Dynamic studies of multiple configurations of CERN’s
Antiproton Decelerator Target core under proton beam impact

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Abstract

Antiprotons, like many other exotic particles, are produced by impacting high energy proton beams onto fixed targets. At the European Organization for Nuclear Research (CERN), this is done in the Antiproton Decelerator (AD) Facility.

The engineering challenges related to the design of an optimal configuration of the AD-Target system derive from the extremely high energy depositions reached in the very thin target core as a consequence of each proton beam impact. A new target design is foreseen for operation after 2021, triggering multiple R&D activities since 2013 for this purpose. The goal of the present Master Thesis is to complement these activities with analytical and numerical calculations, delving into the phenomena associated to the dynamic response of the target core. In this context, two main studies have been carried out. First, the experimental data observed in targets subjected to low intensity proton pulses was cross-checked with analytical and computational methods for modal analysis, applied under the assumption of completely elastic responses of the material. The analysis then focused on the flexural modes of vibration of the rods, excited by off-axis proton beam impacts. In the second part, hydrocode simulations of the response of two potential prototypes for the future AD-Target core are presented. For a deep understanding of the phenomena governing the dynamic response of the target core in operation, the simulations were performed gradually increasing the complexity of the models. The results of the analysis were finally applied to quantify and understand the effects of changes of the core geometry on its dynamic response and tensile pressures reached.
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Introduction

Antiprotons, like many other exotic particles, are produced by impacting high energy proton beams onto fixed targets. At the European Organization for Nuclear Research (CERN), this is done in the Antiproton Decelerator (AD) Facility. Here, after the collision of a 26 GeV/c proton beam with a high-Z material target, antiprotons of 3.5 GeV/c are collected and magnetically conducted to the AD Complex, where they are slowed down to a momentum of 100 MeV/c by electron and stochastic cooling with the aim of confining them for the production of anti-hydrogen and antimatter research experiments[12].

The engineering challenges related to the design of an optimal configuration of the AD-Target system derive from the extremely demanding requirements that need to be satisfied for antiproton production. The target, in particular, needs to be very compact in order to avoid anti-proton re-absorption in the surrounding material and to be as close as possible to a point-like source for the magnetic horn, the particle collector system placed downstream. This compactness requirement results in extremely high energy depositions reached in a very thin target core as a consequence of each proton beam impact.

A new target design is foreseen for operation after 2012, triggering multiple R&D activities since 2013 for this purpose. These activities [12, 13], both numerical and experimental in nature, were mainly focused on the study of the dynamic response of the target core which is subjected to a fast rise of temperature of 2000°C in less than 0.43 μs every time it is impacted by the primary proton beam. The goal of the present Master Thesis is to complement these studies with analytical and numerical calculations, delving into the phenomena associated to the dynamic response of the target core. In this context, two main studies have been carried out:

1. First, an analysis of the modes of vibration observed in rods subjected to low intensity pulses during the HiRadMat27 experiment is presented. In the experiment, carried out in November 2015, 13 rods of high density materials designed to experience similar conditions to those reached in the AD-Target core under proton beam impact. The response to low intensity pulses was found to be particularly interesting because the low level of damping and the absence of cracks inside the material allowed the cross-check of the experimental data with analytical and computational methods for modal analysis obtained assuming completely elastic responses of the material. The analysis then focused on the flexural modes of vibration of the rods, excited by off-axis proton beam impacts.
2. In the second part, hydrocode analyses of the response of two versions of the current prototype of the future AD-Target core are presented. The study, building upon the knowledge gathered in the first part, gives important new insights on the dynamic response of the tantalum rods that make up the target core. The analysis is conducted considering models with increasing complexity following a similar approach to the one adopted in [13]. First, the dynamic response of isolated tantalum rods assuming perfectly elastic material behavior is considered. The second step considers the effects of the introduction of the expanded graphite matrix surrounding the rods and, finally, in the third step, the possibility of plastic deformation of the materials is implemented with the introduction of appropriate strength models. The results of the analysis are finally applied to quantify and understand the effects of changes in diameter and length of the rods on their dynamic response.
Part I

Modes of vibration of the rods in the HiRadMat27 experiment
Chapter 1

Introduction to the HiRadMat27 experiment

1.1 Motivation and scientific goals

When impacted by the Proton Synchrotron (PS) beam with a momentum of 26 GeV/c and a nominal intensity of \(1.5 \cdot 10^{13}\) protons per pulse, the AD-target experiences a temperature increase of up to 2000 K in just 0.43 \(\mu\)s. This nearly instantaneous temperature increase causes a rapid rise in pressure that triggers complex and extreme vibrational phenomena.

Ever since the 1980s, when the designing process of the first version of the target started, great effort was put into trying to understand the response of the material to this intense process. In the last years, in particular, through the work of C. Torregrosa [13] the application of hydrocodes has been shown to be a powerful tool for the analysis of the conditions reached in the AD-Target.

The reliability of numerical simulations, however, is deeply dependent on the reliability of the material constitutive models that are adopted. Some of the most common strength models used in hydrocodes have been historically obtained for and applied to the study of projectile impacts, regimes in which temperatures and strain rates, albeit high, are substantially lower than the ones experienced in the component at study. When using these material models for the resolution of the conditions reached in the AD-Target, they have to be extrapolated, therefore causing relevant uncertainties on the reliability of the computed results. It was clear that, in order to validate the result of the simulations of the dynamic response of the AD-Target, the reliability of the adopted material models had to be tested on components exposed to similar extreme conditions.

After an extensive review of the possible experimental approaches, it was concluded that the only possible way to achieve fast and high temperature rises in the bulk material like those reached in the AD-Target core would require the use of proton beam impacts.

For this purpose an experiment called HRMT27-RodTarg [14] took place in 2015. The
The scope of the experiment was to impact high-intensity and high-energy proton pulses onto 13 thin rods of high-density materials with the goal of reproducing similar conditions to the ones reached in the core of the AD-Target during its operation.

The scientific objectives of the experiment can be summarized in two main points:

1. Validation of the numerical approach (the use of hydrocodes and the extrapolated material strength and failure models) to the analysis of the target core when impacted by the proton beam. This objective was achieved by measuring significant amounts of data (temperature and velocity) on the surfaces of the target rods when impacted by the proton beam and by cross-checking the measurements with the results of the simulations.

2. Reduction of the uncertainties in the response of the AD-Target core to the primary proton beam impact and selection of the future antiproton target material. This was done by comparing the measured responses of sample rods of different high-Z candidate materials (such as W, Ir, Mo, Ta, ...), designed in such a way that they would experience the same conditions reached in the AD-Target.

### 1.2 Experiment design and layout

The HiRadMat [6] (High Radiation to Materials) facility at CERN, in activity since 2012, designed to test material samples and accelerator component assemblies by impacting them with high-intensity proton beams in a controlled area, was selected as the ideal place to conduct the experiment.

The 440 GeV/c proton beam used in the facility is extracted from the CERN Super Proton Synchrotron (SPS) with an adjustable pulse length. The difference in primary proton beam momentum between PS and SPS required an extensive study [14] in order to design the experimental targets in such a way that they would experience similar conditions to those reached in the AD-Target core while producing a response at their surface within the instrumentation limitations. The final design consisted of targets with a diameter of 8 mm and variable length depending on the material, ranging from 140 to 240 mm.

The experiment was designed in a modular way to facilitate assembly and dismantling. Three main parts are distinguishable in Figure 1.2.1:

1. Standard HiRadMat table, provided by the HiRadMat facility to make every experiment compatible with the available space, cooling and electrical connections.

2. Motorized support table with vertical and horizontal degrees of freedom. These degrees of freedom were used in the alignment procedures of the rods with the incoming beam, whose position was measured by means of a beam position monitor BPKG/BTV attached to the tank.
Figure 1.2.1: 3D CAD model of the full HRMT27 experiment assembly [14].

3. Vacuum tank: as seen in Figure 1.2.2a, the 13 sample rods are placed in an aluminum sample holder that could move vertically during the experiment to put every rod in the corresponding on-beam position. A beam position monitor BPKG/BTV is attached to the upstream side of the tank. The rods are positioned inside the sample holder by means of small V-shaped graphite supports that are pressed against the rods by springs, as can be seen in Figure 1.2.2b.

The choice of the instrumentation to be put inside the tank was made taking into account the harsh conditions, in terms of temperature, velocity and radiation, reached on the surfaces of the targets. Four optical heads were placed inside the tank, pointing directly to small mirrors machined on the targets, as can be seen in Figure 1.2.3. Two heads corresponded to an interferometer with a maximum range of measurement of $2 \text{ m/s}$.

One head corresponded to a Laser Doppler Vibrometer (LDV), with a maximum range of measurement of $24 \text{ m/s}$. Simulations showed that points close to the longitudinal center of the target could reach velocities of up to $40 \text{ m/s}$. This is why the designated points of measurement of the LDV for the sample rods were placed close to the upstream end of the target, where the maximum velocities are much lower. The fourth head corresponded to a pyrometer.

The only instruments that were directly in contact with the targets were 26 thermocouples (2 per target). These instruments, with a much lower acquisition rate, were useful for monitoring the temperatures during the experiment, even though many of them got detached because of the high speeds at which the rods were vibrating during the experiment.
An example of a rod with all the indicated points of measurement can be seen in Figure 1.2.3.

1.2.1 Dynamic response of the TZM rod at low intensity pulses

For the purpose of the analysis conducted in this thesis, the dynamic response of the TZM (a dispersion strengthened Mo alloy that has great stability at high temperatures) rod at low intensity pulses is perhaps the most interesting among the ones recorded during the experiment.

The displacement shown in Figure 1.2.4 is obtained through the internal integration of the velocity measured by the LDV on a TZM rod in the 3 ms following a low intensity impact of $1.15 \times 10^{11}$ ppp (for reference, according to the simulations carried out in [14], the intensity required to reproduce similar conditions to the ones reached in the AD-Target was shown to be $1.67 \times 10^{12}$, one order of magnitude more than the impact considered in this section), leading to a computed maximum temperature rise of 45°C.

Upon closer inspection of the recorded displacement, it is apparent that the target response consists of the superposition of three distinct waves, with periods of 1.8 µs, 86 µs and $\sim 1.6$ ms (since the recorded time was only 3 ms, it is hard to obtain a better estimation of the period of this wave). This type of response was exhibited by most targets, even though it was most apparent in the ones impacted by low intensity pulses.

The case of the TZM rod is particularly interesting because it is the only one in which
Figure 1.2.3: Drawing of one of the 140 mm targets indicating the points of measurement of the instruments previously introduced [14]

the three waves are clearly distinguishable and are not immediately dampened. The very low level of damping (the radial and longitudinal wave were still present in the recorded data even in the last instants of the measurement after, respectively, more than 1600 and 35 complete cycles) was interpreted as a proof of the predominantly elastic behavior of the rod after the impact.

The analytical and computational methods used to investigate the nature of these waves are exposed in detailed in the following chapter.
Figure 1.2.4: Displacement obtained by integration of the velocity signal recorded by the LDV in the 3 ms after a $1.15 \cdot 10^{11}$ ppp impact on the TZM rod. As it can be seen by examining the full response (a) and the closeups (b) - (c), the signal is obtained as a superposition of three distinct waves with periods of 1.8 µs, 86 µs and $\sim 1.6$ ms.
Chapter 2

Modal analysis of the HRMT27 targets

The analysis [14] of the stress state in the sample rods of the HRMT27 experiment contributed to an identification of the components that make up the displacement shown in Figure 1.2.4, measured during the dynamic response of the TZM rod to a low intensity pulse:

- The 1.8 \( \mu s \) and the 86 \( \mu s \) period waves were identified as waves traveling through the material, respectively, in the radial and longitudinal directions.

- The 1.6 \( ms \) period wave was thought to correspond to the excitation of a flexural mode of vibration by the proton beam. This hypothesis was suggested by previous studies carried out on targets impacted off-axis by beams in the CNGS experiment [2] and by the fact that, as it can be seen in Figure 2.0.1, several targets were noticeably bent after the experiment.

The presence of radial and longitudinal waves was also observed in the analysis carried out via hydrocode simulations of the dynamic response of the AD-Target core under proton beam impact [13]. Two aspects presented in this study needed further clarification:

- The result of the division of the radius of the target by the period of the radial wave is different from the one of the division of the length of the target by the period of the longitudinal wave.

- The analysis of the axial displacement at different longitudinal positions of the target core after the proton beam impact shows how all the parts of the body move with the same frequency and with a fixed phase relation.

These two aspects suggest that the dynamic response of the targets of the HRMT27 experiment and of the AD-Target core may not be governed by waves propagating in the material. The second aspect, in particular, seems to be coherent with the definition of modes of vibration. The validation of this hypothesis was then faced following two distinct approaches: an analytical one, detailed in Section 2.1, and a computational one, taking advantage of FE methods, detailed in Section 2.4.
Figure 2.0.1: Picture taken during the dismounting phase of the experiment. As it can be seen, many targets present noticeable levels of residual flexural deformation. An interesting case is the one of the top target, made of Iridium (the same material that constitutes the core of the current AD-Target), that exhibited massive fragmentation under a pulse with an intensity of $1.71 \cdot 10^{12}$ ppp
2.1 Analytical approach to radial and longitudinal modes of vibration of cylinders

2.1.1 Coupled modes of vibration of a rod

The analytical approach detailed in this Section follows the work presented in [10], which introduced an analytical method to study the coupled vibrations of cylinders and disks of finite dimensions.

The steps required to obtain the formulation for the natural frequencies of vibration of the rods are reported below for clarity.

In a polar coordinate system whose Z-axis coincides with the direction of the height of the cylinder, Equations 2.1.1 - 2.1.8 can be obtained, based on the theory of elastic dynamics. This theory is applicable because, as already mentioned, the TZM rod exhibited clear signs of predominantly elastic behavior when subjected to low intensity pulses. Let the height and radius of the cylinder be \( l \) and \( R \):

\[
\epsilon_{rr} = \frac{\sigma_{rr} - \nu(\sigma_{\theta\theta} + \sigma_{zz})}{E} \quad (2.1.1)
\]

\[
\epsilon_{\theta\theta} = \frac{\sigma_{\theta\theta} - \nu(\sigma_{rr} + \sigma_{zz})}{E} \quad (2.1.2)
\]

\[
\epsilon_{zz} = \frac{\sigma_{zz} - \nu(\sigma_{\theta\theta} + \sigma_{rr})}{E} \quad (2.1.3)
\]

\[
\rho \frac{\partial^2 u_r}{\partial t^2} = \frac{\partial \sigma_{rr}}{\partial r} + \frac{\sigma_{rr} - \sigma_{\theta\theta}}{r} \quad (2.1.4)
\]

\[
\rho \frac{\partial^2 u_z}{\partial t^2} = \frac{\partial \sigma_{zz}}{\partial z} \quad (2.1.5)
\]

\[
\epsilon_{rr} = \frac{\partial u_r}{\partial r} \quad (2.1.6)
\]

\[
\epsilon_{\theta\theta} = \frac{u_r}{r} \quad (2.1.7)
\]

\[
\epsilon_{zz} = \frac{\partial u_z}{\partial z} \quad (2.1.8)
\]

Where:
• $\varepsilon_{rr}$, $\varepsilon_{\theta\theta}$, $\varepsilon_{zz}$ and $\sigma_{rr}$, $\sigma_{\theta\theta}$, $\sigma_{zz}$ are respectively the radial, tangential and axial stresses and strains

• $u_r$ and $u_z$ are the radial and axial displacements

• $E$, $\nu$ and $\rho$ are Young’s Modulus, Poisson’s ratio and the density of the rod material

If we define the Mechanical coupling coefficient as

$$\alpha = \frac{\sigma_{zz}}{\sigma_{\theta\theta} + \sigma_{rr}}$$

Equations 2.1.1, 2.1.2, 2.1.3 can then be rearranged as:

$$\varepsilon_{rr} - \varepsilon_{\theta\theta} = \frac{(1 + \nu)(\sigma_{rr} - \sigma_{\theta\theta})}{E} \quad (2.1.9)$$

$$\varepsilon_{rr} + \varepsilon_{\theta\theta} = \frac{(1 - 2\nu - 2\alpha)(\sigma_{rr} + \sigma_{\theta\theta})}{E} \quad (2.1.10)$$

$$\varepsilon_{zz} = \frac{(1 - \frac{\nu}{\alpha})\sigma_{zz}}{E} \quad (2.1.11)$$

By examining the previous equations, one can realize that the vibration of the rod can be reduced to the superposition of two equivalent one-dimensional vibrations:

• An equivalent axial vibration, governed by Equations 2.1.5 and 2.1.11

• An equivalent planar radial vibration, governed by Equations 2.1.4, 2.1.9 and 2.1.10

It is clear, however, that these equations (and, therefore, these vibrations) are not independent as they are coupled to each other by the mechanical coupling coefficient $\alpha$. The two vibrations will be analyzed in depth in the next two subsections.

