

# ENERGY MEASUREMENT BY RESONANT DEPOLARIZATION

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## Abstract

Resonant depolarization can be used to precisely measure the average beam energy of electron-positron storage rings. We discuss how the polarization of the beam is measured by Compton scattering of laser light. Results of transverse polarization measurements at LEP, and of longitudinal polarization at SLAC are presented. Calibration of the LEP energy by this technique has a precision of order  $10^{-5}$ ; fluctuations observed at the level of  $\pm 5 \times 10^{-5}$  are attributed to the distortion of the LEP lattice by tidal effects. We conclude with a brief mention of the connection between transverse polarization and Hawking-Unruh-Davies radiation.

## 1 POLARIZATION IN STORAGE RINGS AND SPIN PRECESSION

It is known that in storage rings electrons or positrons tend to become transversely polarized. This effect first proposed by Ternov, Loskutov and Korovina [1] is due to spin flip during the emission of synchrotron radiation favoring the spin orientation of lowest energy in the external field. In a seminal paper Sokolov and Ternov [2] showed that the polarization of the beam is given by

$$P_e = P_o(t - e^{-t/\tau_0}) \quad (1)$$

where the maximum (asymptotic) polarization is

$$P_o = \frac{8}{5\sqrt{3}} = 0.9238 \quad (2)$$

and the polarization time (constant) is

$$\tau_0 = \frac{8}{5\sqrt{3}} \frac{1}{\alpha} \left( \frac{mc^2}{\hbar c} \right)^2 \frac{\rho^3}{c\gamma^5} \left( \frac{R}{\rho} \right) \quad (3)$$

In Eq.(3)  $\rho$  is the bending radius and  $R$  the circumference radius of the ring,  $\gamma = E_e/mc^2$  and  $\alpha = 1/137$  is the fine structure constant. The probability for spin-flip is relatively small so that  $\tau_0$  is long. For instance for LEP  $\tau_0 \sim 340$  minutes.

The observed polarization is in general less than  $P_o$  due to depolarizing effects. These arise because of the precession of the spin about non-vertical components of the magnetic field and are discussed in detail in Dr. Koutchouk's lecture [3]. The precession of the spin of a relativistic particle was given by Bargman, Michel and Telegdi [4].

$$\frac{d\vec{s}}{dt} = -\frac{e}{\gamma m} \left[ \left( 1 + \gamma \frac{(g-2)}{2} \right) \vec{B}_\perp + \frac{g}{2} \vec{B}_\parallel \right] \times \vec{S} \quad (4)$$

Thus in an ideal storage ring ( $B_{\parallel} = 0$ ) the spin precession frequency  $\omega_s$  is related to the rotation frequency  $\omega_c = eB/\gamma m$  by

$$\omega_s = \omega_c \left[ 1 + \gamma \frac{(g-2)}{2} \right] \quad (5)$$

the anomalous part of the electron's magnetic moment is known to great accuracy [5]

$$(g-2)/2 = 1.1596521884(43) \times 10^{-3} \quad (6)$$

We note that at LEP where  $\gamma \sim 10^5$  the spin precesses much faster than the momentum vector, making approximately a hundred turns in one revolution around the storage ring.

Baier and Orlov [6] pointed out that even a small radial magnetic field (see Fig. 1) can lead to depolarization if the ratio of the spin precession frequency to the rotation frequency  $\omega_s/\omega_c$  is an integer. It is customary to define the spin tune

$$\nu_s = \gamma \frac{(g-2)}{2} = \frac{\omega_s}{\omega_c} - 1 \quad (7)$$

For LEP at the  $Z^0$  mass ( $E = 45.6$  GeV)  $\gamma = 8.924 \times 10^4$  and  $\nu_s = 103.48$ . Evidently if one can measure  $\nu_s$ , one can obtain the energy through Eq.(7).

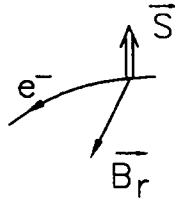


Figure 1. Sketch of radial field component and of a transverse spin orientation.

Depolarization will occur even if  $\nu_s$  is not an integer, provided we add a weak depolarizing field at frequency  $\omega_D$ , so as to satisfy the resonance condition

$$n_s \nu_s \pm n_x Q_x \pm n_y Q_y \pm n_z Q_z \pm (\omega_D/\omega_c) = n \quad (8)$$

Here  $Q_x, Q_y$  are the betatron tunes and  $Q_z$  the synchrotron tune; the  $n$ 's are arbitrary integers. It is desirable to work in the lowest mode namely  $n_s = 1, n_x = n_y = n_z = 0$  so that

$$\nu_s \pm (\omega_D/\omega_c) = n \quad (9)$$

For instance referring to the previous example at LEP where  $\nu_s = 103.48$ , we have

$$f_D = f_c \delta \nu_s \sim 5 \text{ kHz} \quad (10)$$

As will be seen later, the depolarizing resonance is narrow so that  $f_D$  can be measured to good accuracy. In view of the large value of  $\nu_s \sim 100$ , the accuracy in  $\nu_s$  and thus in the machine energy is 100 times better than the accuracy in  $f_D$ .

## 2 COMPTON POLARIMETRY

Polarization of electron/positron beams in storage rings was first reported by LeDuff *et al* [7] who measured the spin dependence of Touschek scattering; this is intrabeam electron-electron scattering. Other groups used the asymmetry in  $e^+e^- \rightarrow \mu^+\mu^-$  [8] to deduce the polarization of SPEAR. Another method for measuring the polarization of high energy electron beams is Möller scattering from thin foils of magnetized iron. However the most generally used technique is Compton scattering of laser photons from the stored electron or positron beam [9,10,11]; first applied at SPEAR in 1979 [12].

