Measurement of the Proton-Proton to ZZX Cross Section at a Center of Mass Energy of 8 TeV

Dissertation

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Abstract

A measurement of the $ZZ$ production in the $\ell^+\ell^-\ell'^+\ell'^-$ channel with $pp$ collisions at $\sqrt{s} = 8$ TeV using the full 2012 dataset amounting to an integrated luminosity of $20$ fb$^{-1}$ collected by the ATLAS experiment is presented in this paper. An event is selected when exactly four reconstructed, isolated leptons are found in the final state. Each pair of leptons reconstructed to form a $Z$ boson must have opposite sign and same flavor and have an invariant mass between 66 and 116 GeV. The background contribution is determined using a data-driven technique due to limited Monte Carlo statistics. Limits on the $f_4^\gamma, f_4^Z, f_5^\gamma, f_5^Z$ anomalous triple gauge couplings are also presented.
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Chapter 1
INTRODUCTION

During 2012, the ATLAS detector collected 20.04 fb\(^{-1}\) of \(pp\) data at a center of mass energy of 8 TeV. Combining the highest energy collisions ever recorded at a human made accelerator with the large amount of data collected allows incredibly precise measurements to be made that can probe the validity of the Standard Model. In the Standard Model (SM) there exist three massive vector bosons that mediate the Weak interaction: the \(W^\pm\) and \(Z\) bosons with respective masses of 80.4 GeV and 91.2 GeV. This paper will focus specifically on the \(pp \rightarrow ZZX \rightarrow l^+l^-l^+l^-\) process, which in the past, due to its very small rate, has been measured with low precision. The high luminosity achieved at the LHC presents a unique opportunity to measure with high precision the cross section of this process and also, with some extrapolation, the inclusive \(pp \rightarrow ZZX\) cross section. That being said, the process \(pp \rightarrow ZZX \rightarrow l^+l^-l^+l^-\) is significant at the LHC for multiple reasons. A precise measurement of the cross section compared with the SM prediction will provide further verification of the SM’s validity as a theory describing fundamental interactions observed in nature. There also exists the opportunity to set tighter limits on the anomalous triple gauge couplings associated with the \(pp \rightarrow ZZX \rightarrow l^+l^-l^+l^-\) process, thus limiting the range of beyond SM physics.

This paper will begin with an overview of the SM theory. The physical phenomena it describes including the fundamental interactions and their corresponding particles are discussed along with a brief description of the Higgs mechanism that, when included in the SM, gives rise to particle masses. Also discussed is the Electroweak interaction, of which
the $W$ and $Z$ bosons mediate.

A description of the ATLAS detector and its subsystems is also provided. The characteristics of all systems are mentioned, including but not limited to dimensions, materials and overall purpose.

Lastly, the analysis is presented. Object ($\mu, e$) selection, event selection and the exact process used to reconstruct $Z$ bosons will be discussed in great detail. The tools used to ensure the Monte Carlo matches the data are also discussed. The results of this analysis will be presented along with theoretical predictions and previous measurements.
Chapter 2
Theory

2.1 The Standard Model

The SM describes physical phenomena observed in nature via three fundamental interactions: the strong, weak and electromagnetic. It is represented by the $SU(3)_{\text{strong}} \times SU(2)_{\text{weak}} \times U(1)_{\text{EM}}$ symmetry groups. All three forces are mediated by gauge bosons. The gluon is the force carrier for the strong interaction that is only felt by particles with color charge such as quarks. The $W^\pm$ and $Z$ bosons are the force carriers for the weak interaction. The $W^\pm$ bosons are responsible for changing leptons into their corresponding neutrinos and vice versa and quark flavor changes and the $Z$ boson is responsible for neutral current interactions. The photon ($\gamma$) mediates the electromagnetic interaction. The SM includes three generations of fundamental particles (Fig. 2.1): leptons: electrons, muons and taus, in order of increasing mass and their corresponding neutrinos, quarks: up/down, charm/strange and top/bottom and the force carriers that are described above.

The SM Lagrangian is defined in Eq. 2.1 and contains a QCD term describing the strong interactions, while all other terms describe the Electroweak interactions (unified description of the electromagnetic and weak sectors). This paper will focus on the Electroweak sector as that is where the $pp \rightarrow ZZX \rightarrow l^+l^-l^+l^-$ process resides. Included in the SM Lagrangian is the Higgs scalar field $\phi$, whose presence is required to allow the appropriate particles to acquire mass. A more detailed explanation of the Higgs mechanism is provided in Appendix A. After some algebra and all the appropriate covariant derivatives are applied the physical gauge bosons ($W^\pm, Z, \gamma$) appear along with their masses.
\[
\mathcal{L} = -\frac{1}{4} F^i_{\mu\nu} F^{i\mu\nu} + \sum_r \bar{q}_r i D_\alpha^\beta q_r^\beta + \frac{1}{4} F^{(\text{EW})}_{\mu\nu} F^{(\text{EW})\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} + \text{QCD} + \text{Higgs}
\]

\[
(D^\phi)^\dagger (D_\phi) - \mu^2 \phi^\dagger \phi - \lambda (\phi^\dagger \phi)^2 + \sum^\infty_{m=0} (q_{mL}^0 i D_{q_{mL}}^0 + \bar{q}_{mL}^0 i D_{\bar{q}_{mL}}^0 + \phi \bar{q}_{mR}^0 i D_{\bar{q}_{mR}}^0 + \bar{q}_{mR}^0 i D_{\bar{q}_{mR}}^0) - \sum^\infty_{m,n=1} \left[ \Gamma^{u}_{mn} q_{mL}^{0} \phi^{n}_{u} + \Gamma^{d}_{mn} q_{mL}^{0} \phi^{n}_{d} + \Gamma^{e}_{mn} q_{mL}^{0} \phi^{n}_{e} \right]
\]

where

\[
F^i_{\mu\nu} = \partial_\mu G^i_\nu - \partial_\nu G^i_\mu - g s f_{ijk} G^j_\mu G^k_\nu
\]

\[
F^{(\text{EW})}_{\mu\nu} = \partial_\mu W^i_\nu - \partial_\nu W^i_\mu - g_\epsilon f_{ijk} W^j_\mu W^k_\nu
\]

\[
B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu
\]

\[
\tilde{\phi} \equiv i \tau^2 \phi^\dagger
\]

\[
q_{mL}^0 = \left(\begin{array}{c}
u_m^0 \\ d_m^0 \\ u_m^0 \end{array}\right)_L, \quad \bar{q}_{mL}^0 = \left(\begin{array}{c}e_m^0 \\ \nu_m^0 \\ l_m^0 \end{array}\right)_L
\]

\[
D_\phi^\alpha q_r^\beta = \gamma_\mu (\partial^\mu \delta^\alpha_\beta + i g s G^{i\mu} L^i_{\beta}) q_r^\beta
\]

The \(\Gamma_{mn}\) matrices describe the Yukawa couplings of the Higgs field to the various flavors of leptons and quarks and \(u_{mR}^0, \phi_{mR}^0, \epsilon_{mR}^0\) are the right handed singlet fields (for quarks and
The leptons. The $W^i$ and $B^i$ fields are the gauge fields for the $SU(2)$ and $U(1)$ groups.

Quark interactions are represented by the “QCD” term, self coupling of the gauge bosons is represented by the “Electroweak” term, fermion interactions are represented by the “fermions” term and the Yukawa term represents the Higgs-fermion coupling.

The $Z^\mu$ field identified as the $Z$ boson is defined as

$$Z^\mu = \frac{gW^3_\mu - g' B^\mu}{\sqrt{g^2 + g'^2}} \equiv \cos \theta_W W^3_\mu - \sin \theta_W B^\mu$$ (2.3)

and the $A_\mu$ field identified as the photon is defined as

$$A_\mu = \frac{g'W^3_\mu + g B_\mu}{\sqrt{g^2 + g'^2}} \equiv \sin \theta_W W^3_\mu + \cos \theta_W B_\mu$$ (2.4)

where the Weinberg angle is defined as:

$$\cos \theta_W = \frac{g}{\sqrt{g^2 + g'^2}}$$ (2.5)

and the $W^1_\mu, W^2_\mu$ fields are identified as the $W^\pm$ boson. As the symmetry is spontaneously broken the $W$ and $Z$ bosons acquire unique masses defined in Eq. 2.6 and Eq. 2.7 respectively.

$$M^2_W = \frac{g^2 \eta^2}{2}$$ (2.6)

$$M^2_Z = \frac{g^2 \eta^2}{2 \cos^2 \theta_W} = \frac{M^2_W}{\cos^2 \theta_W}$$ (2.7)

where $\eta$ is the Higgs potential vacuum expectation value of 246 GeV.

Using the relationship defined in Equation 2.7, if one experimentally measures $\theta_W$ (Weinberg angle) and $M_W, M_Z$ is predicted to be 89.3 GeV which is in agreement with the measured $Z$ mass of 91.2 GeV. The same can be said for $M_W$ and $\theta_W$. This agreement between predicted and experimentally measured values is a remarkable success of electroweak theory.

### 2.2 The Higgs Boson

The $pp \to ZZX \to l^+l^-l^+l^-$ process is an irreducible background to a search for a heavy Higgs boson and is therefore important to measure. In July of 2012 ATLAS and CMS both
observed an excess of events at a mass of approximately 126 GeV (Figs. 2.2 and 2.3). As of March 6th, 2013 the combined signal strength \( \mu = \frac{\sigma}{\sigma_{SM}} \) where \( \sigma_{SM} \) is the cross section predicted by the SM) of all Higgs channels was measured to be \( 1.43 \pm 0.16 \pm 0.14 \) at a mass of 125.5 GeV [4]. As recently as April 16th, 2013 a spin study was performed yielding results showing that the resonance observed resembles a spin 0 state, with an exclusion of the spin 1 state due to the \( H \rightarrow \gamma\gamma \) process and Landau-Yang theorem, and an exclusion of the spin 2 state at a confidence level above 95 % [5]. So far this resonance appears to resemble the SM Higgs boson. Since this particle has a mass less than twice that of the Z boson it falls outside of the measured region defined in this paper.
2.3 Parton Distribution Functions

At the LHC, protons are accelerated and collide with one another at high energies. It is therefore important to be able to accurately model these proton-proton interactions. The proton is made of two up quarks ($u$) and one down quark ($d$) however there exist within the proton gluons and virtual pairs of quarks known as sea quarks. This means that during a proton-proton collision there are many different interactions that can occur.

Parton distribution functions (PDFs) describe the probability of finding a parton of flavor $i$ (quark or gluon) carrying a fraction $x$ (Bjorken scaling variable) of the proton’s momentum with the energy scale of the collision being defined as $Q$. The PDFs can be measured using Deep Inelastic Scattering processes such as $e^+p \rightarrow e^+X$ to probe the substructure of hadrons. Structure functions ($f_i(x, Q^2)$) are then defined for gluons and each flavor of quark (sea and valence) present within the hadrons. Each structure function can
be used to determine the contribution to the total cross section due to each interacting quark and/or gluon. The parton distribution function is then the sum of all the structure functions.

2.4 Standard Model $pp \rightarrow ZZ \rightarrow l^+l^-l'^+l'^-$ Production

The SM electroweak sector described by the non-abelian gauge group $SU(2)_L \times U(1)_Y$ has proven to successfully describe experimental observations. That being said, the process $ZZ \rightarrow l^+l^-l'^+l'^-$ is a significant process at the LHC for multiple reasons. A precise measurement of the cross section compared with the SM prediction will provide further verification of the SM’s validity as a theory describing fundamental interactions observed in nature. In the past a precise measurement of this process was unfeasible as it was suppressed by the branching fraction of the Z bosons decaying into leptons. With the current data collected at ATLAS this is no longer a limitation. The primary amplitudes of ZZ production are seen in figure 2.4.
Figure 2.4: The tree level feynman diagrams of primary modes of ZZ production at the LHC. The $q\bar{q}$ initial state production modes (figures (a) and (b)) account for approximately 94% of the total ZZ cross section while the $gg$ modes (figures (c) and (d)) account for the rest of the cross section.

Figure 2.5: The diagram above shows ZZ s-channel production via an anomalous triple gauge coupling vertex which is currently forbidden in the SM.

The $pp \rightarrow ZZX \rightarrow l^+l^-l^+l^-$ process can also help probe new physics beyond the SM. It could be a final state of a heavy Higgs boson where $m_{Higgs} > 2m_Z$. It is affectionately referred to as the “golden channel” as it is an incredibly clean final state with little background. It is also a final state of the light Higgs discovered in July of 2012 as well, but since $m_{LightHiggs} < 2m_Z$ one Z must be off-shell and does not fall in the scope of this paper. Other beyond SM processes that couple to this process include a doubly-charged Higgs $[6]$, radion resonances $[7]$ and supersymmetric particles. Anomalous triple gauge
couplings, as shown in figure 2.5, are also another possibility where this final state proves to be a sensitive probing mechanism and helps determine limits on the associated couplings.

Currently the predicted cross section for the process \(pp \rightarrow ZZ\) at \(\sqrt{s} = 8\) TeV with next to leading order corrections and including the \(gg\) contribution is \(7.23^{+0.30}_{-0.22}\) pb (the cross section is calculated using the CT10 pdf)\[8\]. The OPAL, ALEPH and DELPHI collaborations at LEP probed \(Z\) boson pair production using the process \(e^+e^- \rightarrow l^+l^-l^+l^-\). The D0 experiment at the Tevatron collider has also measured the \(ZZ\) cross section via the \(ZZ \rightarrow l^+l^-l^+l^-\) channel at \(\sqrt{s} = 1.96\) TeV using \(6.4\) fb\(^{-1}\) of data to be \(1.40^{+0.43}_{-0.37}\) (stat.)\(\pm 0.14\) (sys.) pb. The CDF experiment measured the \(ZZ\) cross section to be \(1.47^{+0.7}_{-0.6}\) pb (Fig. 2.7) where the quoted errors are the statistical and systematic errors added in quadrature.

2.5 Anomalous Triple Gauge Coupling (aTGC) Theory and Introduction

Figure 2.4 shows the allowed SM ZZ production modes and figure 2.5 shows the forbidden s-channel mode that contains an aTGC vertex. ATLAS is not the first experiment to probe aTGC enhancements. The D0 experiment set limits on anomalous \(ZZZ\) and \(ZZ\gamma^*\) couplings.
using 1.9 fb$^{-1}$ of data [9]. ATLAS [10] and CMS [11] have also set limits on aTGC couplings using 5 fb$^{-1}$ of data at $\sqrt{s} = 7$ TeV.

The approach taken here to describe the aTGC enhancement to the SM ZZ cross section is to use an effective Lagrangian. In the case where both Z’s in the final state are on-shell there are four couplings in the effective Lagrangian: two $ZZZ$ couplings and two $ZZ\gamma$ couplings. The aTGC Lagrangian that yields the vertex function in Figure 2.8 is as follows [12]

$$\mathcal{L} = -\frac{e}{M_Z} \left[ f_4^V \left( \partial_\mu V^\nu \right) Z_\alpha (\partial^\alpha Z_\beta) + f_5^V \left( \partial^\sigma V_\sigma \right) \tilde{Z}_\mu \tilde{Z}_\nu \right]$$

(2.8)

where $V = Z, \gamma$, $V_{\mu \nu} = \partial_\mu V_{\nu} - \partial_\nu V_{\mu}$, $Z$ is the $Z$ boson and $\tilde{Z}_\mu = \frac{1}{2} \epsilon_{\mu \beta \rho \sigma} Z^\rho Z^\sigma$. 

Figure 2.7: Predicted and measured $ZZ$ cross section versus center of mass energy.
Figure 2.8: The general vertex for anomalous triple gauge boson couplings where the final state contains two Z bosons and internal line $V_{\mu}$ can be either a Z or $\gamma$.

The general form of this particular triple gauge vertex (Figure 2.8) is written as \[ \begin{align*}
g_{ZZV}^\alpha \Gamma_{ZZV}^{\alpha \beta \mu} &= e \frac{P^2 - M_Z^2}{M_Z^2} \left[ i f_4^V (P^\alpha g^{\mu \beta} + P^\beta g^{\mu \alpha}) + i f_5^V \epsilon^{\mu \alpha \beta \rho} (q_1 - q_2) \right] 
\end{align*} \]

(2.9)

The couplings $f_4^V$ and $f_5^V$ are complex functions of $q_1^2, q_2^2$ and $P^2$ \[ \text{[13][14]} \] ($q_1, q_2$ and $P$ are the momenta of the particles in Figure 2.8) and are also dimensionless. The $f_4^V$ couplings are CP violating while the $f_5^V$ couplings are CP conserving. All couplings are C odd.

As $\sqrt{\hat{s}}$ ($\hat{s}$ is the center of mass energy squared) increases, these couplings innately grow. In order to preserve partial wave unitarity a form factor with $\hat{s}$ dependence is introduced to the couplings. The form factor is defined as

\[ \frac{1}{(1 + \frac{\hat{s}}{\Lambda_{FF}^2})^n} \]

(2.10)

This yields the following expression for the couplings which now have an $\hat{s}$ dependence

\[ f_i^V(\hat{s}) = \frac{f_i^V}{(1 + \frac{\hat{s}}{\Lambda_{FF}^2})^n} (i = 4, 5) \]

(2.11)

where $f_i^V$ are the values of $f_i^V(M_Z^2, M_Z^2, 0)$ at $\hat{s} = 0$ and $\Lambda_{FF}$ is the energy scale at which
physics beyond the SM becomes observable. In this analysis $n = 0$.

The presence of non-trivial aTGC couplings has the effect of an enhanced ZZ cross section at high center of mass energy and large scattering angles. Therefore, the spectra of the invariant mass of the diboson system and the transverse momenta of the Z bosons (Fig. 2.9) are most sensitive to the effects the aTGC enhancement.

