Mott Scattering as a Probe of Long Range QCD

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\textbf{ABSTRACT}

We investigate the possibility of using the Mott scattering between identical nuclei to assess the existence of long range QCD, e.g., a color Van der Waals interaction, as suggested recently. Among other effects which were not considered before, the tail of the nuclear potential, emission of radiation by Bremsstrahlung, atomic screening, emission of delta-electrons, and the quasi-molecule binding are included in our calculations. We show that the sum of these effects can explain the observed shifts in the Mott oscillations in a recent experiment.
Since the Coulomb force between heavy ions is very well known, Rutherford scattering can be used as a probe of long range small interactions between the nuclei. Rutherford backscattering has been used, e.g., by Brügger et al. [1] to search for supermassive matter with strange nuggets. The method is more efficient if one uses the scattering of identical nuclei, say $^{208}\text{Pb} + ^{208}\text{Pb}$. Then, due to the interference between the scattering amplitudes for forward and backward angles, the effects of small corrections are seen as shifts in the Mott oscillations. This method has been used successfully by Vetterli et al. [2] to test the effect of vacuum polarization on sub-Coulomb scattering. With this method the effect could be identified on a 7% level.

Recently, it has been proposed the use of sub-Coulomb scattering as a probe for a hypothetical color van der Waals (CVDW) force between hadrons [3]. This suggestion has been investigated experimentally in GANIL [4]. One of the experimental difficulties is to produce a completely stripped ion beam for the experiment. As we show in this article, the presence of the atomic electrons makes it very difficult, if not impossible, to test the existence of a CVDW force. The data were collected for the system $^{208}\text{Pb} + ^{208}\text{Pb}$ at $E_{\text{lab}} = 873$ MeV and 1130 MeV. The main interaction at these energies is the Coulomb force. The other interactions can be taken as small perturbations:

$$V(r) = \frac{Z_1 Z_2 e^2}{r} + \Delta V(r).$$ (1)

Among the small effects contributing to $\Delta V(r)$ we consider (a) relativistic corrections, (b) nuclear dipole polarizability, (c) nuclear quadrupole polarizability, (d) vacuum polarization, (e) color van der Waals interaction, (f) tail of the nuclear interaction, (g) atomic screening, (h) Bremsstrahlung, (i) emission of delta-electrons, and (j) the formation of a quasi-molecule. The corrections (a)-(e) were considered in ref. [3]. In this article we show that the other corrections are important, especially the last one. The experiment of ref. [4] can be reasonably well described without need to resort to a CVDW interaction.

The main idea in ref. [3] is that a color van der Waals interaction of the form
given by [5] (we use $\hbar = c = 1$)

$$V_{CVDW} = -(\alpha_6/r_0) (r_0/r)^6, \quad \text{or} \quad V_{CVDW} = -(\alpha_7/r_0) (r_0/r)^7, \quad (2)$$

results in sizeable shifts in the Mott oscillations. This is because the estimates of the color van der Waals force due to multigluon exchange [5] yield astonishing large values $\alpha_6 \simeq 1 \text{ fm}$ and $\alpha_7 \simeq 100 \text{ fm}$ (with $r_0 = 1 \text{ fm}$), respectively. Especially in the latter case, the shifts in the Mott oscillations are appreciable. The existence of such a long range QCD interaction has also been strongly criticised theoretically [6]. However, an experimental test of the theoretical predictions is worth to be done [4].

The Mott symmetrized cross section including the potentials $\Delta V(r)$ is given by [3]

$$\frac{d\sigma}{d\Omega} = \frac{d\sigma_d}{d\Omega}(\theta) + \frac{d\sigma_d}{d\Omega}(\pi - \theta) + 2 \left[ \frac{d\sigma_d}{d\Omega}(\theta) \frac{d\sigma_d}{d\Omega}(\pi - \theta) \right]^{1/2} \cos \left\{ 2[(\delta(\theta) - \delta(\pi - \theta))] \right\} \quad (3)$$

where the phase shifts $\delta$ are calculated with the WKBJ approximation. At these energies and for heavy systems it has been shown [7] that this approximation works extremely well. Using the Coulomb phase shift in eq. (3) yields the well known Mott oscillations for the scattering of identical nuclei. The potentials $\Delta V(r)$ in eq. (1) introduce corrections to $\delta(\theta)$ which leads to shifts in the Mott oscillations [3].

In eq. (3), $\theta$ is the scattering angle. The classical cross sections $d\sigma_d/\Omega$ are Rutherford cross sections including corrections arising from the same set of potentials considered, i.e., $\Delta V(r)$. But the corrections on the magnitude of $d\sigma_d/\Omega$ are irrelevant. They damp the Mott oscillations, but do not shift them. The shift is essentially caused by the corrections $\Delta\delta(\theta) - \Delta\delta(\pi - \theta)$ in the last term of equation (3).

