (center), and $\phi$ (right) in each centrality bin (top) and the corresponding contamination ($\lambda$) (bottom) generated from the HIJING event generator with GEANT3 [129]. Filter 768 corresponds to the hybrid track selection for the 2011 Pb–Pb data in the ALICE software framework.

Figure 4.7: Tracking efficiency ($\epsilon$) for ITS-standalone track selection as a function of $p_T$ (left), $\eta$ (center), and $\phi$ (right) in each centrality bin (top) and the corresponding contamination ($\lambda$) (bottom) generated from the HIJING event generator with GEANT3 [129]. Filter 1 corresponds to the ITS-standalone track selection for the 2011 Pb–Pb data in the ALICE software framework.
Figure 4.8: Tracking efficiency ($\epsilon$) for TPC-standalone track selection as a function of $p_T$ (left), $\eta$ (center), and $\varphi$ (right) in each centrality bin (top) and the corresponding contamination ($\lambda$) (bottom) generated from the HIJING event generator with GEANT3 [129]. Filter 128 corresponds to the ITS-standalone track selection for the 2011 Pb–Pb data in the ALICE software framework.

Provided in Table 4.1 and Table 4.2.
### Table 4.1: Summary of event and track selection for p–Pb data

#### Proton-lead

<table>
<thead>
<tr>
<th>Dataset</th>
<th>√s_{NN} = 5.02 TeV p–Pb and Pb–p in 2013</th>
</tr>
</thead>
<tbody>
<tr>
<td>Event selection</td>
<td>MB, low-p_{T} and high-p_{T} muon triggers</td>
</tr>
<tr>
<td></td>
<td>−7 &lt; z_{vtx} &lt; 7 cm</td>
</tr>
<tr>
<td></td>
<td>V0S multiplicity classification</td>
</tr>
<tr>
<td></td>
<td>Muon track required in each event</td>
</tr>
<tr>
<td>Track selection</td>
<td>Hybrid track selection</td>
</tr>
<tr>
<td></td>
<td>0.5 &lt; p_{T} &lt; 4.0 GeV/c and −1.0 &lt; η &lt; 1.0</td>
</tr>
<tr>
<td>Tracklet selection</td>
<td>Δφ_{h} &lt; 5 mrad</td>
</tr>
<tr>
<td>Muon track selection</td>
<td>0.5 &lt; p_{T} &lt; 4.0 GeV/c and −4.0 &lt; η &lt; −2.5</td>
</tr>
<tr>
<td></td>
<td>17.6 &lt; R_{abs} &lt; 89.5 cm</td>
</tr>
<tr>
<td></td>
<td>Trigger chamber matching and p×DCA cut</td>
</tr>
</tbody>
</table>

### Table 4.2: Summary of event and track selection for Pb–Pb data

#### Lead-lead

<table>
<thead>
<tr>
<th>Dataset</th>
<th>√s_{NN} = 2.76 TeV Pb–Pb in 2011</th>
</tr>
</thead>
<tbody>
<tr>
<td>Event selection</td>
<td>MB, central, semi-central triggers</td>
</tr>
<tr>
<td></td>
<td>−8 &lt; z_{vtx} &lt; 8 cm</td>
</tr>
<tr>
<td></td>
<td>VZERO centrality classification</td>
</tr>
<tr>
<td></td>
<td>Additional selection for uniform tracking efficiency</td>
</tr>
<tr>
<td>Track selection</td>
<td>Hybrid track selection</td>
</tr>
<tr>
<td></td>
<td>0.2 &lt; p_{T} &lt; 5.0 GeV/c and −0.8 &lt; η &lt; 0.8</td>
</tr>
</tbody>
</table>
Chapter 5

Forward-central two-particle correlations in proton-lead collisions

The study of forward-central two-particle correlations in proton-lead collisions initially started from two separate approaches, one using correlations between muon and central barrel tracks [124], and the other using correlations between muon track and SPD tracklets [130]. The two approaches were subsequently combined as they shared the common physics goals, and results were published in [71]. In this section, details and results of the forward-central two-particle correlation analysis in p–Pb collisions are discussed. This analysis is based on the motivation explained in Sec. 2.3 and event and track selections described in Sec. 4.1.1 and Sec. 4.2.1.
5.1 Analysis

5.1.1 Associated yield per trigger particle

Following the discussion in Sec. 2.2 and Sec. 2.3, the first observable used for the two-particle correlation analysis is the associated yield per trigger particle. In this analysis, either tracks or tracklets are used as associated particles and muon tracks from the Forward Muon Spectrometer (FMS) are used as trigger particles. This yield, \( Y \), is a function of \( \Delta \phi \) and \( \Delta \eta \), which are defined by \( \Delta \phi = \phi_t - \phi_a \) and \( \Delta \eta = \eta_t - \eta_a \), where the subscripts “t” and “a” correspond to trigger and associated particles, respectively. Following the conventional technique to estimate the yield \[41,42\], \( Y \) is estimated with

\[
Y(\Delta \phi, \Delta \eta) = \frac{1}{N_{\text{trig}}} \frac{d^2 N_{\text{pair}}}{d\Delta \phi d\Delta \eta} \approx \frac{S(\Delta \phi, \Delta \eta)}{B(\Delta \phi, \Delta \eta)}, \tag{5.1}
\]

in each bin of multiplicity class and \( p_T \) of trigger and associated particles with

\[
S(\Delta \phi, \Delta \eta) = \frac{1}{N_{\text{trig}}^{\text{same}}} \frac{d^2 N_{\text{pair}}^{\text{same}}}{d\Delta \phi d\Delta \eta}, \quad B(\Delta \phi, \Delta \eta) = \alpha \frac{d^2 N_{\text{pair}}^{\text{mixed}}}{d\Delta \phi d\Delta \eta}. \tag{5.2}
\]

In equations of \( Y(\Delta \phi, \Delta \eta) \) and \( S(\Delta \phi, \Delta \eta) \), \( N_{\text{trig}} \) denotes the number of trigger particles in the corresponding bin. \( S(\Delta \phi, \Delta \eta) \) estimates the associated yield with trigger and associated particle pairs in the same event, and \( B(\Delta \phi, \Delta \eta) \) with trigger and associated particle pairs selected in different events. The \( \alpha \) in \( B(\Delta \phi, \Delta \eta) \) is determined to normalize \( B(\Delta \phi, \Delta \eta) \) to unity in the \( \Delta \eta \) region of maximal pair acceptance. \( S \) and \( B \) correspond to the signal and background distributions, respectively, and the method of estimating \( Y \) from \( S/B \) is generally referred to as the mixed-event method. Dividing by the background distribution is expected to correct for pair acceptance and pair efficiency effects, and has been used in most of the two-particle correlation analyses. Recently, some alternative methods for finite-acceptance correction were proposed \[131\] but the mixed-event method is used in this thesis. More detailed discussion on the mixed-event method and newly devised methods that were developed as a part of this thesis can be found in \[131\] and Appendix B. Both

\footnote{New methods were proposed by myself, A. Morsch, C. Loizides, and T. Schuster.}
$S(\Delta \phi, \Delta \eta)$ and $B(\Delta \phi, \Delta \eta)$ are obtained within 1 cm-wide intervals of $z_{\text{vtx}}$ to suppress the additional effects from $z_{\text{vtx}}$-dependent efficiencies, and $Y(\Delta \phi, \Delta \eta)$ is estimated in each $z_{\text{vtx}}$ interval. The final per-trigger yield is calculated with the weighted average of $Y(\Delta \phi, \Delta \eta)$ through $z_{\text{vtx}}$ intervals, using $N_{\text{trig}}$ as a weighting factor. In addition, it should be remarked that $Y$ is insensitive to the single-particle efficiency as its effects are mostly canceled out in the process of dividing the signal distribution by the background distribution. Also, the final observable of the analysis, $v_n$, is observed to be insensitive to the single particle efficiency in [129]. Although single-particle efficiency correction is applied in the muon-track correlation approach, its effects in results are negligible.

The multiplicity class is divided into four bins, 0–20%, 20–40%, 40–60%, and 60–100%, based on V0S event classification (Sec. 4.1.1). Three $p_T$ ranges of trigger particles for muon-track correlations are $0.5 < p_T^t < 1.0 \text{ GeV}/c$, $1.0 < p_T^t < 2.0 \text{ GeV}/c$ and $2.0 < p_T^t < 4.0 \text{ GeV}/c$, and five $p_T$ ranges for muon-tracklet correlations are $0.5 < p_T^t < 1.0 \text{ GeV}/c$, $1.0 < p_T^t < 1.5 \text{ GeV}/c$, $1.5 < p_T^t < 2.0 \text{ GeV}/c$, $2.0 < p_T^t < 3.0 \text{ GeV}/c$ and $3.0 < p_T^t < 4.0 \text{ GeV}/c$. For the approach using the central-barrel tracks, associated particles are tracks with $0.5 < p_T^a < 4.0 \text{ GeV}/c$, while that using tracklets does not have a specific $p_T$ range for associated particles, but is estimated to be $\langle p_T \rangle \simeq 0.75 \text{ GeV}/c$ (Sec. 4.2.1). It should be noted that only one $p_T$ bin is used for associated particles in both muon-track correlations and muon-tracklet correlations.

Figure 5.1 shows some associated yields per trigger particle from muon-track correlations in p–Pb (left column), muon-tracklet correlations in p–Pb (middle column) and muon-tracklet correlations in Pb–p (right column) for $0.5 < p_T^t < 1.0 \text{ GeV}/c$. The top row corresponds to the 0–20% multiplicity class, and the bottom row to the 60–100% multiplicity class. In the 0–20% multiplicity class, associated yields are considerably higher than those in the 60–100%. Also, ridge structures on both near-side ($\Delta \phi \sim 0$) and away-side ($\Delta \phi \sim \pi$) are observed, which indicates that the near-side ridge observed in the mid-rapidity [42] exists even up to $2.5 < |\eta| < 4.0$. However, the near-side ridge is smaller than that on the away-side. In the 60–100% multiplicity class, only the away-side ridge appears, which agrees with the results from the associated yield per trigger particle using mid-rapidity tracks [42].
Figure 5.1: Associated yield per trigger particle, \( Y(\Delta \varphi, \Delta \eta) \), from muon-track correlations in p–Pb (left column), muon-tracklet correlations in p–Pb (center column) and muon-tracklet correlations in Pb–p (right column) measured in 0–20% multiplicity class (top row) and 60–100% multiplicity class (bottom row). Trigger particles are reconstructed in the FMS with \(-4.0 < \eta < -2.5\) and \(0.5 < p_T < 1.0 \text{ GeV/c}\), associated particles are \(0.5 < p_T < 1.0 \text{ GeV/c}\) tracks for muon-track correlations and \(\langle p_T \rangle \simeq 0.75 \text{ GeV/c}\) tracklets for muon-tracklet correlations [71].
Short-range correlations (SRC) are expected to be dominant in this multiplicity class, and the $\Delta \eta$ ranges in these yields are sufficiently separated from the $(\Delta \varphi, \Delta \eta) = (0, 0)$ region, where the near-side jet peak is expected. Lastly, the away-side structures in all multiplicity classes decrease from low-$|\Delta \eta|$ to high-$|\Delta \eta|$, which can be understood by considering the kinematic limit of the di-jet at large $|\Delta \eta|$.

5.1.2 Extraction of $v_2$

As discussed in Sec. 2.2, the physics mechanisms that contribute to the per-trigger yield are classified into short-range correlations (SRC) and long-range correlations (LRC). One way to isolate the LRC is to subtract from the per-trigger yields in higher-multiplicity classes the per-trigger yield in the lowest-multiplicity class [42]. This is called a subtraction method, and based on two assumptions, (1) SRC are the dominant physics mechanism for the per-trigger yield in the lowest-multiplicity class, (2) the level of SRC are uniform in all multiplicity classes. The top three plots in Fig. 5.2 show the per-trigger yields after the subtraction, $Y_{\text{sub}}$, from muon-track correlations in p–Pb (left), muon-tracklet correlations in p–Pb (center column) and muon-tracklet correlations in Pb–p (right column) in 0–20% multiplicity class. Near- and away-side ridges are observed to be comparable in size after the subtraction.

The magnitude of LRC is quantified with Fourier coefficients, $v_n$, and these coefficients are related to the $\Delta \varphi$ structures of the per-trigger yield after the subtraction. In practice, $v_n$ for particles reconstructed in the FMS are extracted through a few steps. It starts from the $\Delta \varphi$ projection of $Y_{\text{sub}}$, which is obtained from a first-order polynomial fit along $\Delta \eta$ for each $\Delta \varphi$ bin, since $Y_{\text{sub}}$ has large statistical fluctuations at the edges (near $\Delta \eta = -1.5$ and $\Delta \eta = -5.0$). The results of $\Delta \varphi$ projection are shown in the bottom row of Fig. 5.2. The near- and away-side amplitudes are similar in the Pb–p case, but the near-side amplitude is smaller than the away-side amplitude in the p–Pb cases. The difference in amplitudes may originate from the residual jet contribution after the subtraction, and taken into account in the systematic uncertainty in the later section. Lines on the bottom plots of Fig 5.2
Figure 5.2: Top row: Associated yield per trigger particle after subtraction of the per-trigger yield in the lowest-multiplicity class, $Y(\Delta \varphi, \Delta \eta)$, from muon-track correlations in p–Pb (left column), muon-tracklet correlations in p–Pb (center column) and muon-tracklet correlations in Pb–p (right column) in 0–20% multiplicity class. Other details are the same as the per-trigger yields in Fig. 5.1. Bottom row: $\Delta \varphi$ projection of the above per-trigger yields. The lines indicate the fit to the data and the harmonic contributions [71].
indicate the fitting with Fourier harmonics, which are from

$$Y_{\text{sub}}(\Delta \varphi) = a_0 + 2a_1 \cos(\Delta \varphi) + 2a_2 \cos(2\Delta \varphi) + 2a_3 \cos(3\Delta \varphi) .$$  \hspace{1cm} (5.3)

$a_n$ are extracted by the above fitting, and $\chi^2$/NDF values for this fit are generally below 1.5. Then the relative modulation, $V_{n\Delta}\{2\text{PC,sub}\}$, is estimated by

$$V_{n\Delta}\{2\text{PC,sub}\} = \frac{a_n}{a_0 + b} ,$$  \hspace{1cm} (5.4)

where $b$ is the baseline of the lowest-multiplicity class calculated from the integral of the per-trigger yield around the minimum. The notation $V_{n\Delta}\{2\text{PC,sub}\}$ indicates that this quantity is from two-particle correlations (2PC) with the subtraction method (sub). In Eq. \ref{eq:5.4} $b$ is introduced due to the fact that the per-trigger yield in higher multiplicity classes are subtracted by the per-trigger yield in the lowest-multiplicity class. Based on the assumption that $V_{n\Delta}$ factorizes into a product of trigger and associated single-particle $v_n$ \cite{48}, $v_n\{2\text{PC,sub}\}$ for particles reconstructed in the FMS is finally obtained from

$$v_n\{2\text{PC,sub}\} = \frac{V_{n\Delta}\{2\text{PC,sub}\}}{\sqrt{V_{n\Delta}^\text{c}\{2\text{PC,sub}\}}} ,$$  \hspace{1cm} (5.5)

where $V_{n\Delta}^\text{c}\{2\text{PC,sub}\}$ is measured with central-barrel tracks or tracklets with the corresponding selection criteria depending on the measurement. Equation \ref{eq:5.5} indicates that the contribution from mid-rapidity $v_n$ in the measurement of $v_n$ in the FMS is canceled out on the right side of the Eq. \ref{eq:5.5}. Therefore, we expect the two $v_n\{2\text{PC,sub}\}$ from muon-track correlations and muon-tracklet correlations to agree with each other due to this cancellation.

Lastly, in addition to the $v_n\{2\text{PC,sub}\}$ itself, the ratio between the $v_n\{2\text{PC,sub}\}$ in Pb–p in p–Pb is measured to quantify the difference.
5.1.3 $v^\mu_n\{2PC,sub\}$ and resolution correction

$v_n\{2PC,sub\}$ values extracted with procedures explained in the previous section correspond to the $v_n$ for muons in the kinematic range of the FMS. Compared to the $v_n$ values in the previous measurements [40,42,49], where hadrons are used for the $v_n$ estimation, $v_n\{2PC,sub\}$ from this analysis is distinct as it uses muons for its estimation. This may cause deviations of the values from those of parent hadrons since muons decay from their parent hadrons. However, it is not possible to correct the measured $v_n$ for muons to those at the level of their parent hadrons due to the species-dependent inefficiencies of the front absorber. Therefore, we denote the measured $v_n$ with $v^\mu_n\{2PC,sub\}$, indicating that these values are measured from decay muons reconstructed in the FMS. Also, due to the large uncertainty in $v_3$ and higher order coefficients, only $v_2$ will be discussed below.

In the reconstruction of muons in the FMS, the true value of $p_T$ and $\varphi$ of a muon do not always agree with the reconstructed values, and this may bias the measured $v_n$ values. This is generally referred to as a resolution effect, and the correctional factors are estimated from a toy MC simulation based on the discussion below. We can imagine particles with their true $p_T$ and $\varphi$ values, $(p_{T,MC}, \varphi_{MC})$, and reconstructed values, $(p_{T,rec}, \varphi_{rec})$. If these particles are distributed with a given $v_2$, each track follows

$$\frac{dN}{d\varphi_{MC}} \sim 1 + 2 v_2(p_{T,MC}) \cos(2(\varphi_{MC} - \Psi_2))$$

for their $\varphi$ distribution, where $v_2(p_{T,MC})$ is the true $v_2$ as a function of $p_{T,MC}$. The $v_2$ extracted from an ensemble of such tracks should be also $v_2(p_{T,MC})$.

At the same time, reconstructed quantities are $p_{T,rec}$ and $\varphi_{rec}$ instead of $p_{T,MC}$ and $\varphi_{MC}$, and each reconstructed track follows

$$\frac{dN}{d\varphi_{rec}} \sim 1 + 2 v_2(p_{T,rec}) \cos(2(\varphi_{rec} - \Psi_2))$$

$$\sim 1 + 2 v_2(p_{T,MC} + \Delta p_T) \cos(2(\varphi_{MC} + \Delta \varphi - \Psi_2)),$$
with $\Delta p_T = p_{T,\text{rec}} - p_{T,\text{MC}}$ and $\Delta \varphi = \varphi_{\text{rec}} - \varphi_{\text{MC}}$. $\Delta p_T$ and $\Delta \varphi$ distributions can be investigated through the MC simulation with an event generator and subsequent GEANT3, and $v_2(p_{T,\text{rec}})$ can be extracted from an ensemble of these reconstructed tracks. Effectively, $\Delta p_T$ accounts for the $p_T$-bin migration and $\Delta \varphi$ accounts for the smearing effect in $\varphi$. The ratio of $v_2(p_{T,\text{MC}})/v_2(p_{T,\text{rec}})$ becomes the correction factor for the resolution effects.

In practice, the DPMJET event generator and GEANT3 were used to estimate the $\Delta \varphi$ distribution (Fig. 5.3) and $p_T$-bin migration (Fig. 5.4). The measured $v_2(p_T)$ from muon-track correlations and muon-tracklet correlations were used as a baseline for $v_2(p_{T,\text{MC}})$, and varied to estimate the systematic uncertainty in the correction factor (Fig. 5.5). Since we do not have information on the true $v_2(p_{T,\text{MC}})$, three input $v_2(p_{T,\text{MC}})$ were used, (1) the measured $v_2(p_T)$ fitted with the third order polynomial (Fig. 5.5 left), (2) increasing the value of the lowest $p_T$ bin by $2\sigma$ and decreasing the value of the highest $p_T$ bin by $2\sigma$, then fitted with the third order polynomial (Fig. 5.5 middle), (3) decreasing the value of the lowest $p_T$ bin by $2\sigma$ and increasing the value of the highest $p_T$ bin by $2\sigma$, then fitted with the third order polynomial (Fig. 5.5 right). The difference from three the different input
Figure 5.4: \((p_{T,MC}, p_{T,\text{rec}})\) distribution of muon tracks. Each bin is normalized to make the sum of each row unity.

Figure 5.5: Variations of input \(v_2\) in muon-tracklet correlations in p–Pb. Measured \(v_2\) (left), increasing the lowest \(p_T\) value by 2\(\sigma\) and decreasing the highest \(p_T\) value by 2\(\sigma\) (center), decreasing the lowest \(p_T\) value by 2\(\sigma\) and increasing the highest \(p_T\) value by 2\(\sigma\) (right).
Figure 5.6: Resolution correction factor for $v_2(p_T)$ from muon-track correlations in p–Pb (top left) with systematic uncertainties [1.1%, 0.7%, 1.8%] for each $p_T$ bin, $v_2(p_T)$ from muon-tracklet correlations in p–Pb (top right) with systematic uncertainties [1.1%, 0.5%, 1.0%, 1.6%, 0.9%], $v_2(p_T)$ from muon-tracklet correlations in Pb–p (bottom left) with systematic uncertainties [0.6%, 0.4%, 1.0%, 1.6%, 0.7%], the ratio of $v_2(p_T)$ from muon-tracklet correlation in Pb–p to $v_2(p_T)$ from muon-tracklet correlation in p–Pb (bottom right) with systematic uncertainties [1.7%, 0.8%, 2.0%, 2.7%, 1.5%].