Figure 2.9: The $p_T$ spectrum of the leading Z boson in a signal event for various TGC point values. As is expected there is a significant enhancement in the higher $p_T$ regime.
Chapter 3

The LHC and the ATLAS Detector

ATLAS (A Toroidal LHC ApparatuS) [15] is one of two general purpose detectors on the LHC ring and is designed to survive the extreme conditions provided by the Large Hadron Collider (LHC) caused by the collisions of two counter-rotating proton beams. Designed with a 14 TeV center of mass energy, design luminosity of $10^{34} \text{ cm}^{-2} \text{ s}^{-1}$ and a bunch spacing of 25 ns the LHC is the most powerful collider in the world.

The ATLAS detector (as seen in Figure 3.1) contains several sub-systems each designed with a particular purpose to help verify the Standard Model and look for physics extending beyond the Standard Model. Going from interaction point (IP) outwards we have the BCM (Beam Conditions Monitor), BLM (Beam Loss Monitor), pixel detector, Semi-Conductor Tracker (SCT), Transition Radiation Tracker (TRT), electromagnetic calorimeter (EMcal), Liquid Argon (LAr/hadronic) calorimeter and the muon tracking system. All sub-systems provide vital information on particle energy/momentum and direction.

3.1 Definitions of Coordinates

The coordinate system in ATLAS defines the z-axis to be along the beampipe and the x-y plane as the transverse plane. The positive x-axis points toward the center of the ring from the interaction point (IP) and the y-axis points upwards. $\phi$ (azimuthal angle) is defined to be around the beam axis and $\theta$ (polar angle) is measured from the beam axis. The rapidity
is defined as
\[ y = \frac{1}{2} \frac{E + p_z c}{E - p_z c} \] (3.1)
and the difference in rapidity (\(\delta y\)), is invariant under boosts. As is common with high energy experiments, \(\eta\) or pseudo-rapidity is used in place of rapidity. It is approximated as \(-\ln \tan(\frac{\theta}{2})\). Separation between tracks is often measured in \(\eta, \phi\) space and is defined as
\[ \Delta R = \sqrt{\delta \eta^2 + \delta \phi^2}. \]

\[ \text{Figure 3.1: The ATLAS detector with all sub-systems listed.} \]

\[ \textbf{3.2 Design Purpose} \]

The design of ATLAS was motivated by certain physics processes. The Higgs boson decay channel \(H \rightarrow \gamma\gamma\) requires a precise and accurate energy measurement whereas the \(H \rightarrow ZZ \rightarrow l^+l^-l^+l^-\) decay mode necessitates a precise and accurate momentum measurement.
Very good lepton identification is a necessity for the latter channel as well. In order to effectively identify electrons and photons very efficient electromagnetic calorimetry was essential. Along with that, full-coverage hadronic calorimeter was necessary to accurately measure jets and missing transverse energy. The tracking systems must be able to accurately reconstruct tracks in both the inner detector and in the muon spectrometer from low to high $p_T$ meaning they must provide high-granularity and very precise measurements. To ensure maximum acceptance at large $\eta$ and nearly full azimuthal coverage is needed.

### 3.3 Magnet System

#### 3.3.1 Central Solenoid

The entire Inner Detector and BCM sit within the ATLAS central solenoid. It is designed to generate a magnetic field of 2 T (1.998 T at the center of the magnet using the nominal 7.730 kA current) [3]. Since the calorimeters sit outside the solenoid, the thickness of the solenoid was kept at an absolute minimum to reduce energy loss of particles traversing the detector, which led to the solenoid having approximately 0.66 radiation lengths incident along the normal. The single-layer coil is made with a high-strength Al-stabilized NbTi conductor that was specifically designed to achieve the high field needed for particle tracking while simultaneously minimizing the thickness of the magnet. The inner and outer diameters of the solenoid are 2.46 m and 2.56 m respectively and measures 5.8 m in the z direction. It has a weight of approximately 5.4 tonnes and stores 40 MJ of energy. This results in a stored-energy-to-mass ratio of 7.4 kJ/kg [3]. It takes 30 minutes to charge and discharge the solenoid. In the case of a quench the energy is absorbed by the cold mass consequently raising the temperature of the cold mass to a maximum of 120 K (which is still deemed “safe”). It can be re-cooled to 4.5 K in about one day. A summary of the specifications of both the solenoid and the toroids is available in figure 3.2 and the bare solenoid can be seen in figure 3.4.
### Figure 3.2: Specifications of the ATLAS magnet system.

<table>
<thead>
<tr>
<th>Property</th>
<th>Feature</th>
<th>Unit</th>
<th>Solenoid</th>
<th>Barrel toroid</th>
<th>End-cap toroids</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Size</strong></td>
<td>Inner diameter</td>
<td>m</td>
<td>2.46</td>
<td>9.4</td>
<td>1.65</td>
</tr>
<tr>
<td></td>
<td>Outer diameter</td>
<td>m</td>
<td>2.56</td>
<td>20.1</td>
<td>10.7</td>
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<tr>
<td></td>
<td>Axial length</td>
<td>m</td>
<td>5.8</td>
<td>25.3</td>
<td>5.0</td>
</tr>
<tr>
<td><strong>Mass</strong></td>
<td>Number of coils</td>
<td></td>
<td>1</td>
<td>8</td>
<td>2 x 8</td>
</tr>
<tr>
<td></td>
<td>Conductor</td>
<td>t</td>
<td>3.8</td>
<td>118</td>
<td>2 x 20.5</td>
</tr>
<tr>
<td></td>
<td>Cold mass</td>
<td>t</td>
<td>5.4</td>
<td>370</td>
<td>2 x 140</td>
</tr>
<tr>
<td></td>
<td>Total assembly</td>
<td>t</td>
<td>5.7</td>
<td>830</td>
<td>2 x 239</td>
</tr>
<tr>
<td><strong>Coils</strong></td>
<td>Turns per coil</td>
<td></td>
<td>1154</td>
<td>120</td>
<td>116</td>
</tr>
<tr>
<td></td>
<td>Nominal current</td>
<td>kA</td>
<td>7.73</td>
<td>20.5</td>
<td>20.5</td>
</tr>
<tr>
<td></td>
<td>Magnet stored energy</td>
<td>GJ</td>
<td>0.04</td>
<td>1.08</td>
<td>2 x 0.25</td>
</tr>
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<td></td>
<td>Peak field in the windings</td>
<td>T</td>
<td>2.6</td>
<td>3.9</td>
<td>4.1</td>
</tr>
<tr>
<td></td>
<td>Field range in the bore</td>
<td>T</td>
<td>0.9–2.0</td>
<td>0.2–2.5</td>
<td>0.2–3.5</td>
</tr>
<tr>
<td><strong>Conductor</strong></td>
<td>Overall size</td>
<td>mm$^2$</td>
<td>30 x 4.25</td>
<td>57 x 12</td>
<td>41 x 12</td>
</tr>
<tr>
<td></td>
<td>Ratio Al:Cu:NbTi</td>
<td></td>
<td>15.6:0.9:1</td>
<td>28:1.3:1</td>
<td>19:1.3:1</td>
</tr>
<tr>
<td></td>
<td>Number of strands (NbTi)</td>
<td></td>
<td>12</td>
<td>38–40</td>
<td>40</td>
</tr>
<tr>
<td></td>
<td>Strand diameter (NbTi)</td>
<td>mm</td>
<td>1.22</td>
<td>1.3</td>
<td>1.3</td>
</tr>
<tr>
<td></td>
<td>Critical current (at 5 T and 4.2 K)</td>
<td>kA</td>
<td>20.4</td>
<td>58</td>
<td>60</td>
</tr>
<tr>
<td></td>
<td>Operating/critical-current ratio at 4.5 K</td>
<td>%</td>
<td>20</td>
<td>30</td>
<td>30</td>
</tr>
<tr>
<td></td>
<td>Residual resistivity ratio (RRR) for Al</td>
<td></td>
<td>&gt; 500</td>
<td>&gt; 800</td>
<td>&gt; 800</td>
</tr>
<tr>
<td></td>
<td>Temperature margin</td>
<td>K</td>
<td>2.7</td>
<td>1.9</td>
<td>1.9</td>
</tr>
<tr>
<td></td>
<td>Number of units × length</td>
<td>m</td>
<td>4 × 2290</td>
<td>8 × 4 × 1730</td>
<td>2 × 8 × 2 × 800</td>
</tr>
<tr>
<td></td>
<td>Total length (produced)</td>
<td>km</td>
<td>10</td>
<td>56</td>
<td>2 x 13</td>
</tr>
<tr>
<td><strong>Heat load</strong></td>
<td>At 4.5 K</td>
<td>W</td>
<td>130</td>
<td>990</td>
<td>330</td>
</tr>
<tr>
<td></td>
<td>At 60–80 K</td>
<td>kW</td>
<td>0.5</td>
<td>7.4</td>
<td>1.7</td>
</tr>
<tr>
<td></td>
<td>Liquid helium mass flow</td>
<td>g/s</td>
<td>7</td>
<td>410</td>
<td>280</td>
</tr>
</tbody>
</table>

3.3.2 Toroid Magnets

The muon spectrometer utilizes three large superconducting air-core magnets covering the pseudorapidity range $0 \leq |\eta| \leq 2.7$. They have an open structure that minimizes the effects on momentum resolution due to multiple scattering. The barrel toroid (BT) magnets measure 25 m in the $z$ direction, an inner diameter of 9.4 m and outer diameter of 20.1 m. The end cap toroids (ETCs) have a length in the $z$ direction of 5.0 m, inner diameter of 1.65 m and outer diameter of 10.7 m.

Each toroidal magnet is made of eight flat coils. They are situated radially and symmetrically about the beam axis. The end cap toroid magnets are situated with an azimuthal offset of 22.5° with respect to the BT which can be seen in figure 3.5. This results in an
overlap between the fields that helps maximize the bending power in the transition region between the barrel and end caps.

![Magnetic field map in the transverse plane](image)

Figure 3.3: Above is a magnetic field map in the transverse plane (x-y plane) in the transition region between the barrel and end cap toroids in the muon spectrometer. The separation between the lines is 0.1 Tm. Also visible are the BT and ECT coils.

### 3.4 Beam Conditions Monitor

A possible worst case scenario for any experiment at the LHC is when the beams scrape the collimators designed to protect the detectors. When this happens, a large spray of particles comes from the beam-collimator interaction. Although the radiation dose associated with these types of events are within the design limits of the detector (integrated dose), the large instantaneous rate of particle flux could damage one or more of the detector subsystems. The BCM was designed to protect the ATLAS experiment from the aforementioned type
3.4.1 Luminosity Measurement

It is paramount to determine the amount of data collected at the LHC. Without this information it would be impossible to determine if experimental observations are in agreement with the theoretical predictions. At particle colliders this can be determined by counting...
the number of particles scattered after a primary collision occurs. The number of scattered 
particles (N) can be written as:

\[ \frac{dN}{d\Omega dt} = L \frac{d\sigma}{d\Omega} \]  

(3.2)

Where \( \sigma \) is the scattering cross section, \( d\Omega \) is the differential solid angle element and L is the luminosity that is most commonly expressed in units of cm\(^{-2}\)s\(^{-1}\). Integrating over \( d\Omega \) gives:

\[ \frac{dN}{dt} = L\sigma \]  

(3.3)

\( \frac{dN}{dt} \) can be thought of as scattering rate and L is the instantaneous luminosity.

The ATLAS approach to luminosity monitoring is through event counting. The number 
of detected particles (\( N_{\text{det}} \)) can be related to the number of particles in a bunch crossing 
(\( N_{BC} \))

\[ N_{\text{det}} = N_{BC}(1 - e^{-BL}) \]  

(3.4)

where \( B \) depends on total cross section and the detector efficiency.

Equation 3.3 can be rearranged to represent the luminosity in terms of \( \langle \mu \rangle \) (average 
interactions per bunch crossing) and \( f_{\text{rev}} \) (the frequency of revolution) as

\[ L = \frac{\langle \mu \rangle f_{\text{rev}}}{\sigma}. \]  

(3.5)

There is a problem however. We do not know the true values of \( \langle \mu \rangle \) and \( \sigma \), but we can 
express 3.5 in terms of the their visible or measured values

\[ \mu_{\text{vis}} = \langle \mu \rangle \epsilon \] 

\[ \sigma_{\text{vis}} = \sigma \epsilon \]  

(3.6)

where \( \epsilon \) is the detector efficiency of a single p-p interaction. The BCM is capable of mea-
suring \( \mu_{\text{vis}} \) so the only unknown remaining is the visible cross section.
Van der Meer Scans

In order to measure the visible cross section a calibration constant specific to the BCM must be measured as well. This is done by performing Van der Meer scans [16]. The Van der Meer approach expresses the luminosity in terms of the beam parameters. The luminosity of a single bunch can be written as

$$ L = f_{\text{rev}} N_1 N_2 \int \rho_1(x,y) \rho_2(x,y) dxdy $$ \hspace{1cm} (3.7)

where $\rho_{1,2}(x,y)$ are the particle density functions for beams 1 and 2. Assuming these are independent in $x$ and $y$, the overlap integrals in $x$ and $y$ can be written as

$$ \Omega_x(\rho_1, \rho_2) = \int \rho_1(x) \rho_2(x) dx \Omega_y(\rho_1, \rho_2) = \int \rho_1(y) \rho_2(y) dy. $$ \hspace{1cm} (3.8)

The overlap integrals can then be related to a measured rate, $R_x \delta$, as a function of beam separation $\delta$ and are shown in Equation 3.9.

$$ \Omega_x(\rho_1, \rho_2) = \frac{R_x(0)}{\int R_x(\delta) d\delta} $$ \hspace{1cm} (3.9)

The luminosity is often expressed in terms of an effective beam width defined as

$$ \Sigma_x(\rho_1, \rho_2) = \frac{1}{\sqrt{2\pi}} \frac{R_x(0)}{\int R_x(\delta) d\delta}. $$ \hspace{1cm} (3.10)

The luminosity is then written as

$$ L = \frac{f_{\text{rev}} N_1 N_2}{2\pi \Sigma_x \Sigma_y}. $$ \hspace{1cm} (3.11)

During the scan, one beam is held fixed (at the central point $x=0$, $y=0$) and the other beam is moved in either the $x$ or $y$ direction only. Measuring the interaction rate as the displacement changes gives the distributions $R_{x,y}(\delta)$. It is also important to note that in this particular case the rates measured by the BCM are actually the average interactions per bunch crossing. Equating Equations 3.5 (using the visible quantities) and 3.11 the visible
cross section is

$$\sigma_{vis} = \frac{2\pi \Sigma_x \Sigma_y \mu_{vis}}{N_1 N_2}. \quad (3.12)$$

Using the visible cross section measurement and $\mu_{vis}$ measured by the BCM, Equation 3.5 will yield a measurement of the instantaneous luminosity.

### 3.4.2 Diamond Modules

The BCM consists of 8 diamond modules (four on each side) approximately 0.6 cm$^2$ (0.81 cm $\times$ 0.81 cm) in area on both sides of the detector. It is centered at $z = 0$ (along the beampipe) with modules at $z = \pm 1.86$ m. Since the BCM is situated in such close proximity (6 cm radially) to the interaction point, a material that can withstand the large radiation dose and fluence associated with that environment must be used. It is for these reasons that diamond was the material chosen to be the active medium.

![Diamond Sensor Schematic](image)

Figure 3.7: A schematic of a diamond sensor in the BCM. A charged particle traverses the diamond and knocks loose electrons which then drift towards the electrode held at positive high voltage.

The diamond sensors are designed to tolerate up to 500 kGy doses of radiation and a fluence of $10^{15}$ charged particles per cm$^2$ over the lifetime of the BCM (10 years). Two
sensors are arranged back to back (double-decker assembly) in order to achieve twice the signal compared to a single module; the noise is measured to be less than twice that of the single module assembly. The sensors are oriented so that particle incidence is at an angle of 45° with respect to the beam line (z-axis). This increases the particle path length and signal by $\sqrt{2}$.

Each diamond module of the BCM outputs rates to two channels: High Gain (Low Threshold) and Low Gain (High Threshold). The High Gain output is sensitive enough to measure individual minimum ionizing particles (MIPs) which is essential to measuring the luminosity with a very low uncertainty. The Low Gain channels are much less sensitive since their purpose is to measure the much larger rates that are associated with dangerous beam losses. The BCM can also distinguish between “collisions” and “background” using the particle time of flight between sides A and C and the interaction point. The BCM uses an event counting algorithm to measure the luminosity and therefore only “collisions” should be used to determine the luminosity.

### 3.5 Inner Detector

The ATLAS Inner Detector (ID) was designed to give robust and hermetic charged-particle tracking. Along with providing excellent momentum resolution it also provides primary and secondary vertex measurements up to $|\eta| < 2.5$. Through the use of the Transition Radiation Tracker (TRT), electron identification is provided up to $|\eta| < 2.0$ and in the energy range of 0.5 GeV to 150 GeV. All three of these systems and the BCM sit within the 2 T solenoidal field.

#### 3.5.1 Pixel Detector

The pixel detector is the innermost tracking system consisting of three layers in both the barrel and the endcaps: the B-layer (closest to the interaction point), layer 1 and layer 2. With 1744 pixel modules on board and 47232 pixels on each module, the detector contains approximately 80 million pixels. Approximately 90% of the pixels have a size of
50 \((r - \phi)\times 400(z)\) \(\mu m^2\) while the remaining have a size of 50 \((r - \phi)\times 600\) \(\mu m^2\).

Figure 3.8: A cross sectional view of the pixel detector in the z-y plane. The \(\eta\) coverage is shown along with the location of the three layers within the detector.