Figure 2 shows the kinematics of photon ( $\omega_0 \sim 2.3$  eV) scattering from a high energy electron ( $E = \gamma/m$ ). In the laboratory frame the photon is backscattered, within a cone of angle  $\theta \sim 1/\gamma$  as a high energy gamma with energy of order  $\omega \sim 4\gamma^2\omega_0$ . It is convenient to view the scattering in the rest-frame of the electron where the photon energy is  $\bar{\omega}_0 = 2\gamma\omega_0$ . Thus the energy of the scattered photon is given by the Compton condition

$$\frac{1}{\bar{k}} - \frac{1}{\bar{k}_0} = \frac{1}{m}(1 - \cos \bar{\theta}) \quad (11)$$

where we used the wave-vector  $\bar{\mathbf{k}}_0, \bar{\mathbf{k}}$  of the photon in the electron rest-frame before and after the scattering;  $\bar{\theta}, \phi$  are the polar and azimuthal angles of the scattered photon (with respect to its original direction). In the laboratory frame the energy of the recoil electron is given by

$$\omega = \frac{4\gamma^2\omega_0 \sin^2(\bar{\theta}/2)}{1 + (2\gamma\omega_0/m)(1 - \cos \bar{\theta})} \quad (12)$$

$$= \frac{4\gamma^2\omega_0}{1 + 2\gamma^2(1 - \cos \theta) + (2\gamma\omega_0/m)(1 + \cos \theta)} \quad (13)$$

where Eq.(12) is in terms of the scattering angle  $\bar{\theta}$  in the electron rest frame, whereas in Eq. (13),  $\omega$  is expressed by the laboratory (back) scattering angle  $\theta$  measured from the electron's direction. Thus  $\bar{\theta} = \pi$  corresponds to  $\theta = 0$ !. Eqs.(12,13) are written in the limit  $\beta \rightarrow 1$ . For LEP or SLAC at the  $Z^0$  mass where  $\gamma = 9 \times 10^4$  using  $\omega_0 = 2.3$  eV ( $\lambda = 537$  nm) gives  $\omega_{\max} = 28.4$  GeV.

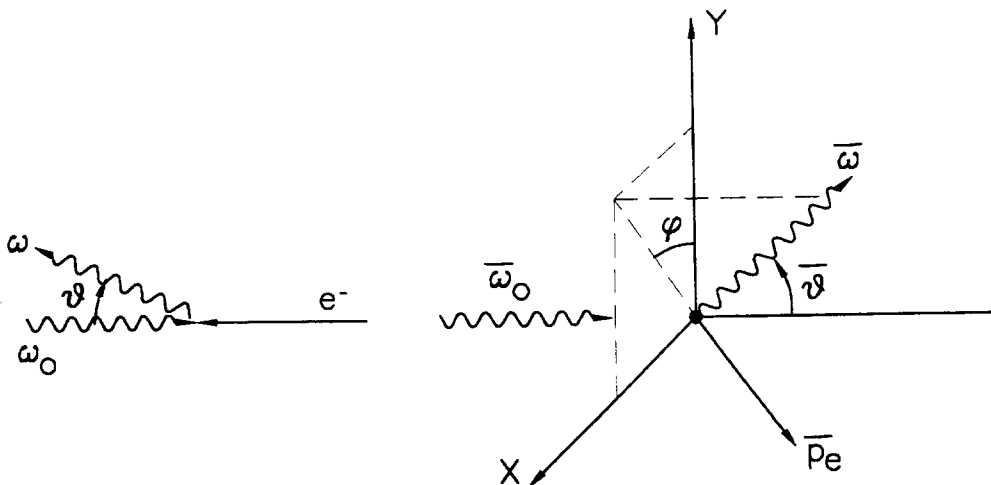


Figure 2. Photon-electron scattering (a) in the laboratory frame, (b) in the electron rest frame.

The spin-dependent Compton cross section for polarized incident light can be found in different forms. We follow Lipps and Tolhoek [13] and give the differential cross-section in the electron rest-frame.

$$\frac{d\sigma}{d\bar{\Omega}} = \frac{R_0^2}{2} \left( \frac{\bar{\omega}}{\bar{\omega}_0} \right)^2 \left[ (1 + \cos^2 \bar{\theta}) + (\bar{\omega}_0 - \bar{\omega}) \frac{1}{m} (1 - \cos \bar{\theta}) \right. \\ \left. + (\xi_1 \cos 2\phi + \xi_2 \sin 2\phi) \sin^2 \bar{\theta} - \xi_3 (1 - \cos \bar{\theta}) \frac{1}{m} \mathbf{S} \cdot (\bar{\mathbf{k}}_0 \cos \bar{\theta} + \bar{\mathbf{k}}) \right] \quad (14)$$

Here  $r_0 = e^2/mc^2 = 2.82 \times 10^{-13}$  cm is the classical electron radius and  $\mathbf{S}$  is the electron spin.  $\xi_1, \xi_2, \xi_3$  characterize the polarization of the incident photons and are analogous to the Stokes parameters.

$\xi_1 = \pm 1$	Linear polarization along the $Y, X$ axis
$\xi_2 = \pm 1$	Linear polarization at $45^\circ$ to the $Y, X$ axis
$\xi_3 = \pm 1$	Circular right, left, polarization

Note for instance that if  $\xi_1$  or  $\xi_2$  are different from zero, an azimuthal asymmetry results even in the absence of electron polarization.

