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The analysis and performance of a beam position monitor using a re-entrant coaxial cavity are presented. The monitor measures both the horizontal and vertical beam positions independently of each other. The bunched beam excites an evanescent dipole mode $H_{11}$ in the coaxial cavity proportionally to the transverse beam displacement from the centre axis. When the beam is located exactly on the centre axis of the circular beam tube of radius $R$, the beam position can be measured with an accuracy better than 0.004 $R$. The linear error of the beam position measurement is smaller than 1% of the position measured. For beam monitoring, microwave cavities can be used either at the resonant frequency $f_r$ of the cavity with a narrow bandwidth $< 10^{-3} f_r$, or below the resonant frequency with a larger bandwidth $> 10^{-1} f_r$. A broadband beam monitor below resonance provides more signal power than a resonant monitor for single bunches. The broadband monitor has a lower beam coupling impedance than the same monitor at resonance, and causes less beam break-up forces to a long bunch train. The high-order modes of the cavity can be damped by resistive ferrite material at the bottom of the cavity. The principle of operation of a broadband microwave monitor and the theory of the evanescent fields in a microwave cavity are illustrated with the example of the re-entrant coaxial cavity loaded by four outputs of 50 $\Omega$.

Introduction

The design and manufacture of microwave beam position monitors with an accuracy of $< 0.01$ mm is a major challenge for the construction of future linear accelerators. In order to obtain the best possible mechanical precision for the construction and alignment of the beam monitors, cavities with circular geometry, which can be milled on CNC working stations, are preferred.

The electrical precision of the beam position monitor is limited by the quality of the signal feed-throughs from the microwave cavity to the electronic equipment for the measurement of the cavity signals. If the centre position of the beam must be measured with an absolute accuracy of $< 10^{-3}$ of the radius of the vacuum chamber, the cavity must be excited at the lowest dipole resonance frequency, and the position signal is extracted from the cavity by a single feed-through. If several feed-throughs are connected to the cavity, unavoidable electrical asymmetries between the feed-throughs cause additional errors to the measurement of the centre position.

The signal energy $W$ extracted from the beam by a microwave cavity is proportional to the shunt impedance $R_S(\omega)$ of the cavity:

$$ W = \frac{1}{\pi} \int_{0}^{\infty} R_S(\omega) I^2(\omega) \, d\omega $$

(1)

where $I(\omega)$ is the Fourier transform of the beam current $I$ as a function of the angular frequency $\omega$. In linear accelerators, the beam intensity $I(\omega)$ has a rather large frequency spectrum, as the length of the bunches and the bunch train are relatively short. Therefore, a resonant beam monitor with a high shunt impedance covers only a small fraction of the frequency spectrum of the beam.

Away from resonance, the shunt impedance of the cavity is much smaller, but the broader bandwidth of the evanescent mode provides more signal power. The larger bandwidth provides also a better resolution of the beam signals in the time domain, and allows the monitoring of intensity and position variations inside a bunch train.
A cavity design suitable for broadband operation below the fundamental resonance mode is the re-entrant coaxial cavity. The Q-factor of the cavity excited at frequency \( f_0 \) decreases slightly by the factor \( \sqrt{f_0/f_r} \) with respect to the resonant frequency \( f_r \). The standing waves induced by the beam in the cavity below the resonant frequency are strongly attenuated in space and in time. As for the evanescent fields in waveguides below the cutoff frequency, the frequency bandwidth of a cavity below resonance is relatively large. The attenuation constant of the evanescent fields below half of the cutoff frequency is practically constant. The bandwidth of the beam monitor can attain 0.5 \( f_0 \).

**Resonance Modes of the Re-entrant Coaxial Cavity**

The re-entrant coaxial cavity is arranged around the beam tube and forms a coaxial line which is short-circuited at the downstream end. This type of cavity is widely used in klystron tube amplifiers for beam modulation and RF-power extraction. The re-entrant coaxial cavity of a beam monitor consists of three distinct regions (fig. 1): I. beam tube, II. gap, III. coaxial cylinder. It has been shown by Hansen\(^3\), how the electromagnetic fields at the boundaries of these regions can be matched to each other by analytical expressions of the field distributions. Nowadays, computer codes provide the necessary parameters like resonance frequency, Q-factor and shunt impedance for a given geometry of a cavity design.

By appropriate shaping of the gap, a large shunt impedance for the fundamental monopole and dipole modes can be obtained for the re-entrant coaxial cavity. In the transition region of the gap (II), the radial electric field lines of the coaxial cylinder (III) are bent by 90° into the direction of the beam axis, so that the H-modes of the coaxial cavity interact with the beam.

The fundamental resonance mode of the re-entrant coaxial cavity is the \( \lambda/4 \)-resonance of a coaxial line. The resonance wavelength \( \lambda_1 \) of the transverse electromagnetic monopole mode depends only on the length \( a \) of the coaxial cavity: \( \lambda_1/4 = a \). This is the lowest resonance frequency for the coaxial cavity. The effective length of the coaxial cavity is (fig. 1): \( a = \ell + 0.5 \ (b+g) + e \).

At the boundary between the gap of the re-entrant cavity and the beam tube, the beam intensity \( I \) induces a voltage \( U_m \):

\[
U_m(a) = -I j Z_c \tan \left( \frac{2 \pi a}{\lambda_o} \right)
\]

The characteristic impedance \( Z_c \) of the coaxial line with outer diameter \( D \) and inner diameter \( d \) is given by

\[
Z_c = \frac{1}{2 \pi} \sqrt{\frac{\mu_o}{\varepsilon_o}} \ln \left( \frac{D}{d} \right) = 60 \Omega \ln(\frac{D}{d}),
\]

where \( \mu_o = 4 \pi \cdot 10^{-7} \) Vs/Am, \( \varepsilon_o = 1/\mu_o \cdot c = 3 \cdot 10^8 \) m/s, \( \lambda_o = c/\nu \), \( \nu = 1 \).

The monopole voltage \( U_m \) is constant all around the circumference of the cavity and does not depend on beam position.

The next higher resonance frequency is found for the coaxial waveguide mode \( H_{11} \). The coaxial mode \( H_{11} \) has the lowest cutoff frequency of all E- and H-modes. If the ratio \( D/d \) between the outer and inner diameter of the coaxial line is sufficiently small, \( D/d < 1.2 \), the cutoff wavelength \( \lambda_c \) of the \( H_{11} \)-mode is \( \lambda_c = 2 \pi r_m \), where \( r_m \) is the mean radius of the coaxial cavity, \( r_m = (D + d)/4 \). The resonance wavelength \( \lambda_2 \) of the mode \( H_{111} \) of the re-entrant coaxial cavity is given by\(^4\):

\[
\lambda_2 = \frac{2 \pi r_m}{\sqrt{1 + \left( \frac{\pi r_m}{2 a} \right)^2}}
\]
The dipole resonance $H_{111}$ is excited by the beam proportionally to the beam intensity and proportionally to the distance of the beam from the centre axis of the monitor. The polarization plane of the dipole mode in the beam tube and in the coaxial cavity is defined by the plane of the beam displacement (fig. 2). The voltage of the dipole field adds to the monopole voltage in the direction of the beam displacement and subtracts from the monopole voltage on the opposite side. Therefore, the beam position $P(x,y)$ can be measured from the output voltage of a pair of feedthroughs mounted on opposite sides of the cavity on the $x$- and $y$-axis (fig. 1).

$$x = L \left( U_1 - U_3 \right) / \left( U_1 + U_3 \right), \quad y = L \left( U_2 - U_4 \right) / \left( U_2 + U_4 \right)$$

$L$ is a monitor constant studied in the next chapter. The difference voltages $U_1 - U_3$ and $U_2 - U_4$ correspond to the voltage of the dipole field along the $x$- and $y$-axis (fig. 2). The sum voltage $U_1 + U_3 = U_2 + U_4 = 2 U_m (z = \ell)$ is the sum of the TEM-monopole signals proportional to the beam intensity.

**Evanescent Dipole Mode of Coaxial Cavity**

When the beam excites the re-entrant cavity below the cutoff frequency of the dipole mode $H_{111}$, the electromagnetic dipole field of the beam does not penetrate far into the coaxial cavity (III). The field energy is concentrated in the gap (II), and the dipole field in the coaxial cavity is attenuated exponentially with the distance $\xi$ from the boundary between the beam tube (I) and the gap (II). Because of the exponential attenuation of waves below the cutoff frequency of the coaxial line, these fields are called evanescent. The attenuation factor $F$ of evanescent fields along the coaxial line shown in fig. 3 is $F = \exp (-\alpha \xi)$, where $\alpha$ is the attenuation constant of the mode considered:

$$\alpha = \frac{2\pi}{\lambda_e} \sqrt{1 - \frac{\lambda_e^2}{\lambda_o^2}}, \quad \lambda_e \equiv 2\pi r_m$$

(6)

Other modes with higher cutoff frequencies need not to be considered, since the fields of the higher modes are attenuated much more than the fundamental dipole mode. The strong damping of higher modes, the absence of mode coupling and mode degeneration are the major advantages of the broadband microwave beam monitor excited at about half of the fundamental resonant frequency.

The fields of the waveguide modes $H_{mn}$ along a coaxial line have been derived by Borgnis. We transcribe the formulae for the evanescent $H_{111}$ mode:

$$H_z = -\gamma_{111}^2 G \sin(\varphi - \varphi_o) \exp(\Gamma \xi)$$

(7)

$$H_\varphi = -\left( \Gamma / \tau \right) G \cos(\varphi - \varphi_o) \exp(\Gamma \xi)$$

(8)

$$E_r = j(\omega \mu_o / \tau) G \cos(\varphi - \varphi_o) \exp(\Gamma \xi)$$

(9)

where $G$ is the excitation constant and $\xi$ the distance from the open end of the coaxial line. Below the cutoff frequency $f_o < f_c$, the propagation constant $\Gamma$ of the evanescent field is a real attenuation constant:

$$\Gamma = 2\pi j \frac{1}{\sqrt{\lambda_e^2 - \lambda_o^2}} = -\alpha$$

(10)

When the diameter ratio of the coaxial line is sufficiently small, $D/d < 1.2$, the eigenvalue $\gamma_{111}$ determined from the boundary conditions of the electro-magnetic fields is given with good precision ($< 10^{-3}$) by

$$\gamma_{111} = 2\pi / \lambda_e \equiv 1 / r_m = 4 / (D + d)$$

(11)
It follows from equations (7) - (10) that the magnetic field components $H_\varphi$ and $H_z$ are in anti-phase below the cutoff frequency, and the electric field $E_r$ is in quadrature with the magnetic fields.

In a coaxial line with a short-circuit, the evanescent field must satisfy the boundary conditions at the short-circuit end which is at distance $\xi = a$ from the open end of the line (see fig. 3).

$$E_r(\xi = a) = 0, \quad H_\varphi(\xi = a) = 0$$

(12)

The superposition of incident waves proportional to $\exp(-\alpha(a-z))$ and reflected waves from the short-circuited end proportional to $\pm \exp(-\alpha(a+z))$, generates a standing wave on the coaxial line which has the components ($z < l$):

$$E_r(\varphi, r, z) = j(\omega \mu_o / \alpha) \left( r_m / r \right) H_o \cos(\varphi - \varphi_o) \sinh(\alpha z)$$

(13)

$$H_\varphi(\varphi, r, z) = \left( r_m / r \right) H_o \cos(\varphi - \varphi_o) \cosh(\alpha z)$$

(14)

$$H_z(\varphi, r, z) = -\left( I / \alpha r_m \right) H_o \sin(\varphi - \varphi_o) \sinh(\alpha z)$$

(15)

where $H_o = H_\varphi(\varphi_o, r_m, 0)$.

The field impedance $Z_f$ of the standing wave in the coaxial line is

$$Z_f(z) = E_r / H_\varphi = j(\omega \mu_o / \alpha) \tanh(\alpha z)$$

(16)

At the open end, the coaxial line is bent by 90° towards the beam tube. With good approximation, the gap (II) between the beam tube (I) and the coaxial cavity (III) can be considered as prolongation of the coaxial line, if $b = g$ (Fig. 1).

**Magnetic Field induced by Beam**

Ultra-relativistic particles ($\gamma > 10$) have a transverse electro-magnetic field. The longitudinal field component induced by high energy particles decreases by a factor $1/\gamma^2$, where $\gamma$ is the ratio between the total energy of the particle and the particle energy at rest. For an ultra-relativistic beam located at the polar coordinates $(\varphi_0, r_0)$ the azimuthal magnetic field induced at azimuth $\varphi$ and radius $R$ on the inner surface of a circular vacuum tube is given by:

$$H_\varphi(\varphi, R) = \frac{-I}{2\pi R} \cdot \frac{R^2 - r_0^2}{R^2 + r_0^2 - 2Rr_0 \cos(\varphi - \varphi_0)} = \frac{-I}{2\pi R} \left[ 1 + 2 \sum_{n=1}^{\infty} \left( \frac{r_n}{R} \right)^n \cos n(\varphi - \varphi_0) \right]$$

(17)

The electric field of the dipole mode inside the beam tube is shown in fig. 2. The electric field induces a displacement current on the inner surface of the beam tube with a current maximum at azimuth $\varphi = \varphi_0$ determined by the beam position: $\varphi_0 = \arctan(y/x)$.

The magnetic field of the dipole mode inside the beam tube excites the $H_{11}$-mode in the coaxial cavity. The two fields are perfectly matched at the open end of the re-entrant cavity.

$$H_{\varphi 1} = \frac{-I}{2\pi R} \cdot \frac{2r_0}{R} \cos(\varphi - \varphi_0)$$

(18)

If the wall between the coaxial cavity and the beam tube is negligibly thin, the magnetic field $H_{\varphi 1}$ (eq. 18) of the beam is equal to the magnetic field $H_\varphi$ (eq. 14) of the coaxial cavity at the location $r = R, z = a$:

$$-\frac{Ir_0}{\pi R^2} \cos(\varphi - \varphi_0) = \frac{H_o r_m}{pR} \cos(\varphi - \varphi_o) \cosh(\alpha a)$$

(19)
The load factor \( p \) takes into account the modification of the magnetic field in region II caused by the load impedance of the feedthroughs, see appendix.

The excitation constant \( H_0 \) in eq. 13 - 15 is determined by the magnetic field of the beam:

\[
H_0 = -Ir_o p / \pi r_m R \cosh(\alpha a)
\]  

\( \text{(20)} \)

**Transfer Impedance of Beam Monitor**

The transfer impedance \( Z_t \) indicates the output voltage \( U_L \) of the monitor induced by the beam of intensity \( I \) located at the centre axis of the beam monitor: \( Z_t = U_L / I \). The output voltage \( U_L \) is measured across the load resistance of 50 \( \Omega \) presented by the RF-receiver to each of the coaxial feed-throughs of the beam monitor.

For the beam in centre position, only the monopole mode needs to be considered, which has a transverse electromagnetic field in the coaxial cavity. TEM-waves on a coaxial line do not have a cutoff frequency, and propagate along the short length of the coaxial cavity practically without attenuation.

The coaxial feed-through is connected to the coaxial cavity by a launcher (part 14 in fig. 4). The launcher between the outer and inner cavity wall has a self-inductance, which constitutes an appreciable reactance \( jX \) at high frequencies. The reactance \( jX \) comprises also the transition from the launcher to the cavity wall, and is different for the monopole and dipole mode.

Taking into account the reactance \( jX_2 \) of the junction between the feed-through and the cavity for the monopole mode, the load impedance of the cavity is \( Z_2 = (Z_o + jX_2) / 4 \), with \( Z_o = 50 \) \( \Omega \). At the junction with the feed-through, the impedance of the coaxial line towards the short-circuit at distance \( \ell \) amounts to \( Z_3 = jZ_c \tan(2\pi \ell / \lambda_o) \). The coaxial line between the gap and the feed-through is loaded at the junction with the feed-through by \( Z_3 \):

\[
Z_3 = \frac{Z_1 Z_2}{Z_1 + Z_2} = \frac{-Z_1}{1 + m_1},
\]  

\( \text{(21)} \)

\[
m_1 = \frac{Z_1}{Z_2} = \frac{j4Z_c \tan(2\pi \ell / \lambda_o)}{Z_o (1 + jm_2)}, \quad m_2 = \frac{X_2}{Z_o}
\]  

\( \text{(22)} \)

The coaxial line with an approximate characteristic impedance \( Z_c \) and a length \( h = a - \ell \) between the gap and the feed-through transforms the image current \( -I \) of the beam at the gap into the current \( I_3 \) in front of the junction by 5)

\[
-I / I_3 = \cos(2\pi h / \lambda_o) + j(\bar{Z}_3 / Z_c)\sin(2\pi h / \lambda_o)
\]  

\( \text{(23)} \)

The voltage \( U_3 \) at the junction is

\[
U_3 = I_3 \bar{Z}_3 = \frac{-I \cdot \bar{Z}_1}{(1 + m_1)\cos(2\pi h / \lambda_o) + j(\bar{Z}_1 / Z_c)\sin(2\pi h / \lambda_o)}
\]  

\( \text{(24)} \)

The voltage \( U_L \) at the load \( Z_o \) of the feed-through is

\[
U_L = U_3 Z_o / (Z_o + jX_2) = U_3 / (1 + jm_2)
\]  

\( \text{(25)} \)

The transfer impedance \( Z_t \) of the beam monitor is given by

\[
Z_t = \frac{U_L}{-I} = \frac{m_2 Z_o / (4 \cos(2\pi h / \lambda_o))}{1 + m_1 - \tan(2\pi \ell / \lambda_o) \tan(2\pi h / \lambda_o)}
\]  

\( \text{(26)} \)
For the example of the wall current monitor BPA in fig. 4$^9$ ($\lambda_0 = 1.5$ m, $D = 338$ mm, $d = 284$ mm, $h = 34$ mm, $\ell = 155$ mm, $m_2 = 0.4$ ), the theoretical transfer impedance is $Z_t = 6.0$ $\Omega$. In the SPS accelerator, the transfer impedance has been measured from the sum signal $\Sigma = 2 U_m$ of the RF-receivers: $Z_t = 5.9 \pm 0.5$ $\Omega$. For an accurate measurement with beam, the gain of the RF-receivers has been carefully calibrated, and the RF-component of the beam intensity $I$ has been determined from the DC beam intensity and the bunch length.

**Linearity of Beam Position Measurement**

The amplitude of the dipole field induced by the beam is proportional to the beam displacement from the centre axis. The response of a re-entrant coaxial cavity is perfectly linear for ultrarelativistic beams, if there are no other modes than the fundamental monopole and dipole mode. When four orthogonal feed-throughs are mounted on the circumference of the coaxial cavity, the measurements of signal voltage are orthogonal, i.e. independent from the coordinate of the beam position on the orthogonal axis.

It was a surprise, when a nearly perfect linearity and orthogonality were discovered with the first beam position monitor using a re-entrant coaxial cavity excited at 200 MHz$^9$. The linear error of the position measurement $x$ was 0.01 $x$. The zero error of the centre position was less than 0.004 $R$, where $R$ is the radius of the beam tube.

At lower frequencies than 100 MHz, a good linearity is obtained from a re-entrant coaxial cavity, if four longitudinal slits are cut in the inner wall of the cavity$^{10}$.

The sensitivity of a beam position monitor is characterized by the transfer impedance and by the dipole constant. The transfer impedance $Z_t$ defines the sum signal $\Sigma$ of the monitor, which is proportional to beam intensity by $\Sigma = 2 Z_t I$. The dipole constant $L$ of a monitor indicates the ratio between the dipole ($\Delta$) and monopole signals ($\Sigma$). The beam position $x$ is calculated by $x = L \Delta \Sigma$.

The dipole constant $L$ can be measured in the accelerator, when the beam is displaced in the position monitor by orbit bumps of calibrated amplitudes $\pm x$. For the SPS wall current monitor with a beam tube radius $R = 134.5$ mm (fig. 4), the monitor constant measured with beam bumps amounts to $L = 128 \pm 6$ mm.

The electronic equipment and the synchronous RF-detectors for the measurement of the sum ($\Sigma$) and difference ($\Delta$) signals of the beam monitor are described elsewhere$^{11}$.

The dipole constant of a beam position monitor is given by the relative field amplitudes of the dipole and monopole modes at azimuth $\varphi_0$ for the beam located at $F(\varphi_0, r_0)$. The amplitude of the dipole mode $U_d$ at a distance $z \leq \ell$ from the short-circuit plane of the coaxial line amounts to

$$U_d(\varphi, z) = \int_{D/2}^{D/2} E_r(\varphi, r, z) dr = H_0 r_m \ell n(D/d) \frac{jm \mu_0}{\alpha} \sinh(\alpha z) \cos(\varphi - \varphi_0)$$

$$U_d(\varphi_0, \ell) = \frac{-I_r p}{\pi R} \ell n(D/d) \frac{jm \mu_0}{\alpha} \sinh(\alpha \ell) \cosh(\alpha a) \tag{27}$$

The dipole constant $L$ of a linear beam position monitor is defined by $L/r_0 = U_m/U_d$ for $z = \ell$ and $\varphi = \varphi_0$, where $\ell$ is the distance of the feed-throughs from the short-circuit plane and $U_m$ is given by $U_m = -I Z_t$.

$$L = R \cdot \frac{\alpha \lambda_0}{4 \pi \nu} \cdot \frac{Z_t \cosh(\alpha a)}{Z_c \sinh(\alpha \ell)} \tag{28}$$
For the parameters of the SPS wall current monitor\(^9\): \(\lambda_0 = 1.5 \, \text{m}, d = 0.338 \, \text{m}, \Delta d = 0.284 \, \text{m}, r_m = (d + d)/4 = 0.1555 \, \text{m}, R = 0.1345 \, \text{m}, l = 0.155 \, \text{m}, a = 0.189 \, \text{m}, \alpha = (1/r_m) \sqrt{1 - (2\pi r_m/\lambda_0)^2} = 4.895/\text{m}, Z_t = 6.0 \, \Omega, Z_c = 10.4 \, \Omega, p = 0.632\) (see appendix), the theoretical dipole constant given by eq. 28 amounts to \(L = 126 \, \text{mm}\).

The linearity and orthogonality of the re-entrant coaxial cavity can be deduced directly from the equations 29 - 32 for the source voltages \(U_i, i = 1,2,3,4\) at the four feed-through ports of the cavity (fig. 1). The total source voltage between the inner and outer wall of the coaxial cavity is the sum of the monopole (\(U_m\)) and dipole voltage (\(U_d\)) at each of the four ports: \(U_i = U_m + U_d\).

Port 1: \(\varphi_1 = 0\), \(U_1 = U_m + U_o(r_0/R) \cos \varphi_o = U_m + U_o x/R \) (29)
Port 2: \(\varphi_2 = \pi/2\), \(U_2 = U_m + U_o(r_0/R) \sin \varphi_o = U_m + U_o y/R \) (30)
Port 3: \(\varphi_3 = \pi\), \(U_3 = U_m - U_o(r_0/R) \cos \varphi_o = U_m - U_o x/R \) (31)
Port 4: \(\varphi_4 = 3\pi/2\), \(U_4 = U_m - U_o(r_0/R) \sin \varphi_o = U_m - U_o y/R \) (32)

\[
U_o = \frac{1}{\pi} \frac{\mu_0}{\alpha} \frac{\sinh(\alpha d)}{\cosh(\alpha a)} \ln(D/d) \quad (33)
\]

The beam position \(P(x,y)\) can be evaluated directly from the voltage measurements at the four feed-through ports by means of eq. 5. To do this, it is recommended to use a 180° hybrid junction, which provides immediately the difference and sum signals before the monitor signals are further processed by the electronic equipment. In this way it can be avoided that unequal signal attenuation or amplifier gain between the four monopole signals cause a zero error of the position measurement. The zero error of the 180° hybrid junction used for the wall current monitors at the SPS is \(\Delta \Sigma < -50 \, \text{dB}\) for equal input voltages \(U_1 = U_3\) or \(U_2 = U_4\). The subsequent electronic equipment can be calibrated very accurately for zero input voltage of the \(\Delta\)-channel corresponding to zero beam position. For this reason, the homodyne and super-heterodyne RF-receivers \(11\) provide the best precision for the beam position measurements in various accelerators.

**Longitudinal and Transverse Beam Impedance of Re-entrant Coaxial Cavity**

The beam induces a voltage across the gap of the re-entrant cavity, which acts back on the beam and deflects it in the longitudinal and transverse direction of the beam axis. The strength of this interaction between the beam monitor and a continuous bunched beam is expressed by the longitudinal and transverse beam impedance of the monitor. The longitudinal beam impedance \(Z_x\) at frequency \(f\) is defined by the decelerating voltage \(U_d\) experienced by the beam of intensity \(I\).

\[
Z_x(f) = -U_d(f)/I(f) \quad (34)
\]

For ultra-relativistic beams (\(\gamma > 10\)) and for a short gap of length \(g \ll \lambda = v/f\), where \(v\) is the beam velocity, the voltage integral along the beam path on the centre axis \(s\) is equal to the voltage \(U_{mg}\) of the monopole mode across the gap\(12\):

\[
U_g = \int E(s) \exp \left(j2\pi s/\lambda \right) ds = U_{mg} \quad (35)
\]

The longitudinal beam impedance \(Z_x\) is equal to the input impedance \(Z_m\) at the open end of the coaxial line (fig. 1), which is loaded by the impedance \(Z_3\) (eq. 21) at the junction with the four feed-throughs

\[
Z_3(f) = \frac{jZ_c \tan(2\pi f/c)}{1 + m_1(f)} \quad (36)
\]
The impedance $Z_3(f)$ at the junction is transformed by the short piece of coaxial line, which has an approximate characteristic impedance $Z_c \approx 60 \Omega \, \ln(D/d)$ and a length $h = a - l$ into the input impedance $Z_m$ of the re-entrant cavity by $^{57}$:

$$Z_m(f) = \frac{Z_c + jZ_c \tan(2\pi fh / c)}{1 + j(Z_3 / Z_c)\tan(2\pi fh / c)} \tag{37}$$

The impedance $Z_3(f)$ can be plotted by means of a Smith-Chart $^5$. For the SPS wall current monitor in fig. 4 ( $h = 34$ mm, $m_2 = 0.4$, $m_1 = 0.219 + j \cdot 0.547$), the longitudinal beam impedance amounts to $Z_3(f_0) = 2.9 \, \Omega + j \cdot 7.4 \, \Omega$ at the bunch repetition frequency $f_0 = 200 \, $MHz. The longitudinal beam impedance is larger than the transfer impedance because of the inductance of the feed-through launchers.

The longitudinal beam impedance can become excessively large under resonance conditions. In order to damp high order resonances at very high frequencies, an absorber ring of resistive ferrite material should be mounted in the coaxial cavity at its short-circuit end, where the magnetic field is highest at resonance.

The transverse beam impedance $Z_4$ is related to the voltage $U_{d8}$ of the dipole mode across the gap of a circular beam tube by $^{12,13}$:

$$Z_4(f) = \frac{U_{d8} \mu}{-I \times 2\pi \eta R} \tag{38}$$

The input impedance $Z_d$ for the dipole mode of the re-entrant cavity below the cutoff frequency is mainly an inductance. In order to calculate the impedance $Z_d$ of the dipole field across the gap of the beam tube, the characteristic impedance $Z_4$ of the coaxial line for the dipole mode $H_{11}$ and the impedance $Z_f$ of the dipole field at the feed-throughs must be calculated. The line impedance $Z_4$ for the evanescent mode $H_{11}$ is derived from the dipole field given in eq. 8 and 9 for $\phi_0 = 0$:

$$Z_4 = \frac{d^2}{\mu} \int_{-\pi/2}^{+\pi/2} E_r(r, \varphi) dr / \int_{-\pi/2}^{\pi/2} H_r(r, \varphi) r d\varphi = \frac{j \pi \eta l \ln(D/d)}{\alpha} \tag{39}$$

The integral of $H_\Phi$ is taken over the half circumference $-\pi / 2 \leq \phi \leq \pi / 2$ only, because the dipole mode has two symmetric half planes with opposite field polarity. The loads of the orthogonal feed-throughs at $\phi = \pm \pi / 2$ do not affect the dipole field, which has a maximum field strength at $\phi = 0$ and zero field at $\phi = \pm \pi / 2$.

A coaxial line with short-circuit at distance $l$ from the input has an input impedance of $Z_4 \tanh(\alpha l)$ below the cutoff-frequency of the dipole mode. At the feed-through at $\phi = 0$, the load resistance $Z_4 = Z_c + jX_4$ is connected in parallel with the coaxial line, and the impedance $Z_f$ at the feed-through becomes

$$Z_f = \frac{Z_4 Z_4 \tanh(\alpha l)}{Z_4 + Z_4 \tanh(\alpha l)} \tag{40}$$

The short piece of coaxial line of length $h$ between the feed-through and the gap transforms the impedance $Z_f$ into $Z_d$ at the gap.

$$Z_d = \frac{Z_4 + Z_4 \tanh(\alpha h)}{1 + (Z_4 / Z_4) \tanh(\alpha h)} \tag{41}$$
The voltage $U_{d_{e}}$ of the dipole field across the gap at azimuth $\phi = \phi_0 = 0$ is obtained from eq. 14 and 20 for the beam position $x = r_0$:

$$\overline{U}_{d_{e}} = \overline{Z}_{d_{e}} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} H_{\phi}(r, \phi) r d\phi = -2\overline{Z}_{d_{e}} I x / \pi R$$  (42)

The transverse beam impedance $Z_{\perp}$ defined in eq. 38 becomes for frequencies below the cutoff of the lowest dipole mode:

$$\overline{Z}_{\perp}(f) = \frac{\overline{Z}_{d}(f) c}{\pi^2 R^2 f}$$  (43)

For the case of the SPS wall current monitor at $f_0 = 200$ MHz, the line impedance is $\overline{Z}_{d}(f_0) = j 28.1 \, \Omega$, the coaxial dipole impedance over the gap is $\overline{Z}_{d}(f_0) = 2.1 \, \Omega + j 17.9 \, \Omega$. The transverse beam impedance of the SPS wall current monitor according to eq. 43 amounts to $\overline{Z}_{\perp}(f_0) = 17 \, \Omega/\text{m} + j 151 \, \Omega/\text{m}$.

The longitudinal and transverse beam impedance of a cavity are determined by the longitudinal electric field of the monopole and dipoles modes respectively. Since the monopole and dipole modes are related in different ways for different cavity shapes, there is no general relation between the longitudinal and transverse beam impedance. The impedance relation of Sacherer\textsuperscript{13)}

$$\overline{Z}_{\perp}(f) = \frac{\overline{Z}_{d}(f) c}{\pi R^2 f}$$  (44)

is often used as “estimate”, which is valid only for the resistive wall effect of straight circular beam pipes below the cut-off frequency of the beam pipe\textsuperscript{13}).

Conclusions

Due to the simple and precise construction of the circular re-entrant coaxial cavity, the transverse position of the beam can be measured with a very high precision. By scaling the dimensions of the SPS wall current monitor ($R = 134.5$ mm) to a smaller beam tube radius $R = 20$ mm for the CTF beam line, the re-entrant coaxial cavity would provide a beam position measurement with an accuracy of $< 80 \, \mu \text{m}$. Such a high precision is required for the next generation of linear accelerators in order to steer the beam exactly through the centre of the accelerating structures and to avoid the break-up of the beam by transverse RF-fields.

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References


4) F. Borgnis, Die konzentr. Leitung als Resonator, Zeitschrift f. Hochfrequenztechnik 56(1940) 47.


7) N. Marcuvitz, Waveguide Handbook, MIT Radiation Laboratory Series (1951) 77.


Appendix

Evanescent Dipole Mode in a Loaded Coaxial Cavity

It has been pointed out by G. Dôme that the dipole constant $L$ of the beam position monitor is influenced by the load impedance of the feed-throughs. In fact, the characteristic impedance of the coaxial line is different for the TEM monopole mode and for the $H_{11}$-dipole mode. Also, the inductance of the junction between the feed-through launcher (part 14 in Fig. 4) and the cavity is different for the monopole and the dipole mode.

The coaxial feed-through has a characteristic impedance of $Z_0 = 50 \, \Omega$ and is perfectly matched up to 3 GHz. The inductance of the launcher which connects the coaxial feed-through to the inner wall of the coaxial cavity can be evaluated from the inductance of a post in a waveguide below the cutoff frequency. The field of the dipole mode in a coaxial cavity with the dimensions $a >> b$ (Fig. 3) is practically identical to that in a rectangular waveguide with the cross-section $\pi r_m^2 \times b$ (Fig. 3)(14). The reactance $X$ of an inductive post of radius $r$ and length $b$ located in the middle of the waveguide of width $\pi r_m$ is given for a wavelength $\lambda_o$ longer than the cutoff wavelength $\lambda_c = 2 \pi r_m / \lambda_o$ by (14):

$$X = 377 \Omega \sqrt{\frac{\lambda_o^2}{\lambda_c^2} - 1} \frac{b}{\lambda_o} \left( \ln \frac{\pi r_m}{11.5r} + 0.2 \frac{\pi r_m^2}{\lambda_o^2} \right)$$  \hspace{1cm} (A1)

Since the feed-through launcher is mounted close to the open end of the re-entrant coaxial cavity, the reactance $X_4$ of the launcher is doubled with respect to the inductive post in an infinitely long waveguide: $X_4 = 2 \times X$.

For the evanescent dipole mode of the beam position monitor BPA in Fig. 4, the reactance of the feed-through launcher calculated according to Eq. A1 amounts to $X_4 \equiv 45 \, \Omega$ for $\lambda_o = 1.5 \, m$, $r_m = 155.5 \, mm$, $b = 27 \, mm$, $r = 2.5 \, mm$. This value is close to the output impedance of the monitor BPA measured at 200 MHz(9).

For the $H_{11}$-dipole mode, the coaxial line in region III is characterized by the line impedance $Z_\ell$ and by the attenuation constant $-\alpha$ for exponentially damped fields below the cutoff frequency (Eq. 6 - 11, $\phi_0 = 0$):

$$Z_\ell = \left| \frac{D/2}{\psi} \int_{-\pi/2}^{+\pi/2} E_r(r, \phi) dr \int_{-\pi/2}^{+\pi/2} H_\psi(r, \phi) r d\phi \right| \frac{j \psi \psi}{\psi} \ln(D/d)$$  \hspace{1cm} (A2)

$$\alpha = \frac{1}{r_m} \sqrt{1 - \left( \frac{2 \pi r_m}{\lambda_o} \right)^2}$$  \hspace{1cm} (A3)

The coaxial line consists of two anti-symmetric half-planes, which are loaded each by a feed-through with the load impedance $Z_4 = Z_0 + jX_4$. The other two orthogonal feed-throughs do not load the orthogonal dipole mode, because they are located in the symmetry plane of the dipole mode, where the transverse fields are zero.

At the junction of the coaxial line of region II with the feed-through and the coaxial line of region III, which is represented by the impedance $Z_5 = Z_\ell \tanh(\alpha \ell)$, the coaxial line II is loaded by the impedance $Z_\ell$:
\[
\bar{Z}_f = \frac{Z_4 \cdot Z_5}{Z_4 + Z_5} = \frac{Z_5}{1 + m_3} \tag{A4}
\]

\[
m_3 = \frac{Z_5}{Z_4} = \frac{Z_f \cdot \tanh(\alpha \ell)}{Z_0 (1 + j m_4)} , \quad m_4 = X_4 / Z_0 \tag{A5}
\]

At the gap of the cavity with the beam tube, the beam induces in the coaxial line the dipole field \( H_{\phi 1} \) (eq. 18, \( \phi_0 = 0 \)) and the dipole current \( I_g \)

\[
I_g = \int_{-\pi/2}^{+\pi/2} H_{\phi 1}(\phi) Rd\phi = \frac{-2I_o}{\pi R} \tag{A6}
\]

The dipole current \( I_g \) induces a voltage \( U_f \) at the other end of the coaxial line II at the junction with the feed-through:

\[
\bar{U}_f = \frac{I_g \cdot \bar{Z}_f}{\cosh(\alpha h) + (\bar{Z}_f / \bar{Z}_d) \sinh(\alpha h)} \tag{A7}
\]

The dipole voltage \( U_{df} \) across the feed-through load \( Z_0 = 50 \Omega \) is

\[
\bar{U}_{df} = \bar{U}_f \cdot Z_0 / (Z_0 + j X_4) = \bar{U}_f / (1 + j m_4) \tag{A8}
\]

Combining the equations A2 - A8, the dipole voltage \( U_{df} \) can be expressed by

\[
\bar{U}_{df} = \frac{-2I_o}{\pi R (1 + j m_4)} \cdot \frac{Z_f \tanh(\alpha \ell) / \cosh(\alpha h)}{1 + m_3 + \tanh(\alpha h) \tanh(\alpha \ell)} \tag{A9}
\]

The load factor \( p \) used in equations 19, 20, 27 and 28 is given by

\[
p = \frac{U_{df}(Z_0 = 50 \Omega)}{U_{df}(Z_0 = \infty)} = \frac{1}{1 + j m_4} \cdot \frac{1 + \tanh(\alpha h) \tanh(\alpha \ell)}{1 + m_3 + \tanh(\alpha h) \tanh(\alpha \ell)} \tag{A10}
\]

For the wall current monitor BPA (fig. 4) the dipole line impedance is \( Z_L = j 28.1 \Omega \) at \( f_0 = 200 \text{ MHz} \). The load factor \( p \) of the beam position monitor BPA (\( \ell = 0.155 \text{ m}, \ h = 0.034 \text{ m}, \ \alpha = 4.895 / \text{m}, m_4 = 0.9 \)) amounts to \( p = 0.632 \).
Fig. 1  Cross section of Reentrant Coaxial Cavity

Fig. 2  Polarisation Vector $\vec{E}(\phi_o)$ of Dipole Mode H11 in Beam Tube excited by Beam at Position $P(x,y)$. $x = r_o \cos \phi_o$, $y = r_o \sin \phi_o$.

Fig. 3  Coaxial Line terminated by short-circuit plane.