The pixel detector provides tracking information up to \(|\eta| < 2.5\) which can be seen in Fig. 3.8. The sensors used must be able to perform adequately over the lifetime of the detector which necessitates the ability to function continuously in a high radiation environment. The leakage current of the sensors increases with radiation dose. This combined with the n-type doping becoming primarily p-type doping after a neutron equivalent fluence of approximately \(2 \times 10^{13} \text{ cm}^{-2}\) requires the sensors to operate in a temperature range of \(-5^\circ C\) to \(-10^\circ C\). The sensors are oxygenated n-type wafers with a thickness of 250 \(\mu m\) with the readout pixels located on the n\(^+\)-implanted side of the sensor. The n\(^+\) implants and
oxygenated material help maintain a good charge-collection efficiency after type inversion. The oxygenated material also increases radiation tolerance to charged hadrons. At the beginning of operation, the sensors will operate at 150 V which can increase up to 600 V after an extended amount of time (around 10 years).

3.5.2 Semiconductor Tracker

The semiconductor tracker (SCT) is another silicon tracking detector located just outside the pixel detector. Instead of using silicon pixels, the SCT implements the use of silicon micro-strip sensors arranged in four cylindrical double layers (one axial and one at a stereo angle of 40 mrad). Due to cost and reliability, the SCT sensors employ a classic single p-in-n technology. These sensors are currently operated at a voltage of approximately 150V, but after ten years of high radiation operation, a bias voltage between 250 and 350 V will be necessary to provide adequate charge collection efficiency. In the barrel region, the strips have a size of $17 (r - \phi) \times 580 \ (z) \ \mu m^2$ and in the end cap disks a size of $17 (r - \phi) \times 580 \ (r) \ \mu m^2$. The sensors are situated at a pitch of $80 \ \mu m$, however in the end cap disks one set of strips is arranged radially instead of axially. The SCT is also equipped with interferometric monitoring allowing to detect how much the modules have moved over a period of time.

Similar to the pixel sensors the sensors in the SCT must also be functional in the high radiation environment. The same specifications mentioned earlier in the pixel detector subsection apply to the SCT sensors as well.

3.5.3 Transition Radiation Tracker

The Transition Radiation Tracker (TRT) is the outermost subsystem of the Inner Detector. The TRT, like the pixel and SCT detectors, has a barrel region and an endcap region. The TRT barrel uses polyimide drift tubes (straws) oriented axially to provide tracking, whereas the end cap region utilizes the same straws oriented radially to track charged particles. The TRT was designed to be able to track charged particles with $p_T > 5 \text{ GeV}$ and $|\eta| < 2.0$ (the TRT barrel acceptance is $|\eta| < 1.0$, the endcaps extend to 2.0). The gas mixture inside the tubes is comprised of 70% Xe, 27% CO$_2$ and 3% O$_2$. The cathodes are operated at
1530V to give a gain of $2.5 \times 10^4$. Discrimination between electrons and pions is possible since electrons will lose much more energy via transition radiation than pions.

As was mentioned previously, all the sub-systems of the ID play a complimentary role to one another and this can be seen in the overall momentum resolution of the ID (Fig. 3.11 and 3.12). For high-momentum tracks the relative momentum resolution was measured to be $\frac{\sigma_p}{p} = (4.83 \pm 0.16) \times 10^{-4}$ GeV$^{-1} \times p_T$ [17].

### 3.6 Calorimetry

The calorimeters in the ATLAS detector play a hugely important role in the physics program. Among these, some notable processes include the photonic and leptonic final states of Higgs decays, in particular, $H \rightarrow \gamma \gamma$ and $H \rightarrow Z^0 Z^0 \rightarrow e^+ e^- e^+ e^-$. Excellent mass resolution (on the order of 1%) is needed. Accurate energy measurements are absolutely essential for precision electroweak processes such as $Z \rightarrow e^+ e^-$ and $W \rightarrow e^+ \nu_e$. This requires an accuracy over a large range of $p_T$ (from a few GeV to a few TeV).
3.6.1 Electromagnetic Calorimeter

The electromagnetic calorimeter consists of the barrel calorimeter and end cap calorimeters. The barrel calorimeter consists of two half barrels that cover an eta range of $|\eta| < 1.475$, $z = \pm 3.2$ m with inner and outer diameters of 2.8 m and 4.0 m respectively. The material in front of the barrel calorimeter consists of approximately 1.5 $X_0$. It is due to this that a liquid argon presampler detector is situated in front of its inner surface over the full $\eta$ range. Each half barrel has 1024 accordion absorbers interleaved with the electrode readouts. The drift gap on either side of the electrodes is 2.1 mm corresponding to a drift time of approximately 450 ns at a voltage of 2 kV. There are a total of 32 modules in the barrel (16 per half barrel). The thickness of a module equates to at least 22 $X_0$: 22 $X_0$ to 30 $X_0$ over $|\eta| < 0.8$ and 24 $X_0$ to 33 $X_0$ over $|\eta| > 0.8$ and $|\eta| < 1.3$.

$^1 X_0$ is defined as the amount of material an electron needs to traverse until its energy is $\frac{E_0}{e}$ where $E_0$ is the initial energy of the electron.
Each module contains absorbers made of lead plates (1.53 mm for $|\eta| < 0.8$ and 1.13 mm for $|\eta| > 0.8$) that are glued between two 0.2 mm thick stainless steel sheets using resin-impregnated glass fiber fabric. The module’s readout electrodes consist of three copper layers separated by insulating polyimide sheets: the two outer layers are held at high voltage while the inner layer is used to read out the signal.

The electromagnetic end-cap calorimeter (EMEC) is a lead-liquid argon sampling calorimeter that uses interleaved accordion-shaped absorbers and electrodes. It covers an $\eta$ range of $1.375 < |\eta| < 3.2$. The material in front of the end cap calorimeters results in energy loss and requires a liquid argon presampler to be implemented over the $\eta$ range of $1.5 < |\eta| < 1.8$. For construction reasons, there are two coaxial wheels that make up the end caps with the boundary placed at $\eta = 2.5$ to match the tracking coverage of the Inner Detector. There are a total of 1024 absorbers and readout electrodes in the end cap calorimeters (768 in the outer wheel and 256 in the inner wheel). The lead absorbers are
sandwiched between 0.2 mm thick stainless steel layers and are 1.7 mm thick in the outer wheel and 2.2 mm thick in the inner wheel. The active thickness of an end cap is larger than \( 24 X_0 \) up to \( \eta = 1.475 \), \( 38 X_0 \) from 1.475 to 2.5 and \( 36 X_0 \) from 2.5 to 3.2.

Both the barrel and end cap calorimeters are sampling calorimeters and consist of three compartments per module: front, middle and back each with different granularities. The barrel calorimeter has an energy resolution of \( \sigma_E = 10\% \cdot \sqrt{E} \) where \( E \) is the energy of the particle [18].

### 3.6.2 Hadronic Calorimeter

**Tile Calorimeter (Tile Cal)**

Similar to the electromagnetic calorimeters, the hadronic calorimeter consists of a barrel and end cap calorimeter. The central hadronic calorimeter (also known as the Tile Calorimeter) is a sampling calorimeter. It uses steel absorbers and plastic scintillating tiles as the active
The ATLAS calorimetry is shown above. The LAr electromagnetic calorimeter (barrel and endcaps), the hadronic calorimeter (tile calorimeter and end caps) and the LAr forward calorimeter are all shown.

The Tile Calorimeter covers an $\eta$ range of $|\eta| < 1.7$ and is divided into the long barrel (LB) covering $|\eta| < 1.0$ and the extended barrels (EB) covering $0.8 < |\eta| < 1.7$. The long barrel measures 5640 mm ($z$) and the extended barrel measures 2910 mm ($z$), and the inner and outer radii measure 2280 mm and 4230 mm respectively. Both the long and extended barrel regions are divided into 64 wedge modules in $\phi$. The innermost cells (A and BC cells seen in Fig. 3.15) have a granularity of $\Delta \eta \times \Delta \phi = 0.1 \times 0.1$ and $\Delta \eta \times \Delta \phi = 0.2 \times 0.1$ for the outermost layer (D cells). These modules are segmented into three layers with a respective thickness of 1.5, 4.1 and 1.8 $\lambda$ (where $\lambda$ is the average interaction length for a proton) in the long barrel and 1.5, 2.6 and 3.3 $\lambda$ in the extended barrel.
Figure 3.14: A section of the electromagnetic calorimeter is shown above. The lead absorbers assembled in the ”accordion” shape can be seen along with the segmented chambers characteristic of a sampling calorimeter.

**Hadronic End Cap Calorimeter (HEC)**

The hadronic end cap calorimeter (HEC) covers the $\eta$ range $1.5 < |\eta| < 3.2$, uses copper absorbers and liquid Argon as the active scintillating medium and sits directly behind the electromagnetic end cap calorimeter. The HEC consists of two cylindrical wheels in each ATLAS end-cap cryostat referred to as the front wheel (HEC1) and the rear wheel (HEC2). Each wheel contains 32 wedge modules which can be seen in Fig. 3.16. The HEC1 modules are made of 25 copper plates and the HEC2 modules contain 17 copper plates. The plates in HEC1 (HEC2) are 25 mm (50 mm) thick except for the front plates of the front and rear modules which are half the nominal thickness. The distance between all plates is 8.5 mm.
Forward Calorimeter

The calorimeters in ATLAS are meant to work in tandem with the tracking systems to provide better and more information about the particles observed. However, since the tracking provided by the Inner Detector only covers up to $|\eta| < 2.5$ and $|\eta| < 2.7$ for the muon spectrometer, calorimetry is needed in this region.

Processes resulting in missing transverse energy ($E_T$) such as the weak decay of W bosons into a lepton and neutrino will be present in the collisions at the LHC. This missing $E_T$ can be inferred by the large momentum imbalance in the transverse plane and for cases where these final state objects are very forward, calorimetry is the only pragmatic way to get a reliable energy measurement. The Forward Calorimeter (FCal) in ATLAS uses liquid Argon as the active medium and is a sampling calorimeter. It also covers a pseudorapidity range of $3.1 < |\eta| < 4.9$.

As is the case with the other calorimeters, the Forward Calorimeter consists of two halves: FCalA and FCalC. Each of these consists of three modules situated one behind the other. The module located closest to the Interaction Point uses copper as the absorber.
Figure 3.16: The diagram on the right shows the eta coverage of the hadronic end cap calorimeter along with the inner radii for the first 9 plates and the last 16 plates of HEC1 and all plates of HEC2. The image on the left shows the shape of the wedge modules as seen in the $r - \phi$ plane.

Figure 3.17: An enhanced cross-sectional view of an FCal1 electrode. The liquid Argon gap can be seen, along with the cathode (tube) and anode (rod).

The FCal2 and FCal3 modules are mainly comprised of tungsten in order to suppress the longitudinal and transverse spreading of hadronic showers. The front face of FCal1 is 4.7
m from the Interaction Point and the modules it contains measure 2.7 $\lambda$ in depth. The modules in FCal2 and FCal3 are 3.7 $\lambda$ and 3.6 $\lambda$ deep respectively resulting in a total depth of 10 $\lambda$.

### 3.7 Muon Spectrometer

The muon spectrometer has a triggerable volume over the pseudo rapidity region of $|\eta| \leq 2.4$. Resistive Plate Chambers (RPCs) are used in the barrel region and are arranged in three stations. Thin Gap Chambers (TGCs) are used in the end-cap regions. In addition to triggering, the RPCs and TGCs also provide a “second-coordinate” measurement of track coordinates that are orthogonal to the primary precision measurement coordinates. This “second-coordinate” measurement allows for improved and more reliable track reconstruction which indirectly translates into better mass resolution when reconstructing mass resonances from muon tracks.

The Muon Spectrometer measures the momentum of muon tracks by using three superconducting air-core toroid magnets. In the pseudorapidity range $|\eta| \leq 1.0$ the muons are bent by a magnetic field produced a large barrel magnet consisting of eight coils that surround the hadronic calorimeter. In the $\eta$ range $1.0 \leq |\eta| \leq 1.4$ (transition region), the magnetic field is provided by a combination of the barrel and end-cap magnets. This particular configuration was chosen as it produces a field mostly orthogonal to the trajectories that the muons have when traversing the field. It also minimizes resolution degradation due to multiple scattering.

It is designed in a way such that particles originating from the IP pass through multiple stations of chambers. Most precision measurements are made by Monitored Drift Tubes (MDTs), however at larger $\eta$ and close to the Interaction Point, Cathode Strip Chambers (CSCs) are used. The CSCs have a higher granularity and are used since they can sustain the high rate environment close to the IP. In the barrel region track measurements are made by three cylindrical layers (stations) of Monitored Drift Tubes surrounding the beam axis (z-axis) at approximate radii of 5, 7.5 and 10 m. The barrel chamber coverage extends to
$|\eta| < 1$. In the end-cap region the chambers are oriented vertically in four discs and cover the pseudorapidity range $1 < |\eta| < 2.7$. The discs are situated at approximately 7, 10, 14 and 21 m from the IP and are concentrically arranged around the beam axis. MDTs are used as the primary tracking detector in the end caps except for the innermost layer ($2.0 < |\eta| < 2.7$), where CSCs are used. The barrel and end caps combine to provide nearly complete coverage over $0 < |\eta| < 2.7$, excluding a very small region at $\eta = 0$ which is used for the passing of cables for other subsystems of the detector.

3.7.1 Momentum Measurement

The muon spectrometer is designed to provide a momentum resolution of $\Delta p_T/p_T < 1.0 \times 10^{-4} \times p/\text{GeV}$ for muons with large transverse momentum ($p_T > 300$ GeV). For less energetic muons the resolution is on the order of one percent due to multiple scattering (in the magnets) and energy loss in the calorimeters. To obtain the quoted resolution, each measurement made at the three stations must be measured with an accuracy better than 50 $\mu$m. The precision momentum measurement is made in the r-z plane (bending plane), with the z position measured in the barrel and the radial measurement made in the end cap and transition region.

3.7.2 Monitored Drift Tubes

Monitored Drift tubes, as stated previously, form the majority of the tracking detectors used in the muon spectrometer. The tubes are made of aluminum, 30 mm in diameter and have a wall thickness of 400 $\mu$m. Located at the center of the tube is a gold plated tungsten-rhenium wire 50 $\mu$m in diameter held at approximately 3100 V. Each tube can vary from 1 m to 6 m in length. The active ionising gas is a mixture of Ar and CO$_2$ (93% and 7%) pressurized at 3 bars and has a maximal drift time of 700 ns. A single tube spatial resolution of 80 $\mu$m is achievable with the aforementioned specifications. To improve the resolution beyond the single wire value the chambers implement a stacking structure. Each MDT chamber has two layers of tubes stacked four high in the inner station and two layers by 3 high in the middle and outer stations.
The chambers themselves will deform and therefore the deformations must be tracked and accounted for. This is done using the Rasnik, BCam and Sacled [19] alignment systems. A CCD or CMOS sensor takes images of the chambers and corrects for the deformation in real time. It is also necessary to know where the chambers are positioned in space and also with respect to one another. This is accomplished by a complicated alignment system that can measure the position of a chamber with respect to the others with a precision of 0.03 mm. The barrel and end cap alignment systems are different, however they both make use of the Rasnik system described earlier.

3.7.3 Cathode Strip Chambers

The Cathode Strip Chambers are multiwire proportional chambers that use an active ionising medium of Ar and CO₂ (80% and 20%). They are present only in the end caps of the muon spectrometer and cover the high eta region (2.0 < |η| < 2.7). They consist of anode wires to collect the ionized charge in the gas and cathode strips to measure the induced signal giving the position of the hit within the chamber. The CSCs are also segmented using two sets of cathode strips: one set of strips parallel to the anode wires that measures the precision (bending) coordinate and one orthogonal to the anode wires that measures
the transverse coordinate.

![Figure 3.19: A general schematic of the Cathode Strip Chambers used in the ATLAS muon spectrometer. The muons traversing the chambers ionize the ArCO₂ gas causing an avalanche to occur on the various anode wires. This induces a charge distribution on the cathode strips which gives the location of the hit.](image)

The CSCs also experience forces due to the toroidal magnetic field and therefore must be monitored as well. The same alignment system used to monitor the MDTs in the end caps is also used to monitor the CSCs.

### 3.7.4 Trigger Detectors

There are two types of trigger chambers used in the muon spectrometer: Resistive Plate Chambers (RPCs) and Thin Gap Chambers (TGCs). The barrel region exclusively uses RPCs whereas TGCs are used in the high rate environment of the end cap regions.

**Resistive Plate Chambers**

Resistive Plate Chambers are gaseous detectors that measure particle interaction via the ionization of the gas contained within the detector. The RPCs used in ATLAS have 2 mm...
thick plates, a plate separation of 2 mm and a gas mixture comprised of $\text{C}_2\text{H}_2\text{F}_4\cdot\text{C}_4\text{H}_{10}\cdot\text{SF}_6$ (97%, 5%, 0.3%). The potential between the plates is approximately 9.8 kV resulting in a very large electric field (4.9 kV/mm). This high electric field facilitates the avalanche of ionization caused by the initial ionization by the incident charged particle. Each chamber has two sets of 30 mm readout strips: one in $\eta$ and $\phi$.

Figure 3.20: A schematic of a resistive plate chamber used in ATLAS.

Thin Gap Chambers

The second type of trigger detector used in ATLAS is a Thin Gap Chamber. The TGCs have two main purposes: provide triggering capability in the end cap regions and to measure the azimuthal coordinate of incident charged particle tracks. The anode wires in Fig. 3.21 are held at a positive high voltage. The ionization charges caused by the incident particle collect at the anode wires and provide a measurement of the bending coordinate. A charge distribution is induced on radial read out strips providing an azimuthal coordinate measurement. The active gas is a mixture of 55% CO$_2$ and 45% n-pentane (n-C$_5$H$_{12}$).
3.8 Level 1 Trigger System

The ATLAS trigger system is designed to operate at the nominal rate of 40 MHz (\( \frac{1}{25 \mu s} \)) at the LHC. It is a three tiered system consisting of the Level-1 (L1), Level-2 (L2) and event filter (EF) event selection systems. The L1 trigger utilizes hardware (specifically designed electronics), and the L2 and EF triggers (collectively known as the High-Level Trigger (HLT)) are software level triggers.

