Radiative Corrections at HERA

H. Spiesberger\textsuperscript{a}, A. Akhundov\textsuperscript{b}, H. Anlauf\textsuperscript{c}, D. Bardin\textsuperscript{d}, J. Blümlein\textsuperscript{e}, H. D. Dahmen\textsuperscript{f}, S. Jadach\textsuperscript{g}, L. Kalinovskaya\textsuperscript{d}, A. Kwiatkowski\textsuperscript{h}, P. Manakos\textsuperscript{e}, T. Mannel\textsuperscript{e, i}, H.-J. Möhring\textsuperscript{j}, G. Montagna\textsuperscript{k}, O. Nicrosini\textsuperscript{j}, T. Ohl\textsuperscript{c, i}, W. Płaczek\textsuperscript{g}, T. Riemann\textsuperscript{e}, L. Viola\textsuperscript{k}

\textsuperscript{a} II. Institut für Theoretische Physik, Universität Hamburg, FRG
\textsuperscript{b} Academy of Sciences of Azerbaijan, Azerbaijan
\textsuperscript{c} Technische Hochschule Darmstadt, FRG
\textsuperscript{d} Joint Institute of Nuclear Research, Dubna, Russia
\textsuperscript{e} DESY – Institut für Hochenergiephysik, Zeuthen, FRG
\textsuperscript{f} Universität Siegen, FRG
\textsuperscript{g} Institute of Physics, Jagellonian University, Kraków, Poland
\textsuperscript{h} Institut für Theoretische Teilchenphysik, Universität Karlsruhe, FRG
\textsuperscript{i} Deutsches Elektronen–Synchrotron DESY, Hamburg, FRG
\textsuperscript{j} Sektion Physik, Universität Leipzig, FRG
\textsuperscript{k} Dipartimento di Fisica Nucleare e Teorica, Università di Pavia, Italy
\textsuperscript{l} Istituto Nazionale di Fisica Nucleare, Sezione di Pavia, Italy

Abstract: Some basic issues of radiative corrections in deep–inelastic scattering are discussed. Programs for their calculation are presented and compared with respect to their physics input as well as their numerical results.

1 Introduction

This article summarizes the basic theoretical background needed for an understanding of radiative corrections in deep-inelastic lepton-nucleon scattering at HERA. Programs for their calculation are reviewed and compared with one another, from the point of view of both their basic physics input and their numerical output. The aim of this article is to describe the present status of the available programs with respect to their range of application, their limitations, and their technical reliability.

In the following first section we will define our notation, discuss the kinematics of radiative deep-inelastic scattering, and repeat some basic formulas. We will review the characteristic features of radiative corrections in deep-inelastic scattering and give a classification of them. Then we will present a comparative inventory of programs as far as they have been presented and discussed during the workshop on HERA physics, held at DESY in 1991. Finally, we will collect numerical results and discuss their technical and theoretical uncertainties.

2 Features of Radiative Corrections

2.1 Kinematics

We consider the processes
\[ e(l) + p(p) \rightarrow e'(l') + X(p_X), \quad (1) \]
\[ e(l) + p(p) \rightarrow e'(l') + \gamma(k) + X(p_X), \quad (2) \]
where the momenta are given in parentheses.

The traditional way to define kinematical variables for deep-inelastic lepton scattering is to use the momentum of the final-state lepton:
\[ Q^2_\ell = -(l' - l)^2, \quad x_l = \frac{Q^2_\ell}{2p \cdot (l' - l)}, \quad y_l = \frac{p \cdot (l' - l)}{p \cdot l}, \quad (3) \]
with \( Q^2_\ell = x_{q\ell}s \) and \( s = (p + l)^2 \). For the non-radiative process (1) without emission of a photon from the lepton line, \( Q^2_\ell \) determines the momentum transferred to the hadronic system and \( x_l \) can be interpreted, in the parton model, as the fraction of the proton momentum entering the hard scattering process.

For the radiative process (2) and when the photon is emitted from the lepton line (see fig. 1) one has, however, to take into account the fact that the emission of momentum leads to a shift of the kinematical variables. The kinematics at the hadronic vertex has to be described rather by
\[ Q^2_h = -(p_X - p)^2, \quad x_h = \frac{Q^2_h}{2p \cdot (p_X - p)}, \quad y_h = \frac{p \cdot (p_X - p)}{p \cdot l}, \quad (4) \]
It is easy to show that
\[ x_h \geq x_l, \quad y_h \leq y_l. \quad (5) \]
the measurement on the hadronic final-state. More recent studies [3] have shown that in a certain kinematical range one obtains the smallest experimental errors when the measurement of the leptonic final-state is combined with that of the hadron flow. It was proposed [4] to consider a mixed definition of kinematic variables where the momentum transfer is determined from the leptonic measurement and y from the hadronic final-state, thus defining so-called mixed variables:

$$Q^2_m = Q^2_l, \quad y_m = y_A, \quad x_m = \frac{Q^2_m}{y_m b}$$

(7)

This possibility is simple enough to be treated analytically and has been implemented in several analytical programs. Eventually, more refined prescriptions will be necessary, but those can be treated only with the help of Monte Carlo programs.

The distinction between leptonic and hadronic variables, and a precise definition of how the measurement of kinematic variables are performed is essential in the discussion of leptonic radiative corrections and has substantial influence on the numerical results. In most cases we will specify explicitly which definition has to be used. It is only in cases where the difference between various definitions is not relevant, that we will use the generic notation x, y and $Q^2$ without indices l, h, m, or JB.

2.2 Lowest-order cross sections

In order to define our notation, we repeat here the Born-level expressions for the NC and CC cross sections. For scattering of electrons or positrons via the neutral-current, the cross section can be written in the following form:

$$\frac{d^2\sigma_{NC}}{dzdy} = \frac{2\pi\alpha^2}{z y Q^2} \left( Y_1 F_1(x, Q^2) + Y_2 F_2(x, Q^2) - y F_L(x, Q^2) \right),$$

(8)

with

$$Y_s = 1 \pm (1 - y)^2,$$

(9)

and

$$F_L(x, Q^2) = F_{1L}(x, Q^2) - 2 x F_1(x, Q^2).$$

(10)

Upper and lower signs refer to positron and electron scattering, resp. The structure functions $F_{1,2L}(x, Q^2)$ coincide with the usual electromagnetic ones for low $Q^2$, but for large momentum transfer they contain contributions from the weak neutral current described by Z exchange diagrams and the structure function $F_2$ is no longer negligible:

$$F_{1L}(x, Q^2) = \sum_{i=1,2} \chi_i(Q^2) \lambda_i(x^2) \left( \frac{P_L}{P_T} \lambda_i^\nu \lambda_i^\nu - P_T \lambda_i^\nu \lambda_i^\nu \right) F_2^i(x, Q^2),$$

(11)

$$F_2(x, Q^2) = \sum_{\nu=1,2} \chi_i(Q^2) \lambda_i^\nu \lambda_i^\nu \left( \lambda_i^\nu - P_T \lambda_i^\nu \right) F_2^i(x, Q^2).$$

(12)

$P_L$ is the degree of longitudinal polarization of the leptons and as abbreviations we used

$$\chi^\nu = 1, \quad \chi^\nu(Q^2) = \frac{Q^2}{Q^2 + M_Z^2},$$

(12)

and

$$\lambda_i^\nu = v_i^\nu v_i^\nu + a_i^\nu a_i^\nu, \quad \lambda_i^\nu = v_i^\nu v_i^\nu + a_i^\nu a_i^\nu.$$
and $\alpha_s^2$ are the vector and axial vector coupling constants of the fermions $f$ to the $Z$-boson, given by their charge $Q_f$ and isospin $I_f^3$:

$$v_f^v = -Q_f, \quad v_f^a = \frac{I_f^3 - 2s_f^v}{2s_f^{cw}},$$
$$a_f^v = 0, \quad a_f^a = \frac{I_f^3}{2s_f^{cw}}.$$ (14)

The weak mixing angle is determined by the gauge-boson masses:

$$c_w = \cos \theta_w = \frac{M_W}{M_Z}, \quad s_w^2 = 1 - c_w^2.$$ (15)

One can also parametrize the $Z$-boson couplings in terms of the $\mu$ decay constant $G_\mu$ using lowest-order the relation

$$\frac{1}{2s_w^{cw}} = \sqrt{\frac{G_\mu M_Z^2}{2\sqrt{2} \pi^3}}.$$ (16)

For a consistent treatment of electroweak radiative corrections one has to take into account also corrections to the relations between electroweak parameters. This will modify eq. (16) by the inclusion of radiative corrections to the $\mu$ decay constant, parametrized with the help of $\Delta \delta$ [5]. The prescription for the calculation of radiative corrections based on (16) is often called the 'modified on-mass-shell scheme' (MOMS).