### 2.1.1.1 The equivalent planar radial vibration of the cylinder

Through some re-elaboration of Equations 2.1.9 and 2.1.10 it is possible to obtain:

$$\sigma_{rr} = E \left[ \frac{\varepsilon_{rr} - \varepsilon_{\theta\theta}}{1 + \nu} + \frac{\varepsilon_{rr} + \varepsilon_{\theta\theta}}{1 - \nu - 2\nu\alpha} \right] \quad (2.1.12)$$

$$\sigma_{rr} - \sigma_{\theta\theta} = E \frac{\varepsilon_{rr} - \varepsilon_{\theta\theta}}{1 + \nu} \quad (2.1.13)$$

By substituting Equations 2.1.12 and 2.1.13 into Equation 2.1.4, and by using Equations 2.1.6, 2.1.7 and by defining the Equivalent radial elastic constant as:

$$E_r = E \frac{1 - \nu\alpha}{(1 + \nu)(1 - \nu - 2\nu\alpha)} \quad (2.1.14)$$
The following equation for the radial displacement is obtained:

\[
\rho \frac{\partial^2 u_r}{\partial t^2} = E_r \left[ \frac{\partial^2 u_r}{\partial r^2} + \frac{1}{r} \frac{\partial u_r}{\partial r} - \frac{u_r}{r^2} \right] \quad (2.1.15)
\]

In the case of harmonic vibration, by substituting the radial displacement component \( u_r = u_{r0} \exp(j \omega t) \), where \( u_{r0} \) is a function only of position:

\[
\frac{d^2 u_{r0}}{dr^2} + \frac{1}{r} \frac{\partial u_{r0}}{\partial r} - \frac{u_{r0}}{r^2} + K_r^2 u_{r0} = 0 \quad (2.1.16)
\]

Having defined:

- \( K_r = \frac{\omega}{V_r} \), Equivalent radial wavenumber
- \( V_r = \sqrt{\frac{E_r}{\rho}} \), Equivalent radial sound velocity

Equation 2.1.16 is a Bessel’s function of order one, whose solution is:

\[
u_{r0} = A_r J_1(K_r r) + B_r Y_1(K_r r) \quad (2.1.17)
\]

By analyzing this solution, it is clear that, since Bessel functions of the second kind of order one, \( Y_1(x) \), becomes unbounded when \( x \to 0 \), \( B_r = 0 \) is required by the physical characteristics of the problem, thus obtaining:

\[
u_{r0} = A_r J_1(K_r r) \quad (2.1.18)
\]

In order to obtain the constant \( A_r \) we must introduce the boundary condition of a free lateral surface:

\[
\sigma_{rr} |_{r=R} = 0 \quad (2.1.19)
\]

By substituting Equations 2.1.12, 2.1.18 into Equation 2.1.19, the radial frequency equation of the equivalent radial vibration of the rod is obtained:

\[
K_r R J_1(K_r R)(1 - \nu \alpha) - (1 - \nu - 2\nu R)J_1(K_r R) = 0 \quad (2.1.20)
\]

### 2.1.1.2 The equivalent axial vibration of the cylinder

After defining the Equivalent axial elastic constant as:

\[
E_z = \frac{E}{1 - \nu \alpha}
\]

Equation 2.1.11 can rewritten as:

\[
\sigma_{zz} = E_z \varepsilon_{zz} \quad (2.1.21)
\]
By substituting Equations 2.1.21 and 2.1.8 into Equation 2.1.5 we obtain:

\[ \rho \frac{\partial^2 u_z}{\partial t^2} = E_z \frac{\partial^2 u_z}{\partial z^2} \quad (2.1.22) \]

In order to calculate the normal modes, the solution form \( u_z = u_{z0} \exp(j\omega t) \) is assumed, thus obtaining:

\[ \frac{d^2 u_{z0}}{dz^2} + K_z^2 u_{z0} = 0 \quad (2.1.23) \]

Having defined:

- \( K_z = \frac{\omega}{V_z} \), Equivalent axial wavenumber
- \( V_z = \sqrt{\frac{E_z}{\rho}} \), Equivalent axial sound velocity

Equation 2.1.23 can be easily recognized as the equation of an harmonic oscillator, whose solution, as is well known, can be written in the form:

\[ u_{z0} = A_z \sin(K_z z) + B_z \cos(K_z z) \quad (2.1.24) \]

Where \( A_z \) and \( B_z \) are constants that can be determined considering the boundary conditions the rod is subjected to.

For the case of application in this study, as seen in Figure 1.2.2b, the target rods were supported by graphite clamps which were pressed against the rods by springs, with a relatively low stiffness and preload. Taking into account the low coefficient of friction between the rods and the supports, the boundary condition that is most representative of the support system of the rods, from the point of view of the longitudinal vibration, can be considered to be that of a rod which is free to move at its extremities.

This condition can be translated in the following couple of equations:

\[ \sigma_{zz}|_{z=0} = 0, \quad \sigma_{zz}|_{z=l} = 0 \quad (2.1.25) \]

By substituting Equations 2.1.21, 2.1.24 and 2.1.8 in to Equations 2.1.25, the axial frequency equation of the equivalent axial vibration of the rod is:

\[ \sin(K_z l) = 0 \quad (2.1.26) \]

Whose solutions are:

\[ K_z l = k\pi, \quad k \in \mathbb{Z} \quad (2.1.27) \]

The fundamental one being:

\[ K_z l = \pi \quad (2.1.28) \]
That is similar to the classical formula for the longitudinal vibration of a beam, except for the fact that \( K_z \) depends on the mechanical coupling coefficient.

Consequently, this means that the only way to obtain the axial frequency is to solve simultaneously Equations 2.1.28 and 2.1.20. The same applies to the radial frequency.

However, a closer inspection of equations 2.1.28 and 2.1.20 suggests that, if the geometric dimensions satisfy conditions like \( l \gg R \) or \( l \ll R \), the two calculated frequencies are far away from each other. The axial vibration is therefore weakly coupled with the radial vibration, and the two can be considered decoupled.

This is the case for the aforementioned TZM rod: with a length of 240 mm and a radius of 4 mm the rod can be considered slender enough to satisfy the \( l \gg R \) condition. Furthermore, as already introduced in Section 1.2.1, the current candidates for radial and longitudinal mode in the TZM rod had measured periods of, respectively, 1.82\( \mu \)s and 86\( \mu \)s, which is in agreement with the previous statement that considered the two frequencies to be very distant from each other for slender rods.

The decoupled vibration modes of the slender rod are presented in the following sections.

### 2.1.2 Decoupled modes of vibration of a slender rod

#### 2.1.2.1 Planar radial vibration of a slender rod

The following is obtained assuming a condition of plane strain: axial strain is zero. Implementing this condition into the definition of the mechanical coupling coefficient, we obtain:

\[
\alpha = \nu
\]

By substituting this expression into Equation 2.1.14, the expression for the equivalent radial elastic constant relative to this case is:

\[
E_r = E \frac{1 - \nu}{(1 + \nu)(1 - 2\nu)}
\]

(2.1.29)

The frequency equation for the planar radial vibration of the rod thus becomes:

\[
K_r R J_0(K_r R)(1 - \nu) - (1 - 2\nu) J_1(K_r R) = 0
\]

(2.1.30)

Which can be numerically solved for \( K_r R \) through a simple script (in Wolfram© Mathematica, for example), which computes the values of \( K_r \) for which \( f(K_r R) = 0 \).

Having obtained \( K_r \), by remembering its definition given in Section 2.1.1.1, the fundamental frequency for the planar radial vibration of the rod can be obtained as:

\[
f_r = \frac{K_r}{2\pi} \sqrt{\frac{E(1 - \nu)}{\rho(1 + \nu)(1 - 2\nu)}}
\]

(2.1.31)
2.1.2.2 One-dimensional longitudinal vibration of a slender rod

In this case radial and tangential stresses are neglected while the corresponding strains are present. From the definition of the mechanical coupling coefficient we obtain:

$$\alpha = \infty$$

This results in:

$$E_\ell = E, \quad V_\ell = \sqrt{\frac{E}{\rho}}$$ \hspace{1cm} (2.1.32)

Taking into account Equation 2.1.32, through some re-elaboration of Equation 2.1.28, the following formula for the fundamental frequency is obtained:

$$f_\alpha = \frac{1}{2\ell} \sqrt{\frac{E}{\rho}}$$ \hspace{1cm} (2.1.33)

2.2 Analytical approach to flexural Modes of vibration

An analytical approach to find the frequencies and the mode shapes of flexural modes of vibration for straight beams has been part of engineering technique for a long time and will not be analyzed in detail. An in-depth analysis of the subject and an extensive list of formulas for natural frequencies and modes shapes can be found, for example, in [4].

It is interesting, however, to review the assumptions made in order to obtain those formulas:

1. The beam section is constant along the axis.
2. The material the beam is made of is linear, homogeneous, isotropic and elastic.
3. The beam is slender (that is, the characteristic dimensions of the section are much smaller than both the length of the beam and the distance between vibration nodes).
4. The Euler-Bernoulli deformation hypothesis is considered valid: plane sections remain plane and only deformations normal to the axis are considered (the slenderer the beam is, the more corresponding to reality this assumption is).
5. No axial loads are applied to the beam.
6. The plane of vibration is also a plane of symmetry of the beam, so that rotation and translation of the beam are uncoupled.
It is worth noting that not all of these assumptions are completely fulfilled by the case of the TZM rod. Because of the many slots machined in the rod to interface with the instrumentation and the flat surfaces that are in contact with the supports, assumptions 1 and 6 are not exactly fulfilled (proof of this will be shown in Section 2.4). Assumption 5 is also not satisfied, since one of effects of the beam-induced heating could be easily reduced to an equivalent axial load (which is the one that is exciting the longitudinal mode of vibration).

The formulas obtained through these assumptions, even though they were not completely satisfied, were applied to the case of the TZM rod with the same reasoning as before. If the calculated frequency of the flexural mode of vibration were found to compatible with the frequency of the measured wave, this fact would be a first step towards the identification of the wave as a flexural mode of vibration.

These natural frequencies can generally be expressed in the following form:

\[ f_i = \frac{k_i^2}{2\pi l^2} \sqrt{\frac{EJ}{\rho A}}, \quad i = 1, 2, 3, \ldots \]  \hspace{1cm} (2.2.1)

Where:

- \( E \) and \( \rho \) are respectively the Young Modulus and the density of the material the rod is made of.

- \( A \) is the area of the cross section of the rod.

- \( J \) is the area moment of inertia of the cross section of the rod. Particular attention has to be paid to the definition of this geometrical dimension. When analyzing the vibration of the rod, one has to make sure that the coordinate reference system of the rod is chosen so that the \( z \) axis coincides with the axis of the rod and the \( x-y \) axes coincide with the principal axes of inertia of the section (i.e. the eigenvectors of the area moment of inertia tensor). In such a coordinate system the \( 2 \times 2 \) area moment of inertia tensor is diagonal, the two only nonzero values present in the matrix being \( J_x \) and \( J_y \), as defined in Equation 2.2.2. The appropriate geometrical dimension to substitute in the formula is \( J_x \) for flexural modes in the \( y-z \) plane and \( J_y \) for those in the \( x-z \) plane. As is well known, for rods with a circular cross section, \( J_x = J_y \) for every rotation of the coordinate system about the \( z \) axis (Mohr’s circle of the section is reduced to a point). However, as will be shown by the modal analysis carried out using Finite Element analysis in Section 2.4, because of the series of slots machined in the rods, this last assertion is not perfectly true, thus resulting into slightly different frequencies associated to the two perpendicular flexural modes of vibration.

\[ J_x = \int_A y^2 \, dx \, dy, \quad J_y = \int_A x^2 \, dx \, dy \]  \hspace{1cm} (2.2.2)
\* \( k_i \) is a dimensionless parameter, function of the boundary conditions applied to the rod. A different natural frequency and mode shape is associated to every single value of the index \( i \). The value for many types of boundary conditions on beams can be found in books such as [4].

### 2.3 Cross-check of the analytical predictions with the experimental results

After presenting the three formulas for the frequencies of the planar radial, longitudinal and flexural vibrations of a rod, they are applied to the aforementioned case of the TZM target impacted by low intensity proton beams in the HRMT27 experiment. As already introduced, this was the one that exhibited the highest degree of conformity with the assumptions made in the the previous analysis.

#### 2.3.1 The 1.82 \( \mu s \) period wave

The application of Equation 2.1.20 to the case of the TZM rod returns a period of 1.82 \( \mu s \) for the planar radial vibration of the rod. The excellent level of agreement between the measured wave and this value can be taken as a first confirmation of the proposed hypothesis.

The same formula was also applied to the cases of the Iridium, Tungsten and Tantalum rods. These rods, compared to the TZM one, were less ideal candidates for a cross-check due to the fact that they exhibited clear signs of plastic deformation to pulses of similarly low intensity. The results are shown in Table 2.1.

<table>
<thead>
<tr>
<th></th>
<th>Measured</th>
<th>Analytical</th>
</tr>
</thead>
<tbody>
<tr>
<td>Iridium</td>
<td>2.22 ( \mu s )</td>
<td>2.26 ( \mu s )</td>
</tr>
<tr>
<td>Tungsten</td>
<td>2.32 ( \mu s )</td>
<td>2.30 ( \mu s )</td>
</tr>
<tr>
<td>Tantalum</td>
<td>2.78 ( \mu s )</td>
<td>2.76 ( \mu s )</td>
</tr>
</tbody>
</table>

Table 2.1: Cross-check between the periods of the measured waves in the Ir, W and Ta rods and the periods provided by the formulas for the planar radial mode of vibration. The two columns, notwithstanding the plastification of the rods, show once again excellent agreement.

Notwithstanding the plastic deformation of the rods, the two columns show excellent agreement, further confirming the theory.

#### 2.3.2 The 86 \( \mu s \) period wave

The application Equation 2.1.33 to the case of the TZM rod gives a period of 89.8 \( \mu s \) for the longitudinal vibration of the rod. Once again, the agreement between the measured
wave and the value obtained considering a mode of vibration is very good.

The same cross-check that was performed for the Iridium, Tungsten and Tantalum rods for the planar radial vibration could not be performed for the longitudinal one, since this vibration was immediately damped due to plastic deformation.

2.3.3 The 1.6 ms period wave

The main problem regarding the application of Equation 2.2.1 is finding an appropriate value for the dimensionless parameter $k_i$. While it is true that many books, such as the already mentioned [4], have entire chapters filled with $k_i$ values relative to a wide variety of boundary conditions for straight beams, the peculiarity of the clamping system of the rods of the HRMT27 experiment, made it so that it was not possible to find a boundary condition that resembles the physical reality of the problem.

Nevertheless, this complex boundary condition could be simplified to a rod completely free of any boundaries if one neglects the force applied by the graphite clamps. This assumption may not be far from reality by considering that, at respectively 4.1 N/mm and $\sim 0.5 mm$, the stiffness and the precompression of the springs were relatively low.

By substituting the value for $k_i$ corresponding to this condition and the appropriate dimensions for the TZM rod in Equation 2.2.1, a period of 1.51 $\mu$s is obtained, predicting with an accuracy of 5% the period of the measured wave.

The application of the formula for the frequency of the flexural mode of vibration to the cases of the Iridium, Tungsten and Tantalum rods does not yield the same level of accuracy. This is likely due to the fact that a free boundary condition does not accurately describe the boundary conditions perceived by these shorter rods when mounted on the same clamping system.

It will however be shown in Section 3.6 that, through the adoption of FE methods, the flexural response of the rods under the beam impact will be reproduced accurately not only in the case of the TZM rods but also for the other rods, especially the Tantalum and Tungsten ones.

2.4 Modal analysis of the TZM rod via finite element simulations

In order to further test the hypothesis of the radial, longitudinal and flexural modes of vibration, the analytical approach was complemented by a Finite Element analysis performed using the capabilities of the software ANSYS® Workbench.