The potentials (a)-(e), listed before are treated in the same way as in ref. [3]. No relevant discrepancy with respect to that calculation [3] was found. We now proceed to study the additional corrections which were not considered before.

**Bremsstrahlung and delta-electrons.** The emission of radiation in form of Bremsstrahlung is well known theoretically. In ref. [8] a semiclassical calculation of Bremsstrahlung radiation for all multipolarities was performed. There is no dipole radiation for a symmetrical system and we concentrate on the quadrupole radiation. Higher order multipolarities are strongly suppressed. For a collision with scattering angle $\theta$ the
total energy emitted in form of quadrupole Bremsstrahlung are calculated using the
equations obtained in ref. [8]. At $\theta_{cm} = 0^\circ$ the Bremsstrahlung radiation vanishes,
since this corresponds to very large impact parameters for which no appreciable ac-
celeration occurs. The radiation is maximum (1.07 MeV for $Pb + Pb$ at 873 MeV) for $\theta_{cm}$ around 90°, and is not negligible at 180°. This is because at $\theta_{cm} = 180^\circ$ the
movement of the charges is similar to that of an imperfect dipole antenna, which also
emits quadrupole radiation.

To obtain the energy loss by emission of $\delta$-electrons we performed a coupled chan-
nels calculation using the two-center Dirac program developed in ref. [9]. The model
gives the differential probability, $dP_\nu/dE_\nu$, for the emission of a $\delta$-electron with
final energy $E_\nu$. To obtain the energy loss we multiply it by $E_\nu + E_b$ and inte-
grate over $E_\nu$, where $E_b$ is the average binding energy of the delta-electron. We
find that the energy loss increases steadily with the scattering angle and is maximum
for $\theta_{cm} = 180^\circ$ (0.29 MeV for $Pb + Pb$ at 873 MeV). This is because for backward
scattering the ions come closer and $\delta$-electrons are more easily emitted.

The effects considered before [3] are of conservative character. The emission of
Bremsstrahlung and of $\delta$-electrons are not. They induce an attenuation of the elastic
cross sections. For the same reason as stated before, more important is the shift
caused by phase differences for forward and backscattering. We can calculate this
phase shift by using the formula

$$\delta = -i \int \Delta E(\theta, t) \, dt$$

(4)

where $\Delta E(\theta, t)$ is the energy loss (positively defined) by emission of Bremsstrahlung
and delta-electrons as a function of time $t$ along a Rutherford trajectory with scat-
tering angle $\theta$.

Atomic screening and nuclear potential. To account for screening of the electrons
remaining in the nuclei the bare Coulomb potential has to be multiplied by screening
functions, i.e.,

$$\frac{Z_1 Z_2 e^2}{r} \rightarrow \frac{Z_1 \phi_1(r) Z_2 \phi_2(r) e^2}{r}$$

(5)

The screening functions $\phi_1(r)$ and $\phi_2(r)$ are calculated according to ref. [7] for a 23+
projectile and a neutral target. These values correspond to the experiment of ref. [4].

For the tail of the nuclear potential we used the Christensen-Winther parameterization [10] which was shown to describe very well the heavy ion elastic scattering data at low energies. We used the nuclear radii given by \( R_i = (1.233A_i^{1/3} - 0.98A_i^{-1/3}) \) fm and the diffuseness \( a_o = 0.63 \) fm.

**Quasi-molecule effect.** As the nuclei come close together the atomic electrons feel the influence of the Coulomb field of both nuclei. This reduces the energy of the system since the electronic levels become more bound. The effective potential describing this effect is given by

\[
V_{QM}(R, Z_1, Z_2) = \sum_i E_i^{Z_1+Z_2}(R) - \left( \sum_j E_j^{Z_1} + E_k^{Z_2} \right),
\]

which is a function of the internuclear distance \( R(t) \). \( E_i^{Z_1+Z_2}(R) \) is the energy of the i-th electron of the compound system \( Z_1 + Z_2 \) separated by the distance \( R \). \( E_j^{Z_1} \) and \( E_k^{Z_2} \) are the energies of the electrons in the individual atoms. The energy of the compound, or quasi-molecule, system was calculated by solving the two-center Dirac equation for the electrons, neglecting the electron-electron interaction, and in the monopole approximation [11]. A similar approach was used in ref. [12] to calculate the effect of formation of a quasi-molecule on Rutherford scattering of heavy ions.