In the parton model, the structure functions are expressed with the help of quark distribution functions:

$$F_2^d(x, Q^2) = x \sum_f (q_f(x, Q^2) + \bar{q}_f(x, Q^2)) \lambda_f^d,$$
$$F_2^s(x, Q^2) = x \sum_f (q_f(x, Q^2) - \bar{q}_f(x, Q^2)) \lambda_f^s,$$ (17)

and the longitudinal structure function vanishes:

$$F_L(x, Q^2) = 0.$$ (18)

The lowest-order cross section for the charged-current reaction reads, in the parton model:

$$\frac{d^2\sigma_{CC}}{dx dy} = \frac{\alpha^2}{4\pi^2} \frac{x^2 W^2(x, Q^2)}{x y Q^2} \left\{ Y_4 W_4^d(x, Q^2) + Y_4 W_4^s(x, Q^2) \right\}. $$ (19)

Here,

$$\chi^W(Q^2) = \frac{Q^2}{Q^2 + M_W^2},$$ (20)

and the parton distribution functions enter in the combinations

$$W_4^d = d + s + b + \bar{u} + \bar{c},$$
$$W_4^s = u + c + d + \bar{s} + b,$$
$$x W_4^d = -(d + s + b) + \bar{u} + \bar{c},$$
$$x W_4^s = -(u + c) + d + \bar{s} + b.$$ (21)

A possible contribution from $W_L$ is neglected here, as is the effect of quark mixing. The normalization of the charged-current cross section can also be rewritten with the help of the muon decay constant using (in lowest-order)

$$\frac{\alpha^2}{4\pi^2} = \frac{G^2_{\mu} M_W^2}{2\pi}.$$ (22)

2.3 Classification of radiative corrections

The diagrams describing the $\mathcal{O}(\alpha)$ corrections to the NC cross section are shown in fig. 2 (virtual one-loop corrections) and fig. 3 (real single-photon bremsstrahlung). We are not going to present the complete set of formulas for the $\mathcal{O}(\alpha)$ radiative corrections. They can be found in the literature [6]-[9]. Instead we only discuss some important features [11]:

In the case of NC scattering, a classification of the various contributions to the radiative corrections can be obtained in terms of Feynman diagrams:

![Figure 2: One-loop diagrams for neutral-current lepton-quark scattering](image)

![Figure 3: Single-photon bremsstrahlung diagrams for neutral current lepton-quark scattering](image)
i) The leptonic corrections are described by diagrams containing an additional photon attached to the lepton line, i.e. the photonic lepton vertex correction combined with the self energies of the external fermion lines, fig. 2a, and the photon emission from the lepton line, fig. 3a.

ii) The quarkonic corrections, represented by diagrams with an additional photon at the quark line analogous to i) (diagrams of figs. 2b, 3b). This part is proportional to the square of the quark charge.

iii) The lepton–quark interference part consisting of the $\gamma \gamma$ and $\gamma Z$ box diagrams (fig. 2c) and the interference of the bremsstrahlung diagrams of figs. 3a with those of figs. 3b. This contribution is proportional to the quark charge. Finally we have

iv) the purely weak corrections, described by all the other diagrams that do not contain an additional photon. This part is infra-red finite and contains the diagonal $\gamma$ and $Z$ self energies (figs. 2d–e), the $\gamma Z$ mixing (fig. 2f), the weak lepton and quark vertex corrections (fig. 2g), and the boxes with two heavy gauge–bosons (fig. 2h).

Each loop diagram contributing to the $\mathcal{O}(a)$-corrected matrix element can be written in terms of form factors $F_i(s, x, y)$ [12]. Using the common short-hand notation for the products of lepton and quark currents such as $\gamma_\mu \otimes \gamma^\mu = \bar{u}(l') \gamma_\mu u(l) \cdot \bar{u}(q') \gamma^\mu u(q)$, the matrix elements can be written as

$$\mathbf{M} = F_{1\gamma} \otimes \gamma^\mu + F_{2\gamma} \otimes \gamma^\mu \gamma^5 + F_{3\gamma} \gamma_\mu \gamma^\mu + F_{4\gamma} \gamma_\mu \gamma^\mu \gamma^5.$$  \hspace{1cm} (23)

The form factors $F_i$ are gauge-invariant and depend on the process and on kinematical variables. Their presence does not change significantly the structure of the Born cross section. They can be used to define an improved or dressed Born cross section which would then include electroweak loop corrections. There are several ways of grouping these corrections together in the cross-section formula; e.g. one could sum several pieces by writing $(1 + \Sigma_i \delta_i)$ or, instead, one could multiply different correction factors $\Pi_i(1 + \delta_i)$ introducing thereby additional terms which are formally of higher order. Numerically–higher order–terms of this type, from purely weak one–loop diagrams, may reach the level of a few per mille at HERA.

The self-energy diagrams (fig. 2d–f) contain loop diagrams which are built with all particle degrees of freedom that couple to the gauge bosons. Therefore they contain information on the whole theory. They depend on the top mass, the Higgs mass, and on the masses and couplings of eventually existing other unknown particles.

The self energies are dominated by fermion loops. This contribution is sometimes referred to as a QED part. The fermion loops of the photon self energy (the vacuum polarization) can be taken into account by the use of the running fine-structure constant which sums the leading logarithms to all orders:

$$\alpha(Q^2) = \frac{\alpha(0)}{1 - \Pi_f^2(Q^2)};$$  \hspace{1cm} (24)

$\Pi_f^2 = \tilde{\Sigma}_f^2/Q^2$ is the fermionic part of the vacuum polarization derived from the renormalized photon self energy $\tilde{\Sigma}$ and $\alpha(0) = 1/137.036$. Its hadronic part is connected to the total hadronic cross section in $e^+e^-$ annihilation. At $Q^2 \approx M_Z^2$ the value of $\Pi_f^2$, including hadronic and leptonic contributions, is $\Pi_f^2 \simeq 0.06$.

In the amplitude for the photon-exchange diagrams, $\alpha(Q^2)$ has to be used; but of course not in the contributions of the weak amplitudes. There one can combine the fermionic contributions of the $Z$ and $W$ self energies $\Pi_f^Z = \tilde{\Sigma}_f^Z/Q^2 + M_Z^2$ and $\Pi_f^W = \tilde{\Sigma}_f^W/Q^2 + M_W^2$ with the correction $1 - \Delta \tau$ (coming from rewriting the coupling constants in terms of the $\mu$ decay constant) into a running $\rho$ parameter. For the NC reaction one defines

$$\rho_{NC} = \frac{1 - \Delta \tau}{1 - \Pi_f^2(Q^2)};$$  \hspace{1cm} (25)

and, similarly, the charged–current (CC) amplitude is modified by the factor

$$\rho_{CC} = \frac{1 - \Delta \tau}{1 - \Pi_f^2(Q^2)}. $$  \hspace{1cm} (26)

For more details, especially with respect to the treatment of leading higher–order corrections connected to a heavy top quark, see [13].

The photonic corrections include contributions from the bremsstrahlung process (2) which has to be integrated over that phase–space region where the additional photon cannot be identified experimentally. In practice, the definition of the relevant phase–space boundaries is very complicated and requires a detailed understanding of detector properties. For simplicity one can assume that no attempt at all is made in the first stage of a physics analysis to single out radiative events. Therefore we consider the integration over the complete phase–space of process (2).

The leptonic radiation, i.e. the bremsstrahlung of photons from the lepton line, can be described in a model-independent framework using structure functions $F_{1,2,3}$ as input [14]. For radiative corrections to the differential cross section defined in terms of leptonic variables, the corresponding contribution can be written in the form

$$\frac{d^2 \sigma_{lep}}{dy dx dy} = \frac{\alpha_y^2}{\pi} \int \frac{dx_1 dx_2 dx_3}{x_3 \sqrt{-\Delta \Lambda}} \left\{ \frac{\Delta}{\Delta_4} (x_1, Q^2_4) R_1 + F_2(x_2, Q^2_4) R_2 + F_3(x_3, Q^2_4) R_3 \right\};$$  \hspace{1cm} (27)

$\phi_1$ and $\phi_2$ are internal phase–space variables which can be either two of the following: $t' = -Q^2_4, z_1 = 2t' \cdot k, \text{ or } z_2 = 2t' \cdot k$. The Gram determinant $\Delta_4$ has to be expressed in terms of $\phi_1$ and $\phi_2$ accordingly. The coefficient functions $R_i$ describe the emission of a photon from the lepton line and depend on the kinematic variables $\phi_i$ as well as on $x_1, y_1, \text{ and } x_3$.

As is seen from eq. (27), the phase–space integration involves explicitly the arguments of the structure functions (note that $Q_4^2 = Q_4^2 + z_1 - z_2$). Therefore only one integration in (27) can be performed analytically. This complication can be avoided if $x_2$ and $Q_4^2$ are used as fixed external variables, which applies to the calculation of corrections to the cross section $d^2 \sigma/dx_2 dy$, defined in terms of hadronic variables.

In the parton model there is also a bremsstrahlung contribution described by radiation from the quark line. Its calculation reveals the appearance of quark–mass singularities, very much as is the case for gluonic corrections. These singular pieces can be factorized and have to be absorbed into the quark distribution functions [15]. The remaining parts are known exactly to order $\mathcal{O}(a)$ and are small. In the leading-logarithmic approximation the quarkonic corrections lead to an additional contribution

\footnote{Note also that the additional factor $\alpha$ coming with real photonic corrections has to be read as $\alpha(0)$ since there the characteristic momentum squared is that of the real photon.}
to the Altarelli–Parisi splitting functions and result in a small modification of the $Q^2$ dependence of parton distribution functions [11, 16].

The lepton–quark-interference of radiation from the lepton and the quark line, combined with the $\gamma\gamma$ and $\gamma Z$ box diagrams, is non-singular and in general small. This contribution is however dominant in the corrections to the charge asymmetries, since it is odd under charge conjugation.

There is a similar classification of radiative corrections to the charged–current reaction. In this case, the separation of leptonic, quarkonic, and interference parts in terms of Feynman diagrams is possible in a physical gauge only. The separation can be understood most easily by investigating the dependence on the charges of external fermions. A gauge-invariant way to separate photonic and weak corrections has been described in [7].

In the leading-logarithmic approximation, leptonic QED corrections can be further separated into contributions coming from the integration over specific phase-space regions: the region with photons collinear with the initial lepton, $k \cdot l \to 0$, leads to a contribution from 'initial-state radiation', whereas 'final-state radiation' is connected to the situation where photons are emitted collinearly with the final-state lepton, $k \cdot l' \to 0$.

There is a third contribution, the so-called Compton part [17, 18], which is characterized by quasi-on-shell virtual photons $Q_2^2 \to 0$. Due to the photon propagator $1/Q_2^2$ and since the lower kinematical limit for $Q_2^2$ with fixed leptonic variables $x_1$, $Q_1^2$ is of the order of the proton mass $m_p$, the magnitude of this latter contribution is determined by the potentially large logarithm $\ln (Q_1^2/m_p^2)$. The Compton part is especially important when corrections are calculated as a function of the lepton variables. It is an important source of uncertainty in the predictions of radiative corrections, since the experimental knowledge of the proton structure functions for $Q_2^2 \to 0$ is not fully exploited in most of the commonly used parametrizations. The Compton part is characterized by a clean experimental signature. In the limit $Q_2^2 \to 0$, the transverse momentum of the emitted photon is balanced by that of the scattered lepton. Therefore it is hoped that one will have a direct access to its magnitude also by a measurement at HERA.

