SENSITIVITY OF THE LHC TRANSVERSE FEEDBACK SYSTEM TO INTRA-BUNCH MOTION

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Abstract

The LHC Transverse Feedback System is designed to damp and counteract all possible coupled bunch modes between the lowest betatron frequency and 20 MHz. The present study reveals that the analog frontend processing scheme based on down converting the pick-up signal at the LHC RF frequency to baseband considerably extends the detected bunch movements visible to the feedback system to beyond 1 GHz. We develop an analytic model of the signal processing chain to explore the impact of even-symmetric and odd-symmetric intra-bunch movements on the detected beam position as a function of the longitudinal bunch shape. A set of equations is derived suitable for numerical simulations, or as a complement in particle tracking codes to further refine the behavior of the LHC transverse feedback system.

INTRODUCTION

The transverse feedback system (TFB) of CERN’s Large Hadron Collider (LHC) measures bunch-by-bunch transverse displacements and damps oscillatory movements by means of fast electrostatic kickers. In order to detect the time-varying beam position the signals of individual bunches are processed in analog and digital [1], generating one position reading per bunch per turn.

In the following we evaluate analytically the performance of the system’s beam position signal processing scheme for normalized longitudinal bunch profiles, $\lambda(t)$, and transverse oscillation patterns, $x(t)$, as test inputs. For comparison the true movement of the center-of-charges, $\bar{x}$, given as

$$ \bar{x} = \int x(t) \lambda(t) \, dt ,$$

for various excitation frequencies is plotted against the digital representation of the beam normalized transverse position calculated by the LHC TFB.

ANALYTIC MODEL

The block diagram in Fig. 1 outlines the signal processing chain of the beam position hardware. Beam induced signals in a pick-up are passed through an analog acquisition system before they are converted to digital. An FPGA (field programmable gate array) calculates from the data stream a normalized position, bunch-by-bunch, which is independent of the per-bunch intensity or the longitudinal bunch shape.

**Analog Processing Scheme**

The electromagnetic field generated by a circulating bunch with normalized longitudinal profile, $\lambda(t)$, and total charge, $q$, interfaces with a stripline type beam position monitor (BPM), thereby induces a signal into two opposing electrodes (denoted $A$ and $B$), the amplitude of which depends on its transverse position w.r.t. the vacuum chamber, $x(t)$, and the pick-up geometry, $d_x$, (linear approximation for small amplitudes). The BPM output voltage follows from,

$$ V_{A,B}(t) = Z_T \int_{-\infty}^{t} \left( 1 \pm \frac{x(t)}{d_x} \right) \cdot q \lambda(t) \cdot h_{BPM}(t - \tau) \, d\tau ,$$

with $Z_T$ as the transfer impedance, and $h_{BPM}(t)$ the impulse response of the BPM. Note that Eq. (2) denotes a convolution integral of the longitudinal bunch profile with the pick-up response. The position information is encoded in the signal amplitude, which is AM modulated, with a strong common signal and with ideally only a small contribution by offset.

Peak voltages from the two electrodes are sufficiently large to transmit the signals by means of equal-length low-loss coaxial transmission lines from the beam line in the underground tunnel to the electronics located on the surface. The transmission line attenuates the raw pick-up signals to levels acceptable for the hybrid and adds dispersion to the pulse response, represented by $h_{COAX}(t)$.

The first element at the surface is a $180^\circ$ hybrid. It combines the transmitted signals, $\bar{V}_A$ and $\bar{V}_B$, to the sum signal, $V_\Sigma$, common to both pick-up electrodes, and it generates the difference, or $V_\Delta$-signal. The sum signal represents the beam longitudinal profile, i.e. the bunch shape as well as the number of charges, whereas the delta signal holds additional information about the transverse position.

Assuming an ideal hybrid, i.e. no cross-talk between the outputs, then

$$ V_\Sigma(t) = \frac{1}{\sqrt{2}} \left[ \bar{V}_A(t) + \bar{V}_B(t) \right] , \quad (3) $$

$$ V_\Delta(t) = \frac{1}{\sqrt{2}} \left[ \bar{V}_A(t) - \bar{V}_B(t) \right] . \quad (4) $$

Special types of bandpass filters, so called comb-filters, shape both the sum and the delta signal in time-domain to a well-defined wavelet [1]. The filter response is designed for a time-limited rectangular window shorter than the nominal bunch spacing, to ensure no mixing between adjacent bunch signals. These filters have a center frequency of 400.8 MHz, equivalent to the LHC RF frequency.