When compared to the analytical approach, this analysis has several advantages, such as the possibility to obtain natural frequencies and mode shapes even of complex geometries. In this sense it was possible to analyze the free vibration of a realistic geometry of the TZM rod, including the slots machined on its surface, without having to introduce
assumptions such as the decoupling of the radial and longitudinal modes of vibration that 
was made in the previous Section.

The downside to modal analysis through FE methods is the fact that an approach 
based solely on it is not rigorous enough to provide a physical explanation of the phenom-
ena taking place in the AD-Target core. This is because, taking as example the case of the 
radial mode of vibration, nearly 100 modes of vibration are present just in the the 540 to 
560 kHz frequency range. It is for this reason that the analytical approach was adopted 
first, in order to have a clear understanding of the physical reality of the problem. The 
finite element approach was adopted only at a second stage to have a confirmation of the 
results obtained through the formulas and to compute the mode shapes associated to the 

modes of vibration.

Since the three identified frequencies of the oscillations that made up the measured 
wave were separated by several orders of magnitude, at 550kHz, 11.6kHz and 610Hz, it 
was deemed appropriate and efficient to run three different modal analyses, restricting 
the results in each of them to the range of frequencies surrounding the ones that were 
sought.

After obtaining the results, these ranges of frequencies were inspected, looking for 

modes of vibration with frequencies compatible with those of the measured wave and 
with mode shapes that would produce displacement compatible with what was registered 
during the experiment.

The results of this process were very satisfactory:

- Figure 2.4.1 shows a mode of vibration with a 1.80 $\mu$s period (very close to the 
  1.82$\mu$s period of the wave that was identified as a radial mode of vibration through 
  the analytical approach). This mode exhibits a clear expansion and contraction of 
  the cross-section of the rod, especially in its central part, in the area where the LDV 
  that performed the measurement was pointing at.

- Figure 2.4.2 shows another mode of vibration, with a period of 89.2$\mu$s, identified as 
  a longitudinal mode of vibration through the analytical approach. An interesting 
  aspect that is clear form an analysis of the figure is that the presence of the ma-
  chined surfaces on the rod, which make its shape not perfectly cylindrical, causes 
  the extension of the axis of the rod to be coupled with a transverse deformation. 
  This secondary transverse deformation would not be present with a perfectly cylin-
  drical rod. It becomes apparent then that the main component of the 86 $\mu$s period 
  wave measured by the LDV was this secondary transverse vibration, and not only 
  the the contraction and expansion of the cross section due to the coupling between 
  the axial strain and the components of strain in the plane of the cross section due 
  to Poisson's effect.

- Figure 2.4.3 shows the last mode of vibration that was identified or, better, as pre-
  viously anticipated in Section 2.2, two modes of vibration in perpendicular planes, 
  with slightly different periods at respectively, from top to bottom, 1.54 and 1.52ms.
The reason for this slight difference in frequencies lies in the small difference in the values of $J_x$ and $J_y$ introduced in the rod by the surfaces and slots machined on its surface. By comparing the two mode shapes with the direction in which the LDV was measuring it is clear that the mode of vibration that was measured was the top one, with a period of $1.54\,ms$.

Figure 2.4.1: A mode with a period of $1.82\,\mu s$. From top to bottom: the undeformed rod and the two opposite extremes in the vibration. The interesting aspects are two: the mode of vibration shows a clear expansion and contraction of the cross section of the rod that is compatible with the measurement of the LDV. The total length of the beam is, however, unaffected by this process of alternate expansion and contraction of the cross section. This is the only mode that shows this type of response among the modes that have periods of $1.6$ to $2\,\mu s$.

Figure 2.4.2: A mode with a period of $89.2\,\mu s$. From top to bottom the undeformed rod and the two opposite extremes in the vibration. By comparing the undeformed rod with the deformed one, the lengthening and shortening of the longitudinal dimension of the rod is apparent. This process is associated with a transverse deformation introduced by the presence of the surfaces and slots that are machined on the rod.

2.5 Overview and discussion of the results

The results of the two approaches to the modal analysis of the TZM rod are shown, together the periods of the waves measured during the HRMT27 experiment, in Tables 2.2b and 2.2c.
Figure 2.4.3: Two flexural modes of vibration with a period, from top to bottom, of 1.54 and 1.51 ms. The reason for this small difference in period can be attributed to the slight difference in the values of $J_x$ and $J_y$ introduced by the surfaces and slots machined on the rod. By comparing the two directions of deformation with the direction in which the measurement was carried out by the LDV, it is clear that the mode that was being measured was the top one.

<table>
<thead>
<tr>
<th>Measured</th>
<th>Analytical Approach</th>
<th>FE Modal Analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.82µs</td>
<td>Planar radial 1.82µs</td>
<td>1.80µs</td>
</tr>
<tr>
<td>86µs</td>
<td>Longitudinal 89.8µs</td>
<td>89.2µs</td>
</tr>
<tr>
<td>1.6ms</td>
<td>Flexural 1.51ms</td>
<td>1.54ms</td>
</tr>
</tbody>
</table>

Table 2.2: a) Periods of the waves that make up the signal measured by the LDV after the impact on the TZM rod. b) Periods associated to the planar radial, longitudinal and flexural modes of vibration, obtained by analytical approach. c) Periods associated to modes of vibration having mode shapes that are compatible with the measurements, obtained via FE modal analysis.

The formulas obtained in Sections 2.1.2.1, 2.1.2.2 and the one reported in 2.2 for the frequencies of, respectively, the radial, longitudinal and flexural modes of vibration yield periods that are in very good agreement with the periods of the three waves that, superposed, make up the measured response.

In Section 2.4 a modal analysis was performed by means of a Finite Element Analysis. The results of this process are compatible with the results of the analytical approach in terms of the periods of the modes and with the measurement carried out during the experiment in terms of periods and shapes.

These results strongly agree with each other notwithstanding the fact that they were obtained through two independent approaches to the same problem. This aspect, coupled with the evidence that was already collected during the simulation campaign conducted
prior to the experiment [13], are interpreted as the definitive proof that the physical mechanism governing the dynamic response of the rods after being impacted by the proton beam is the vibration obtained as a superposition of a radial, a longitudinal and a flexural mode of vibration of the rod.

Other previously unexplained aspects of the dynamic response of the rods, such as the difference in the speeds of propagation of the radial and longitudinal waves, can also be explained by taking into account this interpretation. The reason for the difference in the sound velocity in the two directions is actually due to the different Equivalent Axial Sound Velocity and the Equivalent radial sound velocity, as defined in Sections 2.1.1.1 and 2.1.1.2.
Chapter 3

Numerical and experimental crosschecks of flexural deformations of the HRMT27 targets impacted by proton beams

In Chapter 2 important new insights on the response of the HRMT27 experimental targets when impacted by proton beam (and, consequently, on the AD-Target) were given by analyzing their modal behavior. The three waves observed in the measurement were identified, from lowest to highest period, as consequences of the excitation of a radial, a longitudinal and a flexural mode of vibration by the extremely fast heating induced inside the rod by the proton beam impact.

An aspect that was common to all the targets impacted during the experiment was a large level of residual flexural deformation accumulated in the same direction. The modal analysis conducted in Chapter 2 contributed to confirm that the origin of this deformation was the excitation of a flexural mode of vibration. However, the cause of this excitation was still not completely clear.

The presence of a flexural mode of vibration was not expected during the design phase of the experiment. Past experience with the CNGS target seemed to suggest that this vibration could be excited by the presence of an offset in the proton beam impact with respect to the axis of the target. This was demonstrated in [2]: through an analytical approach it was possible to reproduce the transverse vibration of the CNGS target by defining an equivalent thermal bending moment corresponding to the offset heating induced by the beam. This confirmed that an offset in the beam impact could cause this type of response.

However, the offset considered for the case of the CNGS target (0.6 times the radius of the rod) was much larger than the offsets measured by the beam position monitors during the HRMT27 experiment (around 0.1 times the radius).

The present chapter details the work carried out in order to gain further insights on the
cause of this type of vibration so it can be mitigated in future experiments. An approach based on Finite Element Methods, instead of an analytical one, was adopted in order to produce a model that is more adherent to the reality of the experiment and to test the assumptions made in the previous study. In these simulations, an energy deposition map calculated by using Monte Carlo simulations is imported into the program and applied as an internal energy load to the ANSYS® Transient Thermal module to compute the thermal response of the rod. The solution of this simulation is then imported into the ANSYS® Transient Structural module, which computes the dynamic response of the component at study. In order to have a model that could effectively mirror the actual behavior of the rod as accurately as possible, it was decided to implement the support system of the rods, comprehensive of the spring system that held them in place.

The case of the low intensity pulse on the TZM rod is, again, presented first. The same computational approach is then applied to the cases of low intensity pulses on the Iridium, Tungsten and Tantalum rods.

3.1 Energy deposition in the rods

The first step required in order to perform simulations on the thermal and structural response of beam impacted devices is to obtain a map of the energy deposition on the component that arises as a consequence of the interaction of the proton beam with matter. These energy deposition maps are obtained by using Monte Carlo simulations, a stochastic approach to solving problems used in a wide variety of fields (from physics, to problems of fluid dynamics, computational chemistry and finance).

The Monte Carlo code used to obtain the energy deposition maps on which the simulations presented in the following sections are based is FLUKA, a software that has been in development at CERN since the 1960s as a result of a CERN-INFN collaboration. The software has been used for calculations of particle transport and interactions with matter in a wide range of applications both in accelerator physics and in other fields (such as radiotherapy) and has been proven to simulate with high accuracy the interaction and propagation in matter of about 60 different particles, over a wide range of energies (from 100eV up to 200 TeV)[7].

The geometric domain of the simulation is divided in cells, or bins, of varying dimensions, according to the expected entity of the local gradient of energy. The energy deposition, necessary to conduct the thermal-structural simulations, is exported as a table that contains the energy deposited in each bin, normalized by the intensity of the pulse, and the position of the bin.

It should be stressed that the connection between FLUKA and the following thermal-structural simulations is only one-way. This effectively means that the reduction in the deposited energy during the pulse as a consequence of the the reduction of the density of the impacted rod is neglected by the simulations. Neglecting this effect is indeed considered to be a valid assumption until the impacted material does not experience a
change of state [12]. This is even more valid at the low intensities of the pulses considered in the present cases.

Since one of the main purposes of the HRMT27 experiment was to check the accuracy of the simulations and the material models used, it was crucial to have an energy deposition map that mirrored as accurately as possible the one generated by the actual beam impact.

In order to achieve this, the characteristics of the beam (that were measured by means of beam position monitors BPKG/BTV) during the experiment were used as input parameters for the FLUKA simulation. For reference, the characteristics of the beam that produced the response of Figure 1.2.4 in the TZM rod are reported in Table 3.1.

<table>
<thead>
<tr>
<th>Target</th>
<th>Intensity [p]</th>
<th>Size at $1\sigma$ (H) [mm]</th>
<th>Size at $1\sigma$ (V) [mm]</th>
<th>Position (H) [mm]</th>
<th>Position (V) [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>TZM2</td>
<td>$1.24 \cdot 10^{11}$</td>
<td>2.04</td>
<td>1.84</td>
<td>0.12</td>
<td>0.42</td>
</tr>
</tbody>
</table>

Table 3.1: Characteristics of the beam that produced the response of Figure 1.2.4 in the TZM Target. The horizontal and vertical size of the of the beam at $1\sigma$ were measured by a BTV while the horizontal and vertical coordinates were measured by a BPKG, considering a reference system whose origin coincides with the axis of the impacted rod. Both instruments were attached on the outer side of the vacuum tank.

### 3.2 Transient thermal analysis

The second step in the computational approach is to perform a transient thermal analysis to calculate the temperature rise in the rods during and after the proton beam impact. The use of hydrocode simulations, adopted for previous analyses of the HRMT27 rods [12, 13], was, however, not optimal for the case at hand for the following reasons:

- Hydrocodes, while well suited for problems with high strain rates that require very small timesteps, would require very long computation times in order to cover the $3\, ms$ of the measurement of the LDV.
- The real state of the V-shaped clamping systems would be really hard to replicate using the the boundary conditions and contact algorithms available in Autodyne®, the hydrocode solver used for the simulations of the HRMT27 rods.

The software that was deemed most appropriate for this type of simulations was ANSYS® Workbench, and in particular the Transient Thermal and Transient Mechanical modules.

A representation of the model adopted in the Thermal and the Structural Analysis for the TZM rod is presented in Figure 3.2.1. In this model, in addition to the TZM rod, the V-shaped graphite supports that clamp the rod against a graphite bar by means of springs are clearly visible. These elements were added to the model in order to recreate as realistically as possible the clamping system that was adopted during the experiment.
As it can be seen in the picture, the rod was sliced with a concentric cylinder in order to apply different mesh settings to the geometry: a finer mesh in the core, where, due to the beam impact, the temperature gradient is expected to be higher, and a coarser one on the outer part, where the temperature gradient is expected to be lower. The two bodies are grouped into a multibody part using the Form New Part tool. This enables the use of Shared Topology among the bodies, allowing for a continuous mesh across the common regions where bodies touch, instead of having to define Contact Regions in the Mechanical application [1].

Figure 3.2.1: Model used for the Transient Thermal and Transient Mechanical simulations of the TZM rod. The springs, visible in the figure, are not added as bodies, but through the Connections panel in Transient Structural model as Spring Elements. These elements allow the introduction of the real spring stiffness and compression adopted in the experiment.

The FLUKA-generated table that contains the energy deposition on the rod is interpolated in the mesh in ANSYS® and applied as an internal heat generation to the rod through an ANSYS® APDL script written by members of the EN-STI-TCD section at CERN. For a proper interpolation, element midside nodes in the mesh were dropped. For reasons that will become even clearer in the following sections, it is important in this process to correctly align the coordinate system used in the thermal simulation with the one used in the FLUKA model to compute the energy deposition.

The following parameters can also be set through the APDL script:

- Pulse parameters: Length, intensity and period between pulses (in case of consecutive pulses).
- Analysis settings: number of cycles to be repeated and number of timesteps in which the heating and cooling load steps are divided.

Two thermal simulations were performed:
Figure 3.2.2: Detail of the mesh in the TZM rod: by slicing the rod into two different bodies, it is possible to assign a fine mesh to the core of the rod, where higher gradients of temperature are expected, and a coarser one to the outer part. The two bodies are then joined together into a Multibody part, which enables the use of Shared Topology among the bodies.
• A thermal simulation considering only the adiabatic rise of temperature of the rod induced by the 0.9\(\mu s\) long pulse. This simulation is used afterwards as a load in the transient structural simulation.

• A thermal simulation considering the beam impact and the subsequent heat diffusion that occurs in the 3\(ms\) in which the LDV was measuring. In this second simulation a radiation to ambient boundary condition on the outer surface of the rod was added, considering a value of 0.1 for the emissivity and an ambient temperature of 22°C. No convection boundary conditions were implemented since the experiment was carried out in a vacuum tank. The purpose of this simulation was to verify if the heat diffusion inside the rod would have an influence on the structural response of the target.

3.2.1 Results of the thermal analysis

The results of the two simulations are shown in Figures 3.2.3 and 3.2.4. Below, some interesting features about the thermal results are described:

• The profile of temperature that the energy deposited by the beam generates in the rod could be quite counter-intuitive for a reader not familiar with the physics behind the interaction of high energy particles with matter. The maximum temperature, in fact, is not reached close to the upstream surface of the rod but in the bulk material, after more than half the length of the TZM rod. This is due to the well known mechanism with which high energy charged particles move through matter. When protons move through matter, in fact, they deposit energy along their path, resulting in a loss of speed. The energy lost by the particles is inversely proportional to the square of their velocity. This results in a peak of the deposited energy well downstream of the first point of penetration in the rod.

• The maximum temperature increase in the rod, taking place at the end of the 0.9\(\mu s\) length pulse is 43°C. This moderate increase of temperature, due to the low intensity of the pulse and the rather low (when compared to some of the other materials tested during the HRMT27 experiment) density of TZM, is the reason why this target, as confirmed by the simulations carried out in [12], did not experience the extensive levels of plastic deformation that other targets did at pulses of similar intensities.