The L1 trigger looks for large \( p_T \) muon, electron/photon, jet and hadronic \( \tau \) decay signatures. It also can trigger on events that have associated large missing transverse energy (\( E_T^{\text{miss}} \)) and/or large transverse energy (\( E_T \)) signatures. It uses reduced-granularity information from the RPCs and TGCs to trigger on high \( p_T \) muons and the calorimeter systems to trigger on events associated with large calorimeter signatures. The maximum L1 accept rate is 75 kHz which can be upgraded to 100 kHz [3] and the L1 decision has to reach the front-end electronics within 2.5 \( \mu s \) after the associated bunch-crossing. A flow diagram of the L1 system is seen in Fig. 3.22.

The L2 trigger uses information from Regions-of-Interest (RoI’s) to further filter the event rate. RoI’s are regions where the L1 trigger has identified possible trigger objects of a particular event. The information includes but is not limited to the coordinates and
energy of the trigger object. The L2 trigger reduces the event rate to below 3.5 kHz with a processing time around 40 ms [3].

Figure 3.22: Flowchart showing the trigger process in ATLAS.
Chapter 4
MEASUREMENT OF THE ZZ CROSS SECTION AND aTGC LIMITS

4.1 Monte Carlo Signal and Data Samples

The $pp \rightarrow ZZX \rightarrow l^+l^-l^+l^-$ process and associated decays are modelled using the CT10 PDF\textsuperscript{[20]} (parton density function) set and the PowhegBox (NLO)\textsuperscript{[21][22][23]} and gg2zz (LO)\textsuperscript{[24]} generators. The gg2zz samples simulate the $gg \rightarrow ZZ$ process at LO which is a NNLO correction to the total cross section. It contributes approximately 5.93\% of the total cross section. The gauge boson decays into electrons, muons and τ leptons with all τ leptons decaying inclusively.

The signal MC samples used in this analysis are listed in Table 4.1. The k-factor ($\sigma_{LO}/\sigma_{NLO}$) listed in Table 4.1 is needed to correct to the full NLO prediction. The PowhegBox and gg2zz samples have a filter applied at the generator level that requires at least 3 leptons ($e, \mu$ flavor) that have $p_T > 5$ GeV and $|\eta| < 10$. This filter mainly serves to reject events with τ leptons. Approximately 0.05\% of $pp \rightarrow ZZX \rightarrow l^+l^-l^+l^-$ events are rejected when requiring that both Z bosons have masses in the range of 66-116 GeV (on-shell fiducial region). The gg2zz samples have a generator level filter that requires each Z to have a $p_T > 7$ GeV.

\textsuperscript{2}Next to Leading Order (NLO), Leading Order (LO) and NNLO (Next to Next to Leading Order).
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<tr>
<th>MCID</th>
<th>Process</th>
<th>Generator</th>
<th>Events</th>
<th>k-factor</th>
<th>$\epsilon_{\text{filter}}$</th>
<th>cross section [pb]</th>
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Table 4.1: All MC samples simulating the $pp \rightarrow ZZX \rightarrow l^+l^-l^+l^-$ process along with number of events, cross section, filter efficiencies, k-factor, MC ID run number and generator used.

### 4.2 Event Triggers

A description of the ATLAS trigger system is given in section 3.8 and therefore will not be discussed here. Every $ZZ \rightarrow l^+l^-l^+l^-$ event must pass the event level trigger preselection.

The triggers used are the lowest $p_T$ un-prescaled triggers spanning the entire 2012 data taking period. The electron trigger requirement is an OR of the $\text{EF}_e24\text{vhi}_\text{medium1}$ and $\text{EF}_e60\text{medium1}$ triggers. The muon trigger requirement is an OR statement of the $\text{EF}_\mu24\text{i}_\text{tight}$ and $\text{EF}_\mu36\text{tight}$ triggers. The $\text{EF}_e60\text{medium1}$ and $\text{EF}_\mu36\text{tight}$ triggers are necessary to maintain high efficiency at higher lepton $p_T$.

The $e^+e^-e^+e^-$ final state must satisfy the electron trigger, the $\mu^+\mu^-\mu^+\mu^-$ final state must satisfy the muon trigger and the $e^+e^-\mu^+\mu^-$ can satisfy either or both triggers. In the case of both triggers the threshold $p_T$ is 24 GeV. The sum of the $p_T$ of the tracks in a cone of $\Delta R < 0.2$ surrounding the lepton must be less than 0.10 $p_T$ for electrons and 0.12 $p_T$ for muons.

Each event must contain a trigger-matched lepton. This is defined to be an electron within a cone of $\Delta R < 0.15$ of the electron that fired the trigger and a muon within a cone of $\Delta R < 0.1$ of the muon that fired the trigger. The trigger-matched object must have a $p_T$ at least 1 GeV above the trigger threshold, meaning in our case an electron must have a $p_T > 25$ GeV.
4.3 Muons

<table>
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<td>3. $\mu$: $\eta$</td>
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<td>4. $\mu$: ID hits</td>
<td>MCP recommendation</td>
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<td>5. $\mu$: $z_0 \sin(\theta)$</td>
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<td>7. $\mu$: track isolation</td>
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<td>\eta</td>
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<tr>
<td>3. $\mu$: ID hits</td>
<td>MCP recommendation</td>
</tr>
<tr>
<td>4. $\mu$: $z_0 \sin(\theta)$</td>
<td>$</td>
</tr>
<tr>
<td>5. $\mu$: $d_0$</td>
<td>$d_0/\sigma_{d_0} &lt; 3.0$</td>
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<td>6. $\mu$: calo isolation</td>
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<td>6. $\mu$: $d_0$</td>
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<td>7. $\mu$: track isolation</td>
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Table 4.2: Muon object selection requirements for the specific categories of muons.

The two muon reconstruction algorithms used in this analysis are the STACO and CaloTrkMuID algorithms. STACO muons are reconstructed using information from a muon track in the ID and a muon track in the MS. The energy loss between the ID and MS is taken into account. There are three categories of STACO muons: Combined, Segment Tagged and Stand Alone. Combined muons are reconstructed using a full ID track and a full MS track whereas Segment Tagged muons use information from a full ID track and MS
track segment. Stand Alone muons have no ID track and are therefore reconstructed from an MS track only. Central and forward muons are required to pass the “loose” identification requirement.

Muons in the $|\eta| < 2.5$ region are referred to as “central muons”, muons with $2.5 < |\eta| < 2.7$ are referred to as “forward muons” and muons with $|\eta| < 0.1$ are “calo muons.” Central muons are required to be either Combined or Segment Tagged muons and have a $p_T > 7$ GeV. As per the MCP recommendation, every central muon track must have at least 1 hit in at least 1 Pixel layer, 4 hits in the SCT and less than 3 holes (no hit in a layer that is traversed by the track) over all silicon layers [25]. Each track must also satisfy TRT hit requirements that depend on the $\eta$ region the track lies in. If the track is in the $0.1 < |\eta| < 1.9$ region, then $(n_{\text{TRT hits}} + n_{\text{TRT outliers}}) > 5$ and $n_{\text{TRT outliers}} < 0.9 \times (n_{\text{TRT hits}} + n_{\text{TRT outliers}})$. If the track $|\eta| \leq 0.1$ or $|\eta| \geq 1.9$ and the $n_{\text{TRT hits}} > 5$ then the $n_{\text{TRT outliers}} < 0.9 \times (n_{\text{TRT hits}} + n_{\text{TRT outliers}})$. A TRT outlier is defined as a TRT straw that shows a signal yet was not crossed by the nearby track or as a set of TRT measurements that did not form a smooth trajectory with the measurements made by the Pixel and SCT [26].

Forward muons must have a $p_T > 10$ GeV and be either Combined or Segment Tagged muons. Each forward muon track must register a hit across all three stations in the muon spectrometer (inner, middle and outer). The hit requirements for forward muons are slightly relaxed in comparison to central muons. Each forward muon must have at least 1 hit in all Pixel layers, 3 SCT hits and less than 3 holes over all silicon layers.

The CaloTrkMuID algorithm tags certain tracks in the inner detector as muons in an attempt to regain efficiency around $\eta = 0$ (specifically $|\eta| < 0.1$). It does so by matching the aforementioned ID tracks with signals in the calorimeters that are inconsistent with electrons, photons or hadrons. Low $p_T$ hadrons will deposit nearly all their energy in the first cell of the hadronic calorimeters while high $p_T$ hadrons will deposit more in the proceeding layers. However, muons will leave very little energy in the calorimeters. Using this information one can tag calo muons. They are by definition “loose” muons and must have a $p_T > 20$ GeV as their identification algorithm is optimized for isolated muons with
\( p_T > 20 \text{ GeV} \) \cite{25}. Every calo muon must satisfy the same ID hit requirements as central muons. No TRT hits are required, but if there are more than 5 then \( n_{\text{TRT}} \) outliers < \( 0.9 \times (n_{\text{TRT}} \text{ hits } + n_{\text{TRT}} \text{ outliers}) \). To reduce the amount of particles faking muons each calo muon must have a CaloMuonIDTag value greater than 10 or a CaloLRLikelihood value greater than 0.9. More detail about these algorithms can be found in \cite{27}.

We expect \( pp \rightarrow ZZX \rightarrow l'^+l'^-l'^+l'^- \) events to come from hard processes and therefore the final state muons should come from the primary vertex (defined as the vertex with the highest \( \sum p_T^2 \) of associated tracks). Requiring the magnitude of the distance of closest approach to the primary vertex, \(|z_0 \sin(\theta)|\) to be less than 0.5 mm and the transverse impact parameter significance \(|d_0|/\sigma_{d_0}\) to be less than 3.0 ensures that this is the case. This is only required for Combined and Segment Tagged muons, since Stand Alone muons do not have an inner detector track.

In order to discriminate between prompt muons originating from \( Z \) bosons and secondary muons coming from hadronic jets the muons are required to be isolated. For central and calo muons the scalar sum of the transverse momenta of tracks surrounding the muon in a cone of \( \Delta R = 0.2 \) is required to be less than \( 0.15 \times p_T \) of the muon.

### 4.4 Electrons

Electrons with \(|\eta| < 2.47\) are referred to as “central” and are the only type considered in this analysis. They must satisfy the “Loose++” identification criteria and have an “author” value of 1 or 3. An author value of 1 means that only the cluster based algorithm has identified the electron object. An author value of 3 means that the object has been identified by the cluster and track-based algorithms. The cluster based algorithm is used primarily to reconstruct isolated electrons while the track based algorithm is used to reconstruct soft (low \( p_T \)) electrons.

In order to avoid energy measurements made in problematic regions of the LAr calorimeter, electrons must pass the object quality check (OQ AND 1446 = 0). The number “1446” is a bitmask indicating that the electron’s cluster is affected by one of the following prob-
Electron Object Selection Cut

Central Electrons:

1. e: type
   author = 1 or 3
2. e: quality
   (OQ AND 1446 == 0)
3. e: ID cut
   “Loose++”
4. e: $E_T$
   $E_T > 7$ GeV
5. e: $\eta$
   $|\eta| < 2.47$ includes crack region: $1.37 < |\eta_{\text{cluster}}| < 1.52$
6. e: $z_0 \sin(\theta)$
   $|z_0 \sin(\theta)| < 0.5$ mm
7. e: $d_0$
   $d_0/\sigma_{d_0} < 6.0$
8. e: track isolation
   $\Sigma p_T(\Delta R < 0.2)/p_T < 0.15$
9. e: Overlap Removal
   a) remove if $\Delta R < 0.1$ to a selected muon.
   b) remove lowest $E_T$ if $\Delta R < 0.1$ of another electron

Table 4.3: Electron object selection requirements.

lems: the existence of a dead front-end board in the first or second sampling layer, the
eexistence of a dead region affecting all three sampling layers, or an associated cell core that
is masked.

Impact parameter cuts requiring $|z_0 \sin(\theta)| < 0.5$ mm and $d_0/\sigma_{d_0} < 6.0$ ensure that
the electrons come from the primary vertex.

The electron energy measurement is always made from the EM calorimeter cluster in-
formation, however if the electron has four or more silicon hits (Pixel + SCT) the $\eta$ and
$\phi$ measurements are made from the track information. If the number of silicon hits is less
than 4, they are taken from the cluster information. In the case of overlap removal and the
$\eta$ requirement, the cluster information is used. Each electron must have $E_T > 7$ GeV.

As is the case with selected muons, each electron must be isolated from other surrounding
tracks by requiring the sum of the transverse momenta of tracks surrounding the electron in
a cone of $\Delta R = 0.2$ must be less than $0.15 \times p_T$ of the electron. If a selected electron track
overlaps with a selected muon in a cone of $\Delta R = 0.1$ it is removed. If a selected electron
overlaps with another selected electron within a cone of $\Delta R = 0.1$, the lower $E_T$ electron
is removed.
4.5 Event Selection

A certain set of conditions must be satisfied by each event for it to be considered a valid event in which to search for four isolated leptons originating from two Z bosons.

1. **Good Runs List** : The event must exist in the Good Runs List.

2. **Triggers** : Each event must fire at least one of the triggers mentioned in Sec. 4.2.

3. **Primary Vertex** : Each event must have a primary vertex with at least three associated tracks.

4. **Event Cleaning** : Each event should not have a LAr error value greater than 1.

5. **Data Corruption** : Each event should be free of data corruption in the tile calorimeter (tileError==2) and have no coreFlags error (coreFlags&0x400000!=0).

4.5.1 $\ell^+\ell^-\ell'^+\ell'^-$ Selection

The $e^+e^-e^+e^-$, $\mu^+\mu^-\mu^+\mu^-$ and $e^+e^-\mu^+\mu^-$ final states represent all final state signatures considered in this analysis. Before continuing it will be helpful to describe the nomenclature and specific requirements of the aforementioned final states.

The Z boson candidates reconstructed from di-lepton pairs are classified in two ways:

- The difference of each Z candidate mass to the Z pole (PDG Z mass). The Z candidate with invariant mass closest to the Z pole is the *primary* Z and the remaining is the *secondary* Z.

- The Z candidate with the higher $p_T$ is referred to as the *leading* Z and the remaining is the *subleading* Z.

Each event is required to satisfy the following conditions:

- **Four Leptons** : Each event must have four and only four leptons in the final state. Each lepton must satisfy the selection criterion described in Sec. 4.3 and 4.4.
• **Trigger Matching**: At least one lepton must be trigger matched. If the trigger matched lepton is an electron it must have a $p_T > 25$ GeV and pass the Medium+++ identification requirement. If the trigger matched lepton is a muon it must have a $p_T > 25$ GeV, be a Combined muon and have $|\eta| < 2.4$.

• **Quadruplet Formation**: Each pair of leptons must be opposite sign and same flavor (OSSF). In general, there are multiple ways to pair the leptons in the final state. The pairing combination that minimizes the sum of the magnitude of the distances to the Z pole for each lepton pair is chosen as the proper combination. In other words the pair minimizing $|m_{12} - m_{PDG}^{Z}| + |m_{34} - m_{PDG}^{Z}|$ is chosen where ‘1’, ‘2’, ‘3’ and ‘4’ are the final state leptons and $m_{PDG}^{Z}$ is the Z mass.

• **Primary Z mass**: The primary Z mass must fall in the window

$$66 < m_{Z}^{primary} < 116$$ GeV.

• **Secondary Z mass**: The secondary Z mass must fall in the window

$$66 < m_{Z}^{secondary} < 116$$ GeV.

• **Lepton Separation**: The leptons must have a $\Delta R$ separation greater than 0.2 ($\Delta R(l,l) > 0.2$).

• **Extension Lepton Limit**: Each event is limited to at most two extension leptons (calorimeter and forward muons) and one of each type. Each extension lepton must be paired with one central lepton.

• **$J/\psi$ Veto**: If any OSSF pair combination results in an invariant mass below 5 GeV the event is vetoed.

4.6 Event Weights

4.6.1 Pileup Reweighting

As was mentioned in Sec. 3.4.1, $\mu$ is a measurement of the number of proton-proton collisions per event. Ranging from a few to many, a collision is considered a reconstructed vertex in
an event. The number of reconstructed vertices increases as the beam luminosity increases.
The $\mu$ value is not the same in every data period (since the luminosity is not always the
same) and must be measured. Using the measured $\mu$ distribution per Bunch Crossing ID
(BCID) in data one can compare that to the distribution produced in Monte Carlo and
calculate a weight. This weight, referred to as the pileup reweight, is applied on an event
level basis to Monte Carlo.

4.6.2 Muons

Multiple scale factors are applied to the selected muon objects falling within our final
state definition. As was mentioned in Sec. 4.3, an identification selection is imposed on
muons. This selection has an efficiency that differs between data and Monte Carlo and must
therefore be corrected. This is done by comparing the efficiency measured in data with that
measured in Monte Carlo using a “tag and probe” method described in detail in [28]. In
this method $Z \rightarrow \mu^+ \mu^-$decays are selected by looking for two oppositely charged, isolated
muon tracks with an invariant mass near the $Z$ mass. One track must be a Combined
muon (“tag”) and the other track (“probe”) must either be a Stand Alone muon if the ID
efficiency needs to be measured or an ID track if the MS efficiency is to be measured. The
ID efficiency is the fraction of Stand Alone “probes” that can be associated to an ID track.
The MS efficiency is the fraction of ID “probes” that can be associated to a Combined or
Segment Tagged track. A Stand Alone “probe” is matched to an ID track if $\Delta R \leq 0.05$ and
an ID “probe” is matched if $\Delta R \leq 0.01$. The overall muon ID scale factor for $\ell^+ \ell^- \ell'^+ \ell'^-$
events is 99.3%. The tool used in this analysis is MuonEfficiencyCorrections-02-01-12.