$v_2(p_{T,MC})$ was addressed in the systematic uncertainty of the correction factors.

Figure 5.6 shows the resolution correction factors for $v_2(p_T)$ from muon-track correlations in p–Pb (top left panel), muon-tracklet correlations in p–Pb (top right panel), muon-tracklet correlation in Pb–p (bottom left panel), and for the ratio of $v_2(p_T)$ from muon-tracklet correlation in Pb–p to $v_2(p_T)$ from muon-tracklet correlation in p–Pb (bottom right panel). The measured $v_2^{\mu}$\{2PC,sub\} and ratio values are divided by these factors, and resulted in a small correction of less than 2%.
5.2 Systematic uncertainty

Systematic uncertainties on $v_2^\mu\{2\text{PC,sub}\}$ and the ratio between the $v_2^\mu\{2\text{PC,sub}\}$ in Pb–p and p–Pb are estimated by varying the measurement conditions as explained in this section. In general, the estimation of the systematic uncertainty follows procedures described in [132]. The systematic variation of the measurement condition is grouped into a few classes.

$z_{\text{vtx}}$ acceptance

The tracking efficiency and acceptance of the tracks and tracklets depend on the position of $z_{\text{vtx}}$. Besides the nominal $z_{\text{vtx}}$ range for the event selection, $|z_{\text{vtx}}| < 7$ cm, observables were measured for events with $|z_{\text{vtx}}| < 5$ cm. As mentioned in 5.1.1, the per-trigger yield from muon-track correlations can be either corrected or not corrected for the single-particle efficiency. It is verified that $v_2^\mu\{2\text{PC,sub}\}$ is insensitive to the single-particle efficiency correction, as it is a relative quantity. This justifies the measurement of muon-tracklet correlations without a single-particle efficiency correction, and no further uncertainty is assigned.

Remaining SRC after the subtraction

The per-trigger yields in high multiplicity classes are subtracted by the yield in the lowest multiplicity class in order to remove the contributions from short-range correlations (SRC). However, this is based on the assumption that the level of SRC is invariant in all multiplicity classes. It is generally considered that the difference in the yields from SRC in different multiplicity classes is due to a bias of the event selection on the jet fragmentation in low-multiplicity events [39]. Thus, the per-trigger yield in 60–100% multiplicity class was scaled before it was used in the subtraction method in this variation. The scaling factor ($f$) for the yield in a certain multiplicity class higher than the lowest multiplicity class was determined as the ratio between away-side yields in that multiplicity class and the 60–100% multiplicity class after the subtraction of the second-order Fourier component [39]. The subtraction of the second-order Fourier component is a common technique to isolate the SRC in per-trigger yields. These procedures, the subtraction after scaling the yield in the lowest multiplicity class, were applied in estimations of $V_{n\Delta}$ and $V_{n\Delta}^c$. The scaling factors were observed to be
larger in p–Pb systems ($f < 1.4$) than Pb–p systems ($f < 1.26$), and became smaller for increasing $p_T$. Besides the nominal measurement with $f = 1$, measurements with a scaling factor in the subtraction process were made.

**Remaining LRC in the lowest-multiplicity class**

In addition to the effects from SRC remaining after the subtraction, there may be another effect that arises from the remaining LRC in the lowest-multiplicity class. The per-trigger yield in the lowest-multiplicity class is assumed to be dominated by the SRC, but there is the possibility that the LRC still exist in the lowest multiplicity class with a given multiplicity classification. Instead of the nominal multiplicity classification (60–100% for the lowest multiplicity bin), measurements were repeated with the subtraction with the per-trigger yields in 70–100% multiplicity class and the difference was included in the estimation of the total systematic uncertainty.

**Calculation of $v_2$**

Besides the nominal extraction of $v_2$, measurements were made with several variations in the $v_2$ extraction: The $\Delta \varphi$ projection of $Y_{\text{sub}}(\Delta \varphi, \Delta \eta)$ was obtained from a weighted average over $\Delta \eta$; instead of fitting $Y_{\text{sub}}(\Delta \varphi)$ with Eq. 5.3, $Y_{\text{sub}}(\Delta \varphi)$ was fitted with only the first and second order Fourier harmonics; instead of baseline $b$ in Eq. 5.4, calculated from the integral of the per-trigger yield around the minimum in the lowest-multiplicity class, $b$ was determined from modeling $Y_{\text{sub}}(\Delta \varphi)$ in the lowest-multiplicity class with a Gaussian for the away-side structure and a constant; instead of Eq. 5.4, $V_{n\Delta \{2PC,\text{sub}\}}$ was estimated by

$$V_{n\Delta \{2PC,\text{sub}\}} = \frac{a_n}{B},$$

where $B$ was determined from the integral of the per-trigger yield around the minimum in the high-multiplicity class.

**Resolution correction**

As shown in Fig. 5.6, correction factors have systematic uncertainty themselves. This uncertainty is added in quadrature for the estimation of the total systematic uncertainty.
Table 5.1: Summary of systematic uncertainties in the estimation of $v_2^\mu\{2\text{PC,sub}\}$ and the ratio between the $v_2^\mu\{2\text{PC,sub}\}$ in Pb–p and p–Pb [71]

<table>
<thead>
<tr>
<th>Systematic effect</th>
<th>Assoc. tracks</th>
<th>Assoc. tracklets</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>p–Pb</td>
<td>Pb–p</td>
</tr>
<tr>
<td>Acceptance ($z_{vtx}$-dependence)</td>
<td>3–4%</td>
<td>0–5%</td>
</tr>
<tr>
<td>Remaining jet after subtraction</td>
<td>4–10%</td>
<td>5–14%</td>
</tr>
<tr>
<td>Remaining ridge in low-multiplicity class</td>
<td>1–4%</td>
<td>1–6%</td>
</tr>
<tr>
<td>Calculation of $v_2$</td>
<td>0–1%</td>
<td>0–1%</td>
</tr>
<tr>
<td>Resolution correction</td>
<td>1%</td>
<td>0–1%</td>
</tr>
<tr>
<td>Sum (added in quadrature)</td>
<td>7–11%</td>
<td>6–14%</td>
</tr>
</tbody>
</table>

The total systematic uncertainty is evaluated by combining deviations to the values from nominal measurement in each variation. The systematic variations are believed to be “typical” [132], each deviation is divided by $\sqrt{2}$. For each class of the variation, only the largest contribution is considered in order to avoid double counting, and different contributions are summed in quadrature. Table 5.1 summarizes the systematic uncertainty from variations in the estimation of $v_2^\mu\{2\text{PC,sub}\}$ and the ratio between the $v_2^\mu\{2\text{PC,sub}\}$ in Pb–p and p–Pb.

5.3 Result and discussion

The $v_2^\mu\{2\text{PC,sub}\}$ for muons in the p-going direction (from the p–Pb configuration) measured with both tracks (muon-track correlations) and tracklets (muon-tracklet correlations) as associated particles are shown in Fig. 5.7 as a function of muon $p_T$. It should be noted that statistical uncertainties are marked with bars and systematic uncertainties are marked using boxes or shaded areas in every figure. The good agreement between the two $v_2^\mu\{2\text{PC,sub}\}$ confirms the validity of factorization of $v_2$ in trigger and associated particles. In addition, similar values of $v_2^\mu\{2\text{PC,sub}\}$ are found in muon-tracklet correlations even if different tracklet selections ($\Delta \varphi_{hn}$) are used. Meanwhile, Fig. 5.8 shows $v_2\{2\text{PC,sub}\}$ measured with mid-rapidity tracks in p–Pb collisions from [42]. Comparing $v_2^\mu\{2\text{PC,sub}\}$ in Fig. 5.7 to $v_2\{2\text{PC,sub}\}$ in Fig. 5.8 directly is not valid as muons used in the $v_2^\mu\{2\text{PC,sub}\}$ estimation are decayed from their parent hadrons and pass through the muon absorber, while charged hadrons are used for the $v_2\{2\text{PC,sub}\}$ estimation. Depending on particle composition and on the $p_T$-dependence of the parent particle $v_2$ distribution, the decay muon $v_2$ can be far

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**Figure 5.7:** $v^\mu_2\{2PC, sub\}$ for $-4.0 < \eta < -2.5$ extracted from muon-track correlations and muon-tracklet correlations in p–Pb collisions at $\sqrt{s_{NN}} = 5.02$ TeV [71].

**Figure 5.8:** $v^\mu_2\{2PC, sub\}$ (black closed symbols) and $v^\mu_3\{2PC, sub\}$ for $-0.9 < \eta < 0.9$ extracted from track-track correlations in p–Pb collisions at $\sqrt{s_{NN}} = 5.02$ TeV [42].
different from the parent particle $v_2$. In the resolution correction described in Sec. 5.1.3, the differences in $p_T$ and $\varphi$ between true and reconstructed muons are taken into account. If $p_T$ and $\varphi$ of the parent particles can be considered instead of those of true muons, the resultant difference must be much larger, although the composition and input $v_2$ depend on the model. However, it is at least observed that the magnitudes of $v_\mu^{2}\{2PC, sub\}$ in Fig. 5.7 and $v_2\{2PC, sub\}$ in Fig. 5.8 are of the same order indicating the existence of similar LRC in the kinematic range of the FMS.

The $v_\mu^{2}\{2PC, sub\}$ for muons in the $p$-going ($p$–Pb) and Pb-going (Pb–$p$) directions from muon-tracklet correlations are shown on the left panel of Fig 5.9. As explained in Sec. 4.2.1, central barrel tracks are not available for the Pb–$p$ configuration and muon-track correlations cannot be used. $v_\mu^{2}\{2PC, sub\}$ in the Pb-going direction is larger than $v_2^{2}\{2PC, sub\}$ in the $p$-going direction throughout the muon $p_T$ range, but they show similar $p_T$ dependence. The difference in the two $v_\mu^{2}\{2PC, sub\}$ is quantified by the ratio, $v_\mu^{2}\{2PC, sub\}$ in the Pb-going direction over $v_2^{2}\{2PC, sub\}$ in the $p$-going direction (right panel of Fig. 5.9). Considering the statistical and systematic uncertainties, the ratio is considered to be independent of the muon $p_T$. The ratio is fitted with a constant function after the statistical and systematic uncertainties are added in quadrature, and the value is found to be $1.16 \pm 0.06$ with a $\chi^2$/NDF =0.4.
The results in Fig. 5.9 cannot be directly compared with predictions in [70], which are based on 3+1D hydrodynamic calculations [55] and the AMPT (A Multi-Phase Transport) model with the string melting mechanism [127] for the same reason that results cannot be compared with $v_2\{2PC,\text{sub}\}$ in Fig. 5.8. However, these two calculations predict qualitatively similar trends including larger $v_2$ in the Pb-going direction than those in the p-going direction. For the comparison, model calculations should include decaying of primary particles into muons, and apply the relative efficiencies of muon reconstruction provided in the right panel of Fig. 4.5. With these requirements, calculations based on AMPT model with the same parameter as in [70] were performed and results are shown in Fig. 5.9 in parallel with measurements in the experiment.

AMPT is a Monte Carlo transport model successfully described some bulk properties of heavy ion collisions, such as results of $v_2$ measurements [127]. It uses the HIJING (Heavy-Ion Jet Interaction Generator [128]) for generating the initial conditions, the Zhang’s Parton Cascade (ZPC) for modeling the partonic scatterings, and A Relativistic Transport (ART) model for treating hadronic scatterings. The string melting version of AMPT treats the initial condition as partons and uses a simple coalescence model to describe hadronization.

In the calculations, generated particles from AMPT were decayed into muons using the PYTHIA decayer [133], and relative efficiencies (right panel of Fig. 4.5) were applied to these muons. Event classifications were done in a similar way as the experimental measurements, using the number of charged particles in $2.8 < \eta < 3.9$ and $-3.7 < \eta < -2.7$, which are the $\eta$ ranges in V0S criteria. The $v_2^{\mu}\{2PC,\text{sub}\}$ values were estimated in the same way as in data, but separately for muons decaying from pions, kaons and heavy-flavor hadrons. Then $v_2^{\mu}\{2PC,\text{sub}\}$ from each decay channel were averaged to calculate the final $v_2^{\mu}\{2PC,\text{sub}\}$, while relative yields of each decay channel were used as a weighting factor in the averaging procedure. It was observed that $v_2$ from heavy-flavor muons is 0 throughout the $p_T$ range. Also, the scaling factor $f$, used to scale the yield in the lowest-multiplicity class in the subtraction method, reached $f = 2$ in the AMPT calculations. Different choice of $f$ values set the lower and upper bound of the $v_2^{\mu}\{2PC,\text{sub}\}$ values, and the possible values of $v_2^{\mu}\{2PC,\text{sub}\}$ within the boundaries are expressed with the shaded area in Fig. 5.9.
As shown in the left panel of Fig. 5.9, AMPT calculations qualitatively agree with trends as observed in the data throughout the considered $p_T$ range. However, as can be most easily seen in the right panel of Fig. 5.9, they are qualitatively different. At $p_T > 2.0 \text{ GeV}/c$, where the composition of parent distribution in this range is dominated by the heavy-flavor particles (left panel of Fig. 4.5), $v_2^{2PC,sub}$ measured in the data are significantly larger than those from AMPT calculations. This observation can be explained in a few physics scenarios: (1) muons from heavy-flavor decay have finite $v_2$, (2) the composition of the parent distribution in AMPT is different from that in the data, (3) the $v_2$ of parent particles in AMPT are different from those in the data.
Chapter 6

Azimuthal collectivity of longitudinal structures in lead-lead collisions

In this section, details and results of the analysis of the azimuthal collectivity of longitudinal structure in lead-lead collisions at $\sqrt{s_{NN}} = 2.76$ TeV are discussed. This analysis is based on the motivation explained in Sec. 2.5 and the event and track selections described in Sec. 4.1.2 and Sec. 4.2.2. Results are compared with Monte Carlo simulations for heavy-ion collisions, including HIJING and AMPT.

6.1 Analysis

The main purpose of this analysis is to find correlations among $\eta$ structures of produced particles at mid-rapidity in different azimuthal regions. The figurative scheme of the analysis is shown in Fig. 6.1 and Fig. 6.2. Based on the second-order event plane (Sec. 2.2) determined from VZERO detectors (Fig. 6.1), the azimuthal angle is divided into four regions with respect to the event plane (Fig. 6.2). The $\eta$ structure in each azimuthal
Figure 6.1: Configuration for event plane determination using VZERO detectors.

Figure 6.2: Azimuthal angle is divided into four regions with respect to $\Psi_2$. Each $\varphi_i$ (in-plane and out-of-plane) region covers $\pi/2$ in azimuth.
region is decomposed with orthogonal polynomials, and coefficients from the decomposition are extracted. Correlations among coefficients from different azimuthal regions reveal the azimuthal collectivity of the longitudinal structures in heavy-ion collisions.

### 6.1.1 Event plane estimation

The first step of the measurements is to determine the second-order event plane $\Psi_2$, in order to divide the azimuthal angle into four separate regions with respect to the $\Psi_2$. As mentioned in Sec. 2.2, the second-order event plane approximates the reaction plane or the participant plane in the measurement. Following procedures described in [27], the second-order event plane $\Psi_2$, is determined by

$$Q_{2,x} = \sum_{i=1}^{n} w_i \cos(2\varphi_i), \quad Q_{2,y} = \sum_{i=1}^{n} w_i \sin(2\varphi_i),$$

(6.1)

and

$$\Psi_2 = \frac{1}{n} \tan^{-1}\left(\frac{Q_{2,y}}{Q_{2,x}}\right).$$

(6.2)

Although there are a few different options for the $Q$-vector and $\Psi_2$ estimations in Eq. 6.1 and Eq. 6.2 such as using central-barrel tracks or signals in the ZDC, the VZERO detectors are used in this analysis. Estimating the $\Psi_2$ with information from VZERO detectors prevents measurements from the auto-correlation between $\Psi_2$ and central-barrel tracks, as the $\eta$ coverage of VZERO detectors are separated sufficiently from mid-rapidity. The separation between the $\eta$ ranges for the $Q$-vector estimation and for other observables has been used in various flow measurements (e.g. in [134]). As explained in Sec. 3.2.3, the VZERO detectors consist of two arrays (VZERO-A and VZERO-C), one on each side of the interaction point. Both VZERO-A ($2.8 < \eta < 5.1$) and VZERO-C ($-3.7 < \eta < -1.7$) have four rings, and each ring is divided into eight sectors of 45 degrees in azimuth. For the $Q$-vector estimation in Eq. 6.1, index $i$ runs over sectors of corresponding detectors, and $w_i$ is the measured multiplicity in sector $i$. $\Psi_2$ is estimated by VZERO-A, VZERO-C, and combined VZERO
detectors (V0C + V0A), separately, and the difference in results attributed to the selection of $\Psi_2$ is included in the systematic uncertainty estimation. For the nominal measurement, $\Psi_2$ from the combined VZERO detectors is used.

Due to the non-uniform efficiency of VZERO detectors, the calibration for the $\Psi_2$ estimation is performed using the procedures described in \cite{27,135}. The calibration consists of two steps, (1) gain equalization of the VZERO detector signal and (2) flattening of the $\Psi_2$ distribution through Q-vector re-centering, twisting, rescaling, and the Fourier flattening. The details and effects of each procedure are explained in \cite{135} and references therein, and the calibration results for the 2011 Pb–Pb collision data measured by ALICE are discussed in \cite{136,137}. As described in Sec. 4.1.2, centrality classes used in the analysis are 0–5%, 5–10%, 10–20%, 20–30%, 30–40%, and 40–50% bins based on the multiplicity in VZERO detectors for the nominal measurement. $\Psi_2$ distributions from calibrated VZERO-A and VZERO-C for each centrality class are shown in Fig. 6.3 as well as the uncalibrated $\Psi_2$ from VZERO-C for comparison. Flat distributions of $\Psi_2$ after calibration indicate the proper correction for the non-uniform efficiency of VZERO detectors.

The azimuthal angle ($\varphi$) with respect to the measured $\Psi_2$ in each event is divided into four regions, two in-plane regions ($\varphi_{in}$) and two out-of-plane regions ($\varphi_{out}$) as shown in Fig. 6.2. Two $\varphi_{in}$ bins cover the $\varphi$ ranges $\Psi_2 - \frac{\pi}{4} < \varphi < \Psi_2 + \frac{\pi}{4}$ and $\Psi_2 + \frac{3\pi}{4} < \varphi < \Psi_2 + \frac{5\pi}{4}$, and $\varphi_{out}$ bins cover $\Psi_2 + \frac{\pi}{4} < \varphi < \Psi_2 + \frac{3\pi}{4}$ and $\Psi_2 + \frac{5\pi}{4} < \varphi < \Psi_2 + \frac{7\pi}{4}$.

6.1.2 Decomposition with Legendre polynomials

As proposed in \cite{81}, the longitudinal structure of produced particles as a function of $\eta$ can be decomposed using orthogonal polynomials. There are several options for the choice of orthogonal polynomials, such as Chebyshev polynomials, Hermite polynomials, Laguerre polynomials, and Legendre polynomials. However, the Legendre polynomials have a particular advantage from their simple weight function equal to 1. Although this choice of orthogonal polynomials may not have large effects on the decomposition of the single particle $\eta$ distribution, it will significantly ease the estimation of coefficients in two- or multi-particle
**Figure 6.3:** Calibrated $\Psi_2$ distributions measured in VZERO-C (red) and VZERO-A (blue), and un-calibrated $\Psi_2$ from VZERO-C (black) for each centrality class with 2011 Pb–Pb data.
correlation functions.

The first few terms of Legendre polynomials are $P_0(x) = 1$, $P_1(x) = x$, $P_2(x) = (3x^2 - 1)/2$ and plotted in Fig. 6.4. As seen in the figure, the first-order coefficient corresponds to the level of forward-backward asymmetry within the $\eta$ range, and the second-order coefficient corresponds to the level of mid-peripheral asymmetry.