• By looking at the evolution over time of the temperature profile in Figure 3.2.4, it can be seen that the maximum temperature in the rod decreases by just 2.6°C due to thermal diffusion during the 3\(ms\) after the pulse impact. This is due to the fact that the 3\(ms\) simulation time is more than a hundred times shorter than the characteristic thermal diffusion time in the rod, defined by Equation 3.2.1. By substituting the value for the thermal diffusivity \(\alpha\) and the radius \(R\) of the rod, the \(t_d\) corresponding to the TZM rod is, in fact, 0.36\(s\).
\[ t_d = \frac{R^2}{\alpha} \tag{3.2.1} \]

The small difference in temperature profile between the end of the 0.9 \( \mu s \) long pulse and after the 3 \( ms \) in which the LDV was measuring, coupled with the fact that importing a 3 \( ms \) long thermal simulation into the mechanical simulation makes the thermal-structural simulation is far more computationally expensive, justified neglecting this thermal diffusion in the structural simulation. An equivalent thermal load based on a first ramp consistent with the progressive beam heating during the 0.9 \( \mu s \) long pulse followed by a constant temperature distribution corresponding to the temperature distribution reached at the end of the pulse, was applied.

The difference in the structural response reached by introducing this assumption was later verified, finding it negligible.

![Temperature profile in a longitudinal section of the rod at the end of the beam impact.](image-a)

![Evolution over time of the maximum temperature reached in the rod.](image-b)

Figure 3.2.3: a) Temperature profile in a longitudinal section of the rod at the end of the beam impact. A maximum temperature of 65°C is reached in the core of the rod (reference temperature was set at 22°C). In the figure, the beam is coming from the left. b) Evolution over time of the maximum temperature reached in the rod. Since the pulse length is several orders of magnitude shorter than the characteristic thermal diffusion time in the rod, temperature at this stage evolves linearly, only as a function of the energy which is progressively deposited by the beam during the 0.9 \( \mu s \) duration of the pulse.
Figure 3.2.4: a) Temperature profile in a longitudinal section of the rod at the end of the 3 ms during which the LDV was measuring. b) Evolution over time of the maximum temperature reached in the rod. The first effects of thermal diffusion can be appreciated, contributing to reduce the peak temperature reached in the rod from 65.4°C to 62.8°C.

3.3 Transient structural simulation

The next and final step in the thermal-structural analysis is to perform a structural simulation using as an imported load the thermal response obtained by the thermal simulation shown in the previous Section.

The following paragraph includes a brief description of the settings used for the Transient Structural analysis:

- Mesh settings were kept unchanged from the Transient Thermal simulation, except the element midside nodes setting, which was previously set as dropped, resulting in linear elements. Element midside nodes were, instead, set as kept for the Transient Structural simulation, resulting in quadratic elements, which usually yield better results at less expense than linear elements.

- Contact type was set to Frictionless for the contact surfaces between the target and graphite clamps. This allows bodies to slide and separate relative to each other without any resistance [1]. This contact algorithm was deemed to be the most physically representative since the force applied by the spring to the graphite clamps and targets was very low. This aspect can be better understood by looking at Figure 3.3.1. Another contact type that allows bodies slide and separate is the Frictional setting. When friction is included, shear forces can develop between the two bodies. The influence of different values for the friction coefficient between the two bodies were investigated, and were found to have a very limited impact on the outcome of the simulation.
Figure 3.3.1: Setting the contact algorithm to Frictionless allows the two bodies separate and or to just be in contact in a small portion of the designated contact surface, thus allowing the rod to bend freely.

- The springs that press the supports against the rod were not introduced in the simulation as bodies, but through the capabilities of ANSYS® Mechanical, which allows the introduction of Spring Connections. The corresponding settings panel allows the introduction of the values of the longitudinal stiffness and preload distance, set consistently with the measured values of free and compressed length of the springs and their stiffness. Indeed, to set this preload in a realistic way was important, since it was observed that small differences in this value had a significant effect on the measured response of the rod.

- Constraints on the supports were assigned to match as accurately as possible the physical reality of the problem. By taking a look at the mounting system of the V-shaped supports, it can be seen that they can only move perpendicularly to the rod within slots machined in the aluminum sample holder. Other degrees of freedom, except a limited possibility to rotate, are prevented by the small clearances present between the support sides and the slot. In order to reproduce as accurately as possible these conditions, the V-shaped supports were constrained with a condition of zero normal displacement on their lateral surfaces and a constraint of zero displacement in the direction of the axis of the rod in the edges highlighted in Figure 3.3.3. A Fixed Support constraint was added to the underlying surface of the graphite bar on the other side of the rod.

- Time step settings are given in Table 3.2. The first step follows the settings used in the Transient Thermal simulation for the beam-induced heating during the pulse impact. The time step is then gradually increased to cover the 3 ms of the simulation.
time in a computationally efficient way. In the last step, that covers the time-frame from 50 \( \mu s \) to 3 \( ms \), the time step is considerably coarser than in the beginning. The 6 \( \mu s \) minimum time step adopted during this step does not allow the resolution of the 0.86 \( \mu s \) period radial mode of vibration. This was considered acceptable simulation this was considered to be acceptable, since the focus was the 1.6 \( ms \) period radial mode. Additional simulations carried out at a later stage that considered the values for the time steps of step 3 to be applied all the way to the end of the 3ms revealed how the results are not substantially affected by this change in the entity of the time steps.

<table>
<thead>
<tr>
<th>Step #</th>
<th>End time</th>
<th>Initial</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.9 ( \mu s )</td>
<td>8</td>
<td>8</td>
<td>8</td>
</tr>
<tr>
<td>2</td>
<td>10 ( \mu s )</td>
<td>0.5 ( \mu s )</td>
<td>0.3 ( \mu s )</td>
<td>0.8 ( \mu s )</td>
</tr>
<tr>
<td>3</td>
<td>50 ( \mu s )</td>
<td>2 ( \mu s )</td>
<td>1 ( \mu s )</td>
<td>8 ( \mu s )</td>
</tr>
<tr>
<td>4</td>
<td>3 ( ms )</td>
<td>10 ( \mu s )</td>
<td>6 ( \mu s )</td>
<td>15 ( \mu s )</td>
</tr>
</tbody>
</table>

Table 3.2: Time step settings adopted in the structural simulation. Time steps can be either be specified by the number of substeps in which the step is divided, like it was done in the first row to reproduce the settings used during the thermal simulation, or directly by the step value. Columns labeled Initial, Min and Max respectively specify the size of the first substep, the minimum and the maximum time step sizes adopted during that step.
Figure 3.3.3: a) Zero displacement in the normal direction (in red) to the highlighted lateral surfaces of the support. b) Zero displacement in the direction of the rod axis (in blue) for the highlighted edge c) Fixed support constraint on the surfaces in blue.

• It is possible to make the results of the thermal simulation available as an imported load in the structural simulation by linking the solution of the Transient Thermal simulation to the Transient Structural one in the Project panel of ANSYS® Workbench, as shown in Figure 3.3.4. It is important to mention that this type of connection, similarly to the connection between the FLUKA simulation and the Transient Structural one, is a one-way only connection. By adopting this type of connection the present problem is effectively considered to be a weakly coupled thermoelastic problem: temperature variations influence the strain field but not the inverse. This was shown to be a valid assumption for the axial and the flexural-related strains in [2]. In this stage, ANSYS Mechanical offers the possibility to control the application over time of the thermal load to the Transient Structural simulation.

• Since the aim of the simulation is to obtain a result that can be cross- checked with the measurement made by the LDV, the User Defined Result option was used to reproduce the measurement on the simulated rod. The nodal displacements in the direction in which the LDV was measuring were retrieved for the numerical-experimental crosschecks.
Figure 3.3.4: The solution of the Transient Thermal simulation is imported into the Transient Structural simulation by linking the thermal solution to the structural setup. This process makes the thermal results available as an Imported Load in the setup of the Structural simulation.

3.4 Discussion of the first results

The result of the first simulation, together with experimental displacement recorded by the LDV, is shown in Figure 3.4.1.

The curve does not resemble the experimental data in any way: no clear oscillation is visible and the maximum absolute value of the measured displacement is under 2 µm, as opposed to the 14 µm of the experimental data.

This numerical-experimental mismatch brought the idea that perhaps the offset considered in the simulations (as it was measured by beam position monitors) was not necessarily the same offset with which, in reality, the proton beam impacted the target.

As a matter of fact, by observing Figure 3.4.2 and by comparing the position of the center of the beam to the direction in which the LDV was measuring, it seems quite plausible that a beam that is mainly offset in the vertical direction is not capable of producing a measurable response in the predominantly horizontal direction of measurement.

The next logical step was to investigate the possibility that small errors in the alignment procedure could be the cause of this unexpected discrepancy between the result of the simulation and the experimental data. These errors could have arisen either in the alignment of the rod with the beam and the beam position monitors or directly in the measurement of the position of the beam.

By inspecting the rod alignment procedure that was adopted during the setup of the experiment, described in [14], it can be seen that the estimated precision of positioning of the targets, with respect to the nominal beam axis, was around 0.2 mm. Other important sources of errors could have also lied in the precision of alignment between the axis of the rod and the zero of the position of the beam position monitors, and in the precision of
Figure 3.4.1: Comparison between the displacement obtained by integrating the velocity measured by the LDV and the measurement recreated in the simulation.

Figure 3.4.2: Downstream view of the profile of temperature induced by the beam heating. The center of the beam, the hottest part, is clearly distinguishable as a yellow circle which is slightly offset, mainly in the vertical direction, with respect to the center of the rod. The direction in which the LDV was measuring during the experiment is also reported in the figure, represented by a nearly horizontal red arrow.
the measurement of these instruments, estimated to be around 0.2 \textit{mm} as well.

3.5 Introduction of an offset in the horizontal direction

A new campaign of simulations was then started in order to investigate the effect of these possible misalignment and measurement errors on the measured dynamic response of the rod. For this new process, the original energy deposition table was used while introducing offsets via the interpolation coordinate system. This was done since having an additional Monte Carlo simulation for each different possibility investigated was not computationally affordable. This simplification relies on the reasonable assumption that, due to the fact that the introduced offsets are very small, the geometry mismatch in the peripheral areas can be neglected.

The introduction of the offset in the thermal simulation can be achieved in two ways:

- By shifting the whole geometry of the model in the direction of the desired offset. This process requires the simulation to be set up from scratch and the geometry to be remeshed every time a new candidate value for the offset is investigated. This process, besides being extremely time ineffective, is also characterized by poor repeatability, due to the fact that the model is meshed again every time.

- By modifying the ANSYS® APDL energy interpolation script in such a way that the reference system that is used for the interpolation process is a user defined one. This approach is the one that was finally used, as it solved the mentioned disadvantages of shifting the whole model geometry.

The first type of misalignment that was considered was the possible presence of an offset in the positive horizontal direction, which is the one that could have more directly caused a vibration of the rod that would have been compatible with the measured data. The fact that, among the other possible types of misalignment (i.e., in the vertical direction or a rotation of the rod) this was the one that was analyzed in more does not necessarily mean that this offset was the only one present. This will be shown in the following section, where a small rotation of the coordinate system will be shown to yield similar results to those shown in the present section.

The curves obtained for some values of imposed offsets, compared to the experimental one and the one obtained in the original simulation are reported in Figure 3.5.1.

It can be seen that the curve obtained by introducing an offset of 0.6 mm (leading to a total horizontal offset between the beam and the axis of the rod of 0.72 mm) has a remarkable level of agreement with the experimental data. The curve representing the experimental data is almost exactly duplicated, especially in the first oscillation, both in terms of period and amplitude. This high level of agreement can be interpreted as a confirmation that a misalignment error of 0.6 mm in the positive horizontal direction could indeed have been the cause for the disagreement between the simulation that was based on the data coming from the beam position monitors and the experimental data.
Figure 3.5.1: Comparison between the experimental data and simulations considering no offset (thus assuming no misalignment errors to be present) and offsets of 0.35 and 0.6mm in the positive horizontal direction.
3.5.1 Effect of small rotations of the coordinate system

Although an offset in the positive horizontal direction is the phenomenon that could have more directly caused the flexural vibration measured during the experiment, it would be naive to assume that this was the only type of misalignment that could have been present during the experiment.

For this reason, the effect of other types of misalignment were taken into consideration. The effect of an offset in the vertical direction was not analyzed more in depth because the original simulation clearly showed that a beam that is offset 0.42 mm in the vertical direction would not produce the horizontal displacement that was measured. Another type of misalignment that could have produced a flexural vibration measurable by the LDV is a rotation of the axis of the rod with respect to the direction of the beam impact.

This misalignment is introduced in the simulations by defining a coordinate system that is rotated with the respect to the one of the rod and by modifying the ANSYS® APDL script so that this rotated coordinate system is used in the simulation. The effect of a really small rotation of the rod about the vertical axis, of just −0.5°, are already evident in Figure 3.5.2 in the downstream part of the rod, due to its slenderness.

![Figure 3.5.2: Effect of a negative rotation of 0.5° about the vertical axis on the temperature profile in a horizontal longitudinal section of the rod. Due to the slenderness of the rod, the effect is very noticeable, especially in its downstream part.](image)

Figure 3.5.3 shows the effect that rotations of 0.3 and 0.5° could have had on the measured response. When compared to the previous results, this oscillation appears slightly distorted and does not present the same level of agreement that was found between the experimental data and the curve that corresponded to the 0.6 mm introduced offset. The level of agreement present in Figure 3.5.3 between the experimental data and the curve corresponding to a rotation of -0.5° is still however good enough that it is possible to think that a combination of an offset in the positive horizontal direction and a negative rotation about the vertical axis could have concurred together to the excitation of the measured flexural mode of vibration.

This idea is also supported by considering that a really small rotation, of 0.1 to 0.3° (that could have been caused, for example, by a small error in the positioning of the whole tank in the tunnel), coupled with a 0.1 to 0.3 mm offset of the rod in the positive horizontal direction, could have easily gone unnoticed in the installation procedure, more so than the 0.6 mm misalignment that would have been required by considering only a pure translation of the rod.
Figure 3.5.3: Comparison between the experimental data and simulations considering no offset (thus assuming no misalignment errors to be present) and the effects of rotations of -0.3 and -0.5° about the vertical axis.

3.6 Other targets

After an extensive examination of the behavior of the TZM rod, the attention was shifted to the other rods that were present in the experiment. Once again, only the dynamic response to low intensity pulses of the rods was analyzed to avoid the massive amounts of plastic deformation (and fracture) experienced with higher intensity pulses.

As already mentioned before, even though the intensities of the respective pulses on the Iridium, Tantalum and Tungsten rods were comparable to the one of the pulse that generated the dynamic response of the rod examined in the TZM rod, these targets experienced more extensive degrees of plastic deformation, proven by the immediate dampening of the longitudinal mode of vibration in all of the rods.

3.6.1 Iridium rod

The characteristics of the beam that impacted the iridium rod are reported in Table 3.3. As it can be seen, the characteristics of this beam are very similar to those of the beam that impacted the TZM rod, both in terms of measured intensity and position. However, as it can be seen in Figure 3.6.1, due to Iridium’s higher density, temperatures reached are much higher than in the case of the TZM rod, leading to the plastic deformation of
the target, proven by the immediate dampening of the longitudinal mode of vibration.

<table>
<thead>
<tr>
<th>Target</th>
<th>Intensity [p]</th>
<th>Size at 1(\sigma) (H) [mm]</th>
<th>Size at 1(\sigma) (V) [mm]</th>
<th>Position (H) [mm]</th>
<th>Position (V) [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ir2</td>
<td>(1.15 \cdot 10^{11})</td>
<td>1.98</td>
<td>1.73</td>
<td>0.12</td>
<td>0.45</td>
</tr>
</tbody>
</table>

Table 3.3: Characteristics of the proton beam, as measured by the beam position monitors, that generated the dynamic response of the Iridium rod analyzed in this Section.

Figure 3.6.1: Profile of temperature generated in a longitudinal section of the Iridium rod at the end of the pulse described by Table 3.3.

The setup of the simulation of the Iridium, Tantalum and Tungsten rods, follows the setup of the thermal-structural simulation of the TZM rod, detailed in Sections 3.2 and 3.3. Material response is modeled as elastic since the plastic deformation of the rod is caused by the radial and longitudinal modes of vibration, phenomena whose accurate simulation would require timesteps far too small to simulate the 3\(ms\) of the response of the rod.

The result of the simulation are shown in Figure 3.6.2.
Figure 3.6.2: Comparison between the displacement measured by the LDV in the Iridium rod during the experiment and the results of simulations considering no additional imposed offset (thus assuming the proton beam to impact the rod in the position indicated by the beam position monitors) and imposed offsets of 0.1 and 0.12 mm in the positive horizontal direction.

Figure 3.6.3: Comparison between the displacement of the Iridium rod measured by the LDV during the experiment and the results of simulations considering no misalignment errors to be present and another one in which a negative rotation of 0.1 degrees of the rod about the vertical axis is introduced.