In the forward region the sum of the transverse energy in a cone of $\Delta R= 0.2$ around
the muon must be less than 0.15 of the muon $p_T$. The energy within the cone is corrected
for contributions from pile-up events by using the number of reconstructed vertices in the
event (MuonIsolationCorrection-01-01).
4.6.3 Electrons

The efficiency corrections applied to electrons are of two specific types: ID (Loose++, Medium++, and Tight++) efficiency and reconstruction efficiency. Both efficiencies are measured in data and Monte Carlo using a Z tag and probe method where the invariant mass of the di-electron system ($M_{ee}$) must be between 80 and 100 GeV. More details on this method can be found in Ref [29]. The tools used to apply these scale factors in this analysis are egammaAnalysisUtils-00-04-17 and ElectronEfficiencyCorrection-00-00-09.

4.7 Momentum Smearing For Monte Carlo Events

The $p_T$ distributions of both electrons and muons differ slightly in data and Monte Carlo. In order to accurately simulate data these distributions must be corrected or “smeared” in Monte Carlo.

The muon $p_T$ in Monte Carlo needs to be corrected to match the data distribution. This is done by looking at muons in events with Z bosons, reconstructing the di-muon mass and comparing the experimental and simulated distributions. Using these corrections the muon $p_T$ is “smeared” while keeping the direction unchanged (MuonMomentumCorrections-00-08-05).

Electrons not only have the $p_T$ smeared in Monte Carlo but an energy scale correction must also be applied to data. This is done using egammaAnalysisUtils-00-04-17.

4.8 Fiducial Region Definition

In order to measure the ZZ cross section it is necessary to define a fiducial region. This region defines the kinematic space that events must exist in in order to be considered valid candidate events and helps separate “signal” processes from “background” processes.

The ZZ → $l^+l^-l^+l^-$ signal region is defined as:

- ZZ → $l^+l^-l^+l^-$ where $l = e, \mu$ and each Z must decay to an opposite sign and same flavor lepton pair (ie. $Z \rightarrow e^+e^-$ or $Z \rightarrow \mu^+\mu^-$).
• $66 < m_{12} < 116$ GeV. $m_{12}$ is the mass of the Z that is reconstructed from the first and second leptons.

• $66 < m_{34} < 116$ GeV. $m_{34}$ is the mass of the Z that is reconstructed from the third and fourth leptons.

• The pair of reconstructed Z’s that minimizes $|m_{12} - m_{Z}^{PDG}| + |m_{34} - m_{Z}^{PDG}|$ is chosen.

• $p_{T} > 7$ GeV.

• $|\eta| < 2.7$.

• Each lepton must have a $\Delta R$ separation of at least 0.2 between the other three leptons in the event ($\min(\Delta R(l,l)) > 2$).

4.9 Background to $pp \rightarrow ZZX \rightarrow l^+l^-l^+l^-$

There are many sources of background for this particular final state. The main contributions come from $Z + jet, Zb\bar{b}, Z + \gamma + jet$, and decays involving top quarks such as $t\bar{t}$ and single top where a jet from a heavy flavor decay is misidentified as an isolated lepton or a lepton from a heavy flavor decay passes the lepton isolation requirement (see sec. 4.4 for electrons and 4.3 for muons). The $Z + jet$ sources have two prompt leptons originating from a $Z$, whereas the $t\bar{t}$ source has the two prompt leptons coming from the $W^{\pm}$ bosons. The remaining two leptons can be “fake” leptons (charged hadrons such as pions misidentified as leptons), real leptons originating from heavy flavor ($b$ or $c$ quark) decays, muons decaying in-flight from pions or kaons or electrons coming directly from photons. Additional sources of background are $WZ + jet$ and $WW + jet$ that have three (two) real, prompt leptons and one (two) “fake” leptons and irreducible backgrounds coming from $t\bar{t}Z$ and $ZZZ/WWZ$ processes (not aTGC processes).

In the case of a $WZ + jet$ ($WW + jet$) event where the bosons decay leptonically, there are 3 (2) prompt leptons from the vector boson decays present. Combine that with a fourth (third and fourth) lepton originating from the various aforementioned sources and a 4 lepton event not originating from a $ZZ$ initial state could pass the selection criteria.
It is appropriate to note that even though the majority of leptons in a jet would fail the object level isolation cuts, the tails of the distribution contain leptons that could pass the isolation and identification requirements of selected leptons. It is therefore very important to estimate the contribution from these particular events. Since it is likely that the MC may not model the tails of the jet distributions well a data-driven method is used to estimate the contribution from this type of background.

4.9.1 Data-driven Method

In order to estimate the background from the aforementioned sources, the identification of events very near to the “signal region” is defined. This is done by inverting one or two of the lepton requirements (sec. 4.3 and 4.4) while all others remain the same. A “fake factor” is used to extrapolate these events near the “signal region” into the signal region. This “fake factor”, referred to as FF, is measured in a separate sample that consists mostly of jets and does not contain real leptons.

The calculation of the fake factors begins by defining “pre-selected” leptons as leptons satisfying some of the requirements listed in sections 4.4 and 4.3. These requirements depend on flavor and will be discussed later in subsequent sections. The leptons are then categorized as “L” objects (selected leptons) that pass all selection requirements, or “J” objects (lepton-like jets) that pass all but specific requirements. The fake factor is defined as

\[ FF = \frac{L}{J} \]  \hspace{1cm} (4.1)

and is calculated from a sample of events that consists mostly of \( Z + \text{jet} \) events. It is therefore assumed that any “L” object is a jet that satisfies the “L” selection requirements and is not a real lepton.
4.9.2 Electron Selection

As was mentioned earlier, we must define what the L and J definitions are for both electrons and muons. For central electrons the sample made of L and J objects satisfy all electron selection requirements up to before the track isolation and electron identification cuts. The pre-selected electrons are categorized as L objects if they pass the aforementioned electron selection requirements and satisfy the track isolation and electron ID requirements. The pre-selected electrons that fail either the track isolation or electron ID requirement but pass all other requirements are categorized as J objects. Pre-selected electrons that fail both the track isolation and ID requirements are discarded (fall outside either category) since they represent an area too far from our selected (“good”) electrons. Additionally, studies have shown that these electrons are not well modeled in Monte Carlo which is yet another reason to discard them.

The specific definition for selected electrons (L) and electron-like jets (J) are summarized in Table 4.4.

<table>
<thead>
<tr>
<th>Flavor</th>
<th>Selected (L)</th>
<th>Lepton-like Jets (J)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Muons:</td>
<td>Track isolation $&lt; 0.15$ and $d_0$ significance $&lt; 3.0$ or ($d_0$ significance $&lt; 3.0$ and Track iso $&gt; 0.15$)</td>
<td>($d_0$ significance $&gt; 3.0$ and Track iso $&lt; 0.15$)</td>
</tr>
<tr>
<td>Electrons:</td>
<td>Track isolation $&lt; 0.15$ and Loose++</td>
<td>(!Loose++ and Track iso $&lt; 0.15$)</td>
</tr>
</tbody>
</table>

Table 4.4: Muon and electron L and J definitions.

To extrapolate to region A, a fake factor must be calculated. The fake factor for electrons is defined as

$$ FF_e = \frac{N_{\text{data\ selected\ electrons}}}{N_{\text{data\ electron-like\ jets}}} - \frac{N_{\text{MC WZZ}}}{N_{\text{MC WZZ}}}. $$

(4.2)

When selecting the leptons used to measure the fake factor, we pull from a sample
Figure 4.1: The definition of selected leptons and lepton-like jets for muons (a) and electrons (b). Objects in region A are leptons used in our analysis that can form candidate Z bosons. Muons in regions B and D are categorized as muon-like jets (J). Electrons in regions B and D are categorized as electron-like jets (J). Objects in region C lie too far from the selected region and are discarded. The fake factor is used to extrapolate from regions B and D to region A in both cases of lepton flavor.

of events with a reconstructed Z boson and search for additional leptons in those events. Events that are associated with a reconstructed Z boson are referred to as “Z-tagged” events. Requiring a tagged Z to be present in each event ensures that the sample of events from which the fake factor is calculated consists largely of $Z + \text{jet}$ events. The “Z-tagged” requirements are listed in Table 4.5.

The fake leptons can come from various sources, with the majority originating from $Z + \text{jet}$ events. However, real leptons from $WZ$ and $ZZ$ events can also contribute fake leptons. It is because of this that they must be removed from the L and J populations via the $N_{\text{MC \, selected electrons}}^{WZ,ZZ}$ and $N_{\text{MC \, electron-like jets}}^{WZ,ZZ}$ terms in eq. 4.2. Histograms of the $p_T$ and $\eta$ of selected leptons and lepton-like jets are created for data, $WZ$ and $ZZ$ Monte Carlo. The $WZ$ and $ZZ$ histograms are then subtracted from the data histograms to eliminate their contribution to the fake factor calculation. The fake factor is then calculated by dividing the
<table>
<thead>
<tr>
<th>Criteria</th>
<th>Selection Requirements</th>
</tr>
</thead>
<tbody>
<tr>
<td>Leptons</td>
<td>2 selected muons or 2 selected electrons</td>
</tr>
<tr>
<td>Z-reconstruction</td>
<td>lepton pair must be oppositely charged and same flavor</td>
</tr>
<tr>
<td></td>
<td>1 lepton must be trigger matched to event</td>
</tr>
<tr>
<td>$E_T^{miss}$</td>
<td>$</td>
</tr>
<tr>
<td></td>
<td>$E_T^{miss} &lt; 25$ GeV</td>
</tr>
</tbody>
</table>

Table 4.5: Reconstructed Z requirements for “Z-tagged” events.

selected lepton histogram by the lepton-like jet histogram. This effectively parameterizes the fake factor in $p_T$ and $\eta$ and the distributions are seen in Fig. 4.2 and 4.3.

![Fake Factor Distribution](image)

Figure 4.2: Fake factor distribution parameterized in $p_T$ for electrons.

When applying the fake factors to extrapolate to the signal region, there is assumed to be no correlation between the fake factors parameterized in $p_T$ and $\eta$. Assuming this the fake factor that is applied is defined as

$$FF_e(p_T, \eta) = \frac{FF_e(p_T) \cdot FF_e(\eta)}{\langle FF_e(p_T, \eta) \rangle} \quad (4.3)$$
Figure 4.3: Fake factor distribution parameterized in $\eta$ for electrons.

where $\langle FF_e(p_T, \eta) \rangle$ is the average fake factor defined as the total number of selected electrons (L) divided by the total number of electron-like jets (J).

4.9.3 Muon Selection

Sources of fake muons include heavy flavor decays ($b$ or $c$ quarks that hadronize), $K/\pi$ decays or hadrons that punch through all other sub-systems into the muon spectrometer. Muon-like jets are defined as objects with $d_0$ significance $> 3.0$ or with track isolation $> 0.15$. As is the case with electrons, muons that fail both the $d_0$ significance and isolation (track isolation for central and calo muons and calorimeter isolation for forward muons) requirements are discarded from the analysis as they are too far from our signal region (regions shown in Fig. 4.1). The muon fake factor is defined as

$$FF_\mu = \frac{N^{data}_{selected \muons} - N^{MC \ W, ZZ \ selected \ muons}}{N^{data}_{muon-like \ jets} - N^{MC \ W, ZZ \ muon-like \ jets}}.$$

The sample of muons used to calculate the fake factor come from events that contain a reconstructed Z boson (“Z-tagged”), the selection of which is summarized in Table 4.5. Any muons found in addition to the Z are categorized as selected muons or muon-like jets.
Figure 4.4: Central muon fake factor parameterized in $p_T$. The poor agreement from 50 GeV to 400 GeV is further justification for using this data driven method to estimate the background.

The real leptons from $WZ$ and $ZZ$ contributions are removed via the $N_{MC WZ,ZZ}$ and $N_{MC WZ,ZZ}$ terms in Eq. 4.4.

Unique to muons, there are three separate fake factors calculated and applied to events with fake muons. As was mentioned in section 4.3, there are three types of muons: central, forward and calo. Since these muons are in three unique areas of the detector, the respective probabilities of being faked are not assumed to be the same. Therefore, their fake factors must be measured individually.

All muon fake factors are parameterized in $p_T$ and $\eta$. The distributions are shown in Fig. 4.4 and 4.6.

The fake factor applied to extrapolate to the signal region is defined as

$$FF_\mu(p_T, \eta) = \frac{FF_\mu(p_T) \cdot FF_\mu(\eta)}{\langle FF_\mu(p_T, \eta) \rangle}$$

(4.5)

where $\langle FF_\mu(p_T, \eta) \rangle$ is the average fake factor defined as the total number of selected muons (L) divided by the total number of muon-like jets (J).

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4.9.4 Calculation of Fake Lepton Background

The event content of the fake lepton background consists mainly of events with two fake leptons (F) and two true leptons (T) and events with one fake lepton and three true leptons. For this reason the number of fake 4 lepton events is described as

\[ N_{4\ell}^{\text{fake}} = N_{TTTF} \times f + N_{TTFF} \times f^2. \]  

(4.6)

where \( f \) is the probability of a fake lepton being reconstructed as a selected lepton (L). Events with three true leptons and one fake lepton can arise from \( WZ + \text{jet} \) events, whereas events with two true and fake leptons can come from \( Z + \text{jet} \) (multiple jets), \( \bar{t}t, tW + \text{jet} \) and \( t + \text{jets} \). However, since in data it is impossible to know what is truly a lepton and what is truly a fake, we must relate \( N_{4\ell}^{\text{fake}} \) to events with selected leptons and lepton-like jets. The probability \( (f) \) in Eq. 4.6 is expressed as

\[ f = \frac{L}{L + J}. \]  

(4.7)

where \( L \) is the number of jets that pass the lepton (L) selection requirements.
Using Eq. 4.1, $FF$ and $f$ can then be expressed as

$$FF = \frac{f}{1-f} \quad \text{and} \quad f = \frac{FF}{1 + FF}. \tag{4.8}$$

The number of events with two true leptons and two fake leptons can be related to events with two selected leptons and two lepton-like jets via:

$$N_{LLJJ} = N_{TTFF} \times (1 - f)^2 \tag{4.9}$$

The number of events with three true leptons and one fake lepton can be related to events with three selected leptons and one lepton-like jet via:

$$N_{LLLLJ} = N_{TTTF} \times (1 - f) + N_{TTFF} \times 2f(1 - f) \tag{4.10}$$

where the factor of 2 arises from combinatorics. When combining Eq. 4.10 and 4.9 the background estimate is written as

$$N_{fake}^{4l} = (N_{TTTF} \times f + N_{TTFF} \times 2f^2) - N_{TTFF} \times f^2 = N_{LLLLJ} \times FF - N_{LLJJ} \times FF^2 \tag{4.11}$$

A correction for the contamination from $ZZ$ events is applied and the final expression for the background estimate is

$$N_{fake}^{4l} = (N_{LLLLJ} - N_{LLLLJ}^{ZZ}) \times FF - (N_{LLJJ} - N_{LLLLJ}^{ZZ}) \times FF^2. \tag{4.12}$$

where $N_{LLLLJ}(N_{LLLLJ}^{ZZ})$ is the number of events with 3 (2) objects that satisfy the “L” selection and 1 (2) objects that satisfy the “J” selection. It is important to mention that in this estimate events containing zero “true” leptons and one “true” lepton are neglected and therefore so are their respective terms in Equation 4.12.
4.9.5 Uncertainties

The statistical uncertainty on the background estimate is calculated by adding the relative statistical error associated with the fake factors in quadrature with the relative statistical error on the number of events found in the data, both per channel and total. The systematic uncertainty is defined as the difference between the nominal background estimate and the estimate using average fake factors per lepton flavor (all fake factors applied are parameterized in \(p_T\) and \(\eta\)).

4.9.6 Data-driven Estimates

<table>
<thead>
<tr>
<th>Event Type</th>
<th>(e!)e!e!e!</th>
<th>(\mu!)\mu!)\mu!)</th>
<th>(e!)e!\mu!)!\mu!)</th>
<th>(l!)l!)l!)l!)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(N_{LLLJ})</td>
<td>164</td>
<td>8</td>
<td>113</td>
<td>285</td>
</tr>
<tr>
<td>(ZZ_{LLLJ})</td>
<td>11.31</td>
<td>3.99</td>
<td>16.56</td>
<td>31.86</td>
</tr>
<tr>
<td>(N_{LLJJ})</td>
<td>645</td>
<td>11</td>
<td>443</td>
<td>1096</td>
</tr>
<tr>
<td>(ZZ_{LLJJ})</td>
<td>1.78</td>
<td>0.07</td>
<td>1.00</td>
<td>2.85</td>
</tr>
</tbody>
</table>

Table 4.6: Number of \(LLLJ\) and \(LLJJ\) events per channel and combined over all channels.

The number of \(N_{LLLJ}, N_{LLJJ}, ZZ_{LLLJ}, ZZ_{LLJJ}\) events is summarized in Table 4.6. The number of \(LJJJ\) events in data totals 264.