In the measurements by the ATLAS collaboration [83], the decomposition is applied to the single particle density ratio $R(\eta)$, defined by

$$R(\eta) \equiv \frac{N(\eta)}{\langle N(\eta) \rangle}, \quad (6.3)$$

where $N(\eta) \equiv dN/d\eta$ is the multiplicity distribution as a function of $\eta$ in a single event and $\langle N(\eta) \rangle$ is the average multiplicity distribution within the corresponding centrality class. In the current analysis, $R_{\varphi_i}(\eta)$ is used instead of $R(\eta)$ with

$$R_{\varphi_i}(\eta) \equiv \frac{1}{\int_{\varphi_i} d\varphi} \int_{\varphi_i}^{\varphi_{i+1}} N(\varphi, \eta) \langle N(\varphi, \eta) \rangle d\varphi, \quad (6.4)$$

where the integral range denoted by $\varphi_i$ is equal to the range of the corresponding $\varphi$ bin.
The definition of $R_{\phi_i}(\eta)$ in Eq. 6.4 not only accounts for the limited azimuthal ranges in the measurement, but also considers the uneven efficiency of the central-barrel tracks in ALICE. If the $\phi_i$ range in Eq. 6.4 covers the whole $2\pi$ in azimuth and the detector efficiency is flat throughout $(\phi, \eta)$ space, Eq. 6.4 and Eq. 6.3 become identical.

Within the considered $\eta$ range $[-Y, Y]$, $R_{\phi_i}(\eta)$ is decomposed with

$$R_{\phi_i}(\eta) \propto 1 + \sum_{n=1} a_n T_n(\eta) , \quad (6.5)$$

where

$$T_n(\eta) = \sqrt{n + \frac{1}{2}} P_n(\eta/Y) , \quad (6.6)$$

while $\sqrt{n + \frac{1}{2}}$ is for normalization, such that

$$\int_{-Y}^{Y} T_n(\eta) T_m(\eta) \, d\eta = \delta_{nm} , \quad (6.7)$$

in the current analysis. $Y$ in the $\eta$ range is 0.8 for the central-barrel tracks. The coefficients $a_n$ of the decomposition contain the information of the event-by-event fluctuations of the longitudinal structure.

It should be noted that either $N(\eta)$ within a given $\phi$ range or $R_{\phi_i}(\eta)$ can be decomposed with orthogonal polynomials, and the resultant coefficients from the decomposition commonly represent the $\eta$ structure for a given event. However, the latter has a few more advantages than the former namely: (1) reduced tracking efficiency effects due to the cancellation in the numerator and the denominator in the integrand of Eq. 6.4, and (2) larger sensitivity in event-by-event differences with respect to the mean multiplicity distribution.

### 6.1.3 $a_n$ measurement

Based on Eq. 6.5 the event-by-event $\eta$ structure of charged particles at mid-rapidity is studied with central-barrel tracks. Tracks are reconstructed based on hybrid track selection...
combining clusters in the TPC and ITS (Sec. 4.2.2), and the \( p_T \) range \( 0.2 < p_T < 5.0 \text{ GeV}/c \) is selected specifically to compare to flow measurements in the transverse direction \cite{19}. At each \( \varphi_i \) bin, the mean multiplicity distribution,

\[
\langle N(\varphi, \eta) \rangle_{\varphi_i} = \left\langle \frac{d^2 N}{d\varphi d\eta} \right\rangle_{\varphi_i}, \tag{6.8}
\]

is measured independently within given run number, centrality and 2 cm interval of \( z \)-vertex bin in order to reduce the tracking efficiency dependence on those variables. Due to the elliptic flow of heavy ion collisions, the two \( \varphi_{\text{in}} \) regions have larger multiplicities than the \( \varphi_{\text{out}} \) regions (Fig. 6.5). Then \( a_n(\varphi_i), a_n \) of the corresponding \( \varphi_i \) bin, are estimated with the central-barrel tracks for each centrality class by

\[
\frac{1}{\int_{\varphi_i}^{T} \int_{\varphi_i}^{T} \langle N(\varphi, \eta) \rangle_{\varphi_i} d\varphi = \alpha \left( 1 + \sum_{n=1}^{\infty} a_n(\varphi_i)T_n(\eta) \right), \tag{6.9}
\]

combining Eq. 6.4 and Eq. 6.5 using \( \langle N(\varphi, \eta) \rangle_{\varphi_i} \) of the corresponding run number, centrality, and \( z \)-vertex position.
The measurement procedures for \( a_n(\varphi_i) \) described in the above are equivalent to those in the event-plane method for the transverse flow measurement \[27\], as they both extract coefficients of the decomposition from the \( \eta \) or \( \varphi \) distribution of tracks in each event. Although the two-particle correlation method in the longitudinal direction \[81\] can be used to determine \( \sqrt{a_n a_m} \) by rejecting most of the statistical fluctuation attributed to the finite number of particles in a single event, it cannot determine \( a_n \) on an event-by-event basis or \( a_n \) from separate \( \varphi \) ranges in a single event. Meanwhile, direct measurement of \( a_n \) from a particle distribution in each event may experience considerable statistical fluctuations, but it enables measurements that the two-particle correlation method cannot. The effects of statistical fluctuations from the finite number of particles in each event will be discussed further in Sec. 6.1.5.

Figure 6.6 shows measured particle distributions normalized by the number of events as a function of \( \eta \) in two \( \varphi \) bins averaged over all runs and \( z \)-vertex. These distributions do not exactly agree with that used in the \( a_n(\varphi_i) \) evaluations, as \( \langle N(\varphi, \eta) \rangle_{\varphi_i} \) depends on the run number and \( z \)-vertex bin, but exhibit general trends of particle distributions in \( \eta \). Although the previous ALICE results on \( dN^{\text{track}}/d\eta \) \[75\] were based on different \( p_T \) ranges compared to the current one, overall shapes are similar except for the dip near \( \eta = 0 \) in Fig. 6.6. This is due to tracking efficiency effects at midrapidity, and the previous results were corrected for them. However, the tracking efficiency correction for tracks on an event-by-event basis is not needed for the current \( a_n(\varphi_i) \) measurements by definition in Eq. 6.4, where the correction factors are cancelled out in the numerator and the denominator of the integrand in Eq. 6.4. Selecting events with stricter conditions, such as limiting run numbers and narrower \( z \)-vertex criteria (Sec. 4.1.2) is required only to have a consistent level of statistical fluctuations in the measurements.

Figure 6.7 shows the measured \( a_n(\varphi_i) \) distributions of events for \( a_1 \) (top row), \( a_2 \) (middle row), and \( a_3 \) (bottom row) for \( \varphi_{\text{in}} \) (left column) and \( \varphi_{\text{out}} \) (right column) bins in different centrality classes. Throughout the analysis, coefficients are considered up to the third order. Each \( 1/N_{\text{ev}} dN_{\text{ev}}/da_n \) is fitted with a Gaussian function, and the widths of Gaussian fits are summarized as a function of centrality in Fig. 6.8. In general, the Gaussian fitting describes
Figure 6.6: Measured $1/N_{ev} \, dN^{\text{track}}/d\eta$ for $\varphi_{\text{in}}$ and $\varphi_{\text{out}}$ bins in each centrality bin.
Figure 6.7: Measured $1/N_{ev}dN_{ev}/da_1$ (top row), $1/N_{ev}dN_{ev}/da_2$ (middle row), and $1/N_{ev}dN_{ev}/da_3$ (bottom row) for $\varphi_{in}$ (left) and $\varphi_{out}$ (right) bins in different centrality classes.
Figure 6.8: Widths of Gaussian fits applied to $1/N_{ev} \frac{dN_{ev}}{d\alpha_n}$ up to the third order for $\varphi_{in}$ and $\varphi_{out}$ bins as a function of centrality. Statistical uncertainties are smaller than marker sizes.
well the shape throughout the centrality classes and the three coefficients of the Legendre decomposition. The measured $dN/da_n$ is more narrowly peaked, or equivalently the width of the Gaussian fit is smaller, in the more central classes. Also, $dN/da_n$ in the $\varphi_{in}$ bins have narrower distributions than those in the $\varphi_{out}$ bins. Since a finite number of particles is involved with the $a_n$ measurements, contributions from statistical fluctuations to $dN/da_n$ cannot be ignored. More detailed discussion about statistical fluctuations will be followed by toy Monte Carlo simulations in Sec. 6.1.5.

### 6.1.4 Conditional $a_n$

Once the $a_n(\varphi_i)$ are extracted in each event, correlations among the coefficients represent the correlations among the longitudinal structures in different azimuthal regions. The correlations are studied through the conditional $a_n$, denoted by $a_n(\varphi_i|a_m(\varphi_j))$. This notation is borrowed from probability theory, and defined as a mean $a_n(\varphi_i)$ value for given events with $a_m(\varphi_j)$. In other words, once events with a certain $a_m$ value in $\varphi_j$ bin are selected, the mean $a_n$ in the $\varphi_i$ bin is estimated for the value of $a_n(\varphi_i|a_m(\varphi_j))$.

Various combinations of $n$, $m$, ($n, m = 1, 2, 3, ...$) and $i$, $j$ ($i, j = \text{in, out}$) can be investigated to reveal features of azimuthal correlations of the longitudinal structures, but only $n = m$ cases up to $n = 3$ are discussed in this thesis. For $(\varphi_i, \varphi_j)$ combinations, four groups of conditional $a_n$ are studied:

- $a_n(\varphi_{\text{opp.in}}|a_n(\varphi_{\text{in}}))$
- $a_n(\varphi_{\text{opp.out}}|a_n(\varphi_{\text{out}}))$
- $a_n(\varphi_{\text{out}}|a_n(\varphi_{\text{in}}))$
- $a_n(\varphi_{\text{in}}|a_n(\varphi_{\text{out}}))$

where “opp.in” and “opp.out” correspond to the opposite in-plane and opposite out-of-plane $\varphi$ bins, respectively. Since there are two $\varphi_{\text{in}}$ bins and two $\varphi_{\text{out}}$ bins, the above observables
are estimated by combining all possible configurations in $\varphi$. For example, there are two configurations for $a_n(\varphi_{\text{opp.in}}|a_n(\varphi_{\text{in}}))$, left $\varphi$ bin to right $\varphi$ bin and right $\varphi$ bin to left $\varphi$ bin in Fig. 6.2 and $a_n(\varphi_{\text{opp.in}}|a_n(\varphi_{\text{in}}))$ combines measurements of two configurations in $\varphi$. Also, $a_1(\varphi_i|a_1(\varphi_j))$ and $a_3(\varphi_i|a_3(\varphi_j))$ are odd functions as a function of $a_1(\varphi_j)$ and $a_3(\varphi_j)$, respectively, and values in positive and negative ranges are supposed to be identical in size and opposite in sign. Thus, additional steps are applied in the evaluation of these observables to have better statistics:

- $a_1(\varphi_i|a_1(\varphi_j)) \equiv a_1(\varphi_i|a_1(\varphi_j)) \oplus (-1) a_1(\varphi_i - a_1(\varphi_j))$
- $a_3(\varphi_i|a_3(\varphi_j)) \equiv a_3(\varphi_i|a_3(\varphi_j)) \oplus (-1) a_3(\varphi_i - a_3(\varphi_j))$

where $\oplus$ in the equation means that conditional $a_n$ on the left side of the equation is averaged over left and right quantities with respect to $\oplus$. This means that conditional $a_1$ (conditional $a_3$) values for positive and negative $a_1(\varphi_j)$ ($a_3(\varphi_j)$) are averaged by folding the two ranges with negative coefficient in the other range.

Figure 6.9 shows one example of conditional $a_n$ results, which are conditional $a_1$ for four groups of $(\varphi_i, \varphi_j)$ combinations. The diagram in each panel represents the $(\varphi_i, \varphi_j)$ combination in the corresponding $a_1(\varphi_i|a_1(\varphi_j))$, where the blue and green areas correspond to the $\varphi_i$ and $\varphi_j$ region, respectively. Although each diagram shows a specific $(\varphi_i, \varphi_j)$ combination, the values of conditional $a_1$ are estimated combining all possible configurations in $\varphi$. $a_1(\varphi_i|a_1(\varphi_j))$ for each centrality is fitted with a linear function, $y = \alpha x$, and slopes $\alpha$ are plotted in Fig. 6.10. In $a_1(\varphi_{\text{opp.in}}|a_1(\varphi_{\text{in}}))$ and $a_1(\varphi_{\text{opp.out}}|a_1(\varphi_{\text{out}}))$, the slopes are negative for all centralities and become more negative for the more central classes. This indicates that when a positive (negative) forward-backward asymmetry in mid-rapidity is observed in one of the $\varphi$-regions, a negative (positive) forward-backward asymmetry is observed as a consequence in the opposite $\varphi$-region. Meanwhile, in the $a_1(\varphi_{\text{out}}|a_1(\varphi_{\text{in}}))$ and $a_1(\varphi_{\text{in}}|a_1(\varphi_{\text{out}}))$, the slopes are slightly negative in central classes, and positive in the peripheral classes. Since the two $\varphi$ bins are in contact for these observables, the positive slope corresponds to the proportional $a_1$ values between the two adjacent $\varphi$ bins.
Figure 6.9: $a_1(\phi_{\text{opp.in}}|a_1(\phi_{\text{in}}))$ (top left), $a_1(\phi_{\text{opp.out}}|a_1(\phi_{\text{out}}))$ (top right), $a_1(\phi_{\text{out}}|a_1(\phi_{\text{in}}))$ (bottom left), and $a_1(\phi_{\text{in}}|a_1(\phi_{\text{out}}))$ (bottom right) with statistical uncertainty only without any correction. Each of conditional $a_1$ is fitted with a linear function.
Figure 6.10: Slopes of conditional $a_1$ as functions of centrality without any correction.

Similarly, conditional $a_2$ and $a_3$ are measured for four groups of $(\varphi_i, \varphi_j)$ combinations. However, the results and corresponding discussions are postponed to Sec. 6.3 where results have been corrected for the event-plane resolution effects and multiplicity effects and systematic uncertainties have been calculated.

6.1.5 Correction procedures

Two correction procedures are applied in sequence to the conditional $a_n$ (e.g. Fig. 6.9) and the related slopes (e.g. Fig. 6.10). Observables are corrected for effects of the event-plane resolution and the finite multiplicity in the measurements in this section. The results are observed to be insensitive to the order in which the corrections are made.
Correction for event-plane resolution effects

The second-order event-plane angle ($\Psi_2$) is determined at the first stage of the analysis, as the azimuthal angle must be divided into four regions with respect to the $\Psi_2$. The event-plane used throughout Sec. 6.1.1 to Sec. 6.1.4 is an observed quantity using clusters in the VZERO detectors. However, $\Psi_2$ may not always agree with $\Phi_2$, which is the true second-order event-plane, since $\Psi_2$ is calculated with a finite number of particles. The difference between $\Psi_2$ and $\Phi_2$ results in certain deviations in the values of observables, and these are generally referred to as event-plane resolution effects. In the transverse-direction flow measurements, parameterized by the Fourier coefficients $v_n$, correction procedures for event-plane resolution effects have been well documented in various publications, such as [27]. However, correction procedures for observables in the current analysis are uniquely devised through mathematical derivations with a few assumptions, as the observables are newly conceived.

In the transverse-direction flow analyses, the observed $v_n = v_n^{\text{obs}}$ from the event-plane method is corrected by a factor $\langle \cos(n(\Psi_n - \Phi_n)) \rangle$, and the corrected $v_n$ is estimated by

$$v_n = \frac{v_n^{\text{obs}}}{\langle \cos(n(\Psi_n - \Phi_n)) \rangle},$$

(6.10)

where $\Phi_n$ and $\Psi_n$ are the true $n$th-order event-plane and the observed $n$th-order event-plane, respectively [27]. $v_n^{\text{obs}}$ is an observable estimated with respect to $\Psi_n$, and the goal of the correction is to have $v_n$ with respect to the $\Phi_n$. In Eq. (6.10) the correction factor $\langle \cos(n(\Psi_n - \Phi_n)) \rangle$ is derivable analytically and links $v_n$ and $v_n^{\text{obs}}$. Also, $\langle \cos(n(\Psi_n - \Phi_n)) \rangle$ can be determined through the two-subevent method or the three-subevent method [27,139].

In the same sense, any physics quantity estimated with respect to $\Psi_2$ needs to be corrected to the corresponding quantity with respect to $\Phi_2$. However, the correction factor, such as $\langle \cos(n(\Psi_n - \Phi_n)) \rangle$ in Eq. (6.10), is not always derivable analytically and largely depends on the observable itself. Although the following derivation is based on the observables and divisional scheme of the azimuthal angle used in the current analysis, it can be modified
for various situations. In other words, the correction procedures derived in this section are
general enough to be applied to other observables based on divided azimuthal regions with
respect to the event-plane angle.

Throughout Chap. 6, the notation for conditional \(a_n, a_n(\varphi_i|a_m(\varphi_j))\), is used for convenience,
and equivalent to \(\langle a_n(\varphi_i|a_m(\varphi_j)) \rangle\), where the bracket corresponds to the average over events
in a given centrality class. But only in the following discussion on event-plane resolution
effects, \(a_n(\varphi_i|a_m(\varphi_j))\) and \(\langle a_n(\varphi_i|a_m(\varphi_j)) \rangle\) are distinguished to facilitate the derivation.
\(a_n(\varphi_i|a_m(\varphi_j))\) is conditional \(a_n\) in an event, and \(\langle a_n(\varphi_i|a_m(\varphi_j)) \rangle\) is the average of conditional
\(a_n\) over events in a centrality class. For the correction of \(\langle a_n(\varphi_i|a_m(\varphi_j)) \rangle\), the four \((i,j)\)
groups introduced in Sec. 6.1.4 need to be classified into two observable classes:

- Class 1: \(a_n(\varphi_{\text{opp.}i}|a_m(\varphi_{\text{in}}))\) and \(a_n(\varphi_{\text{opp.}i}|a_m(\varphi_{\text{out}}))\)
- Class 2: \(a_n(\varphi_{\text{out}}|a_m(\varphi_{\text{in}}))\) and \(a_n(\varphi_{\text{in}}|a_m(\varphi_{\text{out}}))\)

With \(a_n(\varphi_i|a_m(\varphi_j))\) notation, \(\varphi_i\) and \(\varphi_j\) are opposite in \(\varphi\) in class 1, and perpendicular
in class 2. If \(\Theta\) is the center of \(\varphi_j\)-bin in \(\varphi\), \(a_n(\varphi_{\text{opp.}i}|a_m(\varphi_{\text{in}}))\) corresponds to the class 1
observable with \(\Theta = \Psi_2\) and \(a_n(\varphi_{\text{opp.}i}|a_m(\varphi_{\text{out}}))\) corresponds to the class 1 observable
with \(\Theta = \Psi_2 + \pi/2\). Similarly, \(a_n(\varphi_{\text{out}}|a_m(\varphi_{\text{in}}))\) corresponds to the class 2 observable
with \(\Theta = \Psi_2\) and \(a_n(\varphi_{\text{in}}|a_m(\varphi_{\text{out}}))\) corresponds to the class 2 observable with \(\Theta = \Psi_2 + \pi/2\).
This classification reflects the fact that \(a_n(\varphi_i|a_m(\varphi_j))\) is estimated based on the
azimuthal division with respect to \(\Psi_2\), and the notation of \(a_n(\varphi_i|a_m(\varphi_j))\) is replaced by
\(a_n(\varphi_i|a_m(\varphi_j))|_{\text{obs}}\) to specify the usage of \(\Psi_2\) in the estimation. This classification works for
all \((n,m)\) combinations.

As mentioned in the above, the ultimate goal of correction for event-plane resolution effects
is to estimate the observables with respect to \(\Phi_n\) instead of \(\Psi_2\). For the ease of discussion,
slopes of \(\langle a_1(\varphi_i|a_1(\varphi_j)) \rangle\) (Fig. 6.10) will be used as an example. If the slopes of class 1 and
2 observables are assumed to depend only on \(\Theta\), those slopes can be denoted with \(I_1(\Theta)\)
and \(I_2(\Theta)\), respectively. In other words, if a function \(S(\langle a_1(\varphi_i|a_1(\varphi_j)) \rangle)\) is defined to yield
the value of the slope of \(\langle a_1(\varphi_i|a_1(\varphi_j)) \rangle\),
\[ S(\langle a_n(\varphi_{\text{opp.in}}|a_m(\varphi_{\text{in}}))|_{\text{obs}} \rangle) = \langle I_1(\Theta = \Psi_2) \rangle \]
\[ S(\langle a_n(\varphi_{\text{opp.out}}|a_m(\varphi_{\text{out}}))|_{\text{obs}} \rangle) = \langle I_1(\Theta = \Psi_2 + \pi/2) \rangle \]
\[ S(\langle a_n(\varphi_{\text{out}}|a_m(\varphi_{\text{in}}))|_{\text{obs}} \rangle) = \langle I_2(\Theta = \Psi_2) \rangle \]
\[ S(\langle a_n(\varphi_{\text{in}}|a_m(\varphi_{\text{out}}))|_{\text{obs}} \rangle) = \langle I_2(\Theta = \Psi_2 + \pi/2) \rangle , \]

and the goal of the correction is to estimate following:

\[ S(\langle a_n(\varphi_{\text{opp.in}}|a_m(\varphi_{\text{in}})) \rangle) = \langle I_1(\Theta = \Phi_2) \rangle = I_1(\Theta = \Phi_2) \]
\[ S(\langle a_n(\varphi_{\text{opp.out}}|a_m(\varphi_{\text{out}})) \rangle) = \langle I_1(\Theta = \Phi_2 + \pi/2) \rangle = I_1(\Theta = \Phi_2 + \pi/2) \]
\[ S(\langle a_n(\varphi_{\text{out}}|a_m(\varphi_{\text{in}})) \rangle) = \langle I_2(\Theta = \Phi_2) \rangle = I_2(\Theta = \Phi_2) \]
\[ S(\langle a_n(\varphi_{\text{in}}|a_m(\varphi_{\text{out}})) \rangle) = \langle I_2(\Theta = \Phi_2 + \pi/2) \rangle = I_2(\Theta = \Phi_2 + \pi/2) , \]

these equations imply that \( I_1(\Theta) \) and \( I_2(\Theta) \) depend only on \( \Theta - \Phi_2 \), the difference between the reference angle \( \Theta \) and the true event plane \( \Phi_2 \). For example,

\[ \langle I_1(\Theta = \Phi_2) \rangle = I_1(\Theta = \Phi_2) \] (6.11)

is valid because \( I_1 \) is assumed to depend only on \( \Theta - \Phi_2 \). In \( I_1(\Theta = \Psi_2) \), \( \Psi_2 \) does not always agree with \( \Phi_2 \), and \( \langle I_1(\Theta = \Psi_2) \rangle \) is distinguished from \( I_1(\Theta = \Phi_2) \). If the \( \Theta \)-dependence of \( I_1 \) (and \( I_2 \)), is known, it is more probable to analytically find a correction factor. However, the \( \Theta \)-dependence of observables is not mathematically known in most cases. Instead, the correction factor can be derived in a data-driven way.