Measurements of the transverse polarization at LEP have been reported [14,15]. In this case  $\mathbf{S} \cdot \bar{\mathbf{k}}_0$  is zero and since circularly polarized light is used ( $\xi_1 = \xi_2 = 0$ ) the spin-dependent part of the cross-section is

$$\xi_3 P_e \frac{\bar{k}}{m} (1 - \cos \bar{\theta}) \sin \bar{\theta} \cos \phi \quad (15)$$

where  $P_e$  is the magnitude of the electron polarization along the  $Y$  axis (see Fig. (2b));  $P_e = (n_+ - n_-)/(n_+ + n_-)$ . When  $\cos \phi > 0$  the scattering is "up", when  $\cos \phi < 0$  "down" with respect to the plane of the ring as sketched in Fig. 3. Thus it is convenient to use a (strip) detector which measures the scattered photons as a function of the  $y$ -coordinate (integrating over  $x$ ). At LEP the detector is placed at a distance of 275 m from the laser interaction point so that the spin independent cross-section is of the form shown in Fig. 4a. The spin dependent part is shown in (b) of the figure, for 100% electron and photon polarization. As  $\xi_3$  is alternated in sign the centroid of the scattered photons is shifted in  $y$  by an amount  $\langle y \rangle = \kappa \xi_3 P_e$ ; for the LEP polarimeter the analyzing power  $\kappa \simeq 500 \mu\text{m}$ . For  $P_e = 0.1$  a shift of  $50 \mu\text{m}$  must be measured; this requires that a flat orbit be established in the machine.

Results from the LEP measurements [15] are shown in Fig. 5. In (a) of the figure is shown the vertical distribution of the back-scattered photons and in (b) of the figure, the asymmetry as a function of  $y$ ,

$$A(y) = \frac{n_+(y) - n_-(y)}{n_+(y) + n_-(y)} \quad (16)$$

where  $n_{\pm}(y)$  is the number of counts at a given  $y$  for  $\xi_3 = \pm 1$ . The data correspond to a polarization  $P_e \sim 0.1$ .

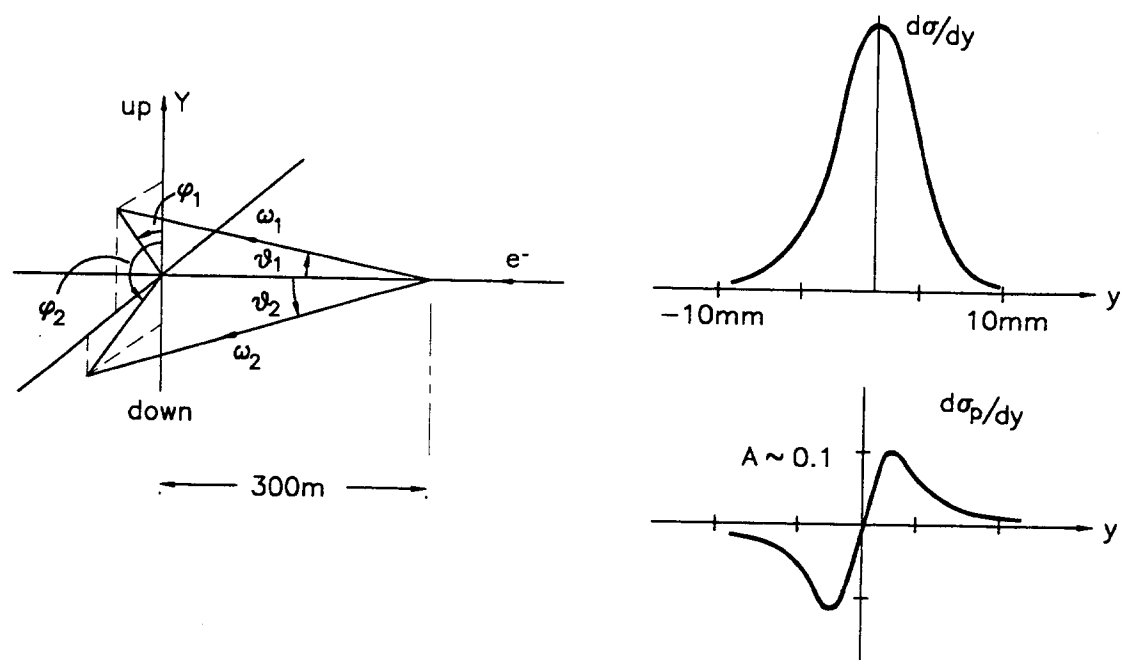


Figure 3. Geometrical arrangement for the detection of backscattered photons. Figure 4. The differential cross-section for the backscattered photons as a function of vertical distance from the median plane. (a) The polarization independent term. (b) The asymmetry for 100% transverse electron polarization and 100% photon circular polarization.

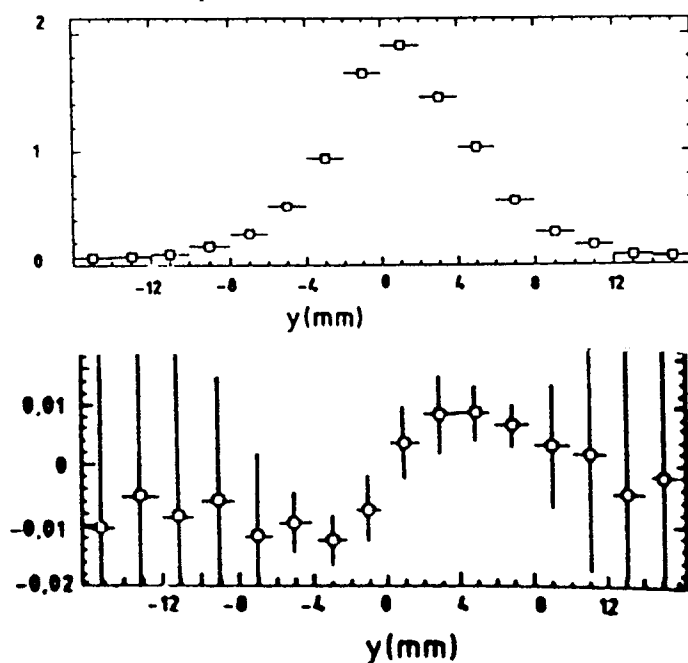


Figure 5. Results from the LEP polarization measurements [Ref. (15)]. (a) Vertical distribution at the detector. (b) Asymmetry as a function of vertical distance.