As it can be seen, once again, a simulation considering the proton beam impacting in
the position measured by the beam position monitors produces a vibration with a much smaller amplitude than the one recorded during the experiment. By introducing an offset in the positive horizontal direction of \(0.12 \text{ mm}\), a vibration with the same peak can be obtained. However, this vibration accurately reproduces the experimental data only in the first part of the oscillation. Numerous trials with different boundary conditions for the rod have been tried, resulting in the same type of output of the simulation. Nevertheless, the accuracy with which the simulation reproduces the first half of the oscillation is good enough to draw conclusions that are analogous to those drawn through the analysis of the TZM rod. Moreover, by observing the results shown in Figure 3.6.3, it is once again demonstrated that the effects of a small negative rotation about the vertical axis can yield effects that are very similar to those caused by the introduction of a displacement in the horizontal direction.

It is worth noting that for the Iridium rod both the displacement and the rotation introduced to reproduce the displacement measured by the LDV are smaller than those introduced for TZM rod. This eliminates the idea that a pure displacement or a pure rotation common to all the rods in the rods in the experiment (due, for example, to an error in the positioning of the whole tank on the beam line) took place. However, it could be the case that a combination of the two types of misalignment, that was common to all the rods, could have concurred together to the excitation of the flexural modes of vibration.

### 3.6.2 Tantalum rod

Most of the considerations made for the Iridium rod apply to the 160 mm long Tantalum rod as well. As it can be seen in Figure 3.6.4, similarly to the case of the Iridium rod, the high density of Tantalum causes higher temperatures than those reached in the TZM rod, even though the rods are subject to proton beams of similar intensities. This fact, coupled with Tantalum’s low yield strength, lead to plastic deformation of the target, proven by the observed dampening of the longitudinal mode of vibration. Nevertheless, the material response in the simulation is modeled as elastic.

<table>
<thead>
<tr>
<th>Target</th>
<th>Intensity [p]</th>
<th>Size at (1\sigma(H)) [mm]</th>
<th>Size at (1\sigma(V)) [mm]</th>
<th>Position (H) [mm]</th>
<th>Position (V) [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ir2</td>
<td>(1.10 \cdot 10^{11})</td>
<td>2.01</td>
<td>1.76</td>
<td>0.12</td>
<td>0.45</td>
</tr>
</tbody>
</table>

Table 3.4: Characteristics of the proton beam, as measured by the beam position monitors, that generated the dynamic response of the Tantalum rod analyzed in this Section.
Figure 3.6.4: Profile of temperature generated in a longitudinal section of the Tantalum rod at the end of the impact of the beam with characteristics described by Table 3.4

Figure 3.6.5: Comparison between the displacement measured by the LDV in the Tantalum rod during the experiment and the results of simulations considering no additional imposed offset and an imposed offset of 0.32 mm in the positive horizontal direction.
Figure 3.6.6: Comparison between the displacement of the Tantalum rod measured by the LDV during the experiment and the results of simulations considering no misalignment errors to be present and other two in which negative rotations of 0.1 and 0.23 degrees of the rod about the vertical axis are introduced.

The result of this process can be seen in Figure 3.6.5. In this case, an offset of 0.32 mm is necessary to reproduce a displacement that is compatible with the one measured during the experiment. The agreement reached between the experimental data and the simulation considering an imposed offset of 0.32 mm in the positive horizontal direction is much better than the one achieved with the Iridium rod.

Figure 3.6.6 shows how similar effects can be achieved by introducing a rotation of 0.23 degrees of the reference system in which the energy deposition is applied.

### 3.6.3 Tungsten rod

Once again, the considerations made for the Iridium and Tantalum rods can be applied to the case of the 140 mm long Tungsten rod which, as it can be seen in Figure 3.6.7, is heated up to 140°C by the beam described in Table 3.5.

<table>
<thead>
<tr>
<th>Target</th>
<th>Intensity $[p]$</th>
<th>Size at $1\sigma(H)$ [mm]</th>
<th>Size at $1\sigma(V)$ [mm]</th>
<th>Position (H) [mm]</th>
<th>Position (V) [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ir2</td>
<td>$1.09 \cdot 10^{11}$</td>
<td>2.07</td>
<td>1.8</td>
<td>$-0.08$</td>
<td>0.47</td>
</tr>
</tbody>
</table>

Table 3.5: Characteristics of the proton beam, as measured by the beam position monitors, that generated the dynamic response of the Tungsten rod analyzed in this Section.

As it can be seen in Figure 3.6.8, an offset of 0.39 mm in the positive horizontal direction is necessary in order to produce a vibration with the same peak and with good agreement with the one measured during the experiment. Once again, as seen in Figure
Figure 3.6.7: Profile of temperature generated in a longitudinal section of the Tungsten rod at the end of the impact of the beam with characteristics described by Table 3.5.

3.6.9, the effect of a displacement in the horizontal direction can be reproduced by considering a rotation of the rod with respect to the axis of the beam which, in this case, corresponds to $-0.25^\circ$.

It is interesting to notice that the values of rotations and displacements considered for the case of the Tungsten rod are very similar to the ones considered for the Tantalum rod.

Figure 3.6.8: Comparison between the displacement of the Tungsten rod measured during the experiment and the results of simulations considering, respectively, a beam impacting in the position indicated by the beam position monitors and another case in which offsets of 0.1 and 0.39mm in the positive horizontal direction are introduced.
Figure 3.6.9: Comparison between the displacement of the Tungsten rod measured during the experiment and the results of simulations considering a beam impacting in the position indicated by the beam position monitors and two cases in which rotations of -0.1 and -0.25 degrees of the rod about the vertical axis are considered.
Part II

Hydrocode simulations for a new design of the AD-Target core
Chapter 4

Conceptual design of the new AD-Target

The second part of this thesis presents the work carried out in the context of the conceptual redesign of a new optimized AD-Target, taking into account the knowledge gained by analyzing the data gathered during the HRMT27 experiment in Part I. The main purpose of this activity was to gain insights on the effect that the geometric parameters involved in the design of the target core have on its dynamic response, with the main goal of optimizing antiproton production while maintaining the component safe during operation.

Figure 4.0.1 shows a section view of one of the potential prototypes for the new AD-Target. Proceeding from the outside towards the core of the target, the following elements are visible:

![Section view of the current structure of the prototypes of the AD-Target](image)

- Pressurized air cooling system consistent of two independent casings made of Ti-6Al-V. This element is of key importance to remove the heat generated by the interaction between the proton beam and the target core. The previous design of the AD-Target, currently in operation, adopts a water cooling system. Adopting a pressurized air cooling system, however, removes the problem of water activation...
and its treatment while reducing the concerns of leaks in the circuit. As shown by the CFD and thermal-mechanical analyses carried out in [12], the air cooling system still maintains good capabilities of cooling down the target core between impacts.

- **Expanded graphite matrix.** This material has been shown in recent years to be an ideal candidate for many components in beam intercepting devices due to its low density (around 1 g/cm³, depending on the compression of the grade), which minimizes the interaction of the material with the beam, and its high thermal conductivity (250 W/mK in the in-plane directions), which is desirable to remove the heat from the rods that make up the target core. The matrix is composed by several hollowed disks of expanded graphite, each having a thickness of 3 mm, that are compressed during the assembly of the target inside the inner titanium casing of the cooling system. This is done in order to ensure better thermal contact between the matrix and both the rods and the cooling system. As it will be detailed in the following sections, a significant problem regarding the use of this material is the presence of great uncertainties in the material response due to the absence in literature of detailed information about the mechanical properties of expanded graphite.

- **Tantalum rods.** The decision to adopt tantalum as the candidate for the new core material is taken based on the results of the HRMT27 experiment. As shown in [12], due to the high ductility of the material, the Tantalum rod was the only rod among the ones tested during the HRMT27 experiment that showed no internal fracture as a consequence of the response of the rod to the sudden beam-induced heating. It is worth mentioning that the Iridium rod, the material the current AD-target core is made of, exhibited massive fragmentation under a proton beam that was supposed to reproduce the same conditions reached in the target during operation, leading to the belief that the current target is fragmented from the very first pulse. Furthermore, unlike the current design, the target core is sliced in small rods of varying diameter. This allows a different response based on the entity of longitudinal energy deposition and it mitigates the effects of the deformation introduced by the flexural modes of vibration that are excited when the beam impacts off-axis, as observed in the analysis of the HRMT27 rods.

The analysis presented in the following sections is carried out following the computational method adopted by C. Torregrosa in [12] to analyze, by means of hydrocode simulations, the dynamic response of the previous versions of the target core and of the rods of the HRMT27 experiment.

All the simulations and findings of this part will be used in the next steps of the prototyping activities for the new AD-Target design. These activities involve the manufacturing of different real scale prototypes and their testing under proton beam impacts using the HiRadMat facility in the so called PROTAD experiment, foreseen to take place in 2018.
Chapter 5

Computational methods

5.1 Limitations of ANSYS® Mechanical

The simulations that were shown in Part I, which focused on the flexural modes of vibrations of the HRMT27 rods, were carried out using ANSYS® Mechanical. This software, which uses an implicit time integration method, while perfectly suitable for simulating that phenomenon, has been proven to be less convenient for simulating phenomena that are characterized by very high strain rates and stress states, such as the ones achieved during the operation of the AD-Target.

The followings subsections summarize the reasons why ANSYS® Mechanical was deemed inadequate for the analysis of this study, while presenting a more convenient computational tool.

5.1.1 Implicit vs explicit

The equation solved by classical solvers in transient analyses can be schematized as the following equation of dynamic equilibrium [3]:

\[ [M] \ddot{U} + [C] \dot{U} + [K] U = F(t) \]  \hspace{1cm} (5.1.1)

Where:

- \([M]\) is the mass matrix of the structure, that can be defined in two formulations: “consistent”, which results in a symmetric and generally full matrix, and “lumped”, which results in a diagonal matrix. This is done by concentrating the mass of the elements in the nodes in a physically acceptable way

- \([C]\) is the dampening matrix

- \([K]\) is the stiffness matrix

- \(\ddot{U}, \dot{U}\) and \(U\) are the vectors of nodal acceleration, velocity and displacement
• F(t) is the vector of the external forces

This differential equation is then solved for \( U \) via a variety of integration methods that can belong to one of the two following types of algorithms:

• Implicit algorithms: The solution at time step \( n + 1 \) is obtained by knowing the solution at step \( n \) and the conditions imposed at step \( n + 1 \). An example of these algorithms is Newmark's Method. For this method, if certain stability conditions are satisfied, the algorithm is \textit{unconditionally stable}: that is, the method is stable regardless of the size adopted for the time step. The choice of the appropriate size for the time step has to be carried out to achieve the convergence of the simulation. The downside to this type of algorithms is that the matrices need to be inverted at every step, resulting in a long computation time required at each step.

• Explicit algorithms: the solution at time step \( n + 1 \) is obtained by knowing the solution \textit{and} the conditions imposed at step \( n \). An example of these algorithms is the Forward Euler's method, which, like every other explicit algorithm, is \textit{conditionally stable}: the stability of the method depends on the size adopted for the time step. If the mass and dampening matrices are diagonal, the calculation at each step is immediate. The downside to this type of algorithms is that the time steps need to be very small to achieve the stability of the method.

Based on these characteristics, the choice for the application of an explicit or implicit method has historically been made according to criteria such as the one presented in Figure 5.1.1. Since, as shown by the simulations done in [12], the strain rates reached in the AD-Target during operation are well above \( 10^4 \text{s}^{-1} \) (the thermal load is applied in just \( 0.4 \mu \text{s} \)), it is clear that the most suitable method to simulate the dynamic response of the component under proton beam impact would be an explicit one.

5.1.2 Strain rate dependence of material response

Another very important factor to consider is the difference in material response that occurs when bodies are subjected to quasi-static or very rapid loads. It is well known, in fact, that strain rate influences the response of the material: metals, for example, tend to exhibit brittle responses when the strain rate is increased.

Since the strain rates reached in the AD-Target are above \( 10^4 \text{s}^{-1} \), the introduction of a strain rate-dependent strength model is a key requirement to perform physically accurate calculations of the target core during operation.

ANSYS® Mechanical, however, an implicit code especially developed to solve static and quasi-static problems only offers the possibility to implement bi-linear or multi-linear kinematic or isotropic hardening models, which are only temperature dependent.

The Johnson-Cook strength model, a temperature and strain rate dependent strength model, was adopted and yielded good agreement for the simulations carried out for the HRMT27 in [12], and will be presented in 5.2.2.
Figure 5.1.1: Typically achieved strain rates in a variety of classes of engineering problems. At low strain rates, the application of implicit codes is suggested because, due to their unconditional stability, the use of larger time steps does not make the simulation unstable. Explicit codes are instead suggested for problems characterized by high strain rates, since the computation time required for each step is very small. [8]

5.2 Introduction to hydrocodes

The previous section outlined some of the reasons why the use of implicit FE codes is not convenient for the simulation of processes characterized by extremely high stresses and strain rates, like those reached in the target core when it is impacted by the proton beam. The work carried out in [12] for the HRMT27 experiment showed how the application of hydrocodes to simulate the dynamic response of the rods in analogous conditions to those of the target core yielded good agreement with the experimental data. This is why the choice for the appropriate solver fell on these tools.

Hydrocodes were born in the late 1950's at Los Alamos National Laboratory to study the impact of steel projectiles on steel targets at velocities of up to 72 km/s. These codes have been developed to study very fast and intense loading on materials and structures [15]. Since the first codes were applied to problems, like impacts, in which the pressures exceeded the strength of the materials by several orders of magnitude, the calculations were first performed assuming hydrodynamic behavior in the material, thus not considering the material strength.

Since then, the capabilities offered by these codes have increased and the calculations are not necessarily performed assuming hydrodynamic behavior anymore, but the name "hydrocodes" has remained the same.

Differently from conventional structural solvers, which, as shown in the previous sec-
tion, solve Newton’s second law (Equation 5.1.1), hydrocodes numerically solve the full system of partial differential equations arising from mass, momentum and energy conservation, as in Equations 5.2.1 - 5.2.5:

\[
\frac{\rho_0 V_0}{V} = \frac{m}{V} \tag{5.2.1}
\]

\[
\rho \ddot{x} = f_x + \frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} + \frac{\partial \sigma_{xz}}{\partial z} \tag{5.2.2}
\]

\[
\rho \ddot{y} = f_y + \frac{\partial \sigma_{yx}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \sigma_{yz}}{\partial z} \tag{5.2.3}
\]

\[
\rho \ddot{z} = f_z + \frac{\partial \sigma_{zx}}{\partial x} + \frac{\partial \sigma_{zy}}{\partial y} + \frac{\partial \sigma_{zz}}{\partial z} \tag{5.2.4}
\]

\[
\dot{\epsilon} = \frac{1}{\rho}(\sigma_{xx}\dot{\epsilon}_{xx} + \sigma_{yy}\dot{\epsilon}_{yy} + \sigma_{zz}\dot{\epsilon}_{zz} + 2\sigma_{xy}\dot{\epsilon}_{xy} + 2\sigma_{xz}\dot{\epsilon}_{xz} + 2\sigma_{yz}\dot{\epsilon}_{yz}) \tag{5.2.5}
\]

It should be noted that no information on material properties is included in these equations: in order to solve them, they have to be coupled with constitutive laws of the material, which will be introduced in the next subsections.

Due to their historical application, hydrocodes have been developed to accurately simulate material response at extreme states, including the response beyond plasticity and fracture. This can be done by implementing a wide variety of material models that can take into account both strain rate and temperature dependence, as will be shown in the following subsections.

Apart from the study of impacts, hydrocodes have been used at CERN ever since 1984 to assess the response of solids and liquid materials to the impact of proton pulses. These, similarly to impact tests, are cases in which extreme stresses and strain rates are reached due to the sudden thermal expansion produced in the material by the particle beam impacts. This aspect becomes even more important when considering that current programs for development of accelerator technologies envision higher and higher particle beam energies and intensities.

Modern hydrocodes decouple the material response for metallic materials into a volumetric and a deviatoric part [15]; the volumetric response is obtained through the use of an Equation of State, as described in Section 5.2.1. The shear response is described through the use of an incremental-elastic plastic formulation, as described in Section 5.2.2.