According to Fig. 6.10, \( \langle a_1(\varphi_{\text{opp.out}}|a_1(\varphi_{\text{out}}))|_{\text{obs}} \rangle \) \( (= I_1(\Theta = \Psi_2 + \pi/2) \) \) is more negative than \( \langle a_1(\varphi_{\text{opp.in}}|a_1(\varphi_{\text{in}}))|_{\text{obs}} \rangle \) \( (= I_1(\Theta = \Psi_2) \) \) in every centrality class. Considering the definitions of event-plane \( \Phi_2 \) and \( I_1(\Theta) \), several characteristics of \( I_1(\Theta) \) can be used for further derivations. Since \( I_1(\Theta) \) depends on \( \Theta - \Phi_2 \), \( I_1(\Theta) \) is assumed to be a continuous and periodic function, i.e.

\[ I_1(\Theta) = I_1(\Theta + n\pi) \] (6.12)
where \( n \) is an integer. Also, \( I_1(\Theta) \) in \( \Phi_2 < \Theta < \Phi_2 + \pi \) is symmetric about \( \Theta = \Phi_2 + \pi/2 \), i.e.

\[
I_1(\Theta = \Phi_2 + \theta) = I_1(\Phi_2 + \pi - \theta).
\]  

(6.13)

Although the mathematical form of \( I_1(\Theta) \) cannot be derived from its continuity and symmetry in Eq. (6.12) and Eq. (6.13), an educated guess for the mathematical form can be made based on these features. In addition, with two boundary conditions, \( \langle I_1(\Theta = \Psi_2) \rangle \) and \( \langle I_1(\Theta = \Psi_2 + \pi/2) \rangle \) in each centrality class (Fig. 6.10), the number of variables in \( I_1(\Theta) \) should be two for these calculations.

The most educated guess for the form of \( I_1(\Theta) \) is

\[
I_1(\Theta) = \alpha \cos(2(\Theta - \Phi_2)) + \beta,
\]

(6.14)

where \( \alpha \) and \( \beta \) are constants that need to be determined with boundary conditions. Eq. (6.14) essentially assumes that the observable \( I_1 \) follows the same azimuthal dependence as the multiplicity distribution in \( \varphi \) originating from the elliptic flow. To determine \( \alpha \) and \( \beta \) in Eq. (6.14), two boundary conditions, \( \langle I_1(\Theta = \Psi_2) \rangle \) and \( \langle I_1(\Theta = \Psi_2 + \pi/2) \rangle \) are used:

\[
\langle I_1(\Theta = \Psi_2) \rangle = \alpha \langle \cos(2(\Psi_2 - \Phi_2)) \rangle + \beta,
\]

(6.15)

\[
\langle I_1(\Theta = \Psi_2 + \pi/2) \rangle = \alpha \langle \cos(2(\Psi_2 + \pi/2 - \Phi_2)) \rangle + \beta = -\alpha \langle \cos(2(\Psi_2 - \Phi_2)) \rangle + \beta.
\]

(6.16)

In Eq. (6.15) and (6.16), \( \langle \cos(2(\Psi_2 - \Phi_2)) \rangle \) is exactly the same as the correction factor for the \( v_n \) correction in the event-plane method, and can be determined by the two-subevent method or the three-subevent method \[27,139\]. Once \( \alpha \) and \( \beta \) are determined, \( I_1(\Phi_2) \) and \( I_1(\Phi_2 + \pi/2) \) can be estimated with

\[
\langle I_1(\Phi_2) \rangle = \alpha + \beta,
\]

(6.17)
\[ \langle I_1(\Phi_2 + \pi/2) \rangle = -\alpha + \beta . \quad \text{(6.18)} \]

Previously, \( \langle \cos(2(\Psi_2 - \Phi_2)) \rangle \) has been measured for 2.76 TeV Pb–Pb collisions in 2010 and reported in [106]. Figure 6.11 shows the corresponding results from [106] and from 2.76 TeV Pb–Pb collisions in 2011 in parallel. For \( \langle \cos(2(\Psi_2 - \Phi_2)) \rangle \) with VZERO detectors in the current data, the three-subevent method is used when \( \Psi_2 \) is determined with exclusive VZERO-A or VZERO-C, and the two-subevent method is used when \( \Psi_2 \) is determined with the combined VZERO-A and VZERO-C detectors. In the two-subevent method, signals in VZERO-A become one subevent, and those in VZERO-C become the other subevent [27].

When \( \Psi_2 \) from VZERO-A and \( \Psi_2 \) from VZERO-C are calculated with the three-subevent method, Eq. (16) in [27] is calculated by

\[
\langle \cos(2(\Psi_2 - \Phi_2)) \rangle_{V0A} = \sqrt{\frac{\langle \cos(2(\Psi_2^{V0A} - \Psi_2^{ring0})) \rangle \langle \cos(2(\Psi_2^{V0A} - \Psi_2^{ring3})) \rangle}{\langle \cos(2(\Psi_2^{ring0} - \Psi_2^{ring3})) \rangle}}, \quad \text{(6.19)}
\]

\[
\langle \cos(2(\Psi_2 - \Phi_2)) \rangle_{V0C} = \sqrt{\frac{\langle \cos(2(\Psi_2^{V0C} - \Psi_2^{ring4})) \rangle \langle \cos(2(\Psi_2^{V0C} - \Psi_2^{ring7})) \rangle}{\langle \cos(2(\Psi_2^{ring4} - \Psi_2^{ring7})) \rangle}}, \quad \text{(6.20)}
\]

while the \( \eta \)-coverage of each ring is specified in Table 3.1.

As an example, slopes of conditional \( a_1 \) measured with the event plane from combined VZERO detectors (VZERO-A + VZERO-C) with and without event-plane resolution corrections are shown on Fig. 6.12. The difference between slopes of conditional \( a_1 \) before and after the correction are shown on Fig. 6.13. It should be noted that

\[ \text{Slope after correction} - \text{Raw slope} \]

is shown on Fig. 6.13 instead of

\[ \frac{\text{Slope after correction}}{\text{Raw slope}} \]
Figure 6.11: (Top) $\langle \cos(n(\Psi_2 - \Phi_2)) \rangle$ of the second-order event plane angle $\Psi_2^{EP}$ extracted from two- and three-detector subevent correlations for TPC, VZERO, FMD, and PMD [106]. (Bottom) Resolution of the second-order event plane angle $\Psi_2$, from VZERO-A and VZERO-C detectors extracted with the three-subevent method and $\Psi_2$ from combined VZERO detectors (VZERO-A + VZERO-C) extracted with the two-subevent method.
Figure 6.12: Slopes of conditional $a_1$ as a function of centrality without (left) and with (right) corrections for resolution effects.

$$\frac{\text{Slope after correction} - \text{Raw slope}}{\text{Raw slope}},$$

because values of raw slopes are near 0 in certain centrality classes. The absolute magnitude of the correction is not so large relative to the size of the statistical uncertainty (Fig. 6.13), especially for the case that the combined VZERO detectors are used for the $\Psi_2$ estimation.

To further investigate the effect of the event-plane resolution correction, slopes of conditional $a_1$, especially class 1 observables ($a_1(\varphi_{\text{opp.in}}|a_1(\varphi_{\text{in}}))$ and $a_1(\varphi_{\text{opp.out}}|a_1(\varphi_{\text{out}}))$), with respect to the $\Psi_2$ measured with the combined VZERO detectors and VZERO-A are compared in Fig. 6.14. Differences between $\langle a_1(\varphi_{\text{opp.in}}|a_1(\varphi_{\text{in}})) \rangle_{\text{obs}}$ and $\langle a_1(\varphi_{\text{opp.out}}|a_1(\varphi_{\text{out}})) \rangle_{\text{obs}}$ is generally smaller for the case that $\Psi_2$ is determined from VZERO-A. This observation agrees with the worse resolution in the $\Psi_2$ estimation with the VZERO-A detector compared to that with the combined VZERO detectors (Fig. 6.11). After the correction for the event-plane resolution effects, the differences are generally reduced. Results of the observables with $\Psi_2$ measured from VZERO-A and VZERO-C are taken into account in the systematic uncertainties.

Eq. (6.14) is used as a nominal form in the correction for event-plane resolution effects, but
Figure 6.13: Difference of slopes of conditional $a_1$ with and without correction for resolution effects.

Figure 6.14: Slopes of class 1 conditional $a_1$ as a function of centrality measured with combined VZERO detectors and VZERO-A without (left) and with (right) the correction for resolution effects.
other forms of $I_1(\Theta)$ can be used for the systematic check of the correction. The simplest form of $I_1(\Theta)$ is a linear function

$$I_1(\Theta) = \alpha |\Theta - \Phi_2| + \beta,$$  \hspace{1cm} (6.21)

within $\Phi_2 - \pi/2 < \Theta < \Phi_2 + \pi/2$, which is then copied in other $\Theta$ ranges using its periodicity. However, the application of this form in practice requires a few more calculations. Mathematical derivations on how to calculate $\langle \cos(2(\Psi_2 - \Phi_2)) \rangle$ have been discussed in [27], but for the linear form of $I_1(\Theta)$ (Eq. 6.21), whereas the $\langle |\Psi_2 - \Phi_2| \rangle$ and $\langle |\Psi_2 - \Phi_2 + \pi/2| \rangle$ need to be calculated instead of $\langle \cos(2(\Psi_2 - \Phi_2)) \rangle$.

The measured flow vector $\vec{Q}_2$, defined in Eq. 6.1, fluctuates around its true vector, $\vec{Q}_2'$. Assuming Gaussian fluctuations, which are true when the multiplicity is large according to the central limit theorem, the dispersion distribution can be written as

$$\frac{dN}{dQ_{2,x} dQ_{2,y}} \sim \exp \left( -\frac{|\vec{Q}_2 - \vec{Q}_2'|^2}{2\sigma^2} \right).$$  \hspace{1cm} (6.22)

In radial coordinates, Eq. 6.22 becomes

$$\frac{dN}{Q_2 dQ_2 d(\Delta \Psi_2)} = \frac{2}{\pi \sigma^2} \exp \left( -\frac{Q_2^2 + Q_2'^2 - 2Q_2Q_2'\cos(2\Delta \Psi_2)}{2\sigma^2} \right),$$  \hspace{1cm} (6.23)

where $\Delta \Psi_2 = \Psi_2 - \Phi_2$, and $2/(\pi \sigma^2)$ is introduced for the normalization of the dispersion distribution with $Q_2$ in the range $[0, \infty]$ and $\Delta \Psi_2$ in the range $[0, \pi/2]$. Integration range with respect to $\Delta \Psi_2$ is determined considering the symmetry and periodicity of the distribution. Then,

$$\frac{dN}{d(\Delta \Psi_2)} = \frac{2}{\pi \sigma^2} \int_0^{\pi/2} dQ_2 Q_2 \exp \left( -\frac{Q_2^2 + Q_2'^2 - 2Q_2Q_2'\cos(2\Delta \Psi_2)}{2\sigma^2} \right)$$

$$= \frac{2}{\pi} e^{-\frac{\chi^2}{2}} \left( 1 + \sqrt{\frac{\pi}{2}} e^{-\frac{\chi^2}{2} \cos^2(2\Delta \Psi_2)} \chi \cos(2\Delta \Psi_2) (1 + \text{Erf}\left[ \frac{\chi \cos(2\Delta \Psi_2)}{\sqrt{2}} \right]) \right),$$  \hspace{1cm} (6.24)

where Erf is an error function and $\chi = \frac{Q_2}{\sigma}$. As verified in [27], $\langle \cos(2(\Psi_2 - \Phi_2)) \rangle$ is
analytically calculable based on Eq. 6.24,

\[
\langle \cos(2\Delta \Psi_2) \rangle = \int_0^{\pi/2} \frac{dN}{d(\Delta \Psi_2)} \cos(2\Delta \Psi_2) d(\Delta \Psi_2) = \frac{\sqrt{\pi}}{2\sqrt{2}} \chi e^{-\chi^2} \left[ I_0\left(\frac{\chi^2}{4}\right) + I_1\left(\frac{\chi^2}{4}\right) \right],
\]

(6.25)

where \(I_n\) is the modified Bessel function of the first kind with order \(n\). Further steps are needed to determine \(\chi\) in Eq. 6.25 but not discussed in this section as they can be found in [27]. Meanwhile, for \(\langle |\Psi_2 - \Phi_2| \rangle\),

\[
\langle |\Psi_2 - \Phi_2| \rangle = \int_0^{\pi/2} \frac{dN}{d(\Delta \Psi_2)} (\Delta \Psi_2) d(\Delta \Psi_2)
\]

(6.26)

is not analytically calculable. Instead, using \(\chi\) values in \(\frac{dN}{d(\Delta \Psi_2)}\), a numerical integration of Eq. 6.26 is conducted as well as a similar numerical integration on \(I_1(\Phi_2 + \pi/2)\). Then \(\alpha\) and \(\beta\) in Eq. 6.21 are evaluated to get \(I_1(\Phi_2)\) and \(I_1(\Phi_2 + \pi/2)\).

For the final results, the cosine form in the procedure (Eq. 6.14) is used for the nominal event-plane resolution correction, and results using the linear form (Eq. 6.21) are included in the estimation of systematic uncertainties.

**Correction for multiplicity effects**

A finite number of particles is involved in the determination of \(\Psi_2\), as well as in the estimation of \(a_n\) and conditional \(a_n\). Its effects in \(\Psi_2\) and its resulting measurements are corrected in the previous discussion, while its effects in conditional \(a_n\) measurements are corrected following the procedures described below.

Due to the elliptic flow, the true multiplicity within the \(\varphi_{in}\)-bin is

\[
N_{in}^{\text{true}} = \int_{\Phi_2-\pi/4}^{\Phi_2+\pi/4} \frac{N_0}{2\pi} \left[ 1 + 2 \sum_{n=1}^{\infty} v_n \cos(n(\varphi - \Phi_n)) \right] d\varphi = \frac{N_0}{4} + \frac{2N_0}{\pi} \sum_{n=1}^{\infty} \left( \frac{v_n}{n} \cos(n(\Phi_2 - \Phi_n)) \sin\left(\frac{n\pi}{4}\right) \right),
\]

(6.27)

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where $N_0$ is the true multiplicity of an event in the corresponding centrality class in the considered kinematic ranges. Then the mean multiplicity becomes

$$\langle N_{\text{true}}^{\text{in}} \rangle = \frac{\langle N_0 \rangle}{4} \left( 1 + \frac{4v_2}{\pi} \right), \quad (6.28)$$

while the mean multiplicity in the $\varphi_{\text{out}}$-bin from similar calculations is

$$\langle N_{\text{true}}^{\text{out}} \rangle = \frac{\langle N_0 \rangle}{4} \left( 1 - \frac{4v_2}{\pi} \right). \quad (6.29)$$

However, the measured multiplicities in $\varphi_{\text{in}}$ and $\varphi_{\text{out}}$ bins are different from the above due to the tracking efficiency and difference between $\Phi_n$ and $\Psi_n$. The measured multiplicities are

$$N_{\text{meas}}^{\text{in}} = \int_{\Psi_2-\pi/4}^{\Psi_2+\pi/4} \frac{N_0^{\text{meas}}}{2\pi} \left( 1 + 2 \sum_{n=1}^{\infty} v_n \cos(n(\varphi - \Phi_n)) \right) d\varphi$$

$$= \frac{N_0^{\text{meas}}}{4} + \frac{2N_0^{\text{meas}}}{\pi} \sum_{n=1}^{\infty} \left( \frac{v_n}{n} \cos(n(\Psi_2 - \Phi_n)) \sin\left(\frac{n\pi}{4}\right) \right), \quad (6.30)$$

$$N_{\text{meas}}^{\text{out}} = \frac{N_0^{\text{meas}}}{4} + \frac{2N_0^{\text{meas}}}{\pi} \sum_{n=1}^{\infty} \left( \frac{v_n}{n} \cos(n(\Psi_2 - \Phi_n)) \sin\left(\frac{n\pi}{4}\right) \right), \quad (6.31)$$

and no further simplification can be done in $\langle N_{\text{out}}^{\text{meas}} \rangle$ and $\langle N_{\text{out}}^{\text{meas}} \rangle$.

The measurements of $a_n$ and conditional $a_n$ through Sec. 6.1.1 to Sec. 6.1.4 are made with $N_{\text{in}}^{\text{meas}}$ and $N_{\text{out}}^{\text{meas}}$. The goal of the correction for the multiplicity effects is to correct deviations attributed to the difference in the measured multiplicity from the true multiplicity. In the above equations, a few parameters are already known, such as $N_{\text{in}}^{\text{meas}}$ and $N_{\text{out}}^{\text{meas}}$ from measurements with data, $v_2$ in each centrality class from previous publication [19], and $\langle N_0 \rangle$ from applying the efficiency correction to the particle distribution as described in Sec. 4.2.2. Although corrections related to $a_1$ are discussed exclusively in the following, the same correction procedures are applied to all orders of $a_n$.

First of all, multiplicity effects in $(1/N_{\text{ev}}) dN_{\text{ev}}/da_1$ in Fig. 6.7 are considered. As more ob-
Previously shown in Fig. 6.8, the widths of the $a_1$ distributions of events are smaller in the more central classes, and smaller in the $\varphi_{\text{in}}$ bin than those in the $\varphi_{\text{out}}$ bin. However, the level of effects from the finite number of particles (i.e. multiplicity effects) depends on the centrality and $\varphi$ bin, as different centrality classes and $\varphi$ bins have different numbers of particles. When a particle distribution with a specific $a_1 = a_{1,\text{in}}$ value is imagined to be detected over many events, the measured $a_1 = a_{1,\text{meas}}$ in each event may not agree with the input $a_{1,\text{in}}$ due to the multiplicity effects and resultant fluctuations in the measurement. Instead, $a_{1,\text{meas}}$ is peaked at $a_{1,\text{in}}$ and distributed around $a_{1,\text{in}}$. The width of the $a_{1,\text{meas}}$ distribution is smaller with higher multiplicity. Thus, one may guess that the difference among widths of $a_1$ distributions of events for centrality classes and $\varphi$ bins are solely attributed to the different multiplicities. To determine the validity of these statements, the $a_{1,\text{in}}$ distribution is investigated for each centrality class and the $\varphi$ bin from a toy Monte Carlo simulation with a few assumptions. As the $(1/N_{\text{ev}}) dN_{\text{ev}}/da_1$ in Fig. 6.7 are nicely described with Gaussian functions, the $(1/N_{\text{ev}}) dN_{\text{ev}}/da_{1,\text{in}}$ are also assumed to be Gaussian functions. Once a Gaussian $(1/N_{\text{ev}}) dN_{\text{ev}}/da_{1,\text{in}}$ in the toy Monte Carlo simulation is combined with the multiplicity distribution of a given centrality class and $\varphi$ bin, the corresponding $(1/N_{\text{ev}}) dN_{\text{ev}}/da_{1,\text{meas}}$ for a given $(1/N_{\text{ev}}) dN_{\text{ev}}/da_{1,\text{in}}$ can be estimated. Various widths of $(1/N_{\text{ev}}) dN_{\text{ev}}/da_{1,\text{in}}$ ($= \sigma_{\text{in}}$) are tested, and the corresponding $(1/N_{\text{ev}}) dN_{\text{ev}}/da_{1,\text{meas}}$ results are shown on Fig. 6.15 for the centrality class 20-30% and $\varphi_{\text{in}}$ as an example. In Fig. 6.15, the $(1/N_{\text{ev}}) dN_{\text{ev}}/da_{1,\text{meas}}$ from data is shown with a red line along with the Monte Carlo results. Then the $\sigma_{\text{meas}}$ as a function of the widths of $(1/N_{\text{ev}}) dN_{\text{ev}}/da_{1,\text{in}}$ ($= \sigma_{\text{meas}}$) are extracted (see Figure 6.16). The $\sigma_{\text{meas}}$ from data is indicated with a vertical red line in Fig. 6.16 and the $\sigma_{\text{meas}}$ as a function of $\sigma_{\text{in}}$ is fitted with the second-order polynomial in order to find the corresponding value of $\sigma_{\text{in}}$ for the data.