The bandpass filter output (denoted with a tilde), applied for the $\Sigma$ signal in Eq. (3) by inserting Eq. (2), results from,

$$ \tilde{V}_\Sigma(t) = q \lambda(t) \ast h_{PU}(t) , $$

where

$$ h_{PU}(t) = \sqrt{2} Z_T \cdot h_{BPM}(t) \ast h_{COAX}(t) \ast h_{BP}(t) . $$

$$ (5) $$

$$ (6) $$
Figure 1: Block diagram of the beam position hardware signal processing scheme.

Equation (6) represents the cascade or convolution (here and in the following indicated with the asterisk notation) of time domain impulse responses, including the beam transfer impedance, signal gain by $\sqrt{2}$ in the hybrid and passive linear elements, shaping the pick-up response in time and frequency domain.

Similarly, the $\Lambda$ signal output follows from Eq. (4) in combination with Eq. (2) as,

$$\tilde{V}_{\Lambda}(t) = \frac{x(t)}{d_x} \cdot q\Lambda(t) \ast h_{PU}(t).$$

(7)

Equations (5) and (7) describe the underlying formalism in time domain, the impact of which can be seen in frequency domain. Namely, convolution in time domain results in a multiplication in frequency domain, or

$$\tilde{V}_{\Sigma}(j\omega) = q\Lambda(j\omega) \cdot H_{PU}(j\omega).$$

(8)

On the other hand, multiplication in time domain as within Eq. (7) results in a convolution in frequency domain, therefore,

$$\tilde{V}_{\Lambda}(j\omega) = \frac{X(j\omega)}{d_x} \ast q\Lambda(j\omega) \cdot H_{PU}(j\omega).$$

(9)

For the case of a sinusoidal excitation where $X(j\omega) = \delta(\pm \omega_\lambda)$ the previous equation states that a transverse oscillation causes a shift in the spectrum of the longitudinal bunch profile to the carrier frequencies at $\pm \omega_\lambda$,

$$\tilde{V}_{\Lambda}(j\omega) = \frac{1}{d_x} q\Lambda \left[ j(\omega \pm \omega_\lambda) \right] \cdot H_{PU}(j\omega).$$

(10)

Within the Beam Position Module a set of mixers demodulate the bandpass filtered signals in in-phase and quadrature components (I/Q pairs for $\Lambda$ and $\Sigma$), followed by optimized low pass filters of Gaussian shape to suppress mirror frequencies and to shape the system response for low output ripples.

The base-band response of the in-phase component (even symmetry) is obtained by multiplication with $c(t) = \cos(\omega L(t))$, whereas the quadrature component (odd symmetry) follows from multiplication with $s(t) = \sin(\omega L(t))$.

Therefore, we obtain from Eq. (5) for the $\Sigma$-signal after low-pass filtering,

$$I_{\Sigma}(t) = k_\Sigma \cdot \left[ q\Lambda(t) \cdot c(t) \right] \ast g(t),$$

$$Q_{\Sigma}(t) = k_\Sigma \cdot \left[ q\Lambda(t) \cdot s(t) \right] \ast g(t),$$

(11)

where

$$g(t) = [h_{PU}(t) \cdot c(t)] \ast h_{LP}(t).$$

(12)

Equation (12) states that the response function of Eq. (6) is demodulated to baseband and subsequently lowpass filtered by $h_{LP}(t)$. At this point it is worth noting that the shape of the baseband response of Eq. (11) is entirely defined by $g(t)$, and only its amplitude being a function of the demodulated longitudinal profile. Signal level adjustments and other coefficients are collected in a single scalar, $k_\Sigma$.

Similarly, the I/Q-demodulation of the bandpass filtered $\Sigma$-signal provided by Eq. (7) evaluates as,

$$I_{\Sigma}(t) = k_\Delta \left( \frac{x(t)}{d_x} \cdot q\Lambda(t) \right) \ast g(t),$$

$$Q_{\Sigma}(t) = k_\Delta \left( \frac{x(t)}{d_x} \cdot q\Lambda(t) \right) \ast s(t) \ast g(t).$$

(13)

Here the order of multiplication is important: (1) Transverse position modulation $x(t)$, (2) Demodulation with $c(t)$ respectively $s(t)$.

Just as for Eq. (11) also for Eq. (13) the shape of the baseband response is defined solely by $g(t)$, with the amplitude now depending also on the excitation frequency and the longitudinal profile.

Digital Position Calculation

Four analog-to digital converters (ADC), clocked beam synchronously, sample the I/Q-pairs, providing each one digitized sample per bunch and signal.