### 5.2.1 Volumetric response

In continuum mechanics the stress state in a material is represented by a symmetric second order tensor which is commonly decomposed in two components, as described in tensor notation in Equation 5.2.6:
\[ \sigma_{ij} = s_{ij} - \delta_{ij} P \quad (5.2.6) \]

Where

\[ P = -\frac{1}{3} \sigma_{kk} \quad (5.2.7) \]

Note the minus sign due to a different sign convention between stress and pressure (which, differently from stress, is positive in compression).

The physical meaning of this decomposition is that the diagonal tensor represents the hydrostatic part of the stress state while the \( \tilde{\sigma} \) tensor represent the deviatoric term.

This decomposition is particularly important to understand how hydrocodes work. As mentioned in the previous section, material laws have to be added to fully solve the system of conservation equations 5.2.1 - 5.2.5. One of the material laws implemented in hydrocodes is the Equation of State (EOS), which is a function that governs the relation between thermodynamic state variables such as pressure (and therefore the hydrostatic component of the stress state), energy and density, as presented in Equation 5.2.8.

\[ P = f(\rho, E) \quad (5.2.8) \]

More generally, it can be said that the EOS is a mathematical description, with some physical basis, of the volumetric behavior of the material for a given set of initial conditions [15].

The simplest EOS that can be considered is just the bulk modulus, which can be thought of as an energy independent EOS that expresses the relation between the volumetric deformation and pressure. From this point, many EOS formulations can be developed and adopted depending on the application.

The EOS that was adopted for all the materials presented in the following simulations was the Mie-Grüneisen EOS. The Mie-Grüneisen EOS belongs to the family of Polynomial EOS, which are described by Equation 5.2.9:

\[ P = C_0 + C_1 \mu + C_2 \mu^2 + C_3 \mu^3 + (C_4 + C_5 \mu + C_6 \mu^2) \quad (5.2.9) \]

Where \( \mu \) is the volumetric parameter defined as:

\[ \mu = \frac{\rho}{\rho_0} - 1 \]

The Mie-Grüneisen EOS is defined as:

\[ P = P_H + \gamma \rho (E - E_H) \quad (5.2.10) \]

Where:

- \( P_H \) and \( E_H \) are pressure and specific energy in a reference state in the Hugoniot curve at the same volume
• \( \gamma \) is the Grüneisen parameter, which in this case is considered independent from pressure and internal energy.

This way, the pressure and internal energy at a point off the Hugoniot are related to the pressure and internal energy in the Hugoniot by the equation at the same volume [11]. It is worth mentioning that the Mie-Grüneisen EOS is not necessarily the best and most accurate EOS formulation for solids [15]. However, it is the one which has had historically most attention and, when properly used, it produces results that are good enough for many practical purposes. Theoretically derived EOS are available only for pure metals, while several of interpolation methods are available to get the needed data for alloys and powders [11]. All the parameters necessary for Equation 5.2.9 can be obtained experimentally by impact tests.

### 5.2.2 Deviatoric response

In order to take into account for the shear response of metals (which was not considered in the very first versions of these codes) an incremental elastic-plastic formulation is used. The velocity strain tensor is decoupled into an elastic and plastic part [9]:

\[
\dot{\varepsilon} = \dot{\varepsilon}^{el} + \dot{\varepsilon}^{pl}
\]  

(5.2.11)

For the plastic part, an incompressibility condition is applied:

\[
\varepsilon_{11}^{pl} + \varepsilon_{22}^{pl} + \varepsilon_{33}^{pl} = 0
\]  

(5.2.12)

In order to predict the onset of yielding, the Von Mises yield criterion is typically used. As outlined in Section 5.1.2, the adoption of a temperature and strain rate dependent constitutive model is of paramount importance in order to perform physically accurate simulations for the AD-Target core, which is subject to a temperature rise of more than 2000°C in just 0.4\( \mu \)s.

In this sense, the simulations used in the following sections adopted the Johnson-Cook (J-C) strength model [9], one of the simplest and most widely used temperature and strain rate dependent strength models. Equation 5.2.13 shows its general form:

\[
\sigma_{eff} = (A + B\varepsilon_{pl}^n) \left(1 + C \ln \frac{\dot{\varepsilon}}{\dot{\varepsilon}_0}\right) \left(1 - \left(\frac{T - T_r}{T_m - T_r}\right)^m\right)
\]

(5.2.13)

Where:

• \( \sigma_{eff} \) is the Von Mises flow stress
• \( \varepsilon_{pl} \) is the equivalent plastic strain
• \( \dot{\varepsilon}_{pl} \) is the plastic strain rate
• \( \dot{\varepsilon}_0 \) is the reference strain rate of 1.0 \( s^{-1} \)
\begin{itemize}
  \item $T_m$ is the melting temperature
  \item $T_r$ is the reference temperature
  \item $A, B, C, n$ and $m$ are obtained from data fitting of experimental $\sigma - \epsilon_p$ curves. $A$
    is the Yield Strength, $B$ and $n$ are work hardening parameters.
\end{itemize}

It is interesting to note that this model can be seen as a modified version of the Hollomon
relation ($\sigma = K\epsilon^n$), included in the first bracket. The second and the third bracket, which
respectively take into account the effect of strain rate and temperature, only imply scaling
and not a shape modification of the curve. The empirical nature of the model allows in
principle its use only in the regimes in which the material data used to fit the parameters
comes from.

This model, far from being the most accurate and comprehensive available, is, however,
the most widely adopted one in commercial codes and has been able to provide results
that were in good agreement with experimental data [14].

\section{Computational method}

In Section 5.1 it was shown how conventional thermal-mechanical simulations using
implicit solvers are not able to produce results in a computationally efficient way due to the
limitations of implicit solvers. Once computed, these results also lack physical accuracy
since they do not take into consideration the effect that temperature and strain rate have
on the material response as well as the response beyond plastic deformation. In Section
5.2 it was shown how the introduction of advanced computational tools like hydrocodes
are more convenient to simulate the extreme dynamic response of the AD-Target core
under proton beam impact. The present section introduces in detail the methodology
used for the simulations of the AD-Target Core.

\subsection{Computational approach}

In the same way as for the simulations of the flexural analysis of the HRMT27 rods,
the first step of the computational approach requires the resolution of an energy deposi-
tion map in the target core material as a consequence of the proton beam-target atoms
interaction. This is once again done by means of FLUKA Monte Carlo simulations.

The previous approach required the energy deposition computed by FLUKA to be
imported first into a Transient Thermal module that computed the thermal response. The
result of this simulation was then linked to a Transient Mechanical module to be used as an
imported load. The second part of this process is not necessary for the simulations carried
out in AUTODYN®, which, by solving the full system of the equations of conservation
of mass, momentum and energy together, can compute the coupled thermal and dynamic
response of the target core.
The energy deposition is applied as an internal energy load to the commercial hydrocode ANSYS AUTODYN® through an internal user subroutine. The energy is applied consistently with the parameters of the beam impacting on the target, as seen in Table 5.1.

<table>
<thead>
<tr>
<th>Pulse intensity [ppp]</th>
<th># of bunches</th>
<th>Bunch spacing [ns]</th>
<th>Bunch length [ns]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.5 \cdot 10^{13}</td>
<td>4</td>
<td>105</td>
<td>30</td>
</tr>
</tbody>
</table>

Table 5.1: PS beam parameters considered for the pulse impacting the target core [12]

As previously mentioned, this energy deposition causes a temperature rise in the target core of up to nearly 2000°C, as can be seen in Figure 5.3.1, which shows the result of a Transient Thermal analysis carried out in ANSYS® Workbench to quickly check the quality of the computed energy deposition map.

As it can be seen, the majority of the energy is deposited in the small tantalum blocks and very little in the expanded graphite matrix. This is due to the big difference in density between the two materials: 16.7 g/cm³ for tantalum and 1.2 g/cm³ for graphite.

It is worth mentioning how, even for the simulations carried out in this part, using ANSYS AUTODYN®, the connection between FLUKA and AUTODYN® is only a one-way coupling. This is considered a valid assumption since the maximum decrease of density during the 430 ns pulse is in the order of 2% of the initial density since the material is not melting.

Just to get a sense of how extreme the process of heating induced by the beam is, each proton bunch deposits an energy of approximately 335 J (for a total of 1.34 kJ over the four bunches), which results in 11.2 GW when considering that it is applied in only 30 ns. Furthermore, the power is applied in the small volume of the target core, producing a staggering $2.87 \cdot 10^{4} \text{TW/m}^3$ [12].

Figure 5.3.1: 1/2 view of the temperature profile in the target core. The result was computed by means of a thermal analysis carried out using ANSYS® Transient Thermal. The simulation considered an equivalent pulse having the same $1.5 \cdot 10^{13}$ intensity of the PS beam that impacts the AD-Target during operation, but concentrated into a single bunch, having the same total length. Beam coming from the left.
5.3.2 Geometry and Computational Domain

The simulations of the AD-Target core were performed using two distinct geometries, referred to, respectively, as V1 and V2, as seen in Figure 5.3.2. In each version, eight tantalum cylinders of diameters ranging from 3 to 8 mm are embedded in the black expanded graphite matrix.

The subroutine of interpolation FLUKA-AUTODYN® requires to have only hexagonal elements in the mesh of the latter. Such type of mesh can be obtained by slicing the geometry in different bodies as shown in Figure 5.3.3. A continuous mesh across the different parts and materials has been used, which avoids the inclusion of contact algorithms. This was done due to the limited capabilities of contact implementation in AUTODYN®. In addition, as it will be shown in Section 6.2, the target core is barely affected by the presence of the expanded graphite matrix. This is mainly due to the big difference in density (more than one order of magnitude) between the two materials.

![Diagram of Version 1](image1)

(a) Version 1

![Diagram of Version 2](image2)

(b) Version 2

Figure 5.3.2: Geometries considered for the two simulations of the AD-Target core. Rods of same length but different diameters are employed in corresponding positions. As it can be seen only 1/4 of the model is considered in order to reduce the computation time.

Since the beam is considered to be aligned with the target’s axis, the computation time for the simulations can be greatly reduced by considering only 1/4 of the original models and by introducing symmetry boundary conditions.

From the point of view of physical constraints, the model considers the outer surface of the expanded graphite matrix unconstrained in every direction. This assumption is
made on the basis of two reasons:

- The lack of physical constraints on the external graphite matrix does not influence the dynamic response of the center core (focus of the study) during the time window of interest. The phenomena that occur in the tantalum rods, such as the radial and longitudinal modes of vibration, in fact, are so fast that they occur at a timescale that is not long enough for the pressure waves to travel and come back to the tantalum rods from the outer surface of the expanded graphite matrix.

- Since the proton beam-induced energy deposition in the target core, applied as an internal energy load by means of a subroutine, is the only load that is applied, there is no need for physical constraints to prevent the rigid body motion of the model. Moreover, differently from implicit solvers, a model does not need to be constrained to converge when using implicit ones, since the equilibrium of the forces is not required by definition.

The preprocessing tool of AUTODYN® allows the insertion of gauges in specific points of the geometry in order to check the time history of the selected variables. The positions of the gauges were selected consistently between the two considered geometries so meaningful comparisons of the results of the simulations could be done. Figure 5.3.4 shows the modeled geometry with its corresponding gauges.

As it was explained in section 5.2, material models have to be added in order to solve the full system of mass, momentum and energy conservation equations considered by the hydrocode. These are implemented in the form of Equations of State (EOS) and Strength Models.

EOS are introduced for both the tantalum and the expanded graphite in the Mindlin-Grüneisen formulation, which, as said, is the one that has historically seen more use in hydrocode simulations.

From the point of view of the strength model two scenarios were analyzed:

- The first scenario considers a fully elastic response of both the tantalum and the graphite. This assumption is clearly physically inaccurate since simulations carried out on previous versions of the AD-Target core have shown the generation of compressive to tensile pressure waves of up to 5 GPa in amplitude. The assumption of elastic behavior is, however, really useful to understand the phenomena underlying the generation of the extreme stresses right after the pulse, since no damping takes place in the material due to the absence of plastic deformation. In this case the used strength models are just the shear moduli of the corresponding materials.

- The second scenario considers the introduction of a Johnson-Cook strength model for tantalum (using experimental parameters from [5]) and a simple Von Mises yield criterion for the expanded graphite. This is due to the fact that the material properties for expanded graphite are very difficult to model, due to its high porosity and anisotropy. No material models for expanded graphite have been found in the
literature so far. This uncertainty in the material properties of expanded graphite is not considered as an issue for the present study, which focuses on the dynamic response of the Tantalum rods. This statement is supported by what will be seen in Section 6.2.

Figure 5.3.3: View of a cross section of the mesh of the target. The edges highlighted in green correspond to the surfaces used for slicing the geometry. The mesh is very fine in the center of tantalum core, where the highest gradients of temperature and pressure are expected, and reproduce the dimensions of the cells used in the FLUKA simulations. It then gets coarser when moving away from the core.

Figure 5.3.4: Position of the gauges in version 1 of the target core. The positions were chosen consistently between the two versions of the target core in order to carry out meaningful comparisons between the results of the two simulations.
Chapter 6

Results of the simulated dynamic response of the target core

In Chapter 5 it was explained how the capabilities offered by conventional implicit finite element solvers like ANSYS® Mechanical are limited for the prediction of the extreme conditions generated in the AD-Target core after the proton beam impact. Previous work on other beam impacted devices and on previous versions of the AD-Target [12] identified hydrocodes as the most convenient computational tool to numerically solve the dynamic response of the AD-Target core.

The results of the simulations carried out on the two configurations of the AD-Target core are presented in the present chapter. First, in Section 6.1, the results of the analysis conducted considering a fully elastic response of a model with isolated tantalum blocks are presented. Then, Section 6.2 shows the effect of including the elastic graphite matrix on the elastic response of the full model. Finally, Section 6.3 presents the result of simulations that take into consideration the more realistic possibility of the materials experiencing plastic deformation.

It must be mentioned that the thermal responses computed by using ANSYS® Mechanical and AUTODYN® show no significant difference, since the former solver is perfectly capable of calculating the increase of temperature in the target caused by the proton beam. Since in ANSYS® Mechanical, unlike in AUTODYN®, it is possible to implement the temperature dependence of physical properties such as the specific heat, it could be claimed that the thermal result computed by the implicit solver is even more accurate than the one computed by the hydrocode. Nevertheless, this aspect is not considered a problem for two reasons:

- The AUTODYN® solver works in terms of internal energy and temperature is merely an output variable, only employed by temperature dependent strength models [12].

- Since the simulated time window is quite short (only the first 30 μs of the dynamic response are generally computed) it is reasonable to assume the absence of ther-
mal diffusion in the target core, in the same way as for the rods of the HRMT27 experiment, in chapter 3.

6.1 Elastic response of the target core

Assuming a perfectly elastic material behavior is far from being physically accurate since the yield strength of tantalum is of around 270 MPa at room temperature and rapidly decreases at higher temperature. Simulations based on this assumption, however, allow a better understanding of the phenomena, such as modes of vibration, that take place in the material because of the absence of damping induced by plastic deformation. Numerical damping implemented in the solver is still present but is negligible during the short simulation time.

The first subsections show the dynamic response computed in simulations that only consider isolated rods. This is a good way to understand the dynamic phenomena taking place in the tantalum rods, since the interaction with the graphite matrix and with other tantalum rods is not present. Section 6.2 will then show that the introduction of the expanded graphite matrix implies only small differences on the dynamic response of the rods.

6.1.1 Elastic response of a \( \phi 3 \, mm \) rod

The present subsection details the dynamic response of the sixth tantalum rod in version 2 of the AD-Target core, as defined in Figure 5.3.2, assuming a perfectly elastic material behavior. The rod has a 3 mm diameter and a length of 10 mm and is subjected to the same energy deposition as it would be in the full AD-Target core. This is achieved by placing the rod at the exact same coordinates as it would be in the full AD-Target core and by applying the FLUKA-generated energy deposition computed for the whole geometry.

Figure 6.1.1a shows the pressure response recorded in the first 5 \( \mu s \) by gauges 4, 10 and 16, which are placed in the positions indicated in Figure 6.1.1b. The first aspect to consider is that, as already mentioned, since the computed pressure response has peaks of up to 5.6 GPa, assuming a perfectly elastic response of the material is by far out of reality. These pressure plots should therefore be interpreted just in a qualitative way.