With the above procedures, $\sigma_{\text{in}}$ for $\sigma_{\text{meas}}$ from data in all centrality classes and $\varphi$ bins are individually extracted and shown in the left panel in Fig. 6.17. The observed trends in Fig. 6.8 smaller widths in more central classes and smaller widths in $\varphi_{\text{in}}$ bins than those in $\varphi_{\text{out}}$ bins, similarly exist even if the $a_{1,\text{in}}$ are used instead of $a_{1,\text{meas}}$. In other words, the trends in the differences among widths of $(1/N_{\text{ev}}) dN_{\text{ev}}/da_{1,\text{meas}}$ from centrality classes and
Figure 6.15: $(1/N_{ev})\,dN_{ev}/da_{1,meas}$ from various widths of $(1/N_{ev})\,dN_{ev}/da_{1,meas}$ combined with the multiplicity in the 20–30% centrality class and $\varphi_{in}$ bin (left panel), and a zoomed-in version of the left plot (right panel). Red line corresponds to the corresponding result from data $(1/N_{ev})\,dN_{ev}/da_{1,meas}$.

Figure 6.16: The width of $(1/N_{ev})\,dN_{ev}/da_{1,meas}$ as a function of the width of $(1/N_{ev})\,dN_{ev}/da_{1,meas}$ for the 20–30% centrality class and $\varphi_{in}$ bin. It is fitted with a second-order polynomial. The width of $(1/N_{ev})\,dN_{ev}/da_{1,meas}$ from data are marked with a vertical red line, and the corresponding width of $(1/N_{ev})\,dN_{ev}/da_{1,meas}$ is estimated in $x$-axis.
\[ \frac{1}{N_{\text{ev}}} \frac{dN_{\text{ev}}}{da_{1,\text{in}}(\varphi_{\text{in}})} \]\

\[ \frac{1}{N_{\text{ev}}} \frac{dN_{\text{ev}}}{da_{1,\text{meas}}(\varphi_{\text{in}})} \]

**Figure 6.17:** The widths of \((1/N_{\text{ev}}) dN_{\text{ev}}/da_{1,\text{in}}\) from the corresponding widths of \((1/N_{\text{ev}}) dN_{\text{ev}}/da_{1,\text{meas}}\) from data as a function of centrality for \(\varphi_{\text{in}}\) and \(\varphi_{\text{out}}\) bins (left panel). Estimated \((1/N_{\text{ev}})/da_{1,\text{in}}(\varphi_{\text{in}})\) (right panel).

\(\varphi\) bins are conserved even without effects of the finite number of particles in the estimation of \(a_1\). Based on the widths of the \(a_{1,\text{in}}\) distribution, \((1/N_{\text{ev}}) dN_{\text{ev}}/da_{1,\text{meas}}\) for \(\varphi_{\text{in}}\) bin and the centrality classes considered are shown as an example in the right panel in Fig. 6.17.

So far, the \(a_{1,\text{in}}\) distributions are estimated with a toy Monte Carlo simulation. Another ingredient for the correction of multiplicity effects in the conditional \(a_1\) is the corrected multiplicity in each of the centrality class and \(\varphi\) bin. Based on the efficiency \(\epsilon\) in Eq. 4.3 as a function of \(p_T\), \(\varphi\), and \(\eta\), the corrected multiplicities of charged particles within the considered kinematic ranges \((= \langle N_0 \rangle)\) in Eq. 6.28 and Eq. 6.29 are estimated and summarized in Table 6.1. For multiplicities in \(\varphi_{\text{in}}\) and \(\varphi_{\text{out}}\) bins, Eq. 6.28 and Eq. 6.29 are used with \(v_2\{2\}\) in [19].

The next procedure of the correction is to combine the \(a_{1,\text{in}}\) distribution with both the measured and corrected multiplicity distributions in the corresponding centrality classes and \(\varphi\) bins. For example, with the 20–30% centrality class and \(\varphi_{\text{in}}\) bin, 2-dimensional distributions as a function of \(a_{1,\text{in}}\) and \(a_{1,\text{meas}}\) with the corrected and measured multiplicities are shown in parallel in Fig. 6.18. Although differences between the two plots in Fig. 6.18 is not so visible in the 2-dimensional plot, a few remarks need to be made. The \(a_{1,\text{meas}}\) for a
Centrality

\[ \langle \frac{dN}{d\eta} \rangle_{|\eta| < 0.8} \mid \eta < 0.8 \]

Class 0.2

\[ \begin{array}{lrr}
0\text{–}5\% & 2321 \pm 121 \\
5\text{–}10\% & 1892 \pm 91 \\
10\text{–}20\% & 1449 \pm 62 \\
20\text{–}30\% & 937 \pm 32 \\
30\text{–}40\% & 605 \pm 16 \\
40\text{–}50\% & 371 \pm 7 \\
\end{array} \]

Table 6.1: Centrality estimated by the multiplicity in VZERO detectors and the corresponding \( \langle \frac{dN}{d\eta} \rangle_{|\eta| < 0.8} \). The \( \langle \frac{dN}{d\eta} \rangle \) values are not corrected for trigger and vertex-reconstruction inefficiencies. Systematic uncertainties are evaluated from varying measurement conditions, such as the track-selection mode (hybrid selection, ITS standalone, and TPC standalone), the number of required clusters in the TPC, and the \( z \)-vertex position for the event selection. Only systematic uncertainties are listed, since the statistical uncertainties are negligible.

Figure 6.18: \( a_{1,\text{in}} \) vs. \( a_{1,\text{meas}} \) for the 20–30\% centrality class and \( \varphi_{\text{in}} \) bin. \( (1/N_{\text{ev}}) dN_{\text{ev}}/da_{1,\text{in}} \) is combined with the measured multiplicity distribution (left panel) and corrected multiplicity distribution (right panel).

given \( a_{1,\text{in}} \) (=\( y \)-axis projection of a given \( x \)-axis bin in Fig. 6.18) is distributed over a wide range of values due to fluctuations in the measurement of \( a_{1,\text{meas}} \). However, it is strictly peaked at \( a_{1,\text{meas}} = a_{1,\text{in}} \), and the \( a_{1,\text{meas}} \) distribution for a given \( a_{1,\text{in}} \) should be narrower with larger multiplicity. On the other hand, the \( a_{1,\text{in}} \) distribution for a given \( a_{1,\text{meas}} \) (=\( x \)-axis projection of a given \( y \)-axis bin in Fig. 6.18) is peaked at a value closer to 0 than that of \( a_{1,\text{meas}} \). Various \( a_{1,\text{in}} \) values can contribute to \( a_{1,\text{meas}} \) due to fluctuations, but the \( a_{1,\text{in}} \) distribution is a Gaussian function peaked at \( a_{1,\text{in}} = 0 \) and therefore more events with an \( a_{1,\text{in}} \) value near 0 contribute to a given \( a_{1,\text{meas}} \) than events with \( |a_{1,\text{in}}| \) larger than \( |a_{1,\text{meas}}| \).

In general, the mean \( a_{1,\text{in}} \) value for a given \( a_{1,\text{meas}} \) is closer to 0 with lower multiplicity or...
Figure 6.19: The mean value of $a_{1,\text{in}}(a_{1,\text{meas}})$ for the 20–30% centrality and $\varphi_a$ bin from Fig. 6.18. Fit results with a linear function are shown.

more narrowly peaked $(1/N_{\text{ev}})dN_{\text{ev}}da_{1,\text{in}}$ distribution.

Figure 6.19 shows the mean $a_{1,\text{in}}$ as a function of $a_{1,\text{meas}} (= a_{1,\text{in}}(a_{1,\text{meas}}))$ for the 20–30% centrality class and $\varphi_{\text{in}}$ bin from Fig. 6.18. The $(1/N_{\text{ev}})dN_{\text{ev}}da_{1,\text{in}}$ is combined with the measured multiplicity distribution and the corrected multiplicity distribution, and the difference in multiplicities causes a difference between two $a_{1,\text{in}}(a_{1,\text{meas}})$. Both $a_{1,\text{in}}(a_{1,\text{meas}})$ are fitted with a linear function, and the $a_{1,\text{in}}(a_{1,\text{meas}})$ with the corrected multiplicity distribution is observed to have a larger slope than those with the measured multiplicity distribution since the corrected multiplicity is larger than the measured multiplicity due to the tracking efficiency. The larger slope indicates that for a given $a_{1,\text{meas}}$ value, $a_{1,\text{in}}$ is closer to the value of $a_{1,\text{meas}}$. The slopes are measured in each centrality class and $\varphi$ bin, and summarized in Fig. 6.20. The value of the slopes is larger in more peripheral classes, as $(1/N_{\text{ev}})dN_{\text{ev}}da_{1,\text{in}}$ is more widely distributed in the more peripheral classes. More importantly, slopes of $a_{1,\text{in}}(a_{1,\text{meas}})$ with the corrected multiplicity distribution are always larger than those with the measured multiplicity distribution. These observations are transferred to the multiplicity correction in the conditional $a_1$ measurements. As an example, the correction in $a_1(\varphi_{\text{opp.in}}|a_1(\varphi_{\text{in}}))$ (Fig. 6.9) is considered, although the same correction pro-
Figure 6.20: Slopes from a linear fit of $a_{1,\text{in}}(a_{1,\text{meas}})$ for two $\varphi$ bins as a function of centrality.

The procedures are applied to all $(\varphi_i, \varphi_j)$ combinations and all orders of $a_n$. The $a_1(\varphi_{\text{opp, in}}|a_1(\varphi_{\text{in}}))$ corresponds to the average $a_1(\varphi_{\text{opp, in}})$ for events with a given $a_1(\varphi_{\text{in}})$ value. I emphasize that the goal of the multiplicity correction is to estimate $a_1(\varphi_{\text{opp, in}}|a_1(\varphi_{\text{in}}))$ with the corrected multiplicity instead of the measured number of particles. If $\alpha_{\text{meas}}$ and $\alpha_{\text{corr}}$ denote the slopes of $a_{1,\text{in}}(a_{1,\text{meas}})$ with the measured multiplicity distribution and the corrected multiplicity distribution, the corrected $a_1(\varphi_{\text{in}})$ value used in $a_1(\varphi_{\text{opp, in}}|a_1(\varphi_{\text{in}}))$ is equal to $(\alpha_{\text{meas}}/\alpha_{\text{corr}}) a_1(\varphi_{\text{in}})$. In other words, in Fig. 6.19

$$a_{1,\text{in}}(a_{1,\text{meas}}) = \alpha_{\text{meas}} a_{1,\text{meas}},$$  \hspace{1cm} (6.32) \\
$$a_{1,\text{in}}(a_{1',\text{meas}}) = \alpha_{\text{corr}} a'_{1,\text{meas}},$$  \hspace{1cm} (6.33) \\

and in the case that $a_{1,\text{in}}(a_{1,\text{meas}}) = a_{1,\text{in}}(a'_{1,\text{meas}})$,

$$a'_{1,\text{meas}} = (\alpha_{\text{meas}}/\alpha_{\text{corr}}) a_{1,\text{meas}}.$$  \hspace{1cm} (6.34) \\

The $a'_{1,\text{meas}}$ corresponds to the value of $a_1$ estimated with the corrected multiplicity. As a result, the $x$-axis in Fig. 6.9 is scaled by $\alpha_{\text{meas}}/\alpha_{\text{corr}}$, and the slopes in Fig. 6.12 are scaled.
by $\alpha_{\text{corr}}/\alpha_{\text{meas}}$. Meanwhile, the value of $a_1(\phi_{\text{opp,in}}|a_1(\phi_{\text{in}}))$ in each $a_1(\phi_{\text{in}})$ is invariant, as
the $a_{1,\text{meas}}$ distribution from a given $a_{1,\text{in}}$ is always peaked at $a_{1,\text{meas}} = a_{1,\text{in}}$. Figure 6.21 shows $a_1(\phi_{\text{opp,in}}|a_1(\phi_{\text{in}}))$ after the event-plane resolution correction and after the subsequent multiplicity correction in parallel. With the multiplicity correction, $x$-axis is scaled by $\alpha_{\text{meas}}/\alpha_{\text{corr}}$.

One assumption for the derivation of the above correction procedures is that $(1/N_{\text{ev}})\,dN_{\text{ev}}\,da_{1,\text{in}}$ is Gaussian. To check the model dependence of the correction, a Lorentzian function,

$$\frac{1}{N} \frac{dN}{da_{n,\text{in}}} \sim \frac{1}{1 + \frac{a_{n,\text{in}}^2}{\sigma^2}}.$$  \hspace{1cm} (6.35)

is used for $(1/N_{\text{ev}})\,dN_{\text{ev}}\,da_{1,\text{in}}$, and the resultant measurements are included in the estimation of the systematic uncertainties.

### 6.2 Systematic uncertainty

Systematic uncertainties are estimated using the same procedures described in Sec. 5.2. The systematic variation of the measurement conditions is grouped into a few classes:

**Event plane estimation**
The analysis starts with an estimation of the second-order event plane $\Psi_2$ in each event in order to divide the azimuthal angle into four regions. The $\Psi_2$ are measured with combined VZERO detectors for the nominal measurements. Measurements were repeated using $\Psi_2$ estimated with the VZERO-A and VZERO-C detectors, separately. In addition, the correction factors for the event-plane resolution effects in the observables were calculated separately in each mode.

**Centrality estimation**

Centrality for the nominal measurement is estimated based on the multiplicity in the VZERO detectors. As central-barrel tracks are used for the $a_n$ measurements, the $\eta$ distance between central barrel tracks and VZERO signals prevents observables from autocorrelations in the estimation of centrality. For a systematic check, centrality was estimated by the number of clusters in the layer 1 of the SPD, and by the number of tracks in the TPC, and applied to the event classification. These variations account for any bias attributed to the centrality estimation.

**The $z_{vtx}$ acceptance**

Besides the nominal $z_{vtx}$ range for the event selection $|z_{vtx}| < 8$ cm, the observables were measured for events with $|z_{vtx}| < 6$ cm and $|z_{vtx}| < 7$ cm. As the Legendre decomposition is applied to Eq. 6.4, which is the ratio of a single-event $\eta$ distribution to the average $\eta$ distribution, effects from tracking efficiency only indirectly influence the measurement through multiplicity effects (Sec. 6.1.5 Correction for multiplicity effects).

**Track selection**

As described in Sec. 4.2.2, central-barrel tracks are selected using the hybrid track selection with $0.2 < p_T < 5.0$ GeV/$c$ and $-0.8 < \eta < 0.8$. In the hybrid track selection, the number of space points in the TPC is required to be more than 70 to ensure track quality. The minimum number of space points was varied to 50 and 80, and measurements were repeated with these new track selection criteria. This variation tests any deviations attributed to the $p_T$ or spatial resolution of central-barrel tracks.
Magnetic field direction
The run numbers for Pb-Pb collisions in 2011 are divided into two groups based on the
direction of the 0.5 T magnetic field in the central-barrel detectors. Any deviation from
asymmetric detector efficiency in the beam-direction is accounted for by comparing results
from the two magnetic field directions. The nominal measurement combines results from
run numbers for both directions, but measurements were repeated on specific run numbers
with the same direction of the magnetic field.

Correction for event-plane resolution effects
In the procedures for the correction for event-plane resolution effects (Sec. 6.1.5, Correction
for event-plane resolution effects), correction factors are not analytically derivable due to
the innate characteristics of the observables. Instead, certain mathematical forms for the
azimuthal dependence of the signal are assumed (Eq. 6.14 and Eq. 6.21) for the correction.
The nominal correction is based on the cosine form (Eq. 6.14), but the measurement was
repeated with the correction for event-plane resolution effects based on the linear form
(Eq. 6.14).

Correction for multiplicity effects
The tracking efficiency results in the smaller number of reconstructed particles than the true
multiplicity in each event. The level of statistical fluctuations in the measurement of \( a_n \)
depends on the number of particles used in the estimation, and deviations attributed to the
difference in the measured multiplicity from its true multiplicity are corrected (Sec. 6.1.5,
Correction for multiplicity effects). In the correction procedures, one of the basic assump-
tions is on the underlying shape of \( (1/N_{ev}) dN_{ev}/da_{1,in} \). For the nominal correction, a
Guassian function is used for \( (1/N_{ev}) dN_{ev}/da_{1,in} \). To check the dependence on this func-
tion, a Lorentzian function (Eq. 6.35) was used and differences in the results were accounted
in the systematic uncertainties. The correction for multiplicity effects scales the \( x \)-axis in
conditional \( a_n \), and \( x \)-axes of different systematic variations are scaled differently. In the
estimation of the total systematic uncertainty, the bin size in \( x \)-axis is required to agree in
different systematic variations. Thus, redefinition of the \( x \)-axis bins in systematic variations
after the correction for multiplicity effects is needed with respect to the \( x \)-axis bin sizes of
the nominal measurements. Redefinition of the $x$-axis bin and estimating the corresponding value is done by taking the weighted average of the conditional $a_n$ values within the range of the corresponding bin. In the new bins, statistical uncertainties are assigned from the fit function of the statistical uncertainty as a function of $a_n(\varphi_j)$ in $a_n(\varphi|a_n(\varphi_j))$ in the original $x$-axis binning, where the fit function is the sixth-order polynomials.

Following the same procedures described in Sec. 5.2, the total systematic uncertainty is evaluated by combining deviations to the values from the nominal measurement in each variation. It should be noted that the level of contributions from different effects varies depending on the observables and centrality. Summaries of systematic uncertainties in each observable are provided in Appendix D.

### 6.3 Results and discussion

The conditional $a_1$ of four $(\varphi_i, \varphi_j)$ combinations, $a_1(\varphi_{\text{opp.in}}|a_1(\varphi_{\text{in}}))$, $a_1(\varphi_{\text{opp.out}}|a_1(\varphi_{\text{out}}))$, $a_1(\varphi_{\text{out}}|a_1(\varphi_{\text{in}}))$, and $a_1(\varphi_{\text{in}}|a_1(\varphi_{\text{out}}))$, for different centrality classes are shown in Fig. 6.22. Following the notation in Sec. 5.3, statistical uncertainties are marked with bars and systematic uncertainties are marked using shaded areas in every figure. Similar to Fig. 6.9, the diagram in each panel represents the $(\varphi_i, \varphi_j)$ combination in the corresponding $a_1(\varphi|a_1(\varphi_j))$. The smaller order of the $y$-axis than that of $x$-axis may indicate that in $a_1(\varphi|a_1(\varphi_j))$, the mean value of the input $a_1(\varphi_j)$ is smaller than the observed $a_1(\varphi_j)$ value, as shown in the Monte Carlo simulations in the discussion of the correction for multiplicity effects in Sec. 6.1.5. The $a_1(\varphi_{\text{opp.in}}|a_1(\varphi_{\text{in}}))$ and $a_1(\varphi_{\text{opp.out}}|a_1(\varphi_{\text{out}}))$ are observed to have negative slopes in all centrality classes, and the magnitude of the slope is generally larger in more central classes. The negative slope indicates that when the positive (negative) $a_1$ is observed in one $\varphi$ bin, the longitudinal distribution of the opposite $\varphi$ bin has negative (positive) $a_1$. Meanwhile, $a_1(\varphi_{\text{out}}|a_1(\varphi_{\text{in}}))$ and $a_1(\varphi_{\text{in}}|a_1(\varphi_{\text{out}}))$ are nearly flat in the most central class, and have positive slopes in the peripheral classes. The positive slope in these conditional $a_1$ corresponds to the same sign and exhibits proportionality of $a_1$ in two adjacent $\varphi$ bins.
Figure 6.22: The \( a_1(\varphi_{\text{opp.in}}|a_1(\varphi_{\text{in}})) \) (top left), \( a_1(\varphi_{\text{opp.out}}|a_1(\varphi_{\text{out}})) \) (top right), \( a_1(\varphi_{\text{out}}|a_1(\varphi_{\text{in}})) \) (bottom left), and \( a_1(\varphi_{\text{in}}|a_1(\varphi_{\text{out}})) \) (bottom right) measured with charged particles with \( 0.2 < p_T < 5.0 \text{ GeV/c} \) and \( |\eta| < 0.8 \) in different centrality classes.
Each conditional $a_1$ is fitted with a linear function, and the corresponding slope is extracted from the fit. Fig. 6.23 shows the slopes extracted from the conditional $a_1$ as a function of centrality.