We can estimate the statistical accuracy of the measurement from the expected counting rate. Typically a NdYAG laser is used which operates in the infrared ( $\lambda = 1054$  nm) and is frequency doubled to the the green ( $\lambda = 532$ ). The laser is Q-switched to deliver 50 mJ in a  $\Delta t_\omega = 12$  ns long pulse at a repetition rate of 30 Hz; the focal spot has a diameter  $d_\omega = 2$  mm and the crossing angle is  $\alpha = 3^\circ$ . We take the electron beam parameters as  $\sigma_h = \sigma_v = 1$  mm with  $10^{11}$  electrons/bunch in a  $\Delta t_e = 10$  mm long bunch, to obtain

$$N_S = \sigma_C \frac{N_\omega N_e}{\pi(d_\omega/2)^2} \frac{d_\omega}{\sin \alpha} \frac{1}{c \Delta t_\omega} \sim 1.4 \times 10^3 / \text{pulse} \quad (17)$$

where we used  $\sigma_C = 3 \times 10^{-25}$  cm<sup>2</sup>. The error in the asymmetry at each value of  $y$  is  $\delta A(y) = 1/\sqrt{n_s}$  with  $n_s$  the counts in each bin. One can reach  $\delta A(y) \sim 0.01$  in 3 seconds. Thus for a polarization  $P_e = 0.1$  corresponding to a peak asymmetry  $A = 0.01$ , a 10% measurement of the polarization can be obtained in 5 minutes.

The case of longitudinal polarization is of interest at the SLC in which case the spin-dependent term (see Eq. 14) is

$$\xi_3 P_e (1 - \cos \bar{\theta}) \cos \bar{\theta} (\bar{k}_0 + \bar{k}) \quad (18)$$

which becomes significant with respect to the spin-independent contribution, especially at  $\bar{\theta} = 180^\circ$ . Thus it is possible to measure the total yield of  $\gamma$ 's as a function of  $\xi_3$  or to measure the differential cross section. The latter method is used at SLC where the asymmetry is recorded as a function of the energy of the recoil electrons. The calculated spectrum for 100% polarization and the results obtained at SLC in 1992 [16] are shown in Fig. 6; the data indicate a longitudinal polarization  $P_e = 0.22$ .

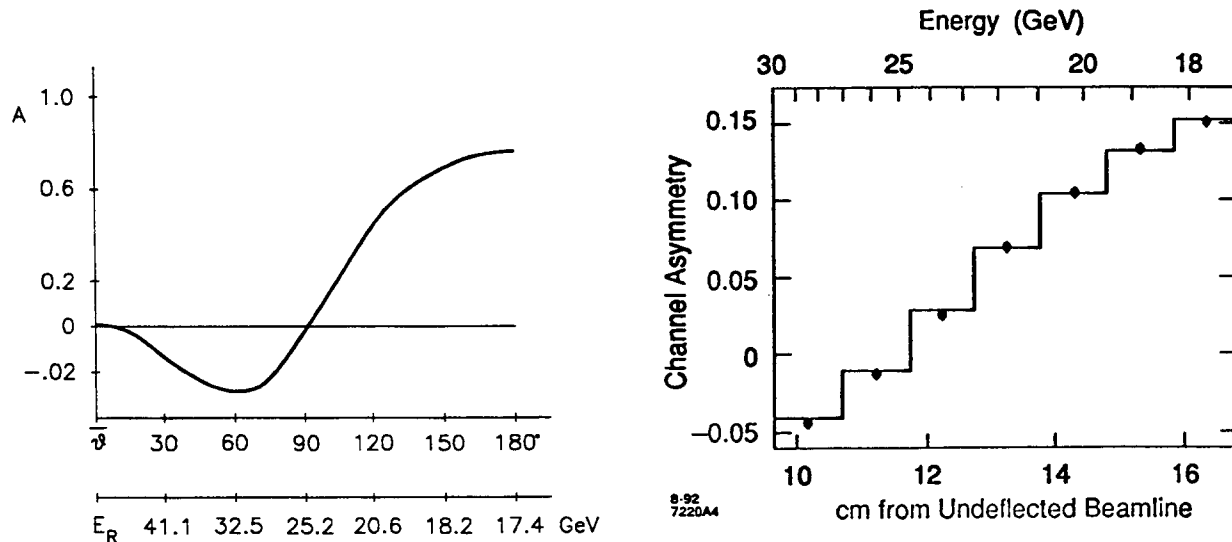


Figure 6. (a) Predicted asymmetry for 100% longitudinal electron polarization and 100% photon circular polarization as a function of recoil electron energy. (b) 1992 data from SLAC [Ref. (16)] corresponding to  $P_e \simeq 22\%$