The leptonic leading logarithms are universal and can therefore actually also be used for an estimate of corrections to processes other than deep–inelastic scattering, to the extent that these processes proceed via one–boson exchange.

3 Programs for the Calculation of Radiative Corrections

3.1 Standards for radiative corrections programs

The best program for the calculation of radiative corrections that can be imagined would have a number of superlative properties: it would be complete with respect to processes that can be described (NC and CC deep–inelastic scattering, jet production, heavy-flavour production, photoproduction, processes beyond the standard model, etc.) and complete with respect to radiative corrections (photon corrections, weak corrections, higher–order contributions, QCD corrections). The 'best' program should combine high speed with high precision, and it should be flexible with respect to input, options, experimental cuts, etc. while being at the same time simple in use. These requirements obviously conflict with one another, and such a program clearly does not exist. Each of the programs described in the following may meet a subset of the above-mentioned properties, but none of them is complete. Each of them has a certain range of application where it is superior to others and in total they complement each other.

There are two types of programs: first we will describe those programs which could be called semi-analytical. They are based on formulas obtained analytically, including an analytical one- or twofold integration over the photon phase–space, and work with some numerical–integration algorithm to obtain the fully integrated radiative cross section. As already mentioned, this final integration involves an integration over the structure functions and can thus only be performed numerically. The second type of programs is Monte Carlo event generators simulating the fully differential cross section. The semi-analytical programs are usually much faster and can reach much higher precision than the Monte Carlo programs, which are more flexible with respect to phase–space cuts.

The program descriptions given below concentrate on the most important aspects relevant for their mutual comparison. More details can be found in the separate manuals contained in volume III of the HERA Workshop Proceedings.

3.2 Analytical programs

3.2.1 TERAD91

TERAD91 is a semi-analytic code for QED and weak corrections to deep–inelastic NC and CC scattering at HERA. Version 2.10 was released on 3 Oct. 1991. The source of TERAD91 originates from four different codes: TERAD, DISEPNC, DISEPPC, and DIZET, which will be discussed in what follows.

TERAD [10]

The first investigations of the TERAD-approach to QED may be found in refs. [20, 21]. The roots of the present code date back to 1986 (TERAD86 [22]): leptonic model-independent QED corrections to fixed-target DIS in leptonic variables with $\gamma$-exchange only.

Some of its recent developments are: the additional possibility i) to use mixed and hadronic variables, including also $Z$-exchange, and ii) to apply some cuts. The output produced by TERAD is the cross section as chosen by flags (see [10, 23]):

$$ \frac{d\sigma}{dE} \sim \frac{2\alpha^3}{\pi} \int d^2T \int d\xi d\eta Q_1^2 d\eta_0 d\xi_0 \frac{1}{Q_1^2} \int d^2S(E, I). $$

(28)

Here, $E$ is a set of two variables on which $d\sigma$ finally depends. In case of leptonic, mixed, hadronic variables:

$$ E = \{y_1, Q_1^2\}, \{y_2, Q_1^2\}, \{y_2, Q_2^2\}. $$

(29)

$I$ is a corresponding set of two variables to be integrated over:

$$ I = \{y_3, Q_3^2\}, \{y_4, Q_4^2\}. $$

(30)
Within TERAD, two (for leptonic variables) or only one (for mixed and hadronic variables) integrations are left to be performed numerically\(^3\). The function \(S(E, I)\) is the result of a one-dimensional analytic integration,

\[
S(E, I) = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{dz_1}{\sqrt{\Delta_k}} S(E, I, z_1, z_2) \tag{31}
\]

\[
= A_1(z_n, Q_1^2) S(E, I) + A_2(z_n, Q_1^2) S(E, I) + A_3(z_n, Q_1^2) S(E, I),
\]

where the variables \(z_1 = 2l \cdot k\) and \(z_2 = 2l' \cdot k\) are related to a photon angle; \(\Delta_k\) is the Gram determinant. The \(A(z_n, Q_1^2)\) are combinations of structure functions \(F_{n,m}\) describing the electroweak interactions of \(m, n = \gamma, Z\) with nucleons as introdused in section 2.2 [see eqs. (11) and (17)].

The double-differential NC cross section may be obtained in leptonic, mixed and hadronic variables. TERAD does not necessarily rely on the quark parton model and calculates QED radiation from the leptonic legs to order \(\alpha\). Soft-photon exponentiation is included. For the cross section in leptonic variables a simultaneous cut on energy and polar angle of the emitted photon may be applied.

Weak corrections can be taken into account through definite modifications of coupling constants in a similar way as in the DISEP part. Longitudinal polarization of the lepton beam is allowed.

DISEPNC [6], DISEPCC [7]

This part calculates the complete electroweak radiative corrections - i.e. QED plus weak loop corrections - to NC and CC scattering in the quark parton model. Output of the DISEP branches are the cross sections as chosen by flags:

\[
d^2\sigma = 2\pi^2a_{W} \sum_{B} \sum_{s \neq Q} c_{Q} K(B) |V(B)| R_{Q}^{B}(B) \tag{32}
\]

where the different sums extend over quarks \(Q\) and anti-quarks \(\bar{Q}\); photons and \(Z\) exchange, \(B = \{(\gamma\gamma), (\gamma Z), (ZZ)\}\); Born cross section \(\langle b = 0, 0, 0\rangle\) and bremstrahlung from leptons, leptonquark interaction, and quarks \(\langle k = \{e, i, q\}\rangle\). For the NC cross section from the \((\gamma, Z)\) part of lepton radiation, e.g. the formula contains the following pieces:

\[
c_{Q} = Q_{Q}^2, \quad V(Q, Z) = v_{Q} v_{Q}(s, Q_{Q}^2) - P_{L} v_{Q}(s, Q_{Q}^2) u_{Q}(s, Q_{Q}^2) - v_{L} u_{Q} - P_{L} u_{Q}, \tag{33}
\]

\[
K(Q, Z) = g_{Q} g_{Q} |F_{Q}(Q_{Q}^2)\chi(Q_{Q}^2) r(s, Q_{Q}^2)\tag{34}\]

The weak corrections, contained in \(P_{L}, v_{Q}, u_{Q}\) and the running \(\alpha\), obtained from DIZET. The longitudinal polarization of the electron beam is \(P_{L}\). The QED (or photonic) corrections are contained in the functions \(R_{Q}^{B}(B)\), e.g.:

\[
R_{Q}^{B}(Q, Z) = R_{Q}(Q, Z, 1, 1 - y_1) + R_{Q}(Q, Z, 1, 1 - y_1), \tag{35}
\]

\[
R_{Q}(B; a, b) = \frac{\alpha}{\pi} \left[ S_{Q}(z, Q_{Q}^2) \right. \]

\[
+ \int_{z_1}^{z_2} dx_1 \left[ T_{a} z_{a Q}(z, Q_{Q}^2) - z_{a Q}(z, Q_{Q}^2) + U_{a} \frac{z_{a Q}(z, Q_{Q}^2)}{z_{a Q}(z, Q_{Q}^2)} \right]\), \tag{36}
\]

\(^3\)In principle, for hadronic variables one could perform the complete threefold integration analytically.

The functions \(S_{Q}, T_{a}, U_{a}\) and \(U_{a}\) depend on \(B, a, b\). They are results of a twofold analytic integration over photonic angles, and \(q\) are the quark distribution functions.

The DISEP branches take into account the QED corrections to the double-differential NC and CC cross sections in leptonic variables, for the NC case with soft-photon exponentiation. The quark parton model is used. Besides leptonic leg radiation, also quark-leg radiation and the interference are included. No cuts may be applied.

Note that in the DISEP approach the factorization scale used in the parton distribution functions is taken to be the lepton momentum transfer. This was chosen in order to enable an analytic treatment of two of the phase-space integrations. Only one integration has to be performed numerically, which makes the DISEP branch of the program very fast. One has to keep in mind, however, that from considerations of higher-order corrections, only the choice \(Q_{Q}^2\) is justified for the part describing leptonic radiation. Numerical differences between the TERAD and DISEP branches will therefore occur and are non-negligible at large \(y_1\) and small \(z_1\).

**Weak corrections library DIZET v.4.04 [24]**

Both branches of the package may use the recently updated weak library DIZET. It allows the calculation of \(\Delta r\) and the complete \(O(\alpha)\) corrections (with the inclusion of some leading higher-order terms connected with the top-quark) to both the \(Z, W\) widths and the weak form factors for cross sections in the \(s\)- and \(t\)-channels. These form factors depend on the kinematics and on the fermion type of the scattering particles.

3.2.2 EPRC91

EPRC91 is a package of programs for the calculation of the complete \(O(\alpha)\) electromagnetic and weak corrections to NC and CC deep-inelastic e+p scattering. It is based on earlier publications [8, 9, 25] and was developed further during the Workshop. The prescriptions realized in EPRC91 rely on the parton-model approach. The kinematic variables are defined from the final-state lepton momentum. In EPRC91, a twofold numerical integration over the full photon phase-space is performed, allowing for \(Q^2\)-dependent parton distribution functions. For the charged-current reaction, exponentiation of soft-photon corrections can be included. The polarization of the lepton beam can be non-zero.

The package includes routines for the evaluation of NC and CC cross sections, the ratio \(R_{NC/CC}\) of NC to CC cross sections, and charge or polarization asymmetries, including total or partial electroweak radiative corrections. Routines for the evaluation of leptonic QED corrections in the leading-logarithmic approximation are available. In this case, the user can choose among the leptonic, mixed, hadronic, or Jauquet-Benedel definition of kinematic variables. The package includes also routines for the calculation of the leading-logarithmic QED corrections to second order [26].

The weak corrections library of EPRC91 has become a part of the Monte Carlo generator HERACLES.