In general, physics mechanisms related to the longitudinal structures are classified into two groups, long-range correlations (LRC) and short-range correlations (SRC), similar to the discussions for the two-particle correlation measurements in the transverse direction (Sec. 2.2.2). As mentioned previously in Sec. 2.2.2, particle correlations from SRC are localized in a small range of $(\varphi, \eta)$ space or opposite $\varphi$ regions (from back-to-back jets), while particle correlations from LRC are distributed over wide ranges in $(\varphi, \eta)$ space. One disadvantage of the current observables is its inability to isolate either SRC or LRC. However, the contributions from SRC and LRC can be indirectly distinguished in the results of conditional $a_n$.

The $a_1$ is a parameter quantifying the forward-backward asymmetry in particle production. The forward-backward asymmetry may originate from SRC, e.g. if a jet and its fragmented particles are located in positive $\eta$, they contribute to the positive value of $a_1$. On the other hand, the asymmetry in the number of participating nucleons in two incoming nuclei generates the initial forward-backward asymmetry in the collision geometry, which is later...
transferred to a forward-backward asymmetry of particle distributions through the medium expansion. The forward-backward asymmetry originating from the asymmetric medium expansion is considered to be part of LRC. Due to the limited range of SRC in ($\varphi, \eta$) space, SRC can contribute to either one or two adjacent $\varphi$ bins in the current analysis for most cases, as the size of each $\varphi$ bin is $\pi/2$. For example, fragmented particles from a jet can be included in either a single $\varphi$ bin or two adjacent $\varphi$ bins. If the jet axis is located in the positive $\eta$ region, the corresponding fragmented particles can contribute to the positive $a_1$ in a given $\varphi$ bin or two adjacent $\varphi$ bins simultaneously. More positive values of slopes in $a_1(\varphi_{\text{out}}|a_1(\varphi_{\text{in}}))$ and $a_1(\varphi_{\text{in}}|a_1(\varphi_{\text{out}}))$ than those in $a_1(\varphi_{\text{opp.in}}|a_1(\varphi_{\text{in}}))$ and $a_1(\varphi_{\text{opp.out}}|a_1(\varphi_{\text{out}}))$ are mostly attributed to the involvement of the SRC in the observables. Meanwhile, back-to-back jets may influence the conditional $a_1$ in two opposite $\varphi$ bins. Once a jet of a di-jet pair is found in a given $\eta$, $\eta$ of the jet in the opposite $\varphi$ depends on the kinematics of hard-scattered partons, which generate a di-jet pair. The contribution from back-to-back jets can be observed in the Monte Carlo simulations with the HIJING event generator, where no LRC are expected (Sec. 6.4.2).

Meanwhile, LRC contribute conditional $a_1$ for all ($\varphi_i, \varphi_j$) combinations. Within the framework of LRC, various physics scenarios can contribute to the negative slope observed in $a_1(\varphi_{\text{opp.in}}|a_1(\varphi_{\text{in}}))$ and $a_1(\varphi_{\text{opp.out}}|a_1(\varphi_{\text{out}}))$. The transverse distribution of the asymmetric number of participating nucleons from two incoming nuclei fluctuates event-to-event, and can be considered as a seed for the asymmetric expansion of the medium in the longitudinal direction. Then the observed conditional $a_1$ may originate solely from the initial bias in the transverse distribution of the asymmetric number of participating nucleons. For example, if there are larger number of participating nucleons moving in the positive $z$-direction in $\varphi_{\text{in}}$ bin and larger number of participating nucleons moving in the negative $z$-direction in $\varphi_{\text{opp.in}}$ bin, the measured $a_1$ in two $\varphi$ bins tend to have opposite-sign values. Meanwhile, the negative slope in conditional $a_1$ with two opposite $\varphi$ bins may indicate the momentum conservation of particles created from the expansion of the medium. If there exists a forward-backward asymmetry in the expansion of the medium in a certain azimuthal angle, the opposite side in azimuth of the medium needs to have a subsequent
forward-backward asymmetry in the opposite direction due to momentum conservation of the medium in \((\varphi, \eta)\) space. However, the initial information of the asymmetry in the longitudinal direction may be diluted through expansion of the medium. The transverse expansion in the direction of \(\varphi_{\text{in}}\) and \(\varphi_{\text{opp.in}}\) is more active than that in the direction of \(\varphi_{\text{out}}\) and \(\varphi_{\text{opp.out}}\). If the level of dilution depends on the expansion rate, the difference between \(a_1(\varphi_{\text{opp.in}}|a_1(\varphi_{\text{in}}))\) and \(a_1(\varphi_{\text{opp.out}}|a_1(\varphi_{\text{out}}))\) can be explained by the above discussion. Also, it can explain the difference between \(a_1(\varphi_{\text{out}}|a_1(\varphi_{\text{in}}))\) and \(a_1(\varphi_{\text{in}}|a_1(\varphi_{\text{out}}))\). In conditional \(a_1\), \(a_1(\varphi_i|a_1(\varphi_j))\), events with \(a_1(\varphi_j)\) are selected first. The initial asymmetry in events with \(a_1(\varphi_{\text{in}})\) is different from that with the same value of \(a_1(\varphi_{\text{out}})\), as they probe the different initial asymmetry due to the dependence of the dilution on the expansion rate. The difference between \(a_1(\varphi_{\text{out}}|a_1(\varphi_{\text{in}}))\) and \(a_1(\varphi_{\text{in}}|a_1(\varphi_{\text{out}}))\) increases in more peripheral classes, and it agrees with the increase of \(v_2\) as a function of centrality (Fig. 2.5). In addition to the the above discussion, the interaction between the medium and spectator nucleons of incoming nuclei, which do not participate in the collision, can contribute to the results of conditional \(a_1\). Also, during the thermal-equilibrium stage in the evolution of heavy-ion collisions (Sec. 2.1), hydrodynamic expansion may influence conditional \(a_1\) for all \((\varphi_i, \varphi_j)\) combinations.

Depending on centrality, the level of contributions from SRC and LRC varies. In general, the level of contribution from LRC is larger in more central classes, and the SRC contribution plays a larger role in particle production in the more peripheral classes. The centrality dependence of conditional \(a_1\) reflects these factors.

Compared to the results in [80], where the dependence of the longitudinal distribution on the asymmetric number of participating nucleon in two incoming nuclei is discussed, conditional \(a_1\) measurements reveal that the forward-backward asymmetry in mid-rapidity is not uniform in the azimuthal direction, but shows specific correlations in the azimuthal direction. Also, the existence of different physics mechanisms in the forward-backward asymmetry is indirectly observed with the conditional \(a_1\). Meanwhile, in Fig. 2.16 and [83] reported by the ATLAS Collaboration, the \(a_1\) was observed to have a contribution from both SRC and LRC, although the kinematic range of the tracks are different from the
current analysis. Although a direct comparison is not plausible due to the difference in the observables and kinematic ranges, the order of $\sqrt{\langle a_1^2 \rangle}$ in Fig. 2.16 agrees with the width of the observed $a_1$ distribution in Fig. 6.7.

In [83] and [84], $a_3$ and higher order coefficients are argued to solely attribute to the SRC. The conditional $a_3$, $a_3(\varphi_{\text{opp.in}}|a_3(\varphi_{\text{in}}))$, $a_3(\varphi_{\text{opp.out}}|a_3(\varphi_{\text{out}}))$, $a_3(\varphi_{\text{out}}|a_3(\varphi_{\text{in}}))$, and $a_3(\varphi_{\text{in}}|a_3(\varphi_{\text{out}}))$, for different centrality classes are shown in Fig. 6.24. Throughout the four $(\varphi_i, \varphi_j)$ combinations and centrality classes, conditional $a_3$ more or less remain near 0. In other words, $a_3$ in different $\varphi$ bins are independent from each other regardless of the value of $a_3$. This is a genuine feature of SRC, as the SRC can contribute to the local region in $(\varphi, \eta)$ space. In principle, the SRC can induce proportional $a_3$ values in two adjacent $\varphi$
Figure 6.25: The $a_2(\varphi_{\text{opp.in}}|a_2(\varphi_{\text{in}}))$ (top left), $a_2(\varphi_{\text{opp.out}}|a_2(\varphi_{\text{out}}))$ (top right), $a_2(\varphi_{\text{out}}|a_2(\varphi_{\text{in}}))$ (bottom left), and $a_2(\varphi_{\text{in}}|a_2(\varphi_{\text{out}}))$ (bottom right) measured with charged particles with $0.2 < p_T < 5.0$ GeV/c and $|\eta| < 0.8$ in different centrality classes.

bins, similar to what is observed in $a_1(\varphi_{\text{out}}|a_1(\varphi_{\text{in}}))$ and $a_1(\varphi_{\text{in}}|a_1(\varphi_{\text{out}}))$. In order to have a specific $a_3$ value in a given $\varphi$ bin, multiple signals from SRC are distributed accordingly in $\eta$ within the range of the $\varphi$ bin. However, a proportional $a_3$ in two adjacent $\varphi$ bins requires these signals to be located at the border of two $\varphi$ bins in an event, which is not very probable. The observed $a_3(\varphi_{\text{out}}|a_3(\varphi_{\text{in}}))$, and $a_3(\varphi_{\text{in}}|a_3(\varphi_{\text{out}}))$ in Fig. 6.24 proves the independence of $a_3$ in two adjacent $\varphi$ bins.

Lastly, the conditional $a_2$, $a_2(\varphi_{\text{opp.in}}|a_2(\varphi_{\text{in}}))$, $a_2(\varphi_{\text{opp.out}}|a_2(\varphi_{\text{out}}))$, $a_2(\varphi_{\text{out}}|a_2(\varphi_{\text{in}}))$, and $a_2(\varphi_{\text{in}}|a_2(\varphi_{\text{out}}))$, for different centrality classes are shown in Fig. 6.25. The $a_2$ is a parameter quantifying the mid-peripheral asymmetry, and positive and negative values represent different shapes in the longitudinal structure. Relative to the average $\eta$ distribution in a
given centrality, events with positive $a_2$ have more particles in peripheral regions within the considered $\eta$ range than the central region, and events with negative $a_2$ have more particles in the central region than in peripheral regions. A very distinctive feature is observed in conditional $a_2$ (Fig. 6.25). The value of $a_2(\varphi_i|a_2(\varphi_j))$ significantly increases for positive $a_2(\varphi_j)$ in central classes for all four $(\varphi_i, \varphi_j)$ combinations, while it remains near 0 for all other $a_2(\varphi_j)$. The increase of the $a_2(\varphi_i|a_2(\varphi_j))$ value is more significant in the more central classes, and disappears in the peripheral classes. This indicates that when a negative value of $a_2$ is observed in a given $\varphi$ bin, $a_2$ in other $\varphi$ bins are independent of the former $a_2$. Meanwhile, when a positive value of $a_2$ is observed in a given $\varphi$ bin, $a_2$ in all other $\varphi$ bins tend to have positive $a_2$ in the more central classes. It should be noted that less than 10% of events have $a_2$ larger than 0.1 in one of the $\varphi$ bins, and less than 1% of events have $a_2$ larger than 0.2 in one of the $\varphi$ bins in the 0–5% centrality class.

The $a_2$ is generally related to the amount of nuclear stopping or shift of the effective center-of-mass of the collisions [84]. However, none of the Monte Carlo event generators used in this thesis successfully generate the observed trend in conditional $a_2$ (Sec. 6.4). One possible candidate to explain the azimuthal collectivity of positive $a_2$ in the central classes is a Color Glass Condensate (CGC) model [140,141], which has been used to describe the long-range rapidity correlations in heavy-ion collisions or nucleon-nucleus collisions. The CGC is an effective theory, describing the behavior of small Bjorken $x$ components of the hadronic wavefunction in QCD. At very high energies or small Bjorken $x$, the parton distribution is dominated by gluons and the density becomes very large leading to saturation. The coupling constant of QCD becomes very small in the saturation scale, and the CGC is based on weak-coupling techniques for this regime. In the CGC, a plasma flux tube is generated in the longitudinal direction during the pre-equilibrium stage (Fig. 6.26). The flux tube may interact with the QGP during the thermalization stage, and effectively influences the expansion in both longitudinal and transverse directions. In particular, the initial transverse geometry of the collision is isotropic in central collisions, and the subsequent expansion in the transverse direction tends to be isotropic in $\varphi$. In this situation, the effects from the flux tube may create an isotropic longitudinal structure in $\varphi$, consistent with the similarly
positive $a_2$ values in all $\phi$ bins in Fig. 6.25. Also, other parameters, such as the QGP kinematics in each event, fluctuations in the profile of the flux tube, and fluctuations in the nuclear stopping power may influence the conditional $a_2$, and require more investigation.

6.4 MC simulation

Conditional $a_n$ are measured with various Monte Carlo event generators, including PYTHIA \cite{133}, HIJING \cite{128}, and AMPT \cite{127}. Similar procedures are applied to these simulations in order to have comparable results with data. For the AMPT and HIJING simulations, $\Psi_2$ is measured with tracks in the same $\eta$-ranges ($2.8 < \eta < 5.1$ and $-3.7 < \eta < -1.7$) as the ranges of VZERO detectors with $0.2 < p_T < 5.0$ GeV/$c$. In particular, a Monte Carlo closure test is conducted with HIJING simulations to investigate effects in the observables from reconstruction with the ALICE detectors.

6.4.1 $N_{\text{coll}}$ PYTHIA

PYTHIA is a widely-used event generator to simulate high-energy proton-proton collisions \cite{133}. As a baseline for all other simulations, the conditional $a_n$ are measured based on PYTHIA events. Each of the PYTHIA events used in this section is an event from a
p–p collision with $\sqrt{s} = 2.76$ TeV, and tuned to have back-to-back jets. To simulate Pb–Pb collisions with these PYTHIA events, multiple PYTHIA events are grouped to form a single Pb–Pb event. \[142\] provides the number of binary collisions in each centrality class, and $N_{\text{coll}} = 100, 500, \text{and} 1500$ are included in the centrality class $0$–$5\%$, $20$–$30\%$, and $50\%$, respectively. These three $N_{\text{coll}}$ values are used as the number of PYTHIA events in a single simulated Pb–Pb events. The collection of PYTHIA events essentially has no long-range correlations, collective features and nuclear effects that can be found in heavy-ion collisions, but solely contain short-range correlations due to the jets.

Since there is no transverse flow, $\Psi_2$ in each event is randomly assigned and primary charged tracks with $0.2 < p_T < 5.0 \text{ GeV}/c$ and $|\eta| < 0.8$ are used in the estimation of conditional $a_n$. Results of conditional $a_n$ are shown in Fig. 6.27. Regardless of the $N_{\text{coll}}$, the $a_n(\varphi_{\text{out}}|a_n(\varphi_{\text{in}}))$ linearly increase as a function of $a_n(\varphi_{\text{in}})$, while the $a_n(\varphi_{\text{out}}|a_n(\varphi_{\text{in}}))$ are more or less constant. The positive slope of $a_n(\varphi_{\text{out}}|a_n(\varphi_{\text{in}}))$ confirms that the positive slopes in $a_1(\varphi_{\text{out}}|a_1(\varphi_{\text{in}}))$ and $a_1(\varphi_{\text{in}}|a_1(\varphi_{\text{out}}))$ in the data originate from SRC (Fig. 6.22). The slope is the largest in $a_1(\varphi_{\text{out}}|a_1(\varphi_{\text{in}}))$ and smallest in $a_3(\varphi_{\text{out}}|a_3(\varphi_{\text{in}}))$, and has weak dependence on $N_{\text{coll}}$. It is observed that the $a_n(\varphi_{\text{in}})$ range for the conditional $a_n$ is smaller in larger $N_{\text{coll}}$ PYTHIA events. This is due to the fact that the collection of PYTHIA events tends to have much larger multiplicity and a narrower $dN_{\text{ev}}/da_n$ distribution, even though $N_{\text{coll}}$ used in these simulations are similar to those in the corresponding centrality class in heavy-ion collisions. In addition, the conditional $a_2$ at $a_{\text{in}} = 0$ are not 0 in all $N_{\text{coll}}$ events. This is related to the fact that the mean value of $dN_{\text{ev}}/da_2$ is not equal to 0, but a positive value.

### 6.4.2 HIJING

HIJING (Heavy-Ion Jet Interaction Generator) is a heavy-ion collision event generator that has successfully reproduced many inclusive spectra, two-particle correlations, as well as the observed flavor and multiplicity dependence of the average transverse momentum \[128\]. It combines a QCD-inspired model of jet production using the Lund model for jet frag-
Figure 6.27: The $a_n(\varphi_{\text{op.in}}|a_n(\varphi_{\text{in}}))$ and $a_n(\varphi_{\text{out}}|a_n(\varphi_{\text{in}}))$ from $N_{\text{coll}} = 100$ (top row), $N_{\text{coll}} = 500$ (middle row), and $N_{\text{coll}} = 1500$ (bottom row) PYTHIA events. Conditional $a_1$ (left column), conditional $a_2$ (middle column), and conditional $a_3$ (right column) are shown for all $N_{\text{coll}}$ events.
mentation, and has special emphasis on the role of mini-jets in various collisional systems. However, it does not contain collective properties of heavy-ion collisions, such as elliptic flow. Thus, short-range correlations dominate the particle production in HIJING, similar to that in $N_{\text{coll}}$ PYTHIA events (Sec. 6.4.1).

Following the measurements with the LHC Pb–Pb data, the centrality class is determined based on the multiplicity in the VZERO detectors and six centrality classes are used (0–5%, 5–10%, 10–20%, 20–30%, 30–40%, and 40–50%) for the analysis. Also, primary charged tracks with $0.2 < p_T < 5.0 \text{ GeV}/c$ and $|\eta| < 0.8$ are selected for the measurements. Although there is no elliptic flow in HIJING ($v_2 = 0$ for all centrality classes), an event-plane angle is estimated based on the primary particles with $0.2 < p_T < 5.0 \text{ GeV}/c$ within the $\eta$-ranges of VZERO detectors. Figure 6.28 shows conditional $a_1$ results for all four $(\varphi_i, \varphi_j)$ combinations with statistical uncertainties only. As done in Fig. 6.23, slopes of conditional $a_1$ in Fig. 6.28 are extracted and shown in Fig. 6.29. Slopes from all conditional $a_1$ are commonly observed to have positive values, and have a slight decrease or remain at a similar value as a function of centrality. This observation may indicate that the contribution from LRC shifts the value of slopes into the negative direction, as only SRC are involved in the conditional $a_1$ in HIJING events. In particular, slopes in Fig. 6.23 have smaller or more negative values in the more central classes, where a larger level of contribution from LRC is expected. The centrality dependence of conditional $a_1$ in data indicates that the different level of contributions to the longitudinal structures from SRC and LRC depending on the centrality. Similar to Fig. 6.24 which shows the conditional $a_3$ results from data, results of conditional $a_3$ in HIJING events are observed to remain near 0 (Fig. 6.28). This observation agrees with the statement that the dominant underlying mechanism for $a_3$ is SRC.

Figure 6.31 shows the conditional $a_2$ results. As mentioned in Sec. 6.3, the significant increase of $a_2(\varphi_i | a_2(\varphi_j))$ in positive $a_2(\varphi_j)$ is not found in HIJING events. Positive slopes in $a_2(\varphi_{\text{out}} | a_2(\varphi_{\text{in}}))$ and $a_2(\varphi_{\text{in}} | a_2(\varphi_{\text{out}}))$ are observed, consistent with the results with the $N_{\text{coll}}$ PYTHIA events (Fig. 6.27).