A summary of the fake estimates are shown in Table 4.7 for \(ZZ\) events falling within our fiducial region. The total data-driven background estimate summed over all three channels (\(e\!)e\!e\!e\!, \(\mu\!)\mu\!)\mu\!) and \(e\!)e\!\mu\!)\!\mu\!\)) is

The total data driven fake estimate for on-shell \(ZZ\) events is

\[17.73 \pm 4.21(\text{stat}) \pm 4.00(\text{syst})\]  \hspace{1cm} (4.13)

where the systematic uncertainty is defined as the difference in the data driven estimate using parameterized fake factors and average fake factors. The statistical uncertainty is calculated by adding \(N_{LLLJ}, ZZ_{LLLJ}, N_{LLJJ}\) and \(ZZ_{LLJJ}\) statistical errors in quadrature.
Table 4.7: Summary of data-driven background estimate per channel and combined over all channels. $N_{LLJJ}$ and $N_{LLIJ}$ are events observed in data and $ZZ_{LLJJ}$ and $ZZ_{LLIJ}$ are events in Monte Carlo.

Combining the MC background prediction from irreducible sources gives the total number of fake 4 lepton events in Eq. 4.14

\[ 19.70 \pm 4.21 \text{(stat.)} \pm 4.00 \text{(syst.)}. \]  

(4.14)

4.9.7 Monte Carlo Background Estimation

Due to the fact that the Monte Carlo predictions are limited by statistics and heavily rely on proper modelling of leptons in jets, the event yield of multiple background processes modelled by Monte Carlo is used as a rough cross-check on the data driven background estimation (within an order of magnitude). The background processes used are $Z + \text{jets}$, $WZ/WW$, $t \bar{t}$ and single top and all MC yields are normalized to 20 fb$^{-1}$. All scale factors mentioned in Sections 4.6.2 and 4.6.3 and corrections (smearing) are applied as well.

The dominant background source over all channels is $Z + \text{jets}$ (Table 4.8) with a predicted yield of $2.66 \pm 2.62$ on-shell $ZZ$ events. The total MC background prediction is $7.43 \pm 2.81$ events satisfying the combined $ZZ$ selection requirements which is to be compared to the data driven background estimation of $17.73 \pm 4.21 \text{(stat.)} \pm 4.00 \text{(syst.)}$. A consequence of the limited statistics associated with the MC background prediction and the large luminosity scaling factor of the $Z + \text{jets}$ samples ($\approx 3$) is that the channel predictions which are found to be zero could have a fluctuation upward of a single event resulting in an increase of the MC background prediction of several events.
Table 4.8: Summary of Monte Carlo background predictions for various sources. The $t\bar{t}+V$ and $ZZZ/ZWW$ values are the irreducible background predictions.

The background estimate due to the irreducible sources $t\bar{t}Z$, $ZZZ$ and $ZWW$ broken down per channel can be seen in Table 4.8. A summary of the data driven background estimate and irreducible background estimate per channel is provided in Table 4.9.

Table 4.9: Summary of total $ZZ$ background consisting of the data driven background estimate combined with the MC prediction from irreducible backgrounds. The first set of errors is statistical and the second set (if provided) are systematic errors.

4.9.8 Systematic Uncertainties on Monte Carlo Estimates

Muons

There are three types of systematic uncertainties associated with muon corrections:

1. **Reconstruction Efficiency**: The muon efficiency scale factors (4.6.2) have 1 $\sigma$ variations in both directions that give the associated systematic uncertainty.

2. **$p_T$ Smearing and Scale**: The muon momentum scale correction is determined as
described in section 4.7. Associated with this correction that smears the muon $p_T$ distribution in MC to match the data distribution, are 1 $\sigma$ variations (up and down) that allow the calculation of a systematic uncertainty associated with the smearing correction. For muons, the uncertainty is calculated separately for ID parameter variation and MS parameter variation and then subsequently added in quadrature to give the total smearing systematic uncertainty. The scale correction is varied up and down within 1 $\sigma$ and the resulting event difference between the nominal and varied yield is the associated scale systematic uncertainty.

3. Isolation and IP Scale Factors: The isolation and IP scale factors and the uncertainties are calculated using a $Z$ tag and probe method with the probe muons satisfying the nominal muon selection.

All muon systematics are summarized in Table 4.10.

4.9.9 Electrons

There are multiple types of systematic uncertainties associated with electron corrections:

1. Reconstruction and Identification efficiency: As mentioned in Section 4.6.3, there are scale factors applied to electrons to match the reconstruction and identification efficiencies measured in Monte Carlo to data. The identification and reconstruction efficiency scale factors and the associated uncertainties are dependent on $\eta$, $p_T$ and electron identification algorithm (Loose++, Medium++, Tight++, etc.). The $\eta$ and $p_T$ uncertainties are added in quadrature to give the total identification and reconstruction efficiency scale factor uncertainties. These scale factors are varied 1 $\sigma$ in both directions and the resulting event yield difference is taken as the systematic uncertainty.

2. Energy Scale: The electron energy scale correction factors are varied up and down within 1 $\sigma$ of the nominal values to obtain the systematic uncertainty on the predicted yield.
3. **Energy Reconstruction:** A smearing correction is applied to the electrons in Monte Carlo to match the energy resolution measured in data. The smearing corrections are varied within the measured $1\sigma$ and the resulting predicted yield difference gives the systematic uncertainty associated with the MC signal expectation.

4. **Isolation and IP scale factors:** The isolation and IP scale factors and the uncertainties are calculated using a $Z$ tag and probe method with the probe electrons satisfying the nominal electron selection.

All electron systematics are summarized in Table 4.10.

### 4.10 Comparison of Observed and Expected Events

Using 20.04 fb$^{-1}$ of 2012 data collected at $\sqrt{s} = 8$ TeV, a total of 305 candidate $pp \to ZZX \to l^+l^-l'^+l'^-$ events falling within the fiducial region were observed. The predicted $pp \to ZZX \to l^+l^-l'^+l'^-$ event yield within the same fiducial region using Monte Carlo modelling is $293.55 \pm 0.85\,\text{(stat.)} \pm 11.07\,\text{(syst.)}$ and the background expectation is $19.70 \pm 4.21\,\text{(stat.)} \pm 4.00\,\text{(syst.)}$. A summary of these results broken down per channel can be seen in Table 4.11.

### 4.11 Cross Section Extraction

#### 4.11.1 Cross Section Definition

The fiducial cross section for $pp \to ZZX \to l^+l^-l'^+l'^-$ is defined as

$$\sigma_{pp \to ZZX}^{\text{fid}} \cdot BR(Z \to \ell^+\ell^-)^2 = \frac{N_{\ell^+\ell^-\ell'^+\ell'^-}^{\text{obs}} - N_{\ell^+\ell^-\ell'^+\ell'^-}^{\text{bkg}}}{L \times C_{ZZ}}$$

(4.15)

where $N_{\ell^+\ell^-\ell'^+\ell'^-}^{\text{obs}}$ is the number of events observed in data, $N_{\ell^+\ell^-\ell'^+\ell'^-}^{\text{bkg}}$ is the data-driven estimate in Eq. 4.14, $L$ is the integrated luminosity represented by the data and $C_{ZZ}$ is the reconstruction correction factor defined in Equation 4.18. The branching ratio for each $Z$ decaying into electrons and/or muons used in Equation 4.15 is defined as
<table>
<thead>
<tr>
<th>Source (%)</th>
<th>eeee</th>
<th>µµµµ</th>
<th>eeeµ</th>
<th>llll</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Reconstruction Uncertainties</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$e$ momentum smearing</td>
<td>0.42</td>
<td>0.27</td>
<td>0.16</td>
<td>0.25</td>
</tr>
<tr>
<td>$e$ energy scale</td>
<td>0.00</td>
<td>0.00</td>
<td>0.04</td>
<td>0.02</td>
</tr>
<tr>
<td>$e$ identification efficiency</td>
<td>5.33</td>
<td>0.00</td>
<td>2.58</td>
<td>2.34</td>
</tr>
<tr>
<td>$e$ reconstruction</td>
<td>3.07</td>
<td>0.00</td>
<td>1.51</td>
<td>1.36</td>
</tr>
<tr>
<td>$e$ isolation $z_0$ $d_0$sig</td>
<td>1.77</td>
<td>0.00</td>
<td>0.82</td>
<td>0.76</td>
</tr>
<tr>
<td>$\mu$ momentum smearing</td>
<td>0.42</td>
<td>0.27</td>
<td>0.16</td>
<td>0.25</td>
</tr>
<tr>
<td>$\mu$ energy scale</td>
<td>0.00</td>
<td>0.05</td>
<td>0.03</td>
<td>0.02</td>
</tr>
<tr>
<td>$\mu$ reconstruction</td>
<td>0.00</td>
<td>1.02</td>
<td>0.52</td>
<td>0.58</td>
</tr>
<tr>
<td>$\mu$ isolation $z_0$ $d_0$sig</td>
<td>0.00</td>
<td>3.44</td>
<td>1.73</td>
<td>1.90</td>
</tr>
<tr>
<td>Trigger SF</td>
<td>0.03</td>
<td>0.20</td>
<td>0.09</td>
<td>0.11</td>
</tr>
<tr>
<td><strong>Total Reconstruction Uncertainty ($C_{ZZ}$)</strong></td>
<td>6.43</td>
<td>3.61</td>
<td>3.60</td>
<td>3.46</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>Theoretical Uncertainties</strong></th>
<th></th>
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<th></th>
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</thead>
<tbody>
<tr>
<td>MC Generator Difference ($C_{ZZ}$)</td>
<td>1.7</td>
<td>0.9</td>
<td>1.8</td>
<td>1.5</td>
</tr>
<tr>
<td>Scale ($C_{ZZ}$)</td>
<td>0.1</td>
<td>0.1</td>
<td>0.01</td>
<td>0.03</td>
</tr>
<tr>
<td>PDF ($C_{ZZ}$)</td>
<td>0.02</td>
<td>0.02</td>
<td>0.002</td>
<td>0.002</td>
</tr>
<tr>
<td>PDF and Scale ($A_{ZZ}$)</td>
<td></td>
<td></td>
<td>0.95</td>
<td></td>
</tr>
<tr>
<td>MC Generator Difference ($A_{ZZ}$)</td>
<td></td>
<td></td>
<td>0.78</td>
<td></td>
</tr>
<tr>
<td><strong>Total ($A_{ZZ}$)</strong></td>
<td></td>
<td></td>
<td>1.3</td>
<td></td>
</tr>
<tr>
<td><strong>Total ($C_{ZZ}$)</strong></td>
<td>6.65</td>
<td>3.72</td>
<td>4.02</td>
<td>3.77</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>Luminosity</strong></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>2.8</td>
</tr>
</tbody>
</table>

Table 4.10: Summary of the systematic uncertainties per channel and combined over all channels used in the $pp \rightarrow ZZX \rightarrow l^+l^-l^+l^-$ cross section calculation. The 4$\ell$ systematics are the weighted average of all the individual channel systematics.
Figure 4.6: Subleading Z mass plotted versus leading Z mass. The blue lines represent the Z mass window each Z must satisfy and the dashed green box represents the fiducial region in which every ZZ candidate event must fall.
Table 4.11: Summary of candidate events observed in data and expected events predicted from Monte Carlo in the 3 sub-channels and combined $\ell^+\ell^-\ell^+\ell^-$ channel. Also shown is the expected background contribution which consists of the data-driven estimation combined with the irreducible sources modelled by Monte Carlo.

$$BR(Z \to \ell^+\ell^-) = BR(Z \to e^+e^-) + BR(Z \to \mu^+\mu^-) \text{ where } BR(Z \to e^+e^-) = 0.0363 \pm 0.00004, \ BR(Z \to \mu^+\mu^-) = 0.0366 \pm 0.00007.$$  

The total cross section over the entire phase space is defined as

$$\sigma_{pp \to ZZX}^{tot} = \frac{N_{\ell^+\ell^- e^+e^-}^{obs} - N_{\ell^+\ell^- e^+e^-}^{bkg}}{L \times C_{ZZ} \times A_{ZZ} \times BR(Z \to \ell^+\ell^-)^2}$$ (4.16)

An extrapolation from the measurement done with detector acceptance is needed to extract the total $pp \to ZZX$ cross section. There are theoretical uncertainties associated with this extrapolation and it is because of this that a fiducial cross section is calculated free of the aforementioned uncertainties.

A quantity is defined

$$C_{ZZ} = \epsilon_{trig} \times \epsilon_{event} \times \epsilon_{lep} \times \epsilon_{\alpha}$$ (4.17)

where $\epsilon_{trig}$ is the trigger efficiency, $\epsilon_{event}$ is the efficiency of the event level selection (primary vertex, etc.), $\epsilon_{lep} = \epsilon_{lep1} \epsilon_{lep2} \epsilon_{lep3} \epsilon_{lep4}$ is the product of the individual lepton efficiencies for all four leptons that pass the object selection requirements and $\epsilon_{\alpha}$ is the reconstruction to generator-level fiducial region correction that includes the smearing and resolution corrections. $C_{ZZ}$ is calculated for each channel and is defined as the ratio of number of events which pass all selection requirements in MC to the number of events that fall within the fiducial region at the generator level as seen in Eq. 4.18. MC events that satisfy all selection requirements also have the necessary corrections (efficiency scale factors and smearing, etc)
The scale factor applied to MC events corrects for efficiency discrepancies between data and MC and is defined as

\[
SF = \frac{\epsilon_{\text{data}, \text{data}}}{\epsilon_{\text{MC}, \text{MC}} \cdot \epsilon_{\text{trig}, \text{reco}}}.
\]  

(4.19)

where \( \epsilon_{\text{reco}} = \epsilon_{\text{lep}} \cdot \epsilon_{\text{event}} \). \( C_{ZZ} \) extrapolates the reconstructed fiducial region to the truth fiducial region of each channel, however the branching ratios unique to each channel are necessary to be able to obtain a total cross section measurement.

The primary purpose of \( C_{ZZ} \) is to correct the measured reconstructed fiducial cross section to a truth-level fiducial cross section defined by the volume in which the measurement is made. Correcting to the fiducial volume keeps the measurement free of theoretical systematic uncertainties such as those associated with the PDF set used. This correction factor is essentially the probability that an event is reconstructed within the defined fiducial volume given that at truth-level this event and all associated objects passed all selection and event requirements.

Another correction factor is needed since the selection cuts for electrons and muons are different, thus resulting in the three sub-channels (\( eeee \), \( e\mu \mu \), \( \mu\mu\mu \)) having different geometric acceptances. Therefore a common geometric volume of acceptance is defined to which each sub-channel measurement is extrapolated using this correction factor. The common geometric region is defined in Section 4.8. This factor, \( A_{ZZ} \), corrects the reconstruction-level cross section to the full phase space describing the possible truth-level qualities and is defined as

\[
A_{ZZ} = \frac{N_{\text{MC Fiducial Volume Generated ZZ}}}{N_{\text{MC All Generated ZZ}}}.
\]  

(4.20)

The uncertainties associated with this correction factor come from the PDF set, varying the PDF set, varying the factorization and renormalization scales and the weighted
difference of the calculation of this factor using a $gg \to ZZ$ generator.

The factorization and renormalization scales are varied up and down by a factor of two and the resulting difference in $A_{ZZ}$ is taken as an uncertainty.

$A_{ZZ}$ is calculated using version 6.3 of the MCFM generator. The total cross section extrapolated to is the scenario where both $Z$ bosons have invariant masses between 66 and 116 GeV. Version 6.3 of MCFM includes the gluon-gluon production contribution and amounts to 5.93% of the total NLO cross section.

Using the CT10 PDF set with the renormalization and factorization scales ($\mu_R = \mu_F$) set to $0.5 \cdot m_{ZZ}$ the cross section for on-shell $ZZ$ production at NLO is $7.23^{+0.30}_{-0.22}$ pb.

The cross section results using a zero-width approximation for the $Z$ mass are shown in Table 4.12.

<table>
<thead>
<tr>
<th></th>
<th>$\sigma$ (fb)</th>
<th>Shift (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>CT10 error set</td>
</tr>
<tr>
<td>$e^+e^-\mu^+\mu^-$</td>
<td>10.54±0.01</td>
<td>+2.0</td>
</tr>
<tr>
<td>$l^+l^-l^+l^-$</td>
<td>21.08±0.02</td>
<td>+2.0</td>
</tr>
</tbody>
</table>

Table 4.12: NLO cross sections calculated using MCFM (version 6.3) in QCD using the on-shell approximation. The errors on the central values of the cross sections are due to MC statistics. The cross sections shown are for the $ee\mu\mu$ final state; the $4\ell$ final state is twice that value. The “CT10 error set” values are derived using the 52 CT10 error sets and the difference in central values between CT10 and MSTW2008NLO. The “scale variation” values are the differences between varying the default value of $m_{ZZ}$ up and down by a factor of two.

The values of $A_{ZZ}$ and $C_{ZZ}$ are summarized in Table 4.13 for all channels. The values of $C_{ZZ}$ for different generators are presented in Table 4.14.

### 4.11.2 Cross Section Calculation

The number of observed events in data and expected (MC prediction) number of events along with an estimate on the number of background events is needed to calculate the total
Table 4.13: The fiducial reconstruction correction factors for different generators. The errors quoted are statistical only. The differences are taken as a systematic error on $C_{ZZ}$.

<table>
<thead>
<tr>
<th>Correction</th>
<th>$e^+e^-e^+e^-$</th>
<th>$\mu^+\mu^-\mu^+\mu^-$</th>
<th>$e^+e^-\mu^+\mu^-$</th>
<th>$\ell^+\ell^-\ell^+\ell^-$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Powheg</td>
<td>0.536±0.008</td>
<td>0.815±0.007</td>
<td>0.663±0.006</td>
<td>0.670±0.004</td>
</tr>
<tr>
<td>gg2zz</td>
<td>0.564±0.009</td>
<td>0.815±0.008</td>
<td>0.672±0.009</td>
<td>0.680±0.005</td>
</tr>
<tr>
<td>Powheg+gg2zz</td>
<td>0.538±0.007</td>
<td>0.815±0.006</td>
<td>0.663±0.006</td>
<td>0.671±0.004</td>
</tr>
</tbody>
</table>

Table 4.14: The fiducial reconstruction and total geometric acceptance correction factors per channel and combined over all channels is presented above. For $C_{ZZ}$ the first uncertainty is statistical and the second systematic. The $A_{ZZ}$ uncertainty is comprised of the PDF and scale systematics.