Those parts of EPRC91 needed for the calculation of the complete \(O(\alpha)\) QED corrections are rather time-consuming and are now used for test purposes only. The program package may be useful, however, for the study of purely weak corrections, e.g. the dependence of radiative corrections on electroweak parameters such as the top mass or the Higgs mass. Moreover, the leading-log routines are helpful in obtaining a fast estimate of radiative corrections to \(O(\alpha)\) and \(O(\alpha^2)\).
3.2.3 HELIOS

HELIOS is a program for the calculation of the O(α) electromagnetic corrections to NC and CC deep-inelastic e²-Nucleon scattering in the leading-logarithmic approximation. It is based on already published work [4, 18], and a detailed description of the program is given in [27].

The contributions from initial- and final-state lepton bremsstrahlung to the leading-logarithmic corrections are derived from

\[
\frac{d^2\sigma^{(i)}}{dx dy} = \frac{\alpha}{2\pi} \int_0^1 \int_0^1 z^2 \left\{ \theta(z - z_0)|J(x_1, y_1, z_2)| - \frac{d\sigma^{\alpha_0}}{dz dy} \right\}
\]

where \(d\sigma^{\alpha_0}/dz dy\) denotes the respective Born cross section and \((x_1, y_1)\) is one of the sets of kinematic variables of eqs. (3), (6), (7). The lower limit \(z_0\) of the \(z\) integration is a function of \(x_1, y_1\) and depends on the choice of the kinematic variables; \(J(x_1, y_1, z)\) is the Jacobian relating the variables \(x_1, y_1\) to \(x_2, y_2\) [4, 18]. The Compton contribution to the NC cross section is described by

\[
\frac{d^2\sigma^{C}}{dx dy} = \frac{\alpha^2}{8\pi^2} \int_0^1 \int_0^1 \frac{z^2 + (x_1 - z_2)^2}{x_1(1 - y_1)}.
\]

Here it is assumed that the photon content of the proton factorizes into quark distribution functions \(q_i(z, Q^2)\) and the \(q \to q'\) splitting function (the Altarelli-Parisi splitting function); \(\Lambda\) is the QCD scale; the Compton term can also be parametrized by \(\ln s/m_p^2\), instead of \(Q^2/\Lambda^2\).

The program offers the optional calculation of the leading-logarithmic QED corrections to \(d^2\sigma/dx dy\) for collisions of unpolarized and polarized electrons or positrons with protons and deuterons. The parton distributions may be chosen in a wide variety of parametrizations [28–29].

For the calculation of \(d\sigma/dx dy\), the contribution of the longitudinal structure functions for NC and CC reactions may be included in first-order QCD also [29]. Furthermore, also the full first order QCD corrections in the DIS scheme are available [29]. In order to account for the typical behaviour of the scattering cross sections as \(Q^2 \to 0\) two options, following [30, 31], are accessible.

One may choose different definitions of the kinematical variables \(x_1\) and \(y_1\) for the radiatively corrected cross section: \(i\) measurement of the kinematical variables from the scattered lepton, \(ii\) from the full electromagnetic final-state for NC reactions (i.e. excluding final-state radiation and the Compton–peak contribution), \(iii\) from a measurement of the hadronic final-state using the Jaquet–Blondel method, and \(iv\) using the mixed variables \((Q^2, x_1 y_1)\).

Since only lepton-leg corrections are treated, any e²N reaction proceeding via the exchange of one gauge–boson may be investigated with this program on the basis of user-supplied differential CC and NC scattering cross sections. As an illustration the program includes the QED corrections in the leading-logarithmic approximation to neutral-current \(QQ\) production in the Weizsäcker–Williams approximation.

3.2.4 APHRODITES

In APHRODITES, version 1.00 [32], higher–order QED leptonic corrections to deep-inelastic electron–proton scattering at HERA energies are implemented according to the structure–function formalism. In this approach the neutral–current corrected cross-sections can be obtained by folding the corresponding Born cross section with the electron structure functions as [33]:

\[
\frac{d\sigma}{dx dy}(z, y_1) = \int_0^1 dz_2 D(z_2, Q^2) D_1(z_1, z_2) \cdot C(E_F(z_1, z_2)) \cdot \Phi(cuts).
\]

We list here only the meaning of the terms entering eq. (39). For their explicit expressions see [33].

(i) \(d\sigma_\alpha/dz dy\) is the lowest–order cross section evaluated with the reduced variables \(z_1\) and \(y_1\) the latter depend on the longitudinal momentum fractions \(z_1\) and \(z_2\) of the \(z_1\) and \(y_2\) incoming and outgoing electrons, respectively, after photon radiation. The fractions \(z_1\) and \(z_2\) range inside the radiation phase–space \(\Omega\) delimited by kinematical and experimental cuts (\(\Phi(cuts)\)).

(ii) \(J\) is the Jacobian of the transformation from reduced \((z_1, y_1)\) to observable \((z, y)\) variables.

(iii) \(D(z, Q^2)\) are the QED structure functions for the initial– and final–state electron giving the probability of finding, inside a parent electron, an electron with longitudinal momentum fraction \(z\) and virtualness \(Q^2\). For \(D(z, Q^2)\) we use the expression obtained in [33]. They include the following QED corrections:

- Complete \(O(\alpha)\) leptonic radiation;
- Soft–photon exponentiation;
- Isotropic hard bremsstrahlung at \(O(\alpha^2)\) in the leading–log approximation.

For the scale \(Q^2\) entering the electron structure functions the leptonic squared momentum transfer \(Q_1^2\) is chosen.

(iv) The function \(C(E_F, \delta)\) describe hard collinear photons emitted by the final electron of energy \(E_F\) within a cone of half-opening angle \(\delta\) and which cannot be resolved in a calorimetric experiment because of the finite angular resolution. APHRODITES uses for \(C\) an exponentiated form of the expression derived in [34].

Compton subprocesses are implemented in APHRODITES, as explained in [26].

APHRODITES is installed on the VAX 6410 at the University of Pavia and is available on request from the authors. A more complete version of the program, including weak loops and the QED corrections due to fermion–pair production, is planned. The extension to CC scattering is also under consideration.
3.3 Monte Carlos

The Monte Carlo programs presented in the following have been developed in order to solve the specific problems related to radiative corrections at HERA. They do not focus on a description of the hadronic final-state in deep-inelastic scattering. Early versions of them were based on the parton model and generated events at the parton level, leaving the final-state quark unhadronized. Only later, for two of them, have interfaces been developed; these call routines from LEPTO and JETSET for the fragmentation and hadronization of the scattered quark and the proton remnant. Recent versions of HERACLES and KRONOS have an option to calculate the leptonic QED corrections in a model-independent framework where the cross section is described by structure functions $F_{1,2,3}$. With these options, the programs are suited for a simulation of an electron-inclusive measurement and also for very small $z_l$ and $Q_l^2$, where a Monte Carlo treatment of the hadronic final state is not straightforward.

All programs apply a rejection technique to the cross section written in terms of variables in which there is a smooth dependence. The most important differences between the programs result from different choices and ordering of the independent variables. HERACLES starts the event generation by choosing the leptonic variables $z_l$ and $Q_l^2$. Therefore the phase space can be restricted by cuts on these variables without losing efficiency and the program is suited for a calculation of radiative corrections to the differential cross section in terms of leptonic variables. In contrast to this, LESKO-F and KRONOS start with hadronic variables $z_h$ and $Q_h^2$. The latter programs are therefore better suited for a calculation of corrections as a function of the hadronic variables, but they lose efficiency when they are used for a calculation of corrected differential cross sections in terms of leptonic variables, very much like HERACLES would become less efficient when used for a calculation of corrections in terms of hadronic variables.

3.3.1 HERACLES

The event generator HERACLES, for the simulation of NC and CC deep-inelastic lepton-nucleon scattering, includes the leptonic corrections, as well as radiation from the quark line and interference of leptonic and quarkonic radiation, and the complete one-loop virtual corrections using the results of refs. [8, 9, 25]. Version 4.1 of HERACLES is described in these proceedings [35] (see also [36]).

The most important capabilities of the generator that are relevant for the discussion of radiative corrections are:

- It allows integration of the differential cross sections for $ep \rightarrow eX$ and $ep \rightarrow e\gamma X$ over kinematical regions which can be given in terms of the leptonic variables $z_l$, $y_l$, $Q_l^2$ and the mass of the hadronic final-state. Also, in the calculation of the cross section for $ep \rightarrow e\gamma X$ it is possible to limit the phase-space by requiring a minimal value for the photon energy $E_\gamma$. These kinematical boundaries are implemented in such a way that no efficiency is lost. Of course, any other phase-space restrictions can be applied. However, since these other kinematical restrictions are realized by rejecting events, the efficiency of the algorithm is reduced.

- The structure of the program allows a separate treatment of the Born term and several parts of the QED corrections (comprising soft and hard bremsstrahlung and the corresponding virtual corrections). These parts describe, for the neutral-current, leptonic initial-state radiation, leptonic final-state radiation, a contribution called Compton part, and quarkonic radiation. In the case of the charged-current reaction, there is at present only one radiative channel describing radiation from the initial lepton.

- Several switches allow the O($\alpha_s$) electroweak virtual corrections (self energies, vertex corrections, and box diagrams) to be included into the non-radiative cross section. If only a moderate accuracy is required, one can turn off the weak contributions thereby saving some CPU-time. There is also a switch that determines whether the electroweak parameters are calculated from fixed $Z$ and $W$ boson masses $M_Z$, $M_W$, or from fixed $M_Z$ and the $\mu$ decay constant $G_\mu$.

- The program describes electron as well as positron scattering, and allows for polarized leptons.

- The user can choose from a set of parametrizations for input quark distributions used in the parton-model approach or for structure functions in a model-independent description of leptonic corrections. In particular, a non-zero longitudinal structure function $F_L$ can be included. Note that in this case it is not possible to include quarkonic radiation and its interference with leptonic radiation. The quark distribution functions may be $Q^2$-dependent and thus include leading-logarithmic QCD corrections. Other $O(\alpha_s)$ corrections are not included, however.