To test the effects in the observables from reconstruction with detectors, the generated
Figure 6.28: The $a_1(\phi_{\text{opp.in}}|a_1(\phi_{\text{in}}))$ (top left), $a_1(\phi_{\text{opp.out}}|a_1(\phi_{\text{out}}))$ (top right), $a_1(\phi_{\text{out}}|a_1(\phi_{\text{in}}))$ (bottom left), and $a_1(\phi_{\text{in}}|a_1(\phi_{\text{out}}))$ (bottom right) in generator-level HI-JING events with statistical uncertainties only.
events with HIJING are passed to GEANT3 \textsuperscript{126} for the simulation of particle transport through the detectors. An analysis based on the tracks or other information created by the event generator is generally referred to as a generator-level analysis, while an analysis based on the reconstructed tracks or other reconstructed information by detectors is referred to as a detector-level analysis. For the detector level analysis with HIJING and GEANT3, the same track and event selection criteria are applied as the measurements with the LHC Pb–Pb data. However, in order to isolate the effects from the reconstruction to the central-barrel tracks, the generator-level particles in the $\eta$ ranges of the VZERO detectors are used in the estimation of $\Psi_2$. In other words, the conditional $a_n$ are measured with the generator-level tracks and the detector-level tracks, separately, but with respect to the same $\Psi_2$ in each event. For the detector-level analysis, the same correction procedures for multiplicity effects are applied as in data (Sec. [6.1.5] Correction for multiplicity effects). As an example, slopes of conditional $a_1$ from generator-level HIJING events and detector-level HIJING events are compared in Fig. [6.32] In Sec. [4.2.2] contamination from secondary particles are observed to make up approximately 3% of produced particles in the reconstructed tracks, and it may cause the differences in Fig. [6.32] The difference between slopes from different levels is less than 4% on average, and considered to be acceptable for the results.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure6_29.png}
\caption{Slopes of conditional $a_1$ as a function of centrality in generator-level HIJING events with statistical uncertainties only.}
\end{figure}
Figure 6.30: The $a_3(\phi_{\text{opp.in}}|a_3(\phi_{\text{in}}))$ (top left), $a_3(\phi_{\text{opp.out}}|a_3(\phi_{\text{out}}))$ (top right), $a_3(\phi_{\text{out}}|a_3(\phi_{\text{in}}))$ (bottom left), and $a_3(\phi_{\text{in}}|a_3(\phi_{\text{out}}))$ (bottom right) in generator-level HIJING events with statistical uncertainties only.
Figure 6.31: The $a_2(\varphi_{\text{opp.in}}|a_2(\varphi_{\text{in}}))$ (top left), $a_2(\varphi_{\text{opp.out}}|a_2(\varphi_{\text{out}}))$ (top right), $a_2(\varphi_{\text{out}}|a_2(\varphi_{\text{in}}))$ (bottom left), and $a_2(\varphi_{\text{in}}|a_2(\varphi_{\text{out}}))$ (bottom right) in generator-level HIJING events with statistical uncertainties only.

Figure 6.32: Slopes of conditional $a_1$ in generator-level HIJING events (left) and detector-level HIJING events (right).
6.4.3 AMPT

AMPT has been introduced in Sec. 5.3. Contrary to the other generators used previously, it contains collective features of heavy-ion collisions, and the AMPT simulation used in this section includes string melting. Instead of VZERO multiplicities for the centrality class determination, the actual cross-section for each collision is used as a centrality estimator. Similar to the measurements with generator-level HIJING events, primary charged tracks with $0.2 < p_T < 5.0\,\text{GeV}/c$ and $|\eta| < 0.8$ are selected for the measurements. Corrections for the event-plane resolution effects described in Sec. 6.1.5 are applied, based on $v_2$ values from the AMPT simulation \cite{143}.

Conditional $a_1$ and subsequent slopes of conditional $a_1$ are shown in Fig. 6.33 and Fig. 6.34, respectively. Slopes of conditional $a_1$ in AMPT are observed to have similar centrality dependence in data (Fig. 6.23), but generally shifted in the negative direction. This can be interpreted with different level of contributions from LRC and SRC to the data. Meanwhile, conditional $a_3$ remains near 0 in all $(\varphi_i, \varphi_j)$ combinations (Fig. 6.35). AMPT does not reproduce the trend of conditional $a_2$ from data, but values remain near 0 similar to the HIJING event generator (Fig. 6.36).
Figure 6.33: The $a_1(\varphi_{\text{opp.in}}|a_1(\varphi_{\text{in}}))$ (top left), $a_1(\varphi_{\text{opp.out}}|a_1(\varphi_{\text{out}}))$ (top right), $a_1(\varphi_{\text{out}}|a_1(\varphi_{\text{in}}))$ (bottom left), and $a_1(\varphi_{\text{in}}|a_1(\varphi_{\text{out}}))$ (bottom right) in generator-level AMPT events with statistical uncertainties only.
Figure 6.34: Slopes of conditional $a_1$ as a function of centrality in generator-level AMPT events with statistical uncertainties only.
Figure 6.35: The $a_3(\phi_{\text{opp.in}}|a_3(\phi_{\text{in}}))$ (top left), $a_3(\phi_{\text{opp.out}}|a_3(\phi_{\text{out}}))$ (top right), $a_3(\phi_{\text{out}}|a_3(\phi_{\text{in}}))$ (bottom left), and $a_3(\phi_{\text{in}}|a_3(\phi_{\text{out}}))$ (bottom right) in generator-level AMPT events with statistical uncertainties only.
Figure 6.36: The $a_2(\varphi_{\text{opp.in}}|a_2(\varphi_{\text{in}}))$ (top left), $a_2(\varphi_{\text{opp.out}}|a_2(\varphi_{\text{out}}))$ (top right), $a_2(\varphi_{\text{out}}|a_2(\varphi_{\text{in}}))$ (bottom left), and $a_2(\varphi_{\text{in}}|a_2(\varphi_{\text{out}}))$ (bottom right) in generator-level AMPT events with statistical uncertainties only.
Chapter 7

Conclusion and Outlook

In this thesis, two analyses based on geometrical correlations of produced particles in relativistic heavy ion physics measured with the LHC accelerator and the ALICE detectors are presented. In an extension of the previously-measured double-ridge structure in p–Pb collisions in midrapidity, two-particle angular correlations between trigger particles in the forward-rapidity range $2.5 < |\eta| < 4.0$ and associated particles in the midrapidity range $|\eta| < 1.0$ in p–Pb collisions are studied at a nucleon-nucleon center-of-mass energy of 5.02 TeV. The trigger particles are muon tracks selected in the Forward Muon Spectrometer, while associated particles utilize either central-barrel tracks from charged particles or tracklets generated with the information of the primary vertex position and the two hits on the SPD layers. The consistency between correlation results from two different selections of associated particles indicates a good factorization of correlation parameters $v_2$, which is a fundamental assumption in the extraction of $v_2$ with two-particle correlations. The double-ridge structure is observed in the forward pseudorapidity ranges covered by this analysis in both proton-going and lead-ion-going directions in high-multiplicity events, and the second-order Fourier coefficients $v_2$ for muons are estimated as a function of the transverse momentum $p_T$. The $v_2$ in the proton-going direction and in the lead-ion-going direction are found to have similar $p_T$ dependence, but the value of $v_2$ in the lead-ion-going direction is larger by $16\pm6\%$ independent of $p_T$ within the uncertainties of the measurement.
The results are compared with calculations based on the AMPT model, which generates qualitatively different values of $v_2$ especially for $p_T > 2.0\text{GeV/c}$.

In the second analysis, correlations among the longitudinal structures of produced particles at midrapidity and different $\varphi$ ranges are investigated in Pb–Pb collisions at a nucleon-nucleon center-of-mass energy of 2.76 TeV. This analysis can be considered as an attempt to understand the QCD medium expansion in 3-dimensional space, and the overall observables are newly devised. The longitudinal structure in each of the divided azimuthal regions is decomposed with the Legendre polynomials, and the corresponding coefficients $a_n$ are extracted up to the third order. Correlations between the coefficients from different $\varphi$ bins are investigated in the form of conditional $a_n$, and they are measured within each of the centrality classes. While the previous pseudorapidity correlation measurements are limited to the longitudinal direction, conditional $a_n$ expand the span of the measurement into $(\varphi,\eta)$ space on an event-by-event basis. The results of conditional $a_1$, which quantify the forward-backward asymmetry in the longitudinal distribution, show the different level of contributions from short-range correlations and long-range correlations depending on the centrality. Various physics mechanisms are considered, such as fluctuations in the transverse distribution of the initial asymmetry in the number of participating nucleons in the two incoming nuclei and the subsequent medium expansion. The calculations based on the HIJING and AMPT event generators are made for the same observables, and the AMPT model is observed to overestimate the contribution from long-range correlations in conditional $a_1$. While the results of conditional $a_3$ confirm that the longitudinal structure quantified by the third-order Legendre coefficient originates from short-range correlations, the results of conditional $a_2$ are observed to have a distinctive feature compared to the calculations from the HIJING and AMPT event generators. The positive $a_2$ value, which corresponds to the longitudinal distribution with more particles in the edges than in the center relative to the mean longitudinal distribution, is collectively observed in all $\varphi$ regions only in central classes, and this $a_2$ behavior disappears in peripheral classes. These results represent new constraints for the 3-dimensional heavy-ion collision models, and require further theoretical investigation.
Collective features in p–Pb collisions have been confirmed with various measurements by different experiment collaborations. For further understanding of the collectivity in proton-ion collisions with precision, there are several possible ways starting from the correlation results in p–Pb collisions in this thesis. As discussed at the end of Chap. 5, some ambiguity in the interpretation of results from data and AMPT can be addressed by measuring yields of pion, kaon and D-meson in the forward-rapidity region. These yields have been measured at midrapidity \cite{144,145}, and the discrepancy in the ratio of the D-meson yield to pion or kaon yield is already observed in AMPT. In addition, to have a comparable $v_2$ at forward rapidity to those at midrapidity, charged particles instead of decayed muons in the forward-rapidity region can be used in future experiments. On the other hand, new theoretical models may produce comparable predictions applying the relative efficiencies for muon decays from pions and kaons (Fig. 4.5). The predicted values may justify the validity of the model of collectivity in proton-ion collisions.

The conditional $a_n$ measurements in Pb–Pb collisions are one of the first methods to study the 3-dimensional expansion of the QCD medium on an event-by-event basis. The 3-dimensional approach is particularly important, as the observables in the transverse direction depend on the parameters of the longitudinal expansion, and the observables in the longitudinal direction depend on the parameters of the transverse expansion. For a full understanding of heavy-ion collisions, the paradigm in relativistic heavy ion physics should be moved from the previous dimensions, either exclusively in the transverse direction or in the longitudinal direction into overall ($\phi, \eta$) space. The conditional $a_n$ measurements have potential to be further improved in both the methodological aspect and the characterization aspect. As the transverse flow measurements have advanced with new measurement techniques over the past two decades, from the initial concept of the Fourier decomposition of the azimuthal structures to the multi-particle correlation method, new measurement techniques in the ($\phi, \eta$) space can be developed in the future. Meanwhile, based on the current conditional $a_n$ method, the dependence on $p_T$, particle species, or collision energy can be investigated to further characterize the physics mechanisms behind the azimuthal collectivity of the longitudinal structures. This is particularly plausible with the LHC run 2
data, which has larger statistics and higher collision energy. Also, the existing heavy-ion models need to be tested with these observables to verify their credibility in $(\phi, \eta)$ space.
Appendices

A Abbreviations

- AGS Alternating Gradient Synchrotron
- ALICE A Large Ion Collider Experiment
- AMPT A Multi-Phase Transport model
- ART A Relativistic Transport
- ATLAS A Toroidal LHC Apparatus
- a.u. Arbitrary unit
- BNL Brookhaven National Laboratory
- CERN European Organization for Nuclear Research
- CGC Color Glass Condensate
- CMS Compact Muon Solenoid
- CTP Central Trigger Processor
- DCA Distance of Closest Approach
- DPMJET Dual Parton Model Jet
- DSL, DSR Dispersion suppressor cells
- EFT Effective Field Theory
- EMC European Muon Collaboration
- EMCAL Electro-Magnetic CALorimeter
- FB Forward-Backward
- FMD Forward Multiplicity Detector
- **FMS** Forward Muon Spectrometer
- **GEANT3** GEometry ANd Tracking-3
- **HBT** Hanbury-Brown Twist
- **HIJING** Heavy Ion Jet INteraction Generator
- **HMPID** High-Momentum Particle Identification Detector
- **ITS** Inner Tracking System
- **L0, L1, L2** Level 0, Level 1, Level 2
- **LEIR** Low Energy Ion Ring
- **LEP** Large Electron-Positron Collider
- **LHC** Large Hadron Collider
- **LINAC** LINear ACcelerator
- **LRC** Long-Range Correlations
- **LSS** Long Straight Section
- **MB** Minimum-Bias
- **MTR** Muon TRigger
- **MWPC** MultiWire Proportional Chamber
- **NLO** Next-to-Leading-Order
- **NNLO** Next-next-to-Leading-Order
- **p–Pb** proton-lead
- **Pb–Pb** lead-lead
- **p–p** proton-proton
- **PDFs** Parton Distribution Functions

- **PHENIX** Pioneering High Energy Nuclear Interaction eXperiment

- **PHOS** PHOton Spectrometer

- **PMD** Photon Multiplicity Detector

- **PS** Proton Synchrotron

- **PP** Participant Plane

- **pQCD** perturbative QCD

- **PSB** Proton Synchrotron Booster

- **QCD** Quantum ChromoDynamics

- **QED** Quantum ElectroDynamics

- **QGP** Quark-Gluon Plasma

- **RHIC** Relativistic Heavy Ion Collider

- **RF** Radio Frequency

- **RPC** Resistive Plate Chamber

- **SDD** Silicon Drift Detector

- **SPD** Silicon Pixel Detector

- **SPS** Super Proton Synchrotron

- **SRC** Short-Range Correlations

- **SSD** Silicon Strip Detector

- **STAR** Solenoidal Tracker At RHIC

- **TOF** Time-Of-Flight
- **TPC** Time Projection Chamber

- **TRD** Transition Radiation Detector

- **ZDC** Zero Degree Calorimeters

- **ZPC** Zhang’s Parton Cascade
B Mathematical discussions on finite-acceptance effects and new correction methods in two-particle correlations

As described in Chap. 5, the mixed-event technique has been used as a standard method to correct for finite-acceptance effects in various two-particle correlation analyses \cite{41,42,44,146,148}, as well as in the forward-central two-particle correlation analysis in this thesis. However, there are multiple papers pointing out the drawbacks of the conventional mixed-event technique and proposing new correction methods, e.g. \cite{131,149,150}. A two-particle correlation analysis may reveal various underlying physics mechanisms depending on how and in what circumstances it is used, and the validity of the mixed-event technique varies depending on its usage. In this appendix, a mathematical review of the finite-acceptance effects and mixed-event method is provided, and new methods for the correction for finite-acceptance effects are summarized following reference \cite{131}. Mathematical discussion of the finite-acceptance effects is based on the case that two-particle correlations are used to study jet-like correlations, and new methods work more precisely than the conventional mixed-event method for the corresponding signals.

In relativistic heavy ion physics, geometrical densities of trigger and associated single-particles are described by functions of azimuthal angle and pseudorapidity, \( \rho_t(\varphi_t, \eta_t) \) and \( \rho_a(\varphi_a, \eta_a) \), where the subscript “t” (“a”) stands for trigger (associated) particles. The two-particle density of trigger and associated particle pairs is denoted by \( \rho_{a,t}(\varphi_a, \eta_a; \varphi_t, \eta_t) \) following the notation in \cite{150,151}. The mathematical definitions of these density functions are

\[
\rho_t(\varphi_t, \eta_t) = \frac{d^2 N_t}{d\varphi_t d\eta_t}, \quad \rho_a(\varphi_a, \eta_a) = \frac{d^2 N_a}{d\varphi_a d\eta_a}, \quad \rho_{a,t}(\varphi_a, \eta_a; \varphi_t, \eta_t) = \frac{d^4 N_{a,t}}{d\varphi_a d\eta_a d\varphi_t d\eta_t}.
\] (A-1)

On the other hand, two-particle correlation functions are distinguished from the two-particle density function, and generally constructed by the simultaneous measurement of pairs of
particles in each event. It may have various forms, and the typical correlation function,
\[ C_{2,R}(\eta_a \eta_t; \varphi_a \varphi_t), \]
is
\[ C_{2,R}(\varphi_a, \varphi_t; \eta_a, \eta_t) = \frac{\rho_{a,t}(\varphi_a, \eta_a; \varphi_t, \eta_t)}{\rho_a(\varphi_a, \eta_a) \rho_t(\varphi_t, \eta_t)} - 1, \tag{A-2} \]
where “R” stands for ratio. In case of rotational invariance of the correlated signal in azimuth,
\[ C_{2,R}(\varphi_t - \varphi_a; \eta_a, \eta_t) = \frac{\rho_{a,t}(\varphi_t - \varphi_a; \eta_a, \eta_t)}{\rho_a(\eta_a) \rho_t(\eta_t)} - 1, \tag{A-3} \]
not considering the dependence in \( \varphi \) in single-particle densities.

Experimentally, \( C_{2,R}(\varphi_t - \varphi_a; \eta_a, \eta_t) \) is further simplified into \( C_R(\Delta \varphi, \Delta \eta) \), defined by
\[ C_R(\Delta \varphi, \Delta \eta) = \frac{S(\Delta \varphi, \Delta \eta)}{B(\Delta \varphi, \Delta \eta)} - 1, \tag{A-4} \]
where \( \Delta \varphi = \varphi_t - \varphi_a, \Delta \eta = \eta_t - \eta_a \), and \( S(\Delta \varphi, \Delta \eta) = \frac{d^2 N_{\text{pair}}}{d \Delta \varphi d \Delta \eta} \) is the two-particle distribution constructed in the same event, while \( B(\Delta \varphi, \Delta \eta) \) is from different events. Also, the per-trigger normalized associated particle yield (per-trigger yield),
\[ C_{\text{yield}}(\Delta \varphi, \Delta \eta) = \frac{1}{N_{\text{trig}}} \frac{d^2 N_{\text{pair}}}{d \Delta \varphi d \Delta \eta}, \tag{A-5} \]
is often used as in the forward-central two-particle correlation in this thesis. In the mixed-event method, \( C_{\text{yield}}(\Delta \varphi, \Delta \eta) \) is approximated by
\[ \frac{1}{N_{\text{trig}}} \frac{d^2 N_{\text{pair}}}{d \Delta \varphi d \Delta \eta} \simeq C_{\text{trig,R}}(0,0) \frac{S(\Delta \varphi, \Delta \eta)}{B(\Delta \varphi, \Delta \eta)} \tag{A-6} \]
where
\[ S(\Delta \varphi, \Delta \eta) = \frac{1}{N_{\text{trig}}} \frac{d^2 N_{\text{same}}}{d \Delta \varphi d \Delta \eta}, \quad B(\Delta \varphi, \Delta \eta) = \frac{1}{N_{\text{trig}}} \frac{d^2 N_{\text{mixed}}}{d \Delta \varphi d \Delta \eta}. \tag{A-7} \]
Comparing \( C_{2,R}, C_R \), and \( C_{\text{trig,R}} \), which commonly share the index “R”, they essentially
contain the same information on how much the correlated production is different from the uncorrelated production as a function of a ratio between the two. In [131], the correct \( C_{\text{yield}}(\Delta \varphi, \Delta \eta) \) is claimed to be distinguished from those ratio functions, and Eq. A-6 is an approximation that its validity largely depends on the property of the correlated signal.

To describe the finite-acceptance effects in two-particle correlations, some definitions of variables are needed first in an axis, \( x \). If the acceptance range of the detector is \([a_1, a_2]\) in \( x \), \( X \), \( f_t(x - X) \), \( f_a(x - X) \), and \( g(X) \) are defined as

\[
X \equiv \text{Common reference point of the trigger and associated particle distributions for each correlated signal, and the center of distributions for convenience.}
\]

\[
f_t(x - X) \equiv \text{Trigger particle distribution in a correlated signal with respect to } x = X \text{ in the range } -b < x - X < b, \text{ where } 2b \text{ corresponds to the size of a correlated signal in } x;
\]

\[
f_a(x - X) \equiv \text{Associated particle distribution in a correlated signal with respect to } x = X \text{ in the range } -b < x - X < b;
\]

\[
g(X) \equiv \text{Distribution of } X \text{ over all events.}
\]

These definitions assume the translational invariance of the signal and the existence of a common reference point in trigger and associated particle distributions. In case of the correlated signal with fragmented particles from a jet, \( X \) corresponds approximately to the jet-axis.