Recently, use of strained GaAs cathodes at SLC has yielded a polarization in excess of 60%. The polarization at the collision point is continuously monitored, as shown

in Fig. 7 [16]. This is because  $P_e$  enters directly in the determination of the left-right asymmetry in  $Z^0$  production

$$A_{LR} = \frac{1}{P_e} \frac{N_L - N_R}{N_L + N_R} \quad (19)$$

where  $N_L(N_R)$  is the flux normalized count of  $Z^0$ 's produced with left(right)-handed electrons. Since

$$A_{LR} = \frac{2(1 - 4 \sin^2 \theta_W)}{1 + [1 - 4 \sin^2 \theta_W]} \quad (20)$$

the error in  $\sin^2 \theta_W$  is  $\delta(\sin^2 \theta_W) = (1/8)\delta A_{LR}$ . The asymmetry error  $\delta A_{LR}$  is found from

$$\delta A_{LR} = \left[ \frac{1}{N} \frac{1}{P_e^2} + A_{LR}^2 \left( \frac{\delta P_e}{P_e} \right)^2 \right]^{1/2} \quad (21)$$

With  $N = N_L + N_R = 5 \times 10^4$ ,  $P_e = 0.65$ ,  $\delta P_e = 0.02$  we find  $\delta A_{LR} = 0.008$  and  $\delta(\sin^2 \theta_W) \simeq 10^{-3}$ . This compares well with the error on  $\delta(\sin^2 \theta_W)$  obtained at LEP.

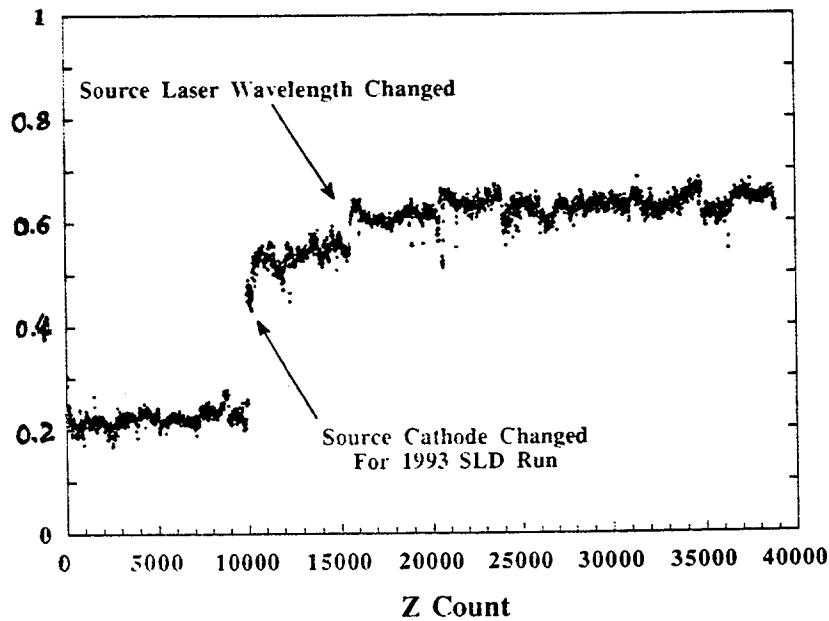


Figure 7. Electron beam polarization for the 1992 and 1993 SLAC runs (Ref. [16]).

### 3 MEASUREMENT OF THE LEP ENERGY

Energy measurements at LEP made in late 1991 have been reported by Arandon *et al* [17]. The nominal energy was  $E = 46.5$  GeV and thus the spin tune was near  $\nu_s = 105.455$ . To depolarize the beam a vertical kicker (radial magnetic field) with field integral  $\int B dl = 3.7$  Gauss-m was used. Note that

$$\nu_s (\int B dl)_{\text{depolarizing}} / (\int B dl)_{\text{guide}} \sim 10^{-4} \quad (22)$$

so that depolarization will occur only after many orbits. Consequently the depolarizing frequency correctly measures the average beam energy. The probability of spin flip by a

depolarizing field was first calculated by Froissart and Stora [18], who give

$$\frac{P(\text{final})}{P(\text{initial})} = 2e^{-\chi} - 1 \quad (23)$$

with

$$\chi = \frac{[\pi\nu_s(\int Bdl)_D/(\int Bdl)_G]^2}{(d\nu_s/dt)} f_0 \quad (24)$$

Here  $f_0(d\nu_s/dt)$  is the rate at which the depolarizing frequency is swept. Clearly for  $\chi \ll 1$  no depolarization is achieved, whereas for  $\chi \gg 1$  one can expect a reversal of the polarization. Using the value of the depolarizing field given in Eq. (22) we find that the sweep rate must be kept in the range of  $d\nu_s/dt \sim 10^{-4}$  Hz/s.

Figure 8 shows that a 9% polarization was established and that it could be destroyed by a static resonance, (introducing a spin bump such that  $\nu_s = 106$ ). When the bump is removed the beam polarizes again and tends to its asymptotic level  $P(t \rightarrow \infty)$  exponentially. The data are fit by using Eq.(1) but with  $P, \tau$  given by

$$P(t \rightarrow \infty) = \frac{P_0}{1 + \tau_0/\tau_D} \quad \tau = \frac{\tau_0}{1 + \tau_0/\tau_D} \quad (25)$$

where  $P_0$  and  $\tau_0$  are the Sokolov-Ternov values given by Eqs. (2,3).

Next the depolarizing kicker was swept and a series of depolarizing resonances are shown in Fig. 9. Note that spin-flip is observed in certain cases. The details of a particular run are shown in Fig. 10. It is important to note that depolarization is recorded within a time interval of  $\sim 16$  s. Given the rate at which the kicker was swept this corresponds to an uncertainty in the beam energy  $\Delta E = 1$  MeV.