Normalized bunch position follows from taking the ratio of the $\Delta$-signal over the $\Sigma$-signal, $X_N = \Delta/\Sigma$, which is independent of the per-bunch intensity. A more elegant way was found by mathematically expanding the ratio with the conjugate complex $\Sigma^*$,

$$X_N = \frac{\Delta \cdot \Sigma^*}{\Sigma \cdot \Sigma^*} = \frac{\Delta}{|\Sigma|^2}.$$  

(14)

By introducing the two phasors, $\Delta = A \cdot e^{ja}$ and $\Sigma = B \cdot e^{j\beta}$,

$$\Delta = A \cos \alpha + j A \sin \alpha \doteq I_\Delta + jQ_\Delta,$$

$$\Sigma = B \cos \beta + j B \sin \beta \doteq I_\Sigma + jQ_\Sigma,$$

(15)

we rewrite Eq. (14) in I/Q-components provided by the sampling,

$$X_N = \frac{I_\Delta I_\Sigma + Q_\Delta Q_\Sigma}{I_\Sigma^2 + Q_\Sigma^2} \pm \frac{j Q_\Delta I_\Sigma - I_\Delta Q_\Sigma}{I_\Sigma^2 + Q_\Sigma^2}.$$  

(16)
For perfect alignment of the two phasors (i.e. $\alpha - \beta = 0$) the first term in Eq. (16) maximizes. Only the real part of $X_N$ is used for the position calculation in the TFB for feedback, whereas the imaginary part provides an indication of head-tail activities and asymmetries in the longitudinal bunch profile.

**RESULTS**

**Simulation Model**

Thanks to the sampling of the continuous-time signals, where only one value is picked, it can be shown that the convolutions in Eqs. (11) and (13) reduce to definite integrals. Hence, the described analytical model further reduces to a more practical implementation which is essentially independent of hardware parameters.

The time-varying transverse position signal across a bunch and the longitudinal beam profile are multiplied with two fixed frequency signals in quadrature,

$$
\begin{align*}
    c(t) &= \cos(\omega_0 t), \\
    s(t) &= \sin(\omega_0 t),
\end{align*}
$$

where $\omega_0/(2\pi) = 400.8$ MHz for the case of the LHC TFB. The longitudinal profile is demodulated to baseband as,

$$
\begin{align*}
    \hat{I}_x(t) &= \int c(t) \lambda(t) \, dt, \\
    \hat{Q}_x(t) &= \int s(t) \lambda(t) \, dt.
\end{align*}
$$

For the case of the delta signal the longitudinal profile is first modulated with the position signal and subsequently demodulated to baseband, as denoted by

$$
\begin{align*}
    \hat{I}_\Delta(t) &= \int c(t) x(t) \lambda(t) \, dt, \\
    \hat{Q}_\Delta(t) &= \int s(t) x(t) \lambda(t) \, dt.
\end{align*}
$$

Finally, the normalization algorithm implemented in the LHC TFB follows from,

$$
x_N = \frac{\hat{I}_x \hat{I}_\Delta + \hat{Q}_x \hat{Q}_\Delta}{(\hat{I}_x)^2 + (\hat{Q}_x)^2}.
$$

**Numerical Simulation**

Figure 2(a) outlines the numerical input of the bunch length simulation, based on Ref. [2]. There, the first notch in the spectrum was found to be at around 1.5 GHz. This profile is modulated with an even-symmetric excitation up to 3 GHz. In Fig. 2(b) the blue trace indicates the result of Eq. (20), and the red curve indicates the true movement of the center-of-charges as given by Eq. (1). Clearly, the damper sensitivity to symmetric intra-bunch motion is a function of the longitudinal beam spectrum and extends significantly beyond the highest betatron frequency from coupled bunch oscillations up to 20 MHz, with the first notch appearing at around 1.9 GHz due to the demodulation.

In Fig. 2(c) the imaginary part of Eq. (16) is evaluated for odd-symmetric excitation (green). For the anti-symmetric case no oscillation amplitude is detected by the normalization algorithm (blue), confirming no movement of the center of charges, and therefore odd modes are not visible to the damper.

**SUMMARY AND CONCLUSION**

As a result of the analytical model a practical set of equations was found which describe the sensitivity of the LHC TFB to intra-bunch motions. Numerical simulations revealed that even-symmetric intra-bunch movements beyond 20 MHz are in fact seen with the current beam position signal processing of the LHC TFB, thereby corrective measures are applied in baseband (up to 20 MHz) by the feedback control. Provided that only one value per bunch is available the information on the excitation frequency is lost.

The damper sensitivity is a function of the longitudinal bunch spectrum (and the oscillation frequency). Notches in the beam spectra will render the damper blind for certain frequencies, however, they do not coincide with the blind frequencies as seen by the signal processing.

As part of these evaluations a modified processing scheme is formulated, which has the potential of indicating anti-symmetric intra-bunch oscillations e.g. for diagnostics.
REFERENCES