The pressure plots show an initial four stepped rise of pressure corresponding to the consecutive impacts of four proton bunches spaced by 120 ns. After this, a high frequency pressure wave response takes place. It can clearly be seen that the waves at the center of the rod (Gauge 4) and the ones towards the periphery (Gauges 10 and 16), are practically in phase and have a period of 1.03 \( \mu s \). The following analysis will prove that the reason for this phenomenon comes from the fact that, since the nominal size of the beam (1 \( \times \) 0.5 mm at 1\( \sigma \)) is comparable with the diameter of the rod, the beam-induced heating affects the entirety of the rod, causing every part of the body to respond simultaneously, thus leading
Figure 6.1.1: a) Pressure response at the center (Gauge 4) and towards the periphery (Gauges 10 and 16) of the 3 mm rod during the first 5 µs after the proton beam impact, assuming a purely elastic material response. The plot clearly shows the presence of a wave with a period of 1.03 µs. b) Temperature profile in the 3 mm rod at the end of the proton beam pulse. Indeed, it can clearly be seen that the beam-induced heating affects the entire section of the rod since the nominal beam size (1 × 0.5 mm at 1σ) is similar to the diameter of the rod.
to the immediate establishment of a mode of vibration.

In order to better investigate this dynamic response, a Fourier transformation was applied to the pressure wave recorded by the gauge in the center. This process should however be done with caution, since the time step (and therefore the sampling rate) chosen by the simulation varies during the computation. The easiest way to bypass this problem is to interpolate the values of the pressure wave at specific query points, with a fixed sampling rate, using linear interpolation (this can be done, for example, using the interp1 function in Matlab®) and then to apply a Fourier transformation to the interpolated signal. The result of this process is shown in Figure 6.1.2.

Two main waves are identified by this process:

- A predominant wave with a frequency of 970 kHz, which corresponds to the 1.03 μs wave that was easily identifiable by inspecting the plots in Figure 6.1.1a

- A secondary wave with a frequency of 140 kHz that was not previously identifiable by a simple inspection of the plots

The experience gained with the analysis of the HRMT27 rods suggests that these two waves are associated to, respectively, radial and longitudinal modes of vibration. In order to find additional evidence that sustains this hypothesis, the analysis can be continued by shifting the attention to displacements instead of pressures.

Figure 6.1.3 shows the displacement in the radial direction measured in gauges 10 and 16. The two curves are once again perfectly in phase, and show how the whole section of the rod expands and contracts simultaneously. By applying a Fourier transformation to the curves using the technique that was described earlier, it can be seen that the two main components of these waves oscillate with the same frequencies found in the pressure wave.

Figure 6.1.4 shows the displacement in the axial direction measured by Gauges 1, 2, 3, 4, 5 and 6. It can be seen how the whole rod is longitudinally expanding in a simultaneous way along its axis, in a similar way to what is happening radially. The reason for this phenomenon is again the fact the temperature rise caused by the energy deposition evenly covers the whole length of the rod and no pressure waves have to travel through the material before the whole rod starts vibrating longitudinally. Once again, the two main components of the waves oscillate with frequencies of 140 and 970 kHz.

It was considered interesting to compare the frequencies to the ones predicted by the analytical formulas presented in Chapter 2. Nevertheless, in order to do so, two aspects shall be taken into consideration:

- The rod, with 3 mm in diameter and 10 mm length, is not slender enough to automatically assume that the two modes of vibration are decoupled.

- The influence of temperature on the mechanical properties (Young’s modulus and density) of the rod must be taken into account. Choosing the appropriate values is not straightforward since temperature varies greatly over the cross section of the
Figure 6.1.2: Fourier Transformation of the pressure response of Gauge 4, shown in Figure 6.1.1a.

Figure 6.1.3: Radial displacement measured by gauges 10 and 16 of the 3 mm rod during the first 30 µs of the simulation, assuming a purely elastic material response.
rod. For a simplified approach, the maximum temperature on the cross section is considered.

The results of this process are summed up in Table 6.1:

- The 970 kHz wave is compatible with a radial mode of vibration, obtained by considering 0.6 as the value of the mechanical coupling coefficient $\alpha$ (as opposed to a value of 0.34 considered for the decoupled planar radial vibration of a slender rod)
- The 140 kHz wave is compatible with a longitudinal mode of vibration, obtained by considering $\infty$ as the value of the mechanical coupling coefficient $\alpha$ (the same value considered for the decoupled longitudinal vibration of the slender rod)

<table>
<thead>
<tr>
<th>Simulation</th>
<th>Analytical approach</th>
</tr>
</thead>
<tbody>
<tr>
<td>970 kHz</td>
<td>Radial $\alpha = 0.6$</td>
</tr>
<tr>
<td>140 kHz</td>
<td>Longitudinal $\alpha = \infty$</td>
</tr>
</tbody>
</table>

Table 6.1: Frequencies of the components of the pressure and displacement waves, shown next to the frequencies of the radial and longitudinal modes of vibration of the rod.

The good agreement between the formulas for the frequencies of radial and longitudinal modes of vibrations and the frequencies of the pressure and radial and axial displacement
waves computed in the hydrocode simulation leads to the confirmation that the dynamic response of the \( \phi 3 \text{ mm} \) tantalum rod is mainly governed by the excitation of a radial and a longitudinal mode of vibration. This theory simultaneously explains why pressure waves along a radius are perfectly in phase, and the the behavior of radial and axial displacements, as shown in Figures 6.1.3 and 6.1.4.

By comparing the Fourier transformation of the pressure response of Figure 6.1.2 and the radial and axial displacements shown in Figures 6.1.3 and 6.1.4, it is interesting to notice how the 140 \( k \text{Hz} \) longitudinal mode of vibration has a big effect on the displacements but has very little influence on the pressure response.

A modal analysis via finite element software of the tantalum rod was also conducted, in the same way as what was done in Part I for the modal analysis of the of the TZM rod, in order to get a second confirmation and a visual representation of the mode shapes associated to the radial and longitudinal modes of vibration.

The results of this modal analysis are the following:

- Figure 6.1.5 shows a mode of vibration with a frequency of 950 \( k \text{Hz} \), the only one in this range of frequencies to exhibit a clear expansion and contraction of the cross-section of the rod. Two aspects are interesting about this mode shape: the waviness induced by the mode shape on the surface of the rod and the small amount of coupling present between the radial and axial expansions. The first aspect, counter-intuitive at first, can be confirmed by running a second AUTODYN\textsuperscript{\textregistered} simulation of the rod with a very fine array of gauges placed on the surface of the rod. By plotting the displacements in the radial direction of several consecutive gauges, the same pattern in the surface can be appreciated. It is also interesting to note how, consistently with what was observed in the analytical analysis, a small degree of coupling is present between the radial and the axial vibration at this frequency.

- Figure 6.1.6 shows a mode of vibration with a frequency of 151 \( k \text{Hz} \) that can be identified as being compatible with the longitudinal vibration of the rod. The lengthening and shortening of the axis of the rod is apparent by comparing the two extremes in the vibration with the undeformed rod. This process is associated with the rod acquiring a "barrel" shape during the process of contraction, which explains the big influence that the longitudinal mode of vibration has on the radial displacement. This was also suggested by the Fourier transformation of the radial displacement. It is interesting to notice how, for this longitudinal mode of vibration, the results obtained via the analytical and the finite element method are both really close to each other and just have slightly higher (around 6\%) frequencies than the one of the longitudinal waves present in the hydrocode simulation. For the analytical analysis this can be solved by considering, instead of the length of the rod at room temperature, its length when heated up by the beam.

The results of the two approaches to the modal analysis of the Tantalum rod are in good agreement with each other and with the results of the hydrocode simulation. This
aspect can be interpreted once again as the definitive proof that the physical mechanism governing the response of the $\phi 3\, mm$ rod after being impacted by the proton beam is the vibration obtained as a superposition of a radial and a longitudinal modes of the rod.

![Image of vibration modes](image_url)

Figure 6.1.5: A mode with a frequency of 950 $kHz$. From top to bottom: the undeformed rod and the two opposite extremes in the vibration. This is the only mode in the range of frequencies compatible with the measured waves showing a clear expansion and contraction of the cross section. This simultaneous compression and expansion compatible both with the displacements in the radial direction (shown in Figure 6.1.3) and with the pressure waves (shown in Figure 6.1.1a).

### 6.1.2 Elastic response of a $\phi 8\, mm$ rod

This subsection presents the dynamic response of the third tantalum rod in version 1 of the AD-Target core. The rod has a $8\, mm$ diameter and a length of $11\, mm$. The same settings for the simulation used for the $3\, mm$ rod were used in this case: the material behavior was assumed perfectly elastic and the rod was subjected to the same energy deposition as it would have been subject to in the AD-Target core. The most interesting aspect of this simulation is to understand the effect of a change in diameter on the dynamic response of the tantalum rods.

Figure 6.1.7a shows the pressure response recorded in the first $5\, \mu s$ by Gauges 4, 10, 16, 22, 28, 34 and 40, which are placed in the positions indicated in Figure 6.1.7b. By comparing this plot to the pressure response in the first $5\, \mu s$ computed for the $\phi 3\, mm$ rod, it is clear that in this case the pressure waves recorded by the gauges placed along the same radius have a higher period and are slightly shifted in phase.

This is due to the fact that, the diameter of the rod big enough with respect to the beam (whose size is $1 \times 0.5\, mm$ at $1\sigma$) that in this case the beam only significantly heats
Figure 6.1.6: A mode with a frequency of 151 kHz. From top to bottom: the undeformed rod and the two opposite extremes in the vibration. The lengthening and shortening of the longitudinal dimension of the rod is apparent. This process is associated with the rod assuming a barrel shape during the process of contraction, which explains the big influence that the longitudinal mode of vibration has on the radial displacement, as testified by the Fourier transformation of the radial displacement, shown in Figure.

up the core of the rod and not the periphery, as what happens for the φ3 mm tantalum rod. The phenomenon that can be appreciated in Figure 6.1.7a can be then interpreted as a redistribution of stress within the body by means of pressure waves. The interesting aspect is that this phenomenon is only transitory. By plotting the pressure response measured by the same gauges in the 30 to 35 µs time window, it can be seen that after an initial transient in which a redistribution of stress occurs, the pressure waves are then reorganized to be practically in phase between the center and the periphery of the rod, just like what was happening for the φ3 mm rod from right after the end of the pulse. Indeed, the stress state is gradually organized into modes of vibration of the rod. This phenomenon also happens for the plots of the radial displacements recorded by the same gauges in the same time frames.

In order to confirm this phenomenon, a Fourier transformation was applied to the pressure at Gauge 4 and to the radial displacement at Gauge 40, after 30 µs. The results can be seen in Figure 6.1.8a. Three main components can be identified: a vibration with a frequency of 114.3 kHz, which has limited effects on pressure, and two vibrations with frequencies of 328.6 and 385.7 kHz.

Once again, a modal analysis of the tantalum rod (considering the mechanical properties of tantalum at 1800°C) was carried out to investigate the possibility that, indeed, the excitation of modes of vibration of the rod could comprehensively explain the dynamic response of the rod.

The results of a modal analysis carried out via finite element methods in ANSYS® Mechanical are the following:
(a) Pressure response in Gauges 4, 10, 16, 22, 28, 34 and 40 in the first 5 μs.

(b) Thermal response of the φ8 mm rod. In this case the rod is big enough that the beam only significantly heats up its core, thus leading to a pressure response that is not initially in phase at the center and the periphery.

(c) Pressure response in Gauges 4, 10, 16, 22, 28, 34 and 40 from 30 to 35 μs.

Figure 6.1.7
(a) Fourier transformation applied to the pressure response measured from 30 to 100\(\mu\)s by Gauge 4

(b) Fourier transformation applied to the radial displacement measured from 30 to 100\(\mu\)s by Gauge 40

(c) Fourier transformation applied to the axial displacement measured from 30 to 100\(\mu\)s by Gauge 1

Figure 6.1.8
• Figure 6.1.9a shows a mode of vibration with a period of $130 \, kHz$ that is compatible with the longitudinal vibration of the rod. The lengthening and shortening of the axis of the rod is apparent by comparing the two extremes in the vibration with the undeformed rod. It is interesting to notice how, in the same way as the longitudinal vibration in the $\phi 3 \, mm$ rod, the mode computed by ANSYS® Mechanical has a slightly higher frequency than the frequency given by the gauges in AUTODYN®.

• Figures 6.1.9b and 6.1.9c show two modes of vibration with frequencies of 324.6 and 385.7 $kHz$ that could be compatible with the two components of the wave having frequencies of 328.6 and 385.7 $kHz$. The two mode shapes are compatible with the pressure and displacement waves both in terms of frequencies and in terms of mode shapes. Both mode shapes present axial displacement in the center of the rod, that is consistent with the Fourier transformation of the axial displacement recorded by Gauge 6.

The good agreement, both in terms of frequencies and modal shapes, between the pressure and displacement signals recorded in the AUTODYN® simulation and the modes of vibration computed via finite element method can be interpreted once again as the definitive confirmation that the physical mechanism governing the response of the tantalum rod is the excitation of longitudinal and radial modes of vibration by the sudden beam-induced heating. In this case, however, due to the bigger size of the rod with respect to the beam, a transitory redistribution of the stress state by means of pressure waves is necessary in order for the rod to start vibrating coherently.

### 6.2 Effect of the presence of the graphite matrix on the elastic response of the tantalum core

In the previous section, the dynamic response of isolated tantalum rods of the target core was presented. It was shown how the key mechanism governing the dynamic response of the rods, especially when the diameter of the rod is comparable to the size of the beam, is the excitation of radial and longitudinal modes of vibration by the sudden beam-induced heating. After understanding these phenomena, another key question was to understand if the presence of the expanded graphite matrix surrounding the rods alters this dynamic response.

Figure 6.2.1a shows the pressure given by Gauge 4 in the two configurations, with and without the presence of the graphite matrix, for the previously analyzed $\phi 3 \, mm$ tantalum rod. Both simulations are still assuming a perfectly elastic material response.

As it can be seen, the introduction of the graphite matrix only affects the amplitude of the pressure response and not its frequency. The reason for this is the progressive transfer of energy from the rod to the graphite matrix during the vibration. The same thing happens to the radial displacement, as seen in Figure 6.2.1b. The fact that the
Figure 6.1.9: a) A longitudinal mode of vibration with a frequency of \(130 \text{kHz}\). b) A radial mode of vibration with a frequency of \(324.6 \text{kHz}\). c) A radial mode of vibration with a frequency of \(385 \text{kHz}\).
frequencies of vibration of the $\phi 8 \text{mm}$ rod are clearly unaltered by the introduction of the graphite matrix can be taken as a confirmation that the rod vibrates with the same modes of vibration regardless of the presence of the graphite matrix.

Figure 6.2.2a shows the pressure recorded by Gauge 4 in the same configurations, with and without the graphite matrix, for the previously analyzed $\phi 8 \text{mm}$ tantalum rod, assuming perfectly elastic material behavior. A small but more apparent change in the pressure response is noticeable in this case: the frequency and the amplitude of the wave are slightly altered. In order to quantify this change, and to see if the introduction of the graphite matrix has an effect on the modes of vibration identified via the analysis of the isolated rod, a Fourier transformation was applied to the pressure wave recorded by Gauge 4 during the interval from 30 to 100 $\mu$s after the start of the pulse, in the case of the rod inserted in the graphite matrix. The result can be seen in Figure 6.2.2b, which shows that the introduction of the graphite matrix has the effect of eliminating the 328.6 $kHz$ radial mode of vibration, which constituted a secondary component of the pressure wave in the dynamic response of the isolated rod, as can be seen in Figure 6.1.8a. The remaining two modes of vibration are unaffected.

The discrepancy between the two configurations is less pronounced from the point of view of the radial displacement on the outer surface of the rod, as it can be seen in Figure, which shows the radial displacement recorded by Gauge 40 in the first 15 $\mu$s after the start of the impact.

After reviewing the results of the comparison between the dynamic response of the tantalum rods when isolated and when embedded the graphite matrix, it can be claimed that, especially in the case of the smaller diameter rods, the response of the rods is fundamentally unaffected by the surrounding graphite matrix: the rods vibrate essentially in the same modes of vibration they would be vibrating in when isolated. This fact is likely due to the high difference in density and compressibility between the two materials.

The analysis of the isolated rods allowed for an easier understanding of the physical mechanisms governing the dynamic response of the tantalum rods caused by the beam impact.

### 6.3 Plastic response of the target core

The results of simulations taking into account the potential plastic deformation in the target materials are shown in the present section. Plastic deformation in the Tantalum core is taken into account by using a Johnson-Cook strength model which, as described in Section 5.2.2, takes into account accumulated plastic deformation, temperature and strain rate. Due to the complexity of modeling the behavior of expanded graphite, the strength model adopted for this material is a simple Von Mises yield criterion.