<table>
<thead>
<tr>
<th>Factor</th>
<th>$e^+e^-e^+e^-$</th>
<th>$\mu^+\mu^-\mu^+\mu^-$</th>
<th>$e^+e^-\mu^+\mu^-$</th>
<th>$\ell^+\ell^-\ell^+\ell^-$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_{ZZ}$</td>
<td>0.644±0.001±0.008</td>
<td>0.644±0.001±0.008</td>
<td>0.644±0.001±0.008</td>
<td>0.644±0.001±0.008</td>
</tr>
<tr>
<td>$C_{ZZ}$</td>
<td>0.538±0.007±0.036</td>
<td>0.815±0.006±0.030</td>
<td>0.663±0.006±0.027</td>
<td>0.671±0.004±0.026</td>
</tr>
</tbody>
</table>

and fiducial cross sections. The number of expected events can be written as

$$N_{exp} = N_{sig} + N_{bkg} \quad (4.21)$$

where $N_{sig}$ is the number of signal events predicted by the MC samples listed in Table 4.1 and $N_{bkg}$ is the data-driven background estimate in Eq. 4.14. The number of signal events can be written as a function of the total cross section as in Eq. 4.22.

$$N_{sig}(\sigma_{pp\rightarrow ZZX}^{tot}) = \sigma_{pp\rightarrow ZZX}^{tot} \times BR(Z \rightarrow \ell^+\ell^-)^2 \times \mathcal{L} \times A_{ZZ} \times C_{ZZ} \quad (4.22)$$

where $BR$ is the branching ratio representing both $Z$ bosons decaying to $\ell^+\ell^-$ (where $\ell = e, \mu$). Eq. 4.22 can be expressed in terms of the fiducial cross section by substituting $\sigma_{pp\rightarrow ZZX}^{tot}$ for $\sigma_{pp\rightarrow ZZX}^{fid}$ and removing $A_{ZZ}$ and $BR$. The systematic uncertainties must be taken into account resulting in the addition of a sum of terms to Eq. 4.22.
\[
N_{\text{sig}}(\sigma_{pp\rightarrow ZZX}^{\text{tot}} , \{x\}) = \sigma_{pp\rightarrow ZZX}^{\text{tot}} \times BR(Z \rightarrow \ell^+\ell^-)^2 \times \mathcal{L} \times A_{ZZ} \times C_{ZZ} \\
\times \left( 1 + x_{AZZ} \sigma_{AZZ} + x_{CZZ} \sigma_{CZZ} + x_{\text{lumi}} \sigma_{\text{lumi}} \right) \tag{4.23}
\]
and
\[
N_{\text{bkg}}(\{x\}) = N_{\text{bkg}} \times \left( 1 + x_{\text{bkg}} \sigma_{\text{bkg}} \right) \tag{4.24}
\]

In Eq. 4.23 and 4.24, each \(x\) is assumed to be normally distributed with a mean of 0 and a variance of 1. The parameters \(\sigma_{AZZ}, \sigma_{CZZ}, \sigma_{\text{lumi}}, \sigma_{\text{bkg}}\) are the standard deviations for each corresponding systematic source.

A minimum log-likelihood technique is used to calculate the cross section (both fiducial and total). Using the information above a likelihood function is defined as
\[
L(\sigma, \{x\}) = \prod_{k=1}^{m} P(N_{\text{data}}, \mu(\sigma, \{x_k\})) \times \frac{1}{(\sqrt{2\pi})^m} e^{-\frac{1}{2}(x_k)^2} \tag{4.25}
\]
where each \(x_k\) is a source of systematic error represented by the \(x\) terms in Equation 4.23 and Equation 4.24. Additionally,
\[
\mu(\sigma, \{x_k\}) = N_{\text{sig}}(\sigma, \{x_k\}) + N_{\text{bkg}}(\{x_k\}). \tag{4.26}
\]

The value of \(m\) in Equation 4.25 is 4 and accounts for the four sources of systematic uncertainty, \(C_{ZZ}, A_{ZZ}\), background estimation and luminosity. The total systematic uncertainties associated with the correction factors are listed in Table 4.10.

The expression \(P(N_{\text{data}}, \mu(\sigma, \{x_k\}))\), in Eq. 4.25 represents the Poisson probability that the number of signal plus background events \((N_{\text{sig}}(\sigma, \{x_k\}) + N_{\text{bkg}}(\{x_k\}))\) will yield the number of events observed in data. The negative log of Eq. 4.25 is minimized with respect to \(\sigma\) and \(x_k\) using Rooftit. The values of the floating parameters are seen in Figure 4.7.

The fiducial cross section is determined to be:
\[
\sigma_{pp\rightarrow ZZX}^{\text{fid}} \cdot BR(Z \rightarrow \ell^+\ell^-)^2 = 20.84^{+1.30}_{-1.25} \text{ (stat.)}^{+0.91}_{-0.82} \text{ (syst.)}^{+0.62}_{-0.55} \text{ (lumi.)} \text{ fb} \tag{4.27}
\]
Figure 4.7: Shown in this figure are the parameter values that minimize the negative log of the likelihood function. Also shown are the correlation and covariance matrices resulting from the fit.
and after extrapolating to the total phase space the total $ZZ$ cross section is:

$$\sigma_{pp\rightarrow ZZX}^{tot} = 7.19^{+0.45}_{-0.43} \text{ (stat.)}^{+0.35}_{-0.31} \text{ (syst.)}^{+0.21}_{-0.19} \text{ (lumi.) pb.}$$  \hspace{1cm} (4.28)$$

## 4.12 Anomalous Triple Gauge Coupling Limits

### 4.12.1 Matrix Element Reweighting

A matrix element reweighting method is used to facilitate the limit extraction of the aTGC couplings. The elements describing the effects of the aTGCs are calculated at NLO using the BHO MC generator and Baur-Rainwater MC generator described in detail in [30] [31] [32]. The magnitude of the effect is calculated using the four vectors and PDG ID codes of the colliding partons and outgoing particles originating from the hard scattering [33].

This reweighting scheme allows an on-the-fly reweighting of existing MC events and is applicable to any ZZ MC sample as long as there is information present regarding the hard process and the colliding partons.

As was stated earlier, there are four unique anomalous couplings present in ZZ production ($f^{V}_{4}$ and $f^{V}_{5}$ where $V = \gamma, Z$). As shown in Eq. 2.8, each coupling appears linearly in the effective Lagrangian. This means that the scattering amplitude and indirectly the cross section will contain linear and quadratic coupling terms. The cross section including the aTGC enhancement can be written as

$$\sigma_{SM+TGC} = F_{00} + f^{\gamma}_{4} F_{01} + f^{\gamma}_{4} F_{02} + f^{Z}_{5} F_{03} + f^{Z}_{5} F_{04}$$

$$+ (f^{\gamma}_{4})^2 F_{11} + f^{\gamma}_{4} f^{\gamma}_{4} F_{12} + f^{\gamma}_{4} f^{Z}_{5} F_{13} + f^{\gamma}_{4} f^{Z}_{5} F_{14}$$

$$+ (f^{Z}_{5})^2 F_{22} + f^{Z}_{5} f^{Z}_{5} F_{23} + f^{Z}_{5} f^{Z}_{5} F_{24}$$

$$+ (f^{Z}_{5})^2 F_{33} + f^{Z}_{5} f^{Z}_{5} F_{34}$$

$$+ (f^{Z}_{5})^2 F_{44}$$  \hspace{1cm} (4.29)$$

where $F_{kl}$ are coefficients describing individual contributions to the total cross section. $F_{00}$ represents the Standard Model contribution and the rest represent the contributions due to the anomalous triple gauge couplings. From equation 4.29, one can subsequently reweight any event from a sample with trivial couplings to an event representing a particular non-
trivial aTGC point. The weight is defined as

\[ \text{Weight}_{\text{event}} = \frac{\sigma_{SM+TGC}}{\sigma_{SM}} \quad (4.30) \]

The \( F_{kl} \) coefficients are entirely governed by the interacting particles specific to the samples being used meaning that they do not change with respect to the anomalous couplings and therefore only needed to be determined once. If we consider a Standard Model coupling, then in principal there are 5 couplings in total. This means that there are 25 coefficients resulting from the calculation of the scattering amplitude, but due to symmetry \( (F_{kl} = F_{lk}) \) this reduces the number to 15 unique coefficients. These coefficients can be determined by considering 15 different cases of coupling values and then solving the linear system of equations.

To clarify a simple example is presented. Consider a case where only a single coupling is present. This reduces equation 4.29 to

\[ \sigma_{SM+TGC} = F_0 + fF_1 + f^2F_2 \quad (4.31) \]

Now let’s consider three different coupling values for \( f: 0, 1, -1 \). This now yields three independent linear equations that may be written in matrix form.

\[
\begin{bmatrix}
\sigma_a \\
\sigma_b \\
\sigma_c
\end{bmatrix} =
\begin{bmatrix}
1 & 0 & 0 \\
1 & 1 & 1 \\
1 & -1 & 1
\end{bmatrix}
\begin{bmatrix}
F_0 \\
F_1 \\
F_2
\end{bmatrix} \quad (4.32)
\]

Here \( F_0 \) is the Standard Model contribution. The \( 3 \times 3 \) matrix from here on will be referred to as \( \hat{A} \) and contains the coupling values for each case. To determine \( \vec{F} \) one simply inverts equation 4.32. Extrapolating this simple example to the case where there are four distinct couplings, \( \hat{A} \) is a \( 15 \times 15 \) matrix and \( \vec{\sigma} \) and \( \vec{F} \) is a 15-dimensional vector.

Using the coefficients derived above extraction of the yield coefficients is now possible.
The yield coefficient is defined as

\[ Y_{kl} = \frac{F_{kl}}{F_{00}} \cdot N_{expected}. \] (4.33)

where \( N_{expected} \) is the number of signal events predicted by Monte Carlo and \( F_{00} = 250.19 \).

Every yield coefficient is determined with all corrections to MC taken into account as this is necessary to achieve an accurate prediction of the number of events expected. The yield coefficients for two scenarios are summarized in Table 4.16 and Tables 4.17, 4.18.

<table>
<thead>
<tr>
<th>( Y_{SM} )</th>
<th>( Y_{f_{\gamma}^4} )</th>
<th>( Y_{f_{\gamma}^5} )</th>
<th>( Y_{f_{Z}^4} )</th>
<th>( Y_{f_{Z}^5} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>0.003 ± 0.009</td>
<td>-0.038 ± 0.019</td>
<td>0.047 ± 0.010</td>
<td>-0.026 ± 0.023</td>
</tr>
<tr>
<td>( Y_{f_{\gamma}^4 f_{\gamma}^4} )</td>
<td>( Y_{f_{\gamma}^4 f_{\gamma}^5} )</td>
<td>( Y_{f_{\gamma}^5 f_{\gamma}^5} )</td>
<td>( Y_{f_{Z}^4 f_{Z}^4} )</td>
<td>( Y_{f_{Z}^4 f_{Z}^5} )</td>
</tr>
<tr>
<td>507.50 ± 6.84</td>
<td>454.4 ± 6.33</td>
<td>-0.272 ± 0.625</td>
<td>-0.257 ± 0.349</td>
<td></td>
</tr>
<tr>
<td>( Y_{f_{\gamma}^5 f_{\gamma}^5} )</td>
<td>( Y_{f_{Z}^4 f_{Z}^4} )</td>
<td>( Y_{f_{Z}^4 f_{Z}^5} )</td>
<td>( Y_{f_{Z}^5 f_{Z}^5} )</td>
<td></td>
</tr>
<tr>
<td>686.3 ± 9.85</td>
<td>-0.258 ± 0.349</td>
<td>-0.729 ± 1.12</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( Y_{f_{Z}^4 f_{\gamma}^5} )</td>
<td>( Y_{f_{Z}^5 f_{\gamma}^5} )</td>
<td>( Y_{f_{Z}^5 f_{Z}^5} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>496.2 ± 6.73</td>
<td>443.3 ± 6.22</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( Y_{f_{\gamma}^5 f_{Z}^5} )</td>
<td>( Y_{f_{Z}^5 f_{Z}^5} )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>669.0 ± 9.69</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 4.15: Each term in the above table represents the enhancement in terms of the Standard Model (no aTGC enhancement) yield for each coupling. They are determined for a 1 bin leading \( Z \) \( p_T \) distribution.

The yield coefficients were calculated using a TGC sample with \( f_{\gamma}^4 = 0.1, f_{\gamma}^5 = 0.0, f_{Z}^4 = 0.0, f_{Z}^5 = 0.0 \). To check the validity of the event weights produced by the matrix element method, the aforementioned TGC sample was reweighted to the Standard Model expectation. The results are seen in Figs. 4.8 and 4.9.

### 4.12.2 Limit Procedure and Method

All limits on anomalous couplings are set at the 95% confidence level using a frequentist method and are determined by minimizing a negative log likelihood function. The system-
Table 4.16: The yield coefficients and their respective errors are seen above which are determined for a 1 bin leading $Z_p T$ distribution. The coefficients give the total number of expected events when multiplying by their respective couplings: i.e. if $f^4_\gamma = 0.1$ and $f^2_\gamma, f^2_\gamma = 0.0$ then the total yield would be $Y_{SM} + Y_{f^4_\gamma} + Y_{f^2_\gamma} Y_{f^2_\gamma} (f^2_\gamma)^2$. Note that $Y_{SM} = 293.55$ events which is the Standard Model predicted yield.

\[
\begin{array}{cccc}
Y_{SM} & Y_{f^4_\gamma} & Y_{f^2_\gamma} & Y_{f^2_\gamma} \\
293.55 & 0.776 \pm 2.58 & -11.02 \pm 5.57 & 13.83 \pm 2.90 & -7.59 \pm 6.63 \\
Y_{f^4_\gamma f^4_\gamma} & Y_{f^2_\gamma f^2_\gamma} & Y_{f^2_\gamma f^2_\gamma} & Y_{f^2_\gamma f^2_\gamma} \\
148976 \pm 5742.2 & 133380 \pm 5162.4 & -79.85 \pm 183.6 & -75.46 \pm 102.37 \\
Y_{f^2_\gamma f^2_\gamma} & Y_{f^2_\gamma f^2_\gamma} & Y_{f^2_\gamma f^2_\gamma} & Y_{f^2_\gamma f^2_\gamma} \\
201466 \pm 7828.2 & -75.77 \pm 102.37 & -214.11 \pm 328.89 & \\
Y_{f^2_\gamma f^2_\gamma} & Y_{f^2_\gamma f^2_\gamma} & Y_{f^2_\gamma f^2_\gamma} & Y_{f^2_\gamma f^2_\gamma} \\
145657 \pm 5618.3 & 130125 \pm 5041.4 & \\
Y_{f^2_\gamma f^2_\gamma} & Y_{f^2_\gamma f^2_\gamma} & Y_{f^2_\gamma f^2_\gamma} & Y_{f^2_\gamma f^2_\gamma} \\
196395 \pm 7640.6 & \\
\end{array}
\]

Gaussian nuisance parameters in the likelihood function.

The likelihood function used to determine the TGC coupling values is the product of a Poisson probability distribution (P) which describes the data and MC measurements and Gaussian terms representing the systematics (nuisance parameters). The nuisance parameters are referred to as $x_{jk}$ and $\sigma_{jk}$ are the standard deviations representing the $k^{th}$ systematic in the $j^{th}$ bin. The systematic uncertainty per bin can be expressed as

\[
\frac{\text{true } N_{\text{sig}}}{N_{\text{sig}}} = \left(1 + x_{AZZ} \frac{\sigma_{AZZ}}{A_{ZZ}} + x_{CZZ} \frac{\sigma_{CZZ}}{C_{ZZ}} + x_{lumi} \frac{\sigma_{lumi}}{L} \right) \\
\frac{\text{true } N_{\text{bkg}}}{N_{\text{bkg}}} = \left(1 + x_{bkg} \frac{\sigma_{bkg}}{N_{bkg}} \right)
\] (4.34) (4.35)
\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|c|}
\hline
 & $Y_{SM}$ & $Y_{f_4^Z}$ & $Y_{f_5^Z}$ & $Y_{f_5^Z}$ \\
\hline
$Y_{f_4^Z}f_4^Z$ & 0.199$\pm$ 0.078 & -0.681$\pm$ 0.151 & 1.02$\pm$ 0.190 & 10.8$\pm$ 0.98 \\
\hline
$Y_{f_5^Z}f_5^Z$ & 278.6$\pm$ 20.9 & 288.7$\pm$ 21.7 & 0.678$\pm$ 0.150 & -0.892$\pm$ 0.176 \\
\hline
$Y_{f_5^Z}f_5^Z$ & 466.5$\pm$ 34.9 & -0.886$\pm$ 0.175 & -2.38$\pm$ 0.319 & \\
\hline
$Y_{f_5^Z}f_5^Z$ & 192.6$\pm$ 14.5 & 200.6$\pm$ 15.1 & & \\
\hline
$Y_{f_5^Z}f_5^Z$ & 326.4$\pm$ 24.4 & & & \\
\hline
\end{tabular}
\caption{The yield coefficients and their respective errors are seen above for two of the four bins in leading $Z$ $p_T$.}
\end{table}

The entire likelihood function is as follows:

\begin{equation}
L(f^V, \{x_{jk}\}) = \prod_{k=1}^{m} \prod_{j=1}^{n} P(N_{data}^j, \mu^{jk}(f^V, \{x_{jk}\})) \times \frac{1}{(\sqrt{2\pi})^{(n+m)}} e^{-\frac{1}{2}(x_{jk})^2},
\end{equation}

where $m = 4$ due to the sources of systematic uncertainty ($C_{ZZ}, A_{ZZ}, \text{lumi}$ and $N_{bkg}$) and $n$ is the number of bins in the $p_T$ distribution of the leading $Z$.

\begin{equation}
\mu^{jk}(f, \{x_{jk}\}) = N_{sig}^{jk}(f^V, \{x_{jk}\}) + N_{bkg}^{jk}(\{x_{jk}\})
\end{equation}
Table 4.18: The yield coefficients and their respective errors are seen above for each of the four bins in leading \( p_T \). The large errors in the highest \( p_T \) bin are due to the small number of events present in the high \( p_T \) regime in the SM Monte Carlo.

where \( f^V \) can be any of the triple gauge couplings \((f_4^\gamma, f_4^Z, f_5^\gamma, f_5^Z)\) and

\[
N_{\text{sig}} = (Y_{\text{SM}} + Y_{f^V} \cdot f^V + Y_{f^V f^V} \cdot (f^V)^2) \cdot L \cdot C_{ZZ}. \tag{4.38}
\]

As was mentioned in Section 4.12.1, the number of signal events \( (N_{\text{sig}}) \) is expressed as a function quadratic in triple gauge coupling. For example, if \( f_4^\gamma = 0.1 \) and all other couplings are set to their SM values of 0, the number of signal events can be expressed as:

\[
N_{\text{sig}} = Y_{\text{SM}} + Y_{f_4^\gamma} f_4^\gamma + Y_{f_4^\gamma f_4^\gamma} f_4^\gamma f_4^\gamma. \tag{4.39}
\]

This equation is plotted in Figure 4.12.
Figure 4.8: Leading Z transverse momentum spectrum of a TGC sample with $f_4^\gamma = 0.1$ reweighted to the Standard Model.