- HERACLES itself generates events at the parton level. The interface DJANGO [37] can be used to call routines from LEPTO 5.2 and JETSET 6.3 [38], for the fragmentation and hadronization of the scattered quark and the proton remnant.

The program has been tested and gives reliable results in the range $10^{-4} \leq z_l \leq 0.9$ and $0.01 \leq y_l \leq 0.99$, excluding the corner of large $z_l$ and small $y_l$. Since exponentiation of soft photonic corrections is not yet included, the program is restricted to not too small values of $y_l$ and not too large $z_l$. In the present version, $z_l$ and $y_l$ must lie in the region $y_l(1 - z_l)^2 \geq 0.004$. (40)

The bremsstrahlung cross section is split into a soft and a hard part. The soft bremsstrahlung is treated analytically and included as a correction in the non-radiative cross section. It describes emission of photons with an energy smaller than a cut-off $E_{\gamma}^{cut}$, which is chosen by the program below the experimental threshold for the detection of photons. $E_{\gamma}^{cut}$ depends on $\mathbf{x}$ and $\mathbf{y}$ and is of the order of a few 10 MeV. The hard bremsstrahlung contribution describes events with photons of an energy above $E_{\gamma}^{cut}$. For the NC process, the hard bremsstrahlung is split into four contributions. Two of them correspond to the collinear peaks from initial- and final-state leptonic radiation. The third one is dominated by the kinematical situation where the electron scatters off an almost real photon, collinear to the incident quark (Compton contribution). These three parts have been obtained by partial a fractioning of the expression describing emission of a photon from the lepton line. Finally, there is a channel for the radiation of a photon from the quark line. This channel is also needed when interference of leptonic and
quarkonic radiation is requested. Since the interference contribution is not positive definite, it cannot be treated separately, but it has to be combined with the other channels. Therefore, it is necessary to turn on all channels when lepton–quark–interference is requested. Quark–mass singularities originating from photon radiation off the quark line are absorbed into the parton distribution functions. The separation of these singularities is performed with the help of cuts on the angles between the photon and the incoming and the outgoing quarks. Details of the treatment of these contributions can be found in ref. [39].

3.3.2 LESKO-F

LESKO-F is a Monte Carlo program for deep-inelastic $e^+p \rightarrow e^+X$ scattering at HERA, version, with collinear bremsstrahlung from the lepton line [40]. It is an upgrade of the previous [41]. The present version 3.0 includes the whole $\mathcal{O}(\alpha)$ QED bremsstrahlung from the process at HERA. The incoming lepton may have non-zero spin polarization. Vacuum polarization corrections to the photon propagator, being significant at HERA energies, are implemented in the program as well. Leptonic decay of the vacuum polarization corrections has been taken from [42], while the hadronic part is calculated using the $Z$ coupling constant calculated with the help of the muon decay constant $G_\mu$. Corrections to the muon decay, $\Delta r$, are included in the above calculation; they are taken from [6].

The main feature of LESKO-F is that it focuses on the description of QED radiative corrections for the deep-inelastic scattering region, where the parton model can be applied. This is guaranteed by the requirement that the hadronic transverse momentum $p_T$ must be greater than some $p_{T,\text{min}}$, which is the main cut implemented in the program. Such that all the accepted events have $Q^2 > p_{T,\text{min}}$. Since the parton model is not valid for $Q^2$ below $Q_{\text{min}}$, the value of $p_{T,\text{min}}$, which is one of the steering parameters of the program, should be greater than $O(1 \text{ GeV})$. The recent version of LESKO-F (version 3.0) gives the possibility to impose additional cuts for the hadronic variables $z_A$ and $Q^2$. Other cuts and/or event selection criteria can be imposed by iteration.

Owing to the $p_T$-cut, it is possible to use hadronic variables (e.g. $z_A$ and $Q^2$) in the event generation. The rejection rate is reasonably small, usually less than 20%. The program is stable for the whole DIS range at HERA. A typical running time is about 1 ms per event for the IBM 3090 computer. The program does not need any external routines or numerical libraries and any extra data sets, so it can be installed on any computer with a FORTRAN 77 compiler.

For each generated event, LESKO-F provides momenta and flavours of final particles at the parton level. They are stored in the standardized HEPEVT event record. An interface for LESKO-F and routines from the program LEPTO 5.2 [38] for parton cascades and fragmentation, called FRANEQ [44], is available. Interfaces for other hadronization programs will be developed in the future.

LESKO-F is simple in use and easy to connect with other programs. It can be run by calling only one routine in three modes: initialization, event generation and post-generation. In the final mode, which is optional, the program prints some useful output and provides the value of the total integrated cross section corresponding to a generated event sample.

The present version of the program is sufficient to describe the deep inelastic scattering process with a precision of better than 5%. In order to improve the above precision higher-order QED radiative corrections must be included. It is planned to include soft–photon exponentiation and to implement the second-order leading–logarithmic corrections in future releases of the program. Another planned improvement is the inclusion of the longitudinal structure function.

3.3.3 Hektor

A Japanese group has developed the Monte Carlo program Hektor for the calculation of $\mathcal{O}(\alpha)$ QED radiative corrections to the neutral–current process at HERA. The program includes leptonic and quarkonic radiation but not yet the lepton–quark interference and no weak corrections. The leptonic corrections can be described in the model-independent approach based on a structure–functions input. Basic kinematical variables are the hadronic ones eq. (4) and the program is therefore suited for the calculation of corrections as a function of $x_A$ and $z_A$. The work has not yet been published and is still in progress, but preliminary numerical results have been communicated [45] which agree with those obtained with TERA&D91.

3.3.4 Kronos

KRONOS was designed for the study of higher-order QED corrections. The program is described in [46, 47].

Features of KRONOS 1.0

The present version of KRONOS is restricted to a description of photon radiation from the initial-lepton state in the leading-logarithmic approximation. Thus the dominant contributions to QED radiative corrections are included, since radiation from the lepton final-state leads only to small corrections for a calorimetric measurement of the energy of the scattered electron. The following three features are unique to KRONOS: firstly, it treats electromagnetic initial-state radiation in the leading-logarithmic approximation bremsstrahlung photons and allows the soft–photon cut off to be set as low as desired. Thirdly, KRONOS includes radiative corrections to the QED Compton processes.

In addition, KRONOS shares the following features with other Monte Carlo for HERA physics: An exhaustive set of parton distribution functions is available using the package PAKPDF [48]. Alternatively, phenomenological parametrizations of the proton structure functions $F_2$ and $F_L$ can be used.

KRONOS has been used for an estimation of radiative corrections to the QED Compton cross section at HERA [49]. Furthermore, a study of multiphoton final states at HERA has been contributed to these proceedings [50].

Future releases of KRONOS after version 1.0 are expected to provide additional features, which are currently under study: interfaces to QCD parton shower and fragmentation Monte Carlos, final-state radiative corrections, higher-order radiative corrections to CC scattering, and the optional generation of suitably weighted event samples.
Algorithm for the generation of multiphoton states

The leading-logarithmic corrections can be summed by using renormalization group techniques that are well known from QCD. These methods are directly applicable to QED inclusive radiative corrections where the outgoing photons are not individually detected. In this approach the cross section with inclusive radiative corrections is obtained by folding the Born cross section with a so-called radiator $\epsilon(\xi, Q^2)$:

$$\sigma(l) = \int_0^1 d\xi \, \epsilon(\xi, Q^2) \sigma_{\text{Born}}(l),$$

(41)

where $l$ is the momentum of the incoming electron, $Q^2$ is the characteristic momentum transfer in the process, and $\xi$ is the fraction of the initial electron momentum left after photon radiation. The radiator $\epsilon(\xi, Q^2)$ is obtained as the solution of the evolution equation

$$Q^2 \frac{\partial}{\partial Q^2} \epsilon(\xi, Q^2) = -\frac{\alpha}{2\pi} \left[ \int_0^\xi d\zeta P(\zeta) \right] \epsilon(\xi, Q^2) + \frac{\alpha}{2\pi} \left[ \int_0^\xi \frac{d\zeta}{\zeta} P(\zeta) \epsilon(\xi, Q^2) \right] \zeta$$

(42)

with the initial condition

$$\epsilon(\xi, m^2) = \delta(1 - \xi)$$

(43)

and the splitting function

$$P(\zeta) = \frac{1 + \zeta^2}{1 - \zeta}. \quad (44)$$

The infra-red regulator $\epsilon \ll 1$ will drop out of the final results.

KRONOS uses a cascade algorithm similar to space-like parton showers in the context of QCD. The branching of the incoming electron is implemented in the following way. First the number of photons is generated, according to a Poisson distribution with mean

$$\bar{n} = \frac{\alpha}{2\pi} \ln \left( \frac{Q^2}{m^2} \right) \int_0^\zeta dz P(z).$$

(45)

The energies of the generated photons are determined according to the splitting function (44), while the transverse components of the photon momenta are determined by a probability distribution according to the pole in the electron propagator $1/2l \cdot k$.

The procedure is repeated as many times as the number of photons in the corresponding event, taking into account the appropriate ordering of virtualities to obtain the leading-logarithmic contribution.

3.4 Comparison of the programs

Table 1 contains an overview of the programs for radiative corrections at HERA as described in the preceding sections. We list there the program names, version numbers, dates of the last release, authors, e-mail contacts, and whether there is a write-up contained in volume III of these proceedings. It is mentioned whether the program needs files for input and output. For details on the use of external files one has to consult the corresponding write-up. Sometimes routines are used which have been taken over from

<table>
<thead>
<tr>
<th>Name</th>
<th>Program</th>
<th>Date</th>
<th>Author(s)</th>
<th>Email contact</th>
<th>Notes</th>
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<td>2.10</td>
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<td>A. Akhmedov, D. Bartel</td>
<td><a href="mailto:aiba@ph.fu-berlin.de">aiba@ph.fu-berlin.de</a></td>
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other sources or other authors. These are mentioned explicitly and are put in parentheses when the corresponding source code is provided as a part of the complete program listing. For example, most of the programs use the package PAKPDF [48] for the calculation of parton distribution functions, but provide the corresponding source file, at least partially. Subroutine names mentioned without parentheses have to be linked from an external library.