With infinite detector acceptance in \( x \) and using the above variables, the per-trigger yield
from a single correlated signal for a given $X$, $C_{\text{inf},\text{single}}$, is defined by the cross-correlation:

$$C_{\text{inf},\text{single}}(\Delta x) = \frac{1}{(N_{\text{trig}})_{\text{inf},\text{single}}} (f_a \ast f_t)_{\text{inf},\text{single}}$$

$$= \frac{1}{(N_{\text{trig}})_{\text{inf},\text{single}}} \int_{-\infty}^{\infty} f_a(x - X - \Delta x) f_t(x - X) \, dx$$

$$= \frac{1}{(N_{\text{trig}})_{\text{inf},\text{single}}} \int_{\min(X+b,X+b+\Delta x)}^{\max(X+b,X-b+\Delta x)} f_a(x - X - \Delta x) f_t(x - X) \, dx$$

$$= \frac{1}{(N_{\text{trig}})_{\text{inf},\text{single}}} \int_{\min(b,b+\Delta x)}^{\max(-b,-b+\Delta x)} f_a(x' - \Delta x) f_t(x') \, dx' , \quad (A-8)$$

where

$$(N_{\text{trig}})_{\text{inf},\text{single}} = \int_{-\infty}^{\infty} f_t(x - X) \, dx = \int_{X-b}^{X+b} f_t(x - X) \, dx$$

$$= \int_{-b}^{b} f_t(x') \, dx' . \quad (A-9)$$

It is observed that $C_{\text{inf},\text{single}}(\Delta x)$ does not depend on $X$, as expected from the assumption of the translational invariance. The per-trigger yield over all events with all signals with infinite acceptance is

$$(N_{\text{trig}})_{\text{inf}} = \int_{-\infty}^{\infty} g(X) \left( \int_{-\infty}^{\infty} f_t(x - X) \, dx \right) \, dX$$

$$= (N_{\text{trig}})_{\text{inf},\text{single}} \int_{-\infty}^{\infty} g(X) \, dX , \quad (A-10)$$

$$C_{\text{inf}}(\Delta x) = \frac{1}{(N_{\text{trig}})_{\text{inf}}} \int_{-\infty}^{\infty} g(X) \left( \int_{-\infty}^{\infty} f_a(x - X - \Delta x) f_t(x - X) \, dx \right) \, dX$$

$$= \frac{1}{(N_{\text{trig}})_{\text{inf},\text{single}}} \int_{-\infty}^{\infty} g(X) \, dX \int_{-\infty}^{\infty} g(X) \, dX \left( \int_{-\infty}^{\infty} f_a(x - X - \Delta x) f_t(x - X) \, dx \right)$$

$$= \frac{1}{(N_{\text{trig}})_{\text{inf},\text{single}}} \int_{-\infty}^{\infty} f_a(x - X - \Delta x) f_t(x - X) \, dx$$

$$= C_{\text{inf},\text{single}}(\Delta x) , \quad (A-11)$$

and is identical to $C_{\text{inf},\text{single}}$.

$^{1}$Cross-correlation: $(f \ast g)(\Delta x) = \int f(x - \Delta x) g(x) \, dx$
Now with finite detector acceptance, \([a_1, a_2]\) in \(x\), parts of the correlated signal may be out of the acceptance depending on \(X\) of the signal. These undetected parts distort the overall per-trigger yield, and need to be corrected for. The per-trigger yield in a single event with given \(X\) and finite acceptance, \(C_{\text{single}}(\Delta x)\), can be written as

\[
C_{\text{single}}(\Delta x) = \frac{1}{(N_{\text{trig}})_{\text{single}}} \int_{-\infty}^{\infty} f_a(x - X - \Delta x) A_a(x - \Delta x) f_t(x - X) A_t(x) \, dx , \tag{A-12}
\]

where

\[
(N_{\text{trig}})_{\text{single}} = \int_{-\infty}^{\infty} f_t(x - X) A_t(x) \, dx , \tag{A-13}
\]

and acceptance operators,

\[
A_t(x) = \begin{cases} 1 & \text{if } a_{1,t} < x < a_{2,t} \\ 0 & \text{otherwise} \end{cases}, \quad A_a(x) = \begin{cases} 1 & \text{if } a_{1,a} < x < a_{2,a} \\ 0 & \text{otherwise} \end{cases} , \tag{A-14}
\]

Then the per-trigger yield over all events with all signals and finite acceptances is

\[
C(\Delta x) = \frac{1}{N_{\text{trig}}} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g(X) f_a(x - X - \Delta x) A_a(x - \Delta x) f_t(x - X) A_t(x) \, dx \, dX , \tag{A-15}
\]

\[
N_{\text{trig}} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g(X) f_t(x - X) A_t(x) \, dx \, dX . \tag{A-16}
\]

Correction for finite-acceptance effects means finding certain formulas to connect \(C_{\text{inf}}(\Delta x)\) and \(C(\Delta x)\), but there exists no analytic formula that relates \(C_{\text{inf}}(\Delta x)\) and \(C(\Delta x)\). However, under specific conditions of the correlated signal, exact formulas can be derived and used as approximations in similar circumstances.

In Eq. [A-15] if the distribution of the signal, \(g(X)\), is constant,

\[
C(\Delta x) = \frac{(A_a \ast A_t)(\Delta x)}{\Delta_t} C_{\text{inf}}(\Delta x) , \tag{A-17}
\]

where \(\Delta_t = a_{2,t} - a_{1,t}\) is the size of trigger particle acceptance. This means that the measured per-trigger yield in the case of a constant signal distribution can be exactly corrected to
the per-trigger yield without finite-acceptance effects by dividing by \( \frac{(A_a \times A_t)(\Delta x)}{\Delta t} \). Another condition that the exact formula can be derived is if \( f_t \) is a \( \delta \)-function. In this case,

\[
C(\Delta x) = \frac{(A_a \times n_{\text{trig}}A_t)(\Delta x)}{N_{\text{trig}}} C_{\text{inf}}(\Delta x) ,
\]

(A-18)

where \( n_{\text{trig}} \) is the trigger particle distribution in \( x \). This condition is similar to the near-side in jet-hadron correlations in a jet.

Similar discussion can be made for the away-side of a correlated signal, and the same correctional factors in Eq. A-17 and Eq. A-18 are derived for certain functions of reference points in the away-side signal. Step-by-step derivations of Eq. A-17 and Eq. A-18 and the test of their validity using the Monte Carlo simulations can be found in [131].
C  Longitudinal two-particle correlations

As discussed in Chap. 6, the $\eta$ distribution of tracks in each event is quantified using the Legendre decomposition. While the decomposition is applied to the $\eta$ distribution within the limited $\phi$ range on an event-by-event basis in the analysis discussed in Chap. 6, it is used differently in the longitudinal two-particle correlation analysis by the ATLAS Collaboration [83]. In principle, both approaches equally quantify the longitudinal structure with

$$R_S(\eta) \propto 1 + \sum_{n=1} a_n T_n(\eta), \quad (A-19)$$

$$T_n(\eta) \equiv \sqrt{\frac{2n+1}{3}} Y P_n \left( \frac{\eta}{Y} \right), \quad (A-22)$$

where

$$R_S(\eta) \equiv \frac{N(\eta)}{\langle N(\eta) \rangle}, \quad (A-21)$$

and $P_n$ are the $n$-th order Legendre polynomials. In Eq. A-21, $N(\eta) = dN/d\eta$ is the $\eta$ distribution of tracks in a single event, and $\langle N(\eta) \rangle$ is the average distribution for a given multiplicity or centrality class. It should be remarked that the $\phi$ range of tracks for the decomposition is different in the two approaches. Instead of the direct extraction of coefficients on an event-by-event basis, the analysis in [83] utilizes the two-particle pseudorapidity correlation function $C(\eta_1, \eta_2)$, defined as

$$C(\eta_1, \eta_2) = \frac{\langle N(\eta_1) N(\eta_2) \rangle}{\langle N(\eta_1) \rangle \langle N(\eta_2) \rangle} \equiv \langle R_S(\eta_1) R_S(\eta_2) \rangle. \quad (A-23)$$
In order to remove the residual multiplicity dependence in the shape of $\langle N(\eta) \rangle$, this correlation function is re-defined as discussed in [84] with

$$
C_N(\eta_1, \eta_2) = \frac{C(\eta_1, \eta_2)}{C_p(\eta_1)C_p(\eta_2)} \tag{A-24}
$$

where

$$
C_p(\eta_1) = \frac{\int_{-Y}^{Y} C(\eta_1, \eta_2) \, d\eta_2}{2Y}, \quad C_p(\eta_2) = \frac{\int_{-Y}^{Y} C(\eta_1, \eta_2) \, d\eta_1}{2Y} \tag{A-25}
$$

are the averages of the $C(\eta_1, \eta_2)$ along the $\eta_2$ or $\eta_1$ direction.

Then this modified two-particle pseudorapidity correlation function $C_N(\eta_1, \eta_2)$ is decomposed with combinations of the Legendre polynomials,

$$
C_N(\eta_1, \eta_2) = 1 + \sum_{n,m=1}^{\infty} a_{n,m} \frac{T_n(\eta_1)T_m(\eta_2) + T_n(\eta_2)T_m(\eta_1)}{2} \tag{A-26}
$$

The two-particle Legendre coefficients can be calculated with

$$
a_{n,m} = \left( \frac{3}{2Y^3} \right)^2 \int_{-Y}^{Y} C_N(\eta_1, \eta_2) \frac{T_n(\eta_1)T_m(\eta_2) + T_n(\eta_2)T_m(\eta_1)}{2} \, d\eta_1 \, d\eta_2, \tag{A-27}
$$

and

$$
a_{n,m} = \langle a_n a_m \rangle. \tag{A-28}
$$

Thus, the measurement via the pseudorapidity two-particle correlations effectively evaluates the root-mean-square of the event-by-event $a_n$, $\sqrt{\langle a_n^2 \rangle}$ or the cross correlation between $a_n$ and $a_m$, $\langle a_n a_m \rangle$. Figure 2.16 shows the results of this measurement by the ATLAS Collaboration.
D Summary of systematic uncertainties in conditional $a_n$

Summaries of systematic uncertainties from conditional $a_1$, slopes in the conditional $a_1$, conditional $a_2$, and conditional $a_3$ are provided in this Appendix. Instead of percentages to show the systematic uncertainties, the actual values of the uncertainties are used in following tables. Only the minimum and the maximum values of the uncertainties with all $(\varphi_i, \varphi_j)$ combinations in $a_n(\varphi_i|a_n(\varphi_j))$ are provided in following tables. It should be remarked that the range of the sum of the systematic uncertainties (i.e. total systematic uncertainties) may not agree with the sum of the minimum or the maximum values of the uncertainties in all variations in quadrature, as the minimum or the maximum may be found in different $(\varphi_i, \varphi_j)$ combinations. Systematic uncertainties vary depending on the observables and the $a_n(\varphi_j)$ value in $a_n(\varphi_i|a_n(\varphi_j))$.

Table 1: Summary of systematic uncertainties in the estimation of conditional $a_1$

<table>
<thead>
<tr>
<th>Systematic effect</th>
<th>0–5% $\times 10^{-4}$</th>
<th>5–10% $\times 10^{-4}$</th>
<th>10–20% $\times 10^{-4}$</th>
<th>20–30% $\times 10^{-4}$</th>
<th>30–40% $\times 10^{-4}$</th>
<th>40–50% $\times 10^{-4}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Centrality estimation</td>
<td>0.073–3.0</td>
<td>0.059–1.5</td>
<td>0.082–11</td>
<td>0.080–5.2</td>
<td>0.19–9.6</td>
<td>0.13–16</td>
</tr>
<tr>
<td>Event-plane estimation</td>
<td>0.19–5.6</td>
<td>0.11–4.9</td>
<td>0.055–3.8</td>
<td>0.14–7.2</td>
<td>0.94–16</td>
<td>0.45–21</td>
</tr>
<tr>
<td>$z_{vtx}$-dependence</td>
<td>0.084–1.5</td>
<td>0.049–1.3</td>
<td>0.097–4.8</td>
<td>0.088–4.2</td>
<td>0.16–5.5</td>
<td>0.39–8.0</td>
</tr>
<tr>
<td>Track selection</td>
<td>0.026–3.6</td>
<td>0.097–1.8</td>
<td>0.17–5.5</td>
<td>0.083–3.3</td>
<td>0.31–5.8</td>
<td>0.28–12</td>
</tr>
<tr>
<td>Magnetic field direction</td>
<td>0.067–3.8</td>
<td>0.065–5.1</td>
<td>0.13–10</td>
<td>0.057–9.3</td>
<td>0.27–12</td>
<td>0.19–29</td>
</tr>
<tr>
<td>Event-plane resolution correction</td>
<td>0.0056–1.2</td>
<td>0.025–0.76</td>
<td>0.0034–3.0</td>
<td>0.076–3.4</td>
<td>0.025–6.3</td>
<td>0.096–7.6</td>
</tr>
<tr>
<td>Multiplicity correction</td>
<td>0.025–4.4</td>
<td>0.036–4.1</td>
<td>0.0067–8.0</td>
<td>0.027–4.4</td>
<td>0.043–9.4</td>
<td>0.11–49</td>
</tr>
<tr>
<td>Sum</td>
<td>0.27–7.3</td>
<td>0.38–7.3</td>
<td>0.82–13</td>
<td>1.6–13</td>
<td>2.6–23</td>
<td>3.0–53</td>
</tr>
</tbody>
</table>
### Table 2: Summary of systematic uncertainties in the estimation of slopes in conditional $a_1$

<table>
<thead>
<tr>
<th>Systematic effect</th>
<th>0–5%</th>
<th>5–10%</th>
<th>10–20%</th>
<th>20–30%</th>
<th>30–40%</th>
<th>40–50%</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(10^{-3})</td>
<td>(10^{-3})</td>
<td>(10^{-3})</td>
<td>(10^{-3})</td>
<td>(10^{-3})</td>
<td>(10^{-3})</td>
</tr>
<tr>
<td>Centrality estimation</td>
<td>0.33 - 0.78</td>
<td>0.22 - 1.3</td>
<td>0.49 - 1.5</td>
<td>0.15 - 0.64</td>
<td>0.25 - 1.2</td>
<td>0.4 - 1.2</td>
</tr>
<tr>
<td>Event-plane estimation</td>
<td>0.49 - 2.3</td>
<td>0.34 - 0.69</td>
<td>0.23 - 0.72</td>
<td>0.14 - 2.1</td>
<td>0.62 - 1.6</td>
<td>0.2 - 0.82</td>
</tr>
<tr>
<td>$z_{vtx}$-dependence</td>
<td>0.18 - 0.84</td>
<td>0.35 - 0.83</td>
<td>0.073 - 0.94</td>
<td>0.077 - 0.61</td>
<td>0.16 - 0.54</td>
<td>0.095 - 0.81</td>
</tr>
<tr>
<td>Track selection</td>
<td>0.078 - 0.88</td>
<td>0.16 - 0.74</td>
<td>0.19 - 0.46</td>
<td>0.081 - 0.45</td>
<td>0.17 - 0.46</td>
<td>0.16 - 0.35</td>
</tr>
<tr>
<td>Magnetic field direction</td>
<td>1.4 - 2.3</td>
<td>0.16 - 2</td>
<td>0.85 - 4.6</td>
<td>0.33 - 2</td>
<td>0.27 - 1.7</td>
<td>0.58 - 3.3</td>
</tr>
<tr>
<td>Event-plane resolution correction</td>
<td>0.001 - 0.78</td>
<td>0.005 - 0.93</td>
<td>0.043 - 1.5</td>
<td>0.15 - 1.9</td>
<td>0.40 - 2.4</td>
<td>0.56 - 2.8</td>
</tr>
<tr>
<td>Multiplicity correction</td>
<td>0.25 - 0.94</td>
<td>0.0026 - 0.64</td>
<td>0.013 - 1.3</td>
<td>0.034 - 1.8</td>
<td>0.13 - 2.2</td>
<td>0.32 - 1.9</td>
</tr>
<tr>
<td><strong>Sum</strong></td>
<td>1.8 - 4.3</td>
<td>0.71 - 3.5</td>
<td>1.2 - 5.3</td>
<td>0.88 - 3.1</td>
<td>1.1 - 3.4</td>
<td>1.5 - 4.1</td>
</tr>
</tbody>
</table>

### Table 3: Summary of systematic uncertainties in the estimation of conditional $a_2$

<table>
<thead>
<tr>
<th>Systematic effect</th>
<th>0–5%</th>
<th>5–10%</th>
<th>10–20%</th>
<th>20–30%</th>
<th>30–40%</th>
<th>40–50%</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>(10^{-2})</td>
<td>(10^{-2})</td>
<td>(10^{-2})</td>
<td>(10^{-2})</td>
<td>(10^{-2})</td>
<td>(10^{-2})</td>
</tr>
<tr>
<td>Centrality estimation</td>
<td>0.0039 - 4.7</td>
<td>0.0022 - 2.2</td>
<td>0.0085 - 4.8</td>
<td>0.00055 - 0.37</td>
<td>0.0014 - 0.45</td>
<td>0.0020 - 0.52</td>
</tr>
<tr>
<td>Event-plane estimation</td>
<td>0.0021 - 1.5</td>
<td>0.00086 - 2.1</td>
<td>0.0045 - 2.2</td>
<td>0.0023 - 0.70</td>
<td>0.0043 - 0.83</td>
<td>0.0080 - 0.96</td>
</tr>
<tr>
<td>$z_{vtx}$-dependence</td>
<td>0.00095 - 2.8</td>
<td>0.0076 - 0.57</td>
<td>0.0023 - 0.65</td>
<td>0.0017 - 0.25</td>
<td>0.0039 - 0.25</td>
<td>0.0019 - 0.38</td>
</tr>
<tr>
<td>Track selection</td>
<td>0.021 - 3.1</td>
<td>0.012 - 4.9</td>
<td>0.0084 - 3.5</td>
<td>0.0016 - 0.48</td>
<td>0.0031 - 0.53</td>
<td>0.0024 - 0.3</td>
</tr>
<tr>
<td>Magnetic field direction</td>
<td>0.00029 - 1.8</td>
<td>0.002 - 1.4</td>
<td>0.002 - 0.75</td>
<td>0.00063 - 0.76</td>
<td>0.0023 - 1.3</td>
<td>0.0032 - 1.2</td>
</tr>
<tr>
<td>Event-plane resolution correction</td>
<td>0.00001 - 0.095</td>
<td>0.0005 - 0.095</td>
<td>0.00011 - 0.061</td>
<td>0.00014 - 0.071</td>
<td>0.00001 - 0.074</td>
<td>0.00022 - 0.073</td>
</tr>
<tr>
<td>Multiplicity correction</td>
<td>0.00027 - 5.1</td>
<td>0.00037 - 2.0</td>
<td>0.00019 - 1.9</td>
<td>0.00001 - 0.43</td>
<td>0.00028 - 0.74</td>
<td>0.00001 - 0.65</td>
</tr>
<tr>
<td><strong>Sum</strong></td>
<td>0.026 - 7.6</td>
<td>0.023 - 6</td>
<td>0.026 - 6.3</td>
<td>0.018 - 1.2</td>
<td>0.024 - 1.6</td>
<td>0.037 - 1.6</td>
</tr>
<tr>
<td>Systematic effect</td>
<td>0–5%</td>
<td>5–10%</td>
<td>10–20%</td>
<td>20–30%</td>
<td>30–40%</td>
<td>40–50%</td>
</tr>
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</tr>
<tr>
<td></td>
<td>$(10^{-4})$</td>
<td>$(10^{-4})$</td>
<td>$(10^{-4})$</td>
<td>$(10^{-4})$</td>
<td>$(10^{-4})$</td>
<td>$(10^{-4})$</td>
</tr>
<tr>
<td>Centrality estimation</td>
<td>0.039 – 1.4</td>
<td>0.054 – 2.5</td>
<td>0.11 – 6.5</td>
<td>0.14 – 7.3</td>
<td>0.10 – 16</td>
<td>0.097 – 17</td>
</tr>
<tr>
<td>Event-plane estimation</td>
<td>0.25 – 3.2</td>
<td>0.028 – 5.3</td>
<td>0.25 – 8.3</td>
<td>0.31 – 7.1</td>
<td>0.18 – 23.9</td>
<td>0.55 – 40</td>
</tr>
<tr>
<td>$z_{\text{vtx}}$-dependence</td>
<td>0.15 – 2.6</td>
<td>0.15 – 5.3</td>
<td>0.1 – 3.1</td>
<td>0.11 – 2.1</td>
<td>0.014 – 10</td>
<td>0.37 – 11</td>
</tr>
<tr>
<td>Track selection</td>
<td>0.032 – 3.0</td>
<td>0.023 – 2.4</td>
<td>0.084 – 3.0</td>
<td>0.095 – 2.8</td>
<td>0.043 – 8.4</td>
<td>0.35 – 9.9</td>
</tr>
<tr>
<td>Magnetic field direction</td>
<td>0.056 – 9.2</td>
<td>0.032 – 4.6</td>
<td>0.59 – 12</td>
<td>0.13 – 7.6</td>
<td>0.13 – 21</td>
<td>0.021 – 72</td>
</tr>
<tr>
<td>Event-plane resolution correction</td>
<td>0.00066 – 0.35</td>
<td>0.0025 – 0.45</td>
<td>0.0051 – 0.71</td>
<td>0.00072 – 1.9</td>
<td>0.0021 – 3.5</td>
<td>0.01 – 4.0</td>
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<td>Multiplicity correction</td>
<td>0.0075 – 0.43</td>
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<td>0.0023 – 2.3</td>
<td>0.012 – 5.3</td>
<td>0.043 – 19</td>
<td>0.032 – 29</td>
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<tr>
<td>Sum</td>
<td>0.46 – 9.9</td>
<td>0.65 – 8.2</td>
<td>1.1 – 17</td>
<td>0.94 – 11</td>
<td>1.6 – 39</td>
<td>2.8 – 73</td>
</tr>
</tbody>
</table>
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