Multiple simulations considering various values for the Yield Stress of the graphite matrix were carried out, showing negligible effect on the dynamic response of the tantalum rods, even changing this value over many orders of magnitude.
(a) Pressure waves recorded by Gauge 4 in the previously analyzed \( \phi 3 \text{ mm} \) rod in simulations considering the rod isolated or with the presence of the expanded graphite matrix.

(b) Radial displacement recorded by Gauge 16 in the \( \phi 3 \text{ mm} \) rod in simulations considering the rod isolated or with the presence of the expanded graphite matrix.

Figure 6.2.1
(a) Pressure recorded by Gauge 4 in the previously analyzed \( \phi 8 \text{ mm} \) rod in two simulations considering the to be isolated or embedded in the graphite matrix

(b) Fourier transformation applied to the pressure wave recorded by Gauge 4 in the \( \phi 8 \text{ mm} \) rod in the configuration considering the rod embedded in the graphite matrix

Figure 6.2.2
Figure 6.2.3: Radial displacement recorded by Gauge 40 in the φ8 mm rod in two simulations considering the rod isolated or embedded in the graphite matrix

Figure 6.3.1 shows the pressure response given by Gauge 4 in the φ3 mm tantalum rod, (R6 in Version 2 of the AD-Target core, according to Figure 5.3.2), assuming in one case a perfectly elastic response in the materials and in the other the possibility of plastification. There are multiple interesting aspects in this plot:

- In the case of the plastic response, the pressure wave experiences a fast dampening, due to the energy loss associated to plastic deformation processes. This dampening has a noticeable effect not only on the decreasing amplitude of the pressure wave, but also on the frequencies of the components that make up the wave. By applying a Fourier transformation to the pressure wave recorded in the simulation assuming the presence of plastic deformation, a clear shift of the components toward lower frequencies can be observed. The frequency of the 970 kHz radial mode of vibration, for example, is decreased to 933 kHz. This shift towards lower frequencies is compatible with the well known relation between undamped and the damped natural frequencies of vibration, shown in Equation 6.3.1. In this formula, \( f_d \) and \( f_n \) are the dampened and undamped frequencies for the harmonic oscillator and \( \zeta \) is the damping ratio. The application of this formula to the radial mode of vibration yields a damping ratio \( \zeta \) equal to 0.27. As it can be seen in Figure 6.3.2, which shows the axial displacement recorded by Gauge 1 and 6, this is not the case for the slower 140 kHz mode of vibration, which is dampened even before the first oscillation.

\[
    f_d = f_n \sqrt{1 - \zeta^2} \tag{6.3.1}
\]

- Another important difference between the elastic and the plastic dynamic response that is common to all the rods is the fact that the pressure response tends to be
shifted down towards tensile pressures. This relevant difference in pressure distribution, which is especially noticeable in the first oscillations, had already been observed in [12]. The reason for this behavior is most probably the limitation of the deviatoric component in the stress tensor by the plastic limit of the material.

Other interesting aspects of the dynamic response of the tantalum core which are relevant for the prototyping activities of the new AD-Target are shown in the following subsections.

### 6.3.1 Effect of a change in the rod diameter

By examining the elastic and plastic dynamic response of several tantalum rods, an aspect that becomes immediately apparent is the strong influence of the rod diameter on the pressure response, and in particular on the maximum reached tensile pressures. This is the reason why simulations of the dynamic response of two configurations of the AD-Target were performed: the effect of a change of diameter can be isolated from other variables (such as temperature and length of the rods) by comparing the response of two rods with the same length and the same deposited energy (belonging to V1 and V2) but with different diameters.

The results for some of the more significant comparisons between the rods in the two versions are shown in the following:

- Figure 6.3.3 shows the pressure response at the center of R3, which has a length of
11 mm, and where the maximum reached temperature is 2150 K. This rod has a diameter of 8 mm in version 1 and of 5 mm in version 2 of the AD-Target core. The decrease of diameter from 8 to 5 mm, in addition to the decrease of the period of the pressure wave, has a significant effect on the increase of the maximum absolute value of the tensile pressure, from $-2.7$ to $-4.8 \, GPa$.

- Figure 6.3.4 shows the pressure response at the center of the R5, which has a length of 10 mm, and where the maximum reached temperature is 1890 K. This rod has a diameter of 6 mm in version 1 and of 4 mm in version 2 of the AD-Target core. The 2 mm decrease in diameter from 6 to 4 mm yields analogous effects to the previous case, among which an increase of the absolute value of the tensile pressure, from $-3.77$ to $-4.9 \, GPa$.

- Figure 6.3.5 shows the pressure response at the center of R7, which has a length of 8 mm, and where the maximum reached temperature is 1590 K. This rod has a diameter of 4 mm in version 1 and of 3 mm in version 2 of the AD-Target core. Differently from the previous cases (in which a decrease of diameter is directly associated with higher tensile pressure) decreasing the diameter from 4 to 3 mm, has the opposite effect. The same phenomenon also happens for the sixth and eighth rods, characterized by the same diameters but different lengths and temperature distributions. The reason for this can be deduced by looking at the pressure responses during the pulse. In the case of the 3 mm diameter rod, the period of the radial mode of vibration becomes short enough that the fourth proton bunch impacts when the pressure is progressing towards tensile states. In contrast, with rods with larger
diameter, the fourth proton bunch impact coincides with the progression of pressure towards compressive states. Therefore, the localized compressive peak of pressure generated by the fourth bunch while the 3 mm rod is already progressing towards tensile states is the reason why in this case a reduction in diameter reduces the value of the maximum tensile pressure.

![Graph showing pressure response](image)

**Figure 6.3.3:** Pressure response at the center of the third rod in version 1 and 2 of the AD-Target core. A 3 mm reduction in diameter, with every other physical and geometric variable unchanged, yields a 2.1 GPa increase in the absolute value of the tensile peak reached in the first oscillation.

The reason for the increase of tensile pressure when decreasing the diameter of the rods down to 4 mm is not easy to pinpoint. A possible explanation could be given by comparing the behavior of the rod in the radial direction with a spring-mass system. As it can be seen in Figure 6.3.6, the pressure and displacement in the radial direction are always in counterphase. This is the same as an elastic spring-mass system. This happens at any point along the same radius.

Moreover, when changing the diameter of the rod between the two versions of the core, the maximum radial displacement is practically unchanged. By noting this, the increase the tensile pressure can be explained as the combination of the following two effects:

- In the same way as we have rods of different diameters, let’s assume two springs with the same characteristics, apart from a different length. It is well known that the shorter spring will be proportionally more stiff than the longer one. This means that to reach the same absolute extension in both springs, the force applied to the shorter spring (the stiffer one) must be higher. This could be extrapolated to the target
Figure 6.3.4: Pressure response at the center of the fifth rod in version 1 and 2 of the AD-Target core. A 2 mm reduction in diameter, with every other physical and geometric variable unchanged, yields a 1.2 GPa increase in the absolute value of the tensile peak reached in the first oscillation.

Figure 6.3.5: Pressure response at the center of the seventh rod in version 1 and 2 of the AD-Target core. Unlike the previous cases, a reduction in diameter yields a decrease in the absolute value of the tensile peak reached in the first oscillation.
Figure 6.3.6: Pressure at the core plotted against the radial displacement on the surface of the seventh rod in version 1 of the Target core. It is clear from this graph how, similarly to what would happen for a spring-mass system, the pressure at the core is always in counterphase with the radial displacement of a point having the same axial coordinate rods, given the absolute radial deformation is similar regardless of the diameter (as it is simply defined by the expansion of the heated material). Under this conditions, the thinner rod behaves like as the shorter rod, i.e., it is stiffer.

- As it was described at length in Section 6.1, and as it can be seen in Figures 6.3.3 - 6.3.5, the period of the radial mode of vibration of the rod is decreased when the diameter is. By noting that the displacement in the radial direction is unaffected by the change in diameter, a more intense amplitude of the pressure wave will be necessary at the center of the rod to carry out a radial oscillation having the same amplitude in a shorter amount of time.

At the end of this analysis it is clear how, from the mechanical point of view, rods with bigger diameter (apart from the case of the increase from 3 to 4 mm) are consistently subject to less extreme peak in tensile pressures and will, therefore, be less likely to fracture during the dynamic response to the proton pulse. As said in the introduction to this part, however, rods with bigger diameters have historically been considered to have lower antiproton yield than smaller rods. FLUKA-based studies are currently ongoing to quantify this effect.

As it can be seen in Figures 6.3.3 - 6.3.5, apart from the case of the 3 mm diameter rod, a change in diameter has no effect on the compressive peak reached at the end of the proton pulse.
6.3.2 Effect of a change in Length of the rod

The previous subsection detailed the effects of core diameter on the pressure response. This analysis was made possible by considering two versions of the AD-Target core in which rods in equivalent positions (therefore subjected to the same energy deposition), have the same length but different diameters. Similarly the influence of the rod length in the tensile pressure has been assessed by comparing the response of R1 and R5 in version 1 of the AD-Target core. The two rods both have a diameter of 6 mm and are subjected to similar energy depositions, reaching 1841 K for R1 and 1886 K for R5, while having different lengths. As it can be seen in Figure 6.3.7, even though the diameter is unchanged and the temperature increase is essentially the same, changing the length of the rod influences both the compressive and tensile pressures reached. The maximum tensile pressure reached is, in fact, increased by 1.53 GPa when the length of the rod is increased from 8 to 10 mm.

This result, unforeseen during the set up of the simulations, justifies the implementation of another campaign of simulations in order to finalize the optimal dimensions of the core rods of the new AD-Target.

![Graph showing pressure response at center of R1 and R5 in version 1 of AD-Target core.]

Figure 6.3.7: Comparison between the pressure response at the center of R1 and R5 in version 1 of the AD-Target core. The longer 10 mm rod, with the same diameter and subject to very similar energy depositions to the shorter 8 mm one, experiences higher pressures, in both the compression and tension during the first oscillation.
6.3.3 Distortion of the pressure wave in the first oscillation with small diameters rods

The last phenomenon that deserves being explained in more detail (and whose presence had already been noted in [12]) can be appreciated in Figure 6.3.8. This figure shows the pressure response and its time derivative on the axis of a 4 mm diameter rod during the first 1.5 µs of the simulation.

![Graph](image)

Figure 6.3.8: Plot of the pressure response (blue) and its time derivative (orange) on the axis of the rod (Gauge 20) during the first 1.5 µs of the simulation. The first part of the plot clearly show the discontinuities in the pressure response associated to the beginning and end of each proton bunch impact. After this, four distortions of the pressure wave are particularly noticeable in its time derivative.

The first part of the plot shows four rises of pressure associated to the four impacting bunches. As it can be seen, at the end of each bunch an abrupt decrease of pressure, due to inertia effects after the sudden expansion, takes place. These abrupt decreases in pressure were referred to as end-of-pulse tensile waves in [13]. One phenomenon that can be noted in the pressure response of the rods with smaller diameters (starting from the 5 mm diameter rods), that could also play a role in in the tensile pressures reached, is the presence of four equally spaced distortions of the wave after 0.5 µs of the response. As it was observed in [12], these distortions are particularly noticeable by observing, instead of the pressure response, its time derivative. It is worth mentioning that this phenomenon is observed for the rods with small diameters both in their elastic (see, for example, Figure 6.1.1a) and in their plastic response, with and without the presence of the surrounding graphite matrix.
In order to understand the origin of this phenomenon, a simulation considering the plastic response of an isolated 4 mm diameter rod (R7 in version 1 of the AD-Target core), was carried out, introducing a total of 20 gauges spaced by 0.1 mm along the radius. The number associated to the gauges increases going from the surface to the center of the rod, where gauge number 20 is positioned.

By examining the time derivatives of the pressure waves recorded by the gauges, shown in Figures 6.3.9, 6.3.10 and 6.3.11, it can be seen that, at the end of each bunch a series of tensile pressure waves are generated on the surface of the rod (Gauges 1 to 4). This can be appreciated by the fact that they are present just after 50 ns of the response in Figure 6.3.9 (corresponding to the gauges in the periphery) while they do not appear before 400 ns in Figure 6.3.11 (corresponding to the gauges in the center). Therefore they travel from the surface to the center of the rod. In addition, the amplitude of the wave gradually increases while traveling towards the center due to the cylindrical geometry of the rod.

The origin of the tensile pressure waves on the surface of the rod is interpreted as the consequence of the reflection of small compression waves generated close to the surface of the rod due to the local energy deposition. These compressive waves are then reflected as tensile pressure waves which travel towards the center of the rod.

![Figure 6.3.9](image)

Figure 6.3.9: Time derivative of the pressure waves recorded close to the surface (Gauges 1 to 7). As it can be seen, right from the end of the first proton bunch, a compressive wave is generated in the proximity of the surface and it starts traveling towards the center of the rod. The other pulses have similar effects, even though they are obviously more difficult to discern.
Figure 6.3.10: Time derivative of the pressure waves recorded from Gauges 9 to 15.

Figure 6.3.11: Time derivatives of the pressure waves recorded close to the center of the target. The behavior at the core of the rod is fundamentally different from the one on its surface. As it can be seen, at the end of the first three proton bunches no compressive waves take place in the center of the rod. After the end of the last one the tensile pressure wave arrives at the center of the rod and it causes the first of the four distortions.
Chapter 7

Conclusions

The work presented in this Master Thesis was carried out in the context of the R&D activities for a new design of the AD-Target.

In the first part it was possible to demonstrate that the dynamic response of thin cylindrical targets subjected to primary proton beam impacts is governed by the excitation of radial, longitudinal and (if the impact is off-axis) flexural modes of vibration. This was demonstrated by analyzing the responses to low intensity pulses of targets during the HiRadMat27 [14] experiment and by cross-checking the experimental data with analytical and finite element methods for modal analysis.

The analysis then focused on the flexural modes of vibration of the rods. This study demonstrated how, when slender targets are subjected to the proton beam impact, the presence of small misalignments in the beam impact position can cause significant flexural deformations. For instance, in Chapter 3, a flexural response similar to the one recorded during the HiRadMat27 experiment was recreated via FE simulations by considering just a 7% offset of the target diameter. This suggests the use of more precise alignment procedures, such as beam based alignments, in future experiments involving thin target rods. Furthermore, for the case of the real target operation, this result encourages the adoption of a sliced target core which could contribute to drastically reduce of the effect of the excitation of these flexural modes of vibration.

In the second part, hydrocode analyses of the response of the current prototypes of the future AD-Target core were presented. For a deep understanding of the phenomena governing the dynamic response of the target core in operation, the simulations were performed increasing gradually the complexity of the models. The study, building upon the knowledge gathered in the first part, gave important new insights on the dynamic response of the tantalum rods that make up the target core, and in particular characterized the influence of the geometry on the dynamic response of the core.

In this sense it was shown how, from the mechanical point of view, the increase of the target rod diameter over 4 mm has a beneficial influence. Above 4 mm, in fact, the larger the diameter, the lower is the tensile pressure reached. For instance, the increase of the rod diameter from 5 mm to 8 mm in rods subjected to the same energy deposition is
associated to a 44% decrease of the tensile pressures reached. Rods with larger diameters, however, have historically been considered to have lower antiproton yield than smaller rods. FLUKA-based studies are currently ongoing to quantify this effect. In addition, below $4\text{ mm}$, an opposite effect has been observed, i.e. the tensile pressure reached in rods of 2 or 3 mm are lower than in $4\text{ mm}$.

Furthermore, an unforeseen result during the set up of the simulations, was that changing the length of the rod also influences both the compressive and tensile pressures reached. This result justifies the implementation of another set of simulations in order to study this effect and finalize the optimal dimensions of the core rods for the new AD-Target.

Finally, the last phenomenon that was explained in this study is related to four equally spaced distortions of the pressure wave after $0.5\,\mu\text{s}$ of the transient, which were also observed in previous works [13] but whose origin had not been explained. The analysis of the present thesis proved that the cause of this perturbation is the presence of tensile waves which are created on the surface of the rod after every proton bunch impact and then travel towards its center. Their origin is thought to be consequence of the reflection of small compression waves generated close to the surface of the rod due to the local energy deposition there.

All the simulations and findings of this thesis will be used in the next steps of the prototyping activities for the new AD-Target design. These activities involve the manufacturing of different real scale prototypes and their testing under proton beam impacts using the HiRadMat facility in the so called PROTAD experiment, foreseen to take place in 2018.
Bibliography