Some of the analytical programs are restricted to a specific choice of kinematical variables, as displayed in the table; others have separate branches which cover several possibilities in this respect. The Monte Carlo programs can in principle treat any choice. However, as already explained, there is always only one set of variables for which cuts can be chosen without loss of efficiency. The corresponding preferred set of variables is given in parentheses in the table. Two of the analytical programs have an option for the application of simple cuts. Some additional comments are given in the last column of the table.

### 4 Comparison of Results

#### 4.1 Input parameters

For a careful comparison of the various programs, we have chosen a common set of input data which is described in the following.

We use the parametrizations of parton distribution functions from Duke and Owens [51], set 1. These parametrizations are valid for $Q^2 \geq Q_0^2 = 4 \text{ GeV}^2$. Since in the calculation of radiative corrections we have to integrate over structure functions down to small values of $Q_2^2$ of the order of $x^2 m_c^2$, we use a special prescription in this region in order not to produce numerical results which just reflect inadequacies of the parametrizations. We decided to use the values of $q(x, Q_2^2 = 4 \text{ GeV}^2)$ for $Q_2^2 \leq 4 \text{ GeV}^2$ and to further modify the low-$Q^2$ behaviour by a multiplicative factor [31]

$$ (1 - \exp(-a^2 Q_2^2)) \quad a^2 = 3.37 \text{ GeV}^{-2}. $$

(46)

This modification ensures that the structure functions vanish in the limit $Q^2 \to 0$. The low-$Q^2$ behaviour, as well as the dependence on $x$ for $x \to 0$, has a considerable influence on the numerical results for leptonic QED corrections. The factorization scale chosen in the distribution functions is usually the hadronic momentum transfer $Q_2^2$.

The electroweak parameters are fixed to [52]

$$ M_Z = 91.175 \text{ GeV}, \quad M_H = 300 \text{ GeV}, \quad m_t = 140 \text{ GeV}. $$

(47)

In this article we do not consider the dependence of radiative corrections on the electroweak parameters, for this we refer the reader to [33]. Since some programs have to use quark masses to regularize mass singularities, we have to specify also the following: $m_c = 62 \text{ MeV}, \quad m_s = 83 \text{ MeV}, \quad m_t = 215 \text{ MeV}, \quad m_q = 1.5 \text{ GeV}, \quad m_b = 4.5 \text{ GeV} [53]$. Numerical results are independent from these parameters, except for numerically small threshold effects from the heavy $b$ quark. The hadronic contribution to the vacuum polarization

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Table 1: Characteristics of programs for the calculation of radiative corrections.

<table>
<thead>
<tr>
<th>Name</th>
<th>QED Code</th>
<th>Weak Code</th>
<th>Kinematical Structure</th>
<th>Complete $O(\alpha)$</th>
<th>Leptonic $O(\alpha)$</th>
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<th>HERAOS</th>
<th>APEPHIDITES</th>
<th>HERACLES</th>
<th>LEEF</th>
<th>KRONOS</th>
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<td>Leptonic $O(\alpha)$</td>
<td>No</td>
<td>No</td>
<td>Complete $O(\alpha)$</td>
<td>Complete $O(\alpha)$</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>HERAOS</td>
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<td>Complete $O(\alpha)$</td>
<td>Complete $O(\alpha)$</td>
<td>Leptonic $O(\alpha)$</td>
<td>No</td>
<td>No</td>
<td>Complete $O(\alpha)$</td>
<td>Complete $O(\alpha)$</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>APEPHIDITES</td>
<td>Complete $O(\alpha)$</td>
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<td>Complete $O(\alpha)$</td>
<td>Complete $O(\alpha)$</td>
<td>Leptonic $O(\alpha)$</td>
<td>No</td>
<td>No</td>
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<td>Leptonic $O(\alpha)$</td>
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<tr>
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<td>Complete $O(\alpha)$</td>
<td>Complete $O(\alpha)$</td>
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<td>No</td>
</tr>
<tr>
<td>KRONOS</td>
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<td>Complete $O(\alpha)$</td>
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<td>No</td>
<td>Complete $O(\alpha)$</td>
<td>Complete $O(\alpha)$</td>
<td>No</td>
<td>No</td>
</tr>
</tbody>
</table>

In the DISPE branches of TERAD91, the leptonic momentum transfer $Q_2^2$ is chosen, see section 3.2.1.
is calculated using Burkhardt’s parametrization [43]. Using this input, the programs DIZET (i.e. TERAD91) or EPRC91 (i.e. also HERACLES) can calculate the W mass and the weak mixing angle. Including leading terms of second order proportional to $m_t$ and $O(\alpha_s)$ corrections to $\Delta r$, both programs are in accordance with each other

$$M_W = 80.100 \, \text{GeV}, \quad s_W = 0.2282.$$  \hspace{1cm} (48)

We will have to make additional remarks concerning the input when we discuss the treatment of the quarkonic QED corrections.

4.2 Leptonic QED corrections

We start with a discussion of results for the purely photonic part of the corrections connected to the leptonic line. Table 2 and fig. 4 show results for the NC corrections $\delta_{NC}(z_i, y_i)$ (in per cent) to the differential cross section $d\sigma/dz_i dy_i$, where the final-state kinematics is defined from the scattered lepton’s momentum, restricted to $Q^2 \geq 4 \, \text{GeV}^2$. The table includes results from TERAD91, HERACLES, HELIOS and APHRODITES. The first two programs calculate the same quantity with exactly the same conventions for all input quantities; their calculations are exact to $O(\alpha)$ and include no higher-order contributions, except those entering via $\Delta r$ in the calculation of $M_W$ and the Z boson couplings. The numerical accuracy of TERAD91 is estimated to be of the order of 0.1% whereas that of HERACLES, being limited by the statistics of the Monte Carlo integration, ranges from 0.1% to about 1.0%, depending on $z_i$ and $y_i$. Within these errors, the agreement between the two programs is perfect, and satisfactory for the HERA experiments since the size of the systematic experimental errors of the differential cross-section measurement is expected to be of the order of 1%. Both programs are based on completely different technical methods, therefore we believe that by this comparison we could demonstrate the technical reliability of the two programs.

The third column in table 2 displays results from HELIOS, i.e. from a calculation including the leading logarithms to first order. It is seen that this approximation gives reasonable results with an accuracy of better than 5% at large $z_i$, the quality of the leading-log approximation becoming worse at smaller $z_i$ and large $y_i$. The Compton contribution, being a genuine part of the leptonic corrections, is included, of course. It must be pointed out that the scale of the logarithm in the Compton contribution cannot be determined within the framework of a leading-log calculation. Its choice has a considerable influence on the results for large $y_i$ and small $z_i$. In the table it was chosen as $\ln (s/m_W^2)$ [6].

Finally, table 2 also contains results from APHRODITES, which include higher-order contributions from soft-photon exponentiation and second-order leading logarithms. As is seen, these higher-order terms lead to additional positive contributions. Especially for small $y_i$ at high $z_i$, this is due to higher-order soft photon corrections. At small $z_i$, where higher-order corrections are also non-negligible, they are due mainly to second-order hard photon radiation. These findings are in agreement with former results [26]. Note that effects from unobserved additional fermion pairs are not included here. These contributions should also be counted as contributing to radiative corrections since, fermion pairs being emitted predominantly in the electron direction, it will not be possible to
observe the effect experimentally. In [36] it was found that additional $f \bar{f}$ pairs can contribute non-negligible corrections for low $x_t$ and very large $y$.\footnote{The authors of APHRODITES do not exclude that the results at large $y$ are affected by numerical instabilities.}

We should like to mention that the results discussed in these paragraphs correspond to a measurement of the bare electron. In reality, the calorimetric measurement of particle energies in the HERA experiments is not able to distinguish a single electron from an electromagnetic cluster consisting of an electron and accompanying photons if the latter are emitted close to the former. This affects also the size of radiative corrections, which change by several per cent. The effect can be studied with the help of Monte Carlo programs, of course, but it is also implemented in the analytic program APHRODITES. Numerical results are presented in a separate contribution in these proceedings.

The results of table 2 were obtained without any cut on the photon phase space. In TERAD91 there is an option which allows to reject all final states with photons of an energy $E_\gamma \geq E_{\gamma}^{\min}$ and an emission angle between $\vartheta_{\gamma \min}$ and $\vartheta_{\gamma \max}$ with respect to the beam axis. Figure 5 shows results from TERAD91, compared with a calculation of HERACLES for the purely leptonic QED corrections, with the leptonic definition of $x$ and $y$ for a specific value of $x_t = 10^{-3}$ and $E_{\gamma}^{\min} = 1$ GeV, $\vartheta_{\gamma \min} = 0.1 \text{ rad}$ and $\vartheta_{\gamma \max} = \pi$. Although these cut values are certainly not applicable in the HERA experiments, this comparison is another check of the programs and demonstrates their reliable performance. Results for corrections with more complicated cuts on the final-state photon have been described in [54].

Fixing the kinematics with the help of the hadronic final state by using hadronic
variables (eq. (4)), Jaquet–Blondel variables (eq. (6)), or mixed variables (eq. (7)), one excludes those phase-space regions where \( Q^2 \) can be much smaller than what is measured externally. Results for these choices obtained by KRONOS, HELIOS, TERAD91, and EPRC91 are shown in figs. 6 and 7 and in table 3. For the purely hadronic definition and with the Jaquet–Blondel definition of kinematic variables (fig. 6), corrections are in general small, of the order of \(-10\%\), depending only slightly on \( x \) and \( y \). The corrections reach a magnitude of \(-20\%\) only for very large \( y \). The leading-log results obtained from EPRC91 are in good agreement with the exact calculation of TERAD91 for the hadronic variables. One should not forget that the hadronic variables as defined in eq. (4) are not accessible by experiment. We included results for them here to demonstrate the sensitivity of radiative corrections to the exact definition of \( x \) and \( y \). The calculations based on the Jaquet–Blondel variables (KRONOS to \( O(x) \) and HELIOS) agree with each other at the level of the statistical accuracy of the KRONOS results, but they differ by several per cent from the hadronic variables case. It is obvious that the calculation of radiative corrections has to be adjusted carefully to the experimental way of extracting kinematic variables. Table 3 contains two columns with results from KRONOS, showing that also higher-order corrections are much smaller than for the case of leptonic variables. In most cases they are smaller than the statistical accuracy of the KRONOS results, which is of the order of 1%. For large \( z_{2,Y} \), however, they may reach several per cent. In fig. 6 the results from HELIOS for the corrections as a function of the Jaquet–Blondel variables are confronted with those from TERAD91 for the corrections as a function of hadronic variables.