The relation between the beam energy and the spin tune was given by Eq. (7); numerically we can write

$$\nu_s = \frac{(g-2)}{2} \frac{E}{mc^2} = \frac{E(\text{GeV})}{0.4406486(1)} \quad (26)$$

The beam energy is continuously monitored from a knowledge of the field of the bending magnets and the r.f. frequency. The best estimate so obtained is referred to as  $E_{FD}$  (for field display). The results of the resonant depolarization measurements established that

$$E_0 - E_{FD} = (-37.1 \pm 1.4)\text{MeV} \quad (27)$$

making an important improvement in the absolute value with which the beam energy is known.

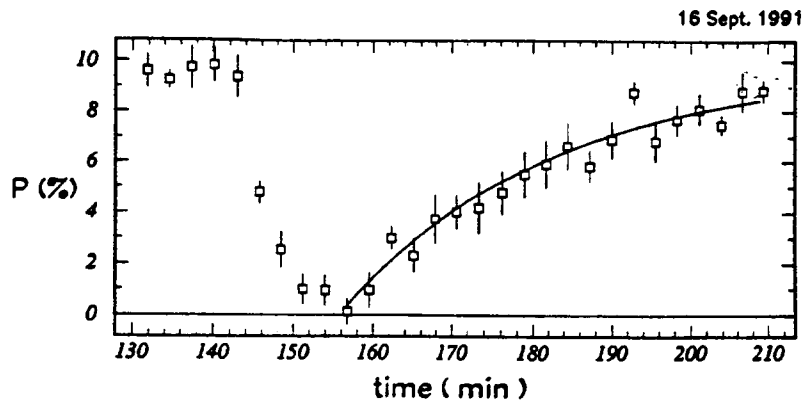


Figure 8. Depolarization of the LEP beam by the introduction of a "spin bump". Note the polarization rise when the bump is removed (Ref. [17]).

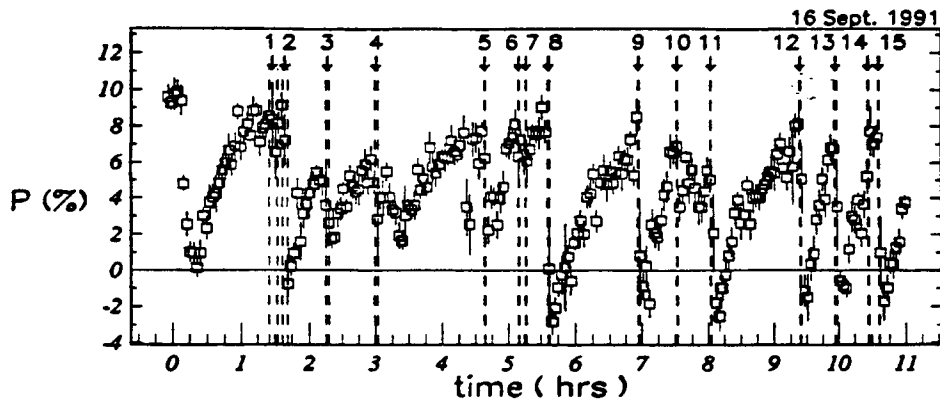


Figure 9. Sequence of resonant depolarizations at LEP. Note that occasionally the polarization can be reversed (Ref. [17]).

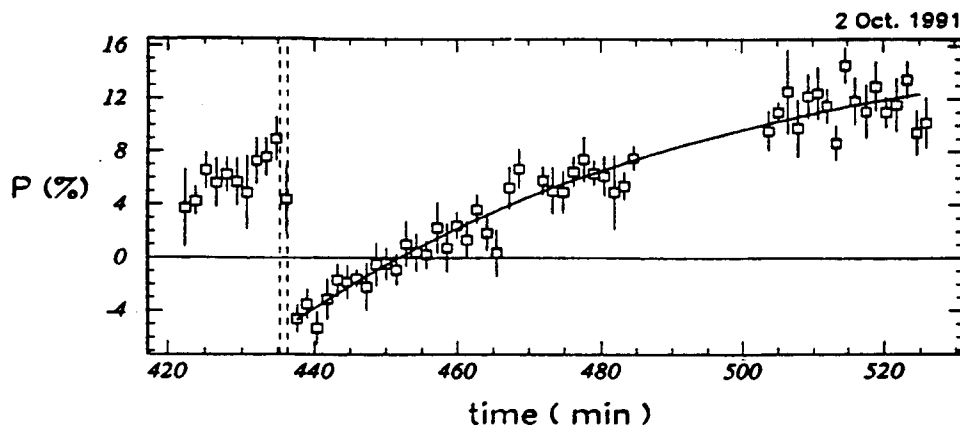


Figure 10. Detailed plot of resonant depolarization at LEP (Ref. [17]).

#### 4 THE EFFECT OF THE TIDES

In reference [17] the authors state that the precision of the energy measurements made at LEP was  $\pm 1.5 \times 10^{-5}$  but that the variability of the results was  $\pm 6 \times 10^{-5}$  "consistent with the expected stability and reproducibility of the machine". The observed energy spread was almost 10 MeV. Only two effects could cause such an energy change: a change in the magnetic field, which was excluded because of the relative stability of the excitation current, or a change in the orbit radius.