In figure 1 we show the calculated potential (6) for the \( Pb+Pb \) quasi-molecule with 141 electrons (59 in the projectile and 82 in the target). We see that the potential is quite large at small separations between the nuclei. But more important for the shift in the Mott oscillations is the form of the potential. It is roughly proportional \( R^{-0.24} \). This slow dependence will cause sizeable shifts of the Mott oscillations, as we shall see next.

**Shifts in the Mott oscillations.** The results of our calculations for the respective shifts in the Mott scattering of \( Pb+Pb \) at \( \theta_{Lab} = 30^\circ \) are shown in table I. We observe from these results that the most relevant effects are the relativistic corrections, dipole polarizability, vacuum polarization, atomic screening, formation of a quasi-molecule and the color van der Waals force with a \( r^{-7} \) dependence.
magnitude less than what is observed experimentally. In this case, the force would seem to be several orders of magnitude different from what is observed experimentally. A different value of dy is also not successful to explain the trend of the data. The contribution of weakly interacting light hypothetical particles, such as axions, to the long range force between nuclei seems to be several orders of magnitude less than what is observed experimentally.

In figure 2 we show all the shifts added (dashed curve), except for the effect of the formation of a quasi-molecule and of a CVDW force. We observe that the shifts are not sufficient to explain the data. When the shifts caused by the formation of a quasi-molecule is added we obtain the solid curve. The agreement with the experimental data is quite reasonable.

The inclusion of a CVDW force overestimates the point obtained at high energy, for a $1/r^7$ interaction. A $1/r^6$ interaction is not relevant, except if a non-realistic value of the constant $\alpha_0$ is taken. A different value of $\alpha_7$ is also not successful to explain the trend of the data. The contribution of weakly interacting light hypothetical particles, such as axions, to the long range force between nuclei seems to be several orders of magnitude less than what is observed experimentally. In this case, the force would

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Table I - The shifts (in degrees) in the Mott scattering oscillations for $Pb + Pb$ scattering at three different laboratory energies and $\theta_{Lab} = 30^\circ$, due to (a) relativistic corrections ($\Delta_R$), (b) nuclear dipole polarization ($\Delta_{DP}$), (c) nuclear quadrupole polarization ($\Delta_{QP}$), (d) vacuum polarization ($\Delta_{VP}$), (e) atomic screening ($\Delta_{AS}$), (f) quasi-molecule ($\Delta_{QM}$), (g) tail of the nuclear interaction ($\Delta_N$), (h) Bremsstrahlung ($\Delta_{Brems}$), (i) production of $\delta$-electrons ($\Delta_{e^-}$), and (j) color van der Waals force ($\Delta_{VDW}^{(r)}$) with a $1/r^r$ dependence.
be of the form [13]
\[ V_\alpha = A_1 A_2 g_\alpha^2 \frac{m_\alpha^2}{4m_n^2} \frac{\exp(-m_\alpha r)}{r} \] (7)
where \( m_n \) is the nucleon mass. With \( m_\alpha \approx 1 \text{ eV} \) this force is of very long range. However, the coupling constant \( g_\alpha \) is very small, \( g_\alpha \approx 10^{-8} \), as compared to the Coulomb constant, \( Z_1 Z_2 e^2 \).

**Conclusions.** We have shown that the shifts in the Mott cross sections due to Bremsstrahlung, emission of \( \delta \)-electrons, tail of the nuclear potential and of quadrupole polarizability are not important. The addition of a hypothetical color van der Waals interaction is not the main cause of the shifts observed in the experiment of ref. [4]. The data indicate that if this force exists it might have a different dependence than those suggested [5] (see eq. 2). We feel however that such questions could be better assessed if an experiment with completely stripped nuclei could be performed. This is because the effects of atomic screening and of the formation of a quasi-molecule are very large. If such experiments could be performed, than a more definite statement about a possible long range force could be done. Such an experiment could perhaps be done with cooled beams intersecting at a small angle [14]. Such beams are currently obtainable at the GSI/Germany facility. With sufficient experimental data the radial dependence of the potential corrections could be obtained by the inverse scattering technique [15].

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**References**
14. P. Egelhof, private communication
Figure Captions

Fig. 1 The total binding energy of 141 electrons in a $Pb + Pb$ quasi-molecule (dashed line) as a function of the distance between the nuclear centers. Extracting from it the binding energies of individual atoms (i.e., at $R = \infty$) yields the quasi-molecule potential (solid line).

Fig. 2 The shifts in the Mott oscillations for the elastic scattering of $Pb + Pb$. The data are from ref. [4]. The dashed curve is a calculation which includes all effects displayed in table I, except for the formation of a quasi-molecule and a CVDW force. When one includes the first effect the solid curve is obtained. The dashed-dotted curve contains both effects.