The negative log likelihood of the function in Equation 4.36 is minimized with respect to $f^V$ (coupling) with one particular coupling solely responsible for the enhancement to the SM cross section. The value of coupling that minimizes the aforementioned function is then used to calculate a profile likelihood test statistic defined as:

$$q(f^V) = \frac{-\ln(L(f^V, x_{jk}^\hat{V}))}{-\ln(L(f^V, x_{jk}))}$$  \hspace{1cm} (4.40)$$

where $x_{jk}^\hat{V}$ is the value of $x_{jk}$ that minimizes the numerator for a fixed coupling value and $x_{jk}$ are the values of $f^V$ and $x_{jk}$ that minimize the denominator. A test statistic value, $q_{\text{obs}}(f^V)$, is determined using the number of events observed in data for each coupling. To determine how often an outcome at least as unlikely as the actual observation is expected to occur, 10,000 pseudo-experiments are performed for each value of $f^V$. The test statistics
Figure 4.9: Invariant mass spectrum of the 4 lepton system of a TGC sample with $f_4^\gamma = 0.1$ reweighted to the Standard Model.

$q_{\text{pseudo}}(f^V)$ are then compared with $q_{\text{obs}}(f^V)$. In each pseudo experiment, the systematic nuisance parameters are fluctuated according to a Gaussian distribution around the mean value of $x_{jk}$ for a fixed coupling value. The number of “observed” events, $N_{\text{pe}}$ is drawn randomly from a Poisson distribution with mean equal to the number of events computed using the corresponding coupling ($f^V$) and systematic values.

The p-value, defined for each value of $f^V$, is defined as the fraction of pseudo experiments whose test statistic, $q_{\text{pe}}$, is smaller than the observed test statistic, $q_{\text{obs}}(f^V)$. By scanning through many values of $f^V$, all values of the coupling for which $p(f^V) \geq 5\%$ can be calculated to define the 95% confidence interval of $f^V$ calculated using data. If the coupling value that minimizes the negative log likelihood function falls within this interval, it is said to be set at the 95% confidence level (CL).
Figure 4.10: The $p_T$ spectrum of the leading Z boson in a signal event for various TGC point values. If coupling value is not specified in legend, it is set to 0. As is expected there is a significant enhancement in the higher $p_T$ regime for non zero aTGCs.

To determine what the expected limits should be using the SM predicted yield (signal plus background) a large number (5,000) pseudo experiments are performed using the SM predicted yield as $n_{\text{obs}}$. The systematics are treated as Gaussian nuisance parameters and the number of “observed” events are sampled from a Poisson distribution with the mean set the SM predicted yield.

4.12.3 Observed and Expected Limits

Two sets of limits are presented using the 20.04 fb$^{-1}$ data set collected at $\sqrt{s} =$ 8 TeV with ATLAS. The limits are determined from event yields in the leading $Z$ $p_T$ distribution. The limits for 1 bin in leading $Z$ $p_T$ are presented in Table 4.19 and 4 bins in leading $Z$
Figure 4.11: Invariant mass spectrum of the 4 lepton system in a signal event for various TGC point values. If coupling value is not specified in legend, it is set to 0. As is expected there is a significant enhancement in the higher mass regime.

$p_T$ are presented in Table 4.20.
Figure 4.12: Event yield as a function of $f_4^\gamma$.

Figure 4.13: Event yield as a function of $f_4^Z$.  

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Figure 4.14: Event yield as a function of $f_5^\gamma$.

Figure 4.15: Event yield as a function of $f_5^Z$. 
<table>
<thead>
<tr>
<th>Coupling</th>
<th>Expected Limit</th>
<th>Observed Limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>No Form Factor</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$f_4^\gamma$</td>
<td>[-0.018, 0.018] ± 0.003</td>
<td>[-0.016, 0.016]</td>
</tr>
<tr>
<td>$f_4^Z$</td>
<td>[-0.016, 0.016] ± 0.003</td>
<td>[-0.014, 0.014]</td>
</tr>
<tr>
<td>$f_5^\gamma$</td>
<td>[-0.018, 0.018] ± 0.003</td>
<td>[-0.016, 0.016]</td>
</tr>
<tr>
<td>$f_5^Z$</td>
<td>[-0.016, 0.016] ± 0.003</td>
<td>[-0.014, 0.014]</td>
</tr>
</tbody>
</table>

Table 4.19: Anomalous Triple Gauge Coupling limits for a 1 bin leading $Z p_T$ distribution.

<table>
<thead>
<tr>
<th>Coupling</th>
<th>Expected Limit</th>
<th>Observed Limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>No Form Factor</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$f_4^\gamma$</td>
<td>[-0.007, 0.007] ± 0.002</td>
<td>[-0.004, 0.004]</td>
</tr>
<tr>
<td>$f_4^Z$</td>
<td>[-0.006, 0.006] ± 0.002</td>
<td>[-0.003, 0.003]</td>
</tr>
<tr>
<td>$f_5^\gamma$</td>
<td>[-0.007, 0.007] ± 0.002</td>
<td>[-0.004, 0.004]</td>
</tr>
<tr>
<td>$f_5^Z$</td>
<td>[-0.006, 0.006] ± 0.002</td>
<td>[-0.003, 0.003]</td>
</tr>
</tbody>
</table>

Table 4.20: Anomalous Triple Gauge Coupling limits for a 4 bin leading $Z p_T$ distribution.
Chapter 5
CONCLUSIONS

With 20.04 fb$^{-1}$ of data collected at $\sqrt{s} = 8$ TeV by the ATLAS detector, there is a unique opportunity to not only precisely measure electroweak processes, and more specifically, the $pp \to ZZX$ cross section, but also search for anomalous triple gauge couplings. In the aforementioned data, 305 $ZZ \to \ell\ell\ell'$ events were observed (summary of events per channel in Table 4.11). The Standard Model expected yield including the signal and background expectations is $313.3 \pm 4.1$(stat.) $\pm 11.8$(syst.). The $pp \to ZZX$ fiducial cross section is measured to be:

$$\sigma_{pp \to ZZX}^{\text{fid}} \cdot \text{BR}(Z \to \ell^+ \ell^-)^2 = 20.84_{-1.25}^{+1.30}(\text{stat.})_{-0.82}^{+0.91}(\text{syst.})_{-0.55}^{+0.62}(\text{lumi.}) \text{ fb.} \quad (5.1)$$

Extrapolating to the total phase space, the total $pp \to ZZX$ cross section is measured to be:

$$\sigma_{pp \to ZZX}^{\text{tot}} = 7.19_{-0.43}^{+0.45}(\text{stat.})_{-0.31}^{+0.35}(\text{syst.})_{-0.19}^{+0.21}(\text{lumi.}) \text{ pb} \quad (5.2)$$

which is found to be in good agreement with the SM predicted total cross section of $\sigma_{SM} = 7.23_{-0.22}^{+0.30}$ pb.

The 1 bin limits set on the $f_4^\gamma, f_4^Z, f_5^\gamma, f_5^Z$ anomalous triple gauge couplings are roughly 3 times tighter and the 4 bin limits are approximately 5 times tighter than the respective limits previously set in the 4.7 fb$^{-1}$ analysis [10].
Figure 5.1: Summary of cross sections for various electroweak processes.

Figure 5.2: Limits on anomalous triple gauge couplings measured by multiple experiments including limits set in this analysis.


[27] https://twiki.cern.ch/twiki/bin/viewauth/atlas/calomuoncontainer.


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Appendix A
THE HIGGS MECHANISM

Starting with a simple example one can gain an understanding of the Higgs mechanism. Assume a Lagrangian defined as

\[ \mathcal{L} = (\partial^\mu \phi^*)(\partial_\mu \phi) - m^2 \phi^* \phi - \lambda(\phi^* \phi)^2 = (\partial_\mu \phi)(\partial^\mu \phi^*) - V(\phi, \phi^*) \quad (A.1) \]

where the \( \lambda \) term represents self-interaction between the scalar field \( \phi \). To obtain the ground state, one must minimize the potential \( V(\phi, \phi^*) \) giving

\[ \frac{\partial V}{\partial \phi} = m^2 \phi^* + 2\lambda \phi^* (\phi^* \phi). \quad (A.2) \]

Assuming \( \lambda > 0 \), if \( m^2 > 0 \) the minimum occurs when \( \phi = \phi^* = 0 \), however if \( m^2 < 0 \) a local maximum exists when \( \phi = 0 \) and the minimum exists when

\[ |\phi|^2 = \frac{m^2}{2\lambda}. \quad (A.3) \]

If a global transformation such as the one described in Eq. A.4 is applied to the scalar field

\[ \phi \rightarrow e^{i\theta} \phi \quad (A.4) \]

one can see that the Lagrangian and the resulting ground state are unchanged when compared to the original Lagrangian (Eq. A.1) and ground state (Eq. A.3). This means the Lagrangian is invariant under that global transformation and the ground state is degenerate in \( \theta \).
If we then define $\phi$ as

$$\phi = \frac{1}{\sqrt{2}}(\phi_1 + i\phi_2) \quad (A.5)$$

and minimize $V$ with respect to $\phi_1$ we find that if $m^2 < 0$ the vacuum expectation value (vev) is

$$(\phi_1^2 + \phi_2^2) = \frac{-m^2}{2\lambda} = a^2. \quad (A.6)$$

If we choose $\langle \phi_1 \rangle_0 = a$, then it follows that $\langle \phi_2 \rangle_0 = 0$. As this choice is made, the symmetry is spontaneously broken. We can expand $\phi$ around $a$ so that it is now defined as

$$\phi = a + \frac{1}{\sqrt{2}}(\phi_1 + i\phi_2). \quad (A.7)$$

The Lagrangian now becomes

$$\mathcal{L} = \frac{1}{2}(\partial^\alpha \phi_1)(\partial_\alpha \phi_1) + \frac{1}{2}(\partial^\alpha \phi_2)(\partial_\alpha \phi_2) - 2\lambda a^2 \phi_1^2 - \sqrt{2}\phi_1(\phi_1^2 + \phi_2^2) - \frac{\lambda}{4}(\phi_1^2 + \phi_2^2)^2. \quad (A.8)$$

As one can see, there is no $\phi_2^2$ term, meaning that $\phi_2$ is a massless field, however there is a $\phi_1^2$ term. Spontaneously breaking the symmetry of the Lagrangian has resulted in one massive field and one massless field.

Consider a Lagrangian that is invariant under the local transformation

$$\phi \rightarrow e^{i\Lambda(x)} \phi. \quad (A.9)$$

In order for the aforementioned Lagrangian to be invariant the covariant derivative is introduced and has the form

$$D_\mu = (\partial_\mu + ieA_\mu) \quad (A.10)$$
resulting in

\[ \mathcal{L} = (\partial_{\mu} + ieA_{\mu})\phi(\partial^{\mu} + ieA^{\mu})\phi^* - m^2(\phi^* \phi) - \lambda((\phi^* \phi))^2 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}. \]  

(A.11)

Using the scalar field definition in Eq. A.7, Eq. A.11 becomes

\[ \mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + e^2a^2A_{\mu}A_{\mu} + \frac{1}{2}(\partial_{\mu}\phi_1)^2 + \frac{1}{2}(\partial_{\mu}\phi_2)^2 - 2\lambda a^2\phi_1^2 + \sqrt{2}eaA_{\mu}\partial_{\mu}\phi_2 + H.O. \]  

(A.12)

The two massive fields in this case are the photon \((A_{\mu})\) and \(\phi_1\) and the massless field is \(\phi_2\). However, there is also a mixed term in \(A_{\mu}\) and \(\phi_2\) indicating that a photon can change into a scalar field which does not make much sense. In fact, \(\phi_2\) can be eliminated entirely by a gauge transformation. For infinitesimal \(\Lambda\) the components of \(\phi\) transform like

\[ \phi_1' = \phi_1 - \Lambda\phi_2 \]
\[ \phi_2' = \phi_2 + \Lambda\phi_1 + \sqrt{2}\Lambda a \]  

(A.13)

A particular value of \(\Lambda\) may be chosen so that \(\phi_2 = 0\) resulting in the Lagrangian having only two fields (Eq. A.14), both of which are massive.

\[ \mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + e^2a^2A_{\mu}A_{\mu} + \frac{1}{2}(\partial_{\mu}\phi_1)^2 - 2\lambda a^2\phi_1^2 + \text{coupling terms} \]  

(A.14)

This method, when applied to the electroweak Lagrangian, gives rise to three massive vector bosons \((W^\pm, Z)\), one massless boson (photon) and one massive scalar field called the Higgs boson.
Appendix B

Additional Kinematic Plots

Figure B.1: $\ell^+\ell^-\ell'^+\ell'^-$ invariant mass distribution.
Figure B.2: $e^+e^-e^+e^-$ invariant mass distribution.

Figure B.3: $\mu^+\mu^-\mu^+\mu^-$ invariant mass distribution.
Figure B.4: $e^+ e^- \mu^+ \mu^-$ invariant mass distribution.

Figure B.5: $\ell^+ \ell^- \ell'^+ \ell'^-$ $p_T$ distribution
Figure B.6: $e^+e^-e^+e^-$ $p_T$ distribution

Figure B.7: $\mu^+\mu^-\mu^+\mu^-$ $p_T$ distribution
Figure B.8: $e^+ e^- \mu^+ \mu^- p_T$ distribution

Figure B.9: $\ell^+ \ell^- \ell'^+ \ell'^-$ leading lepton pair mass distribution. Sub-leading lepton pair has a mass between 66 and 116 GeV.
Figure B.10: $e^+e^-e^+e^-$ leading lepton pair mass distribution. Sub-leading lepton pair has a mass between 66 and 116 GeV.

Figure B.11: $\mu^+\mu^-\mu^+\mu^-$ leading lepton pair mass distribution. Sub-leading lepton pair has a mass between 66 and 116 GeV.
Figure B.12: $e^+e^-\mu^+\mu^-$ leading lepton pair mass distribution. Sub-leading lepton pair has a mass between 66 and 116 GeV.

Figure B.13: $\ell^+\ell^-\ell'^+\ell'^-$ sub-leading lepton pair mass distribution. Leading lepton pair has a mass between 66 and 116 GeV.
Figure B.14: $e^+e^-e^+e^-$ sub-leading lepton pair mass distribution. Leading lepton pair has a mass between 66 and 116 GeV.

Figure B.15: $\mu^+\mu^-\mu^+\mu^-$ sub-leading lepton pair mass distribution. Leading lepton pair has a mass between 66 and 116 GeV.
Figure B.16: $e^+e^-\mu^+\mu^-$ sub-leading lepton pair mass distribution. Leading lepton pair has a mass between 66 and 116 GeV.
Appendix C

Fake Factor and Selected and Lepton-like Jet Distributions

Figure C.1: Forward muon fake factor parameterized in $p_T$. 
Figure C.2: Forward muon fake factor parameterized in $\eta$.

Figure C.3: Calo muon fake factor parameterized in $p_T$. 
Figure C.4: Calo muon fake factor parameterized in $\eta$.

Figure C.5: Electron-like jet $p_T$ distribution.
Figure C.6: Selected electron $p_T$ distribution.

Figure C.7: Electron-like jet $\eta$ distribution.
Figure C.8: Selected electron $\eta$ distribution.

Figure C.9: Muon-like jet $p_T$ distribution.
Figure C.10: Selected muon $p_T$ distribution.

Figure C.11: Muon-like jet $\eta$ distribution.
Figure C.12: Selected muon $\eta$ distribution.

Figure C.13: Forward muon-like jet $p_T$ distribution.
Figure C.14: Selected forward muon $p_T$ distribution.

Figure C.15: Forward muon-like jet $\eta$ distribution.
Figure C.16: Selected forward muon $\eta$ distribution.
Figure C.17: Calo muon-like jet $p_T$ distribution.
Figure C.18: Selected calo muon $p_T$ distribution.

Figure C.19: Calo muon-like jet $\eta$ distribution.
Figure C.20: Selected calo muon $\eta$ distribution.