The required change in orbit radius is very small because of the "momentum compaction",  $(1/\alpha)$ , in strong focussing machines such as LEP

$$\frac{\Delta E}{E} = \left(\frac{1}{\alpha}\right) \frac{\Delta R}{R} \quad \alpha \sim \frac{1}{Q^2} \quad (28)$$

Here  $Q$  is the betatron tune which for LEP is of order  $Q = 70$ , so that  $(1/\alpha) \sim 5 \times 10^3$ . Thus if we take  $\Delta E/E = 2 \times 10^{-4}$  we find  $\Delta R/R = 4 \times 10^{-8}$  or  $\Delta R = 0.15 \text{ mm}$ ! However for a fixed lattice, changes in the mean radius are directly reflected in the frequency of the r.f.

$$\Delta f/f = \Delta R/R \quad (29)$$

since  $f = 352 \text{ MHz}$  at LEP,  $\Delta R/R = 4 \times 10^{-8}$  would imply a frequency shift  $\Delta f = 14 \text{ Hz}$ . Such a shift is at least an order of magnitude greater than the observed stability of the r.f. One concludes that the observed energy fluctuations in LEP cannot be explained by magnet current or r.f. instabilities.

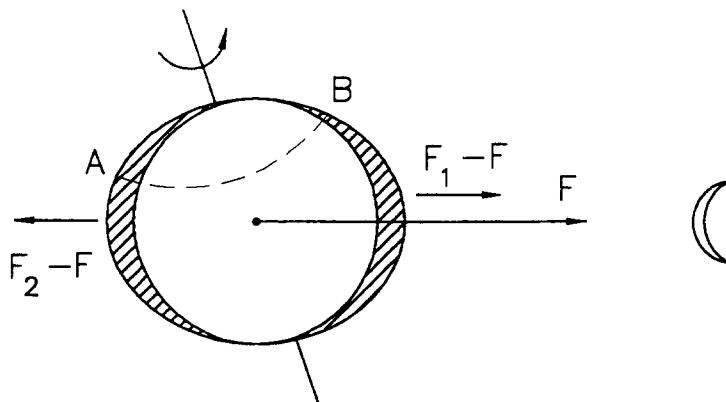


Figure 11. Illustration of the tidal forces acting on the earth due to the moon.

The answer to this puzzle was provided by G. Fisher [19] who suggested that the machine lattice is being deformed by tidal effects. To illustrate that this is possible we calculate the relative change in local gravity,  $g$ , due to the attraction of the moon as shown in Fig. 11. The tidal acceleration is given by

$$\frac{\Delta F}{m} = \frac{1}{2m}(F_1 - F_2) = G \frac{M_{\text{m}} R_{\oplus}}{R_{\oplus}^2 \epsilon R_{\oplus} \epsilon} \quad (30)$$

or

$$\frac{\Delta g}{g} = \frac{M_{\text{m}}}{M_{\oplus}} \left(\frac{R_{\oplus}}{R_{\oplus} \epsilon}\right)^3 \sim 5.7 \times 10^{-8} \quad (31)$$

It is reasonable to assume that the relative change in the effective radius of the orbit is of order  $\Delta g/g$  which reproduces the observed energy variability. Fig. 12 shows the results of a careful measurement of the LEP energy over a 26 hour period [20]. The curve gives the energy change expected from the tidal effects of the sun and moon, based on the above arguments. Further lattice distortions due to ground motion are also observed.

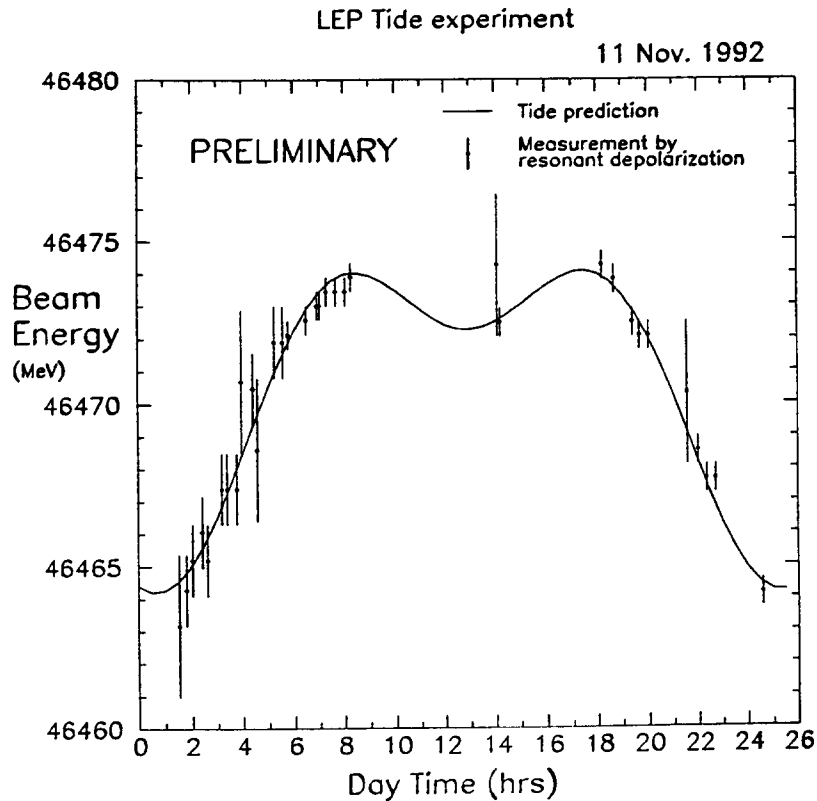


Figure 12. Data from the measurement of the LEP energy over a 26 hour period. The solid curve is the prediction of the tidal distortion of the machine lattice (Ref. [20]).