Also in the case of mixed variables (fig. 7) the corrections are in general small, except for large \( x \), and small \( y \). Here again, \( x_m \) and \( Q^2 \), at large values of \( y \), do not exclude small \( Q^2 \) leading to an enhancement of corrections in this region. The leading-log calculation is again a good approximation at the per cent level, as can be seen from the dashed lines in the same figure.

The Monte Carlos HERACLES and LESKO-F have also been checked against each other in a separate study. As already explained, a direct comparison of corrections either in terms of leptonic or in terms of hadronic kinematic variables with these two programs is very time-consuming. Therefore it was decided to directly compare distributions in various kinematic variables obtained from both programs. HERACLES was run in two different phase-space regions defined by \( 10^{-3} \leq z_1 \leq 0.1 \) and \( 0.1 \leq z_1 \leq 0.5 \), both with the additional condition \( Q^2 \geq 100 \text{ GeV}^2 \), and a subsequent cut on the transverse momentum of the hadronic final state (the scattered quark) of \( p_T^q \geq 10 \text{ GeV} \) and rejected in a second step events with wrong values of \( z_1 \) and \( Q^2 \). In this way the loss of efficiency was distributed between the two programs and could be held at a reasonable level. As an example of the results, we show in figs. 8 and 9 the distributions of events with respect to \( z_1 \) and \( x_1 \). Only these two examples are reproduced here, but we checked a large number of other distributions such as energies and angles, and no systematic deviation was found.

### 4.3 Radiation from the quark line

Radiative corrections resulting from photon emission from the quark line have been calculated in the parton model in [6, 8] and a detailed comparison has been presented in [55, 56]. Since agreement was found there at the per mille level, we do not present the

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Table 3: Results for NC leptonic QED corrections in hadronic and Jaquet–Blondel variables. \((x,y) = (x_2, y_2)\) or \((x,y) = (x_B, y_B)\).

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
<th>Jaquet–Blondel variables</th>
<th>Hadronic variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 10^{-2} )</td>
<td></td>
<td>KRONOS LLA ( O(x) )</td>
<td>HELIOS LLA ( O(x) )</td>
</tr>
<tr>
<td>0.01</td>
<td>0.01</td>
<td>-3.0</td>
<td>-4.5</td>
</tr>
<tr>
<td>0.10</td>
<td>0.10</td>
<td>-4.1</td>
<td>-3.1</td>
</tr>
<tr>
<td>0.30</td>
<td>0.30</td>
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<td>-2.4</td>
</tr>
<tr>
<td>0.50</td>
<td>0.50</td>
<td>-4.1</td>
<td>-6.4</td>
</tr>
<tr>
<td>0.70</td>
<td>0.70</td>
<td>-6.9</td>
<td>-6.5</td>
</tr>
<tr>
<td>0.90</td>
<td>0.90</td>
<td>-9.9</td>
<td>-11.3</td>
</tr>
<tr>
<td>0.99</td>
<td>0.99</td>
<td>-16.1</td>
<td>-20.1</td>
</tr>
</tbody>
</table>

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28
corresponding results separately again. Rather we will present numerical results below for the complete \(O(\alpha)\) corrections, including also the interference of leptonic and quarkonic radiation and the purely weak contributions (see section 4.6).

In [6, 8] no separation of mass singular and finite parts had been performed, but the calculation was done with finite, though small, values for the quark masses. As explained already, a correct treatment requires the separation and absorption of singular parts into the parton distribution functions. The remaining finite pieces are known to be small. It is therefore justified to use a simplified prescription for their calculation: instead of deriving formulas within one of the widely used factorization schemes MS.
into account the non-perturbative fragmentation and hadronization process. The same applies to the phase-space region, with photons which are collinear with the scattered quark. Also events of this type are excluded in HERACLES by applying a corresponding cut. The contribution from those phase-space regions excluded in the event generation has been integrated analytically in the collinear approximation and is combined with virtual photonic corrections, the finite terms being defined such that there is no correction left over from the cone around the initial quark.

4.4 Interference of leptonic and quarkonic radiation

The interference of amplitudes describing radiation from the lepton line and from the quark line are combined with $\gamma\gamma$ and $\gamma Z$ box diagrams to obtain an infrared-finite result. Mass singularities do not appear in this case. Corresponding results have been obtained again in [6] and [8], in agreement with each other (see [55, 56]). The lepton–quark interference results in a correction to the cross section, staying below 1% for $Q^2 \leq 4 \times 10^4$ GeV$^2$, but increasing with increasing $Q^2$. It is dominant in the corrections to charge asymmetries. Again, we do not want to show corresponding results separately, but combine them with other $O(\alpha)$ corrections.
### 4.5 Weak corrections

Numerical results for the genuine one-loop corrections combined with the fermionic contributions to the self-energy diagrams (vacuum polarization) are presented in table 4. They include leading terms of higher orders obtained by the Dyson summation of these latter parts as described in section 2.2. Results are calculated with the programs TERA91 and EPRC91. They reach the 10% level. Genuine weak corrections, i.e. without the vacuum polarization, reach about 2%. The difference between results from TERA91 and EPRC91 at large $Q^2$ could be for instance due to a different grouping of separate parts of corrections into form factors and thus reflect the uncertainty from unknown higher-order weak corrections.

### 4.6 Complete $O(\alpha)$ corrections

In table 5 we present a comparison of the complete $O(\alpha)$ photonic and weak corrections to the NC cross section in terms of leptonic variables obtained from TERA91 and HERACLES 4.1. These results include the parts described in the preceding sections 4.2 to 4.5. Moreover, an additional refined prescription for the calculation of purely leptonic QED corrections has been used here. Since the latter part is large, we decided to base its calculation on an improved Born cross section, including the vacuum polarization by using the running fine structure constant $\alpha(Q^2)$. TERA91 includes in the improved Born cross section used in the evaluation of leptonic corrections also an approximation to the weak form factors, whereas HERACLES does not include this kind of higher-order effect. Note that the quarkonic radiation is included, but it is based on different prescriptions for the separation of mass singular parts in both programs. Apart from purely photonic higher-order corrections, the results in table 5 constitute our best results for radiative corrections to the NC cross section. Comparing tables 2, 4, and 5, and keeping in mind that the results of table 4 are dominated by the vacuum polarization, one finds that for the calculation of $O(\alpha)$ radiative corrections with an accuracy of a few per cent it is sufficient to calculate the purely leptonic QED corrections and use the running fine structure constant.

### Table 4: Results for the purely weak NC one-loop corrections including vacuum polarization. Upper lines: TERA91, lower lines: EPNC91.

<table>
<thead>
<tr>
<th>$x_1$</th>
<th>0.01</th>
<th>0.10</th>
<th>0.30</th>
<th>0.50</th>
<th>0.70</th>
<th>0.90</th>
<th>0.99</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y_1$</td>
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<td>3.38</td>
<td>4.23</td>
<td>4.65</td>
<td>4.94</td>
<td>5.16</td>
<td>5.26</td>
</tr>
<tr>
<td>$y_1$</td>
<td>2.07</td>
<td>3.38</td>
<td>4.23</td>
<td>4.65</td>
<td>4.94</td>
<td>5.16</td>
<td>5.26</td>
</tr>
<tr>
<td>$y_1$</td>
<td>3.38</td>
<td>5.27</td>
<td>6.40</td>
<td>6.96</td>
<td>7.34</td>
<td>7.62</td>
<td>7.73</td>
</tr>
<tr>
<td>$y_1$</td>
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<td>5.27</td>
<td>6.40</td>
<td>6.96</td>
<td>7.34</td>
<td>7.62</td>
<td>7.73</td>
</tr>
<tr>
<td>$y_1$</td>
<td>5.27</td>
<td>7.75</td>
<td>9.00</td>
<td>9.57</td>
<td>9.93</td>
<td>10.18</td>
<td>10.27</td>
</tr>
<tr>
<td>$y_1$</td>
<td>5.27</td>
<td>7.75</td>
<td>9.00</td>
<td>9.57</td>
<td>9.93</td>
<td>10.18</td>
<td>10.27</td>
</tr>
<tr>
<td>$y_1$</td>
<td>7.74</td>
<td>10.30</td>
<td>10.95</td>
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<td>10.55</td>
<td>10.60</td>
<td>10.70</td>
</tr>
<tr>
<td>$y_1$</td>
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<td>9.57</td>
<td>11.98</td>
<td>10.68</td>
<td>10.77</td>
<td>11.16</td>
<td>11.78</td>
</tr>
<tr>
<td>$y_1$</td>
<td>9.55</td>
<td>10.92</td>
<td>10.33</td>
<td>10.33</td>
<td>10.68</td>
<td>11.29</td>
<td>11.75</td>
</tr>
</tbody>
</table>

### Table 5: Results for the complete $O(\alpha)$ NC corrections. Numbers in the upper line are from TERA91, in the second line from HERACLES 4.1. The lower line contains an estimate of the statistical accuracy of HERACLES; $z$ and $y$ are defined from the final-state lepton's momentum. There is no entry from HERACLES for $z_1 = 0.3$, $y_1 = 0.91$ because of the restriction (40).

<table>
<thead>
<tr>
<th>$x_1$</th>
<th>0.01</th>
<th>0.10</th>
<th>0.30</th>
<th>0.50</th>
<th>0.70</th>
<th>0.90</th>
<th>0.99</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y_1$</td>
<td>88.3</td>
<td>106.7</td>
<td>217.4</td>
<td>566.1</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>$y_1$</td>
<td>88.3</td>
<td>106.7</td>
<td>217.4</td>
<td>566.1</td>
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<td>$y_1$</td>
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<tr>
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<td>217.4</td>
<td>566.1</td>
<td></td>
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</table>

### 4.7 Higher-order corrections

In view of the fact that $O(\alpha)$ corrections can be huge at HERA, one has to ask whether higher-order corrections are non-negligible. There are several sources for them:

Purely photonic higher-order contributions can be sizeable if they originate from multiple soft photon (virtual or real) radiation or from radiation of collinear photons. Both effects can be treated theoretically either by exponentiation or by the use of renormalization group techniques. Soft photon exponentiation is implemented in several of the programs discussed in this report: In TERA91 the approach of [57] is realized for the NC process, whereas EPNC91 applies Yennie-Frautschi-Suura exponentiation [58] for the CC process. Also APHODITES includes soft photon corrections in an exponentiated form. Soft–photon exponentiation leads to sizeable positive contributions in those regions of the $(x, y)$ plane where $O(\alpha)$ corrections are large and negative, i.e. for large $x_1$ and small $y_1$, as well as for large $x_2B$ and large $y_2B$.

Higher-order leading-logarithmic corrections arising from collinear radiation of photons have been studied in [26] and are implemented in the program EPNC91. This program includes the effect from two-photon radiation and from fermion–pair production (including $e^+ e^-$ pairs) whereas the present version of APHODITES treats two-photon radiation only. The Monte Carlo KRONOS goes beyond the two-photon production. It has implemented the effect of multiple photon radiation in the collinear approximation; however, it is restricted to the initial–state radiation (see the separate contribution in these...
proceedings). Those higher-order effects treated by APHRODITES and KRONOS have been discussed already in section 4.2, see tables 2 and 3. The additional second-order processes (two-photon radiation, \( f' f\) and \( e'^- e^-\) pair creation) add positive contributions to the radiative corrections. They diverge for \( y_i \to 1\) even when normalized to the \( \mathcal{O}(\alpha)\) corrections. Relative to the first-order corrections they reach 10% for \( x_f = 2.5 \times 10^{-3}\), \( y_i = 0.8 \) [26].

The non-singlet contributions of fermion-pair production has to be combined with fermion-loop insertions in internal photon propagators. It is automatically included by the use of the running fine structure constant. This is done in all programs which take into account self-energy corrections. TERAD91, EPRC91 and HERACLES 4.1 use the running fine structure constant also for the calculation of leptonic corrections. Thus these programs include part of the leading-logarithmic higher-order corrections. The effect of this more refined treatment is non-negligible for values of \( x\) and \( y\) where the \( \mathcal{O}(\alpha)\) corrections are large in magnitude, i.e. for small \( x_f\), large \( y_i\), and for large \( x_f\), small \( y_i\).

Finally we want to comment on purely weak higher-order corrections. The leading terms related to large values of the top mass proportional to \( \alpha^2 m_t^2\) and \( \alpha m_t^2\) are well-known from studies of radiative corrections at LEP/SLC (see [13] and references therein). They are included in TERAD91, EPRC91 and HERACLES 4.1. In most cases they will probably be negligible compared with the experimental accuracy, except in dedicated studies of electroweak effects as for example the measurement of \( R_{NC/GC}\) [13, 59].

### 4.8 Characteristics of radiative corrections at low \( x\)

Radiative corrections at low \( x\) deserve a special discussion for two reasons: first the statistical errors of the NC cross-section measurement at low \( x\) are small, even for a moderate integrated luminosity, and a knowledge of radiative corrections with high precision is needed. Secondly, they are large and need a more careful treatment in order to obtain reliable results.

Technical difficulties are due to the fact that at small \( x\) also terms \( \alpha m_t^2\) or \( m_t^2\) are numerically important since they are enhanced by logarithms of \( x\). The Compton part, i.e. the phase-space region with small \( Q_t^2\) is especially important. This is obvious from the observation that with fixed \( x_2\) and \( y_2\) the kinematical lower limit of the hadronic momentum transfer is \( Q_t^2 \geq x_f m_t^2\). Therefore the leptonic corrections have to be calculated in a model-independent framework using structure functions \( F_2\) and \( F_L\) as input. The radiatively corrected, i.e. measured, cross section is strongly influenced by the behaviour of structure functions at low \( Q_t^2\) extending down to yet unexplored values. Using different input parametrizations for structure functions, the results for \( \mathcal{O}(\alpha)\) radiative corrections may change by tens of per cent. The extraction of structure functions from experimental data must therefore be done with the help of an iterative procedure [60].

The effect of higher-order corrections in the low-\( x\) region has been estimated with the help of the program EPRC91. Results are shown in fig. 10. For this figure, \( F_2\) (i.e. the electromagnetic contribution to \( F_2\), \( F_2^\gamma\)) in the notation of section 2.2) was calculated with two variants of parton distribution functions from [61] which have been modified in the low-\( Q_t^2\) range by the prescription of eq. (46). The figure displays the second-order correction normalized to the first-order-corrected result \( F_2^{(2)} / F_2^{(1)} - 1\), where \( F_2^{(n)}\) includes leading-logarithmic corrections up to nth order. The result is shown as a function of \( x_f\)

![Figure 10: Higher-order leading-logarithmic corrections to \( F_2\) for low \( x\).](image)

For fixed \( Q_t^2 = 20\ \text{GeV}^2\). In this plot, small values of \( x_f\) correspond to large \( y_i\). It is seen that second-order corrections reach the level of 10% (normalized to the \( \mathcal{O}(\alpha)\) corrected result) for \( Q_t^2 = 20\ \text{GeV}^2\) and \( x_f = 2.5 \times 10^{-4}\). Thus they are non-negligible. The second-order corrections depend on the input parametrizations to a smaller extent than the \( \mathcal{O}(\alpha)\) corrections. Variations of \( \mathcal{O}(\alpha^2)\) corrections larger than shown in fig. 10 may however occur when parton distribution functions from other authors are used.

### 4.9 Corrections to the charged-current process

Radiative corrections to the CC process at HERA have been studied in [7] and [25]. Results have been presented there after separating the complete electroweak corrections into leptonic, quarkonic, and interference parts of QED corrections and a purely weak
contribution. The comparison of numerical results of these two references shows good agreement of the photonic contributions at the per mille level. This also confirms results of the leading-log calculation of [18].

The old calculations were updated during this workshop by using the input of eq. (47) for the electroweak parameters and by suppressing the quark-mass logarithms setting \( m_q = 2 \text{ GeV} \) as explained in section 4.3. Corresponding numerical results for the complete electroweak corrections to the CC cross section from TERAD91 and EPRC91 are shown in fig. 10. Here, the kinematic variables \( x \) and \( y \) are defined as in eq. (3), thus assuming that the neutrino momentum was reconstructed. In contrast with the NC case, the corrections do not reach large positive values since the \( W \) boson propagator \( 1/(Q^2 + M_W^2) \) does not allow an enhanced radiative cross section in the case where photon emission leads to a reduced value of \( Q^2 \). The results for other definitions of kinematic variables would resemble the analogous NC corrections. These possibilities are not yet implemented in a program for the full set of \( O(\alpha) \) corrections, but they can be estimated using the leading-log programs of EPRC91 or HELIOS.

The results of TERAD91 and EPRC91 shown in fig. 11 show good agreement in the central region of \( x \) and \( y \) values. Differences of 1 to 2% occur for small \( x \) at small \( y \) and in the region of large \( x \). This situation is not completely satisfactory and has to be improved upon in the future. However, we believe that this level of agreement is sufficient for the first one or two years of data taking at HERA, where the statistical precision of the measurement of the CC cross section will probably not reach the level of 1%.

5 Conclusions

We hope to have shown in this article that the theory of radiative corrections in deep-inelastic lepton-nucleon scattering at HERA is well understood and that programs for their calculation are well established. The comparison of results from several programs, written by various authors and based on different approaches, has shown a good level of agreement, and it was demonstrated that reliable tools for the calculation of radiative corrections are available and ready to be used in the data analysis.

Our article did not cover all aspects of radiative corrections. We demonstrated that their size strongly depends on the way kinematic variables are defined. However, effects of radiative corrections on the reconstruction of event kinematics and on the measurement of structure functions, being closely interrelated to experimental questions, are discussed only in separate contributions to this working group report. Also other working groups have investigated this topic. We mentioned only shortly that radiative corrections strongly depend on the structure functions input, but we did not present a systematic study of this dependence. That this input dependence can in turn be used for a measurement of, for example, the longitudinal structure function \( F_2 \) (using initial-state radiation) [62], or of the low-\( Q^2 \) behaviour of structure functions (using Compton-type radiative events) [63], is discussed elsewhere. Electroweak one-loop corrections and their impact on the measurement of electroweak parameters at HERA are discussed in detail in the report of the working group on electroweak physics [13, 59].

There are still open questions, and in some respect more work is needed. We mentioned the radiative corrections to the CC cross section where we did not find agreement of individual calculations to a completely satisfactory degree. In spite of the fact that higher-order QED corrections are known from several studies, the implementation of these effects in programs that cover the complete electroweak one-loop corrections is still missing. Radiative corrections to processes other than deep-inelastic scattering have not been studied systematically. A treatment of weak loops and of the leading-logarithmic leptonic corrections for a \( Z^\gamma \) search at HERA has been applied in [64] and, in some of the studies of exotic physics, initial-state leptonic corrections have been taken into account [55]. Of course, the leptonic QED corrections in their model-independent approach can be applied to any scattering process mediated by one-boson exchange, provided the corresponding hadronic vertex is parametrized with the help of structure functions \( F_2 \). Especially the leading-logarithmic approximation may serve as a first estimate of radiative corrections in these cases. This would cover heavy-flavour production, photoproduction, or jet cross...
sections. However, no attempt was yet made to calculate the complete electroweak corrections to these processes. Eventually, this would require the investigation of combined QED and QCD corrections of $O(\alpha \cdot \alpha_s)$.

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