## 5 HAWKING-DAVIES-UNRUH RADIATION

In this section we review the possible manifestation of Hawking radiation [21] in the limiting value of the polarization in an electron storage ring. This relationship was first pointed out in a classic paper by Bell and Leinaas [22].

Hawking proved that the fluctuations of the vacuum at the horizon of a black hole result in the emission of electromagnetic radiation with a Planck spectrum at an equivalent temperature

$$kT = \frac{\hbar c}{2\pi} \frac{1}{(4GM/c^2)} \quad (32)$$

where  $M$  is the mass of the black hole. We can illustrate this result by considerations based on the uncertainty principle as shown in Fig. 13. We assume that a virtual pair of photons is produced at a distance  $\epsilon$  from the horizon whose radius is  $r_s = 2GM/c^2$ . If one of the photons enters the horizon of the black hole it can not any more communicate

with the other photon which must therefore become real. Let  $\Delta\tau$  and  $\Delta\varepsilon$  be the lifetime and energy of the fluctuation in its own rest frame, so that

$$\Delta\varepsilon\Delta\tau = \hbar \quad (33)$$

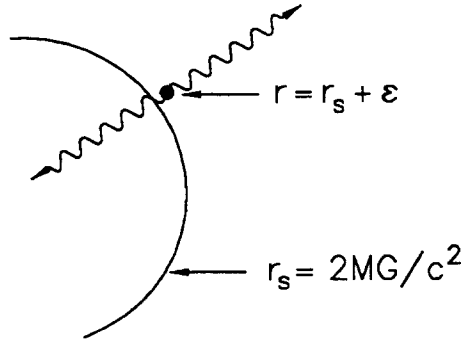


Figure 13. Sketch of virtual photon pair production near the horizon of a black hole.

Because of the strong gravitational field the time  $\Delta\tau$  needed for the photon to traverse the distance  $\varepsilon$  to the horizon, is

$$\Delta\tau = (2/c)\sqrt{\varepsilon(2GM/c^2)} \quad (34)$$

whereas the energy of the other photon when at a large distance from the horizon, is

$$E = \Delta\varepsilon\sqrt{\varepsilon(2GM/c^2)} \quad (35)$$

Combining Eqs. (33-35) we obtain  $E = \hbar c/(4GM/c^2)$ . If we identify  $E/2\pi$  with the mean energy  $kT$  of the Planck spectrum, we reproduce Eq. (32).

The acceleration at the horizon of a black hole is  $a_s = GM/r_s^2 = c^4/(4GM)$ , and this establishes an accelerated frame equivalent to the gravitational field at the horizon. On the basis of this equivalence Davies [23] and Unruh [23] argued that a frame of reference subject to a linear acceleration  $a$ , should be considered as a thermal bath at temperature  $T$ , with

$$kT = \frac{\hbar a}{2\pi c} \quad (36)$$

These ideas remain conjectural and have not been verified by experiment, because the emitted radiation is very weak. For instance the lifetime due to Hawking radiation, of a black hole of one solar mass is  $\tau \sim 10^{54}$  years. Similarly, in the laboratory it is difficult to produce accelerated systems with significant temperature according to Eq. (36). There has however been a considerable theoretical interest in the subject [24,25].

Bell and Leinaas [22] pointed out that because of relativistic effects, the acceleration to which electrons are subject in a storage ring is significant, to the extent that the

influence of the thermal bath of Eq. (36) may be observable [26]. In the rest-frame of the electron

$$a = \frac{\gamma^2 c^2}{R} \quad \text{or} \quad T = \frac{\hbar c}{2\pi k} \frac{\gamma^2}{R} \quad (37)$$

where  $R$  is the orbit radius. For LEP the equivalent temperature is  $T \sim 1200$  K. Considering the electron spins in equilibrium with the thermal bath, the ratio of the population of the two spin states will be given by the Boltzman distribution

$$\rho_+/\rho_- = \exp(-\mu B'/kT) \quad (38)$$

where  $B' = \gamma B$  is the magnetic field in the electron rest-frame. For the magnetic moment  $\mu$  we write  $\mu = g\mu_B = g(e\hbar/2m)$  and note that in Eq. (38) we have assumed a spin-1/2 system. Furthermore  $R = \gamma mc/eB$ , so that the Boltzman exponent reduces to

$$\frac{\mu B'}{kT} = \left( g \frac{e\hbar}{2m} \gamma B \right) \left( \frac{\gamma mc}{eB} \frac{1}{\gamma^2} \frac{2\pi}{\hbar c} \right) = \pi g \quad (39)$$

The above result is remarkable in that it is independent of the electron energy or of the ring parameters. Thus for all storage rings the population ratio  $\rho_+/\rho_-$  should be the same. The beam polarization is a measure of the population ratio,

$$P = \frac{\rho_- - \rho_+}{\rho_- + \rho_+} = \frac{1 - e^{-\pi g}}{1 + e^{-\pi g}} \quad (40)$$

If we use  $g = 1$  in Eq. (40) we find  $P = 0.9174$ . This is to be compared with the maximal polarization  $P_0 = 0.924$  given by Eq. (2). Therefore, the asymptotic polarization in electron storage rings derived by considering the quantum-mechanical spin-flip transitions may be the first tangible manifestation of the Hawking-Davies-Unruh effect. The reader will wonder why we used  $g = 1$  instead of the known value of  $g = 2$  for the electron; this is related to the Thomas precession in the circular orbit. For this point and for estimates of the time constant of the build-up of the polarization one should consult the original paper [22].

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**Plate 1** Nicholas C. Christofilos in 1960. At that time he was group leader in controlled thermonuclear reactions at Lawrence Livermore National Laboratory. Courtesy